Large-scale and infinite dimensional dynamical systems approximation

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Doctorant 3ème année - ONERA/DCSD

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Co-encadrant: Cédric SEREN

1 What about model approximation ?
- Context
- Objectives and problem formulation

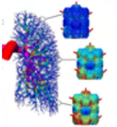
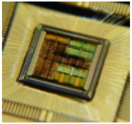
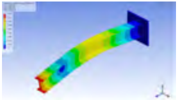
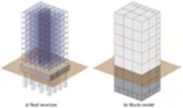
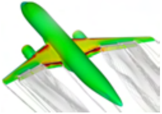
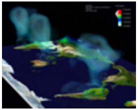
2 My contributions
- Theoretical contribution: model approximation with time-delay structure
- Methodological contribution: stability regions for time-delay systems
- Industrial applications: rhin flow system

3 Conclusions and perspectives

4 Academic outputs

Large-scale systems are present in many engineering fields: aerospace, computational biology, building structure, **VLI** circuits, automotive, weather forecasting, fluid flow. . .



- difficulties with simulation and memory management (*e.g.*, **ODE** solvers);
- difficulties with analysis (*e.g.*, frequency response, norms computation. . .);
- difficulties with controller design (*e.g.*, robust, optimal, predictive, etc.);
- . . . **induce numerical burden;**
- . . . **need for numerically robust and efficient tools.**

Large-scale dynamical models

Physical system

- Highly accurate and/or flexible A/C;
- Spacecraft, launcher, satellites;
- Fluid dynamics (Navier-Stokes);
- High fidelity models.

Large-scale dynamical models

Physical system

fluid mechanics, structure, etc.

Partial Differential Equations (PDEs)

$$-\vec{\nabla} p + \rho \vec{g} = \rho \vec{a}$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho \vec{v}) = 0$$

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Physical system

rigid behaviour

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Differential Algebraic Equations (DAEs) or Rational Functions

$$E\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

$$H(s) = H_1(s) + H_d e^{-\tau s} + \dots$$

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Discretisation

finite elements, finite volume, etc.

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simulation, control, analysis, optimisation, etc.

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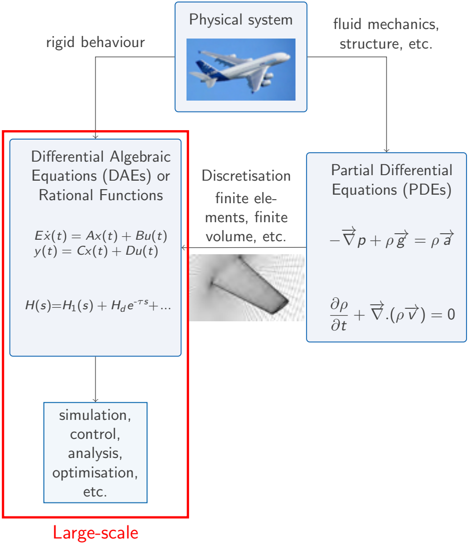
Context
Large-scale dynamical models



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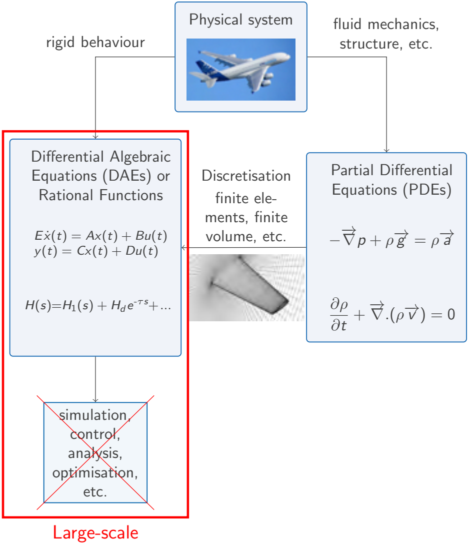

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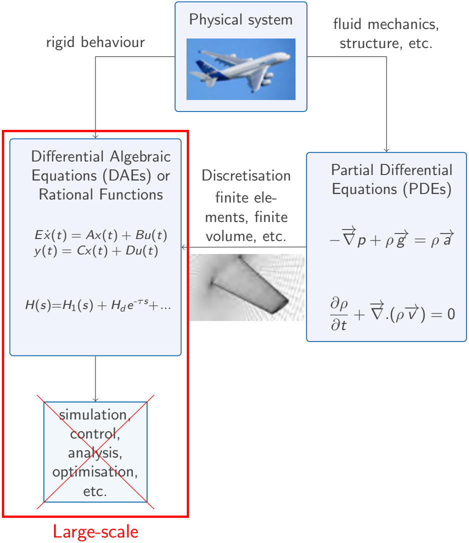

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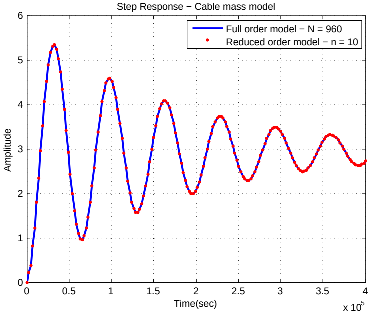
- Spacecraft, launcher, satellites;
- Fluid dynamics (Navier-Stokes);
- High fidelity models.

⇒ **objective:** alleviate numerical burden

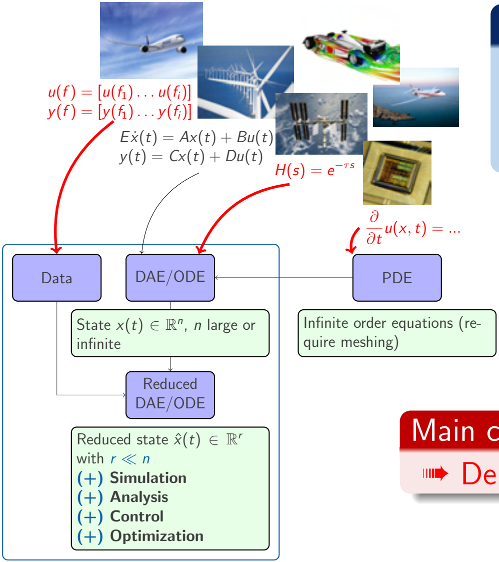
- allows to increase simulation speed while preserving precision.
- allows to apply modern analyses and control techniques.

Example: Cable mass model simulation

- Full model $N = 960$ ➞ Simulation time $\approx$ 23.70s.
- Reduced model $n = 10$ ➞ Simulation time $\approx$ 0.02s.
- Approximation time $\approx$ 4.03s.



Context
Realization-less model approximation



Reference modelings of interest

- [i/o] data-driven models;
- **T**ime-**D**elay **S**ystems (**TDS**);
- **PDE**-based descriptors...

Main concern

- Derive suitable low order models

Objectives

Find a reduced order modeling $\hat{\mathbf{H}}$ for which:

- ✔ the approximation error is small;
- ✔ the stability is preserved. . .

. . . from an efficient and computationally stable procedure.

The quality of the approximation can be evaluated using some mathematical norms. Find

$$\hat{\mathbf{H}} := \begin{cases} \hat{E}\dot{x}(t) = \hat{A}x(t) + \hat{B}u(t) \\ y(t) = \hat{C}x(t) \end{cases}$$

s.t.:

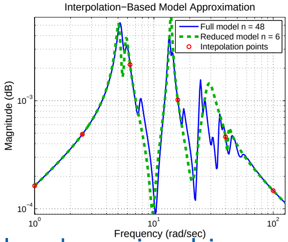
$\|\mathbf{H} - \hat{\mathbf{H}}\|^2$ is minimum → **optimisation problem to solve**

Mathematical formulation

Find $\hat{\mathbf{H}}$ of order $r << n$ which minimizes[a] :

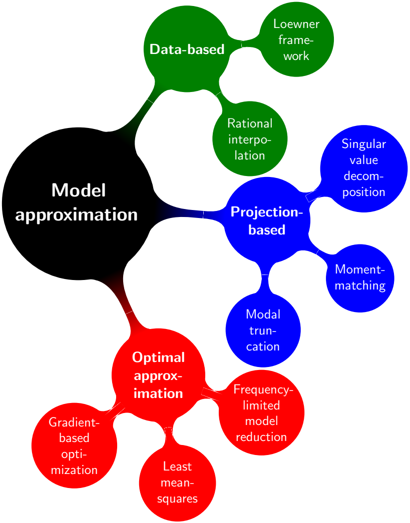
$$\hat{\mathbf{H}} := \underset{\substack{\mathbf{G} \in \mathcal{H}_2^{n_y \times n_u} \\ \dim(\mathbf{G}) = r \in \mathbb{N}^\star}}{\operatorname{argmin}} ||\mathbf{H} - \mathbf{G}||_{\mathcal{H}_2}, \quad (1)$$

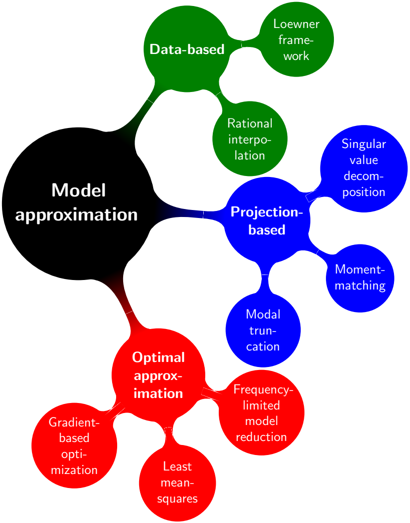
[a] $\mathcal{H}_2$-norm is the "system energy"



➠ Tackle this problem by rational interpolation

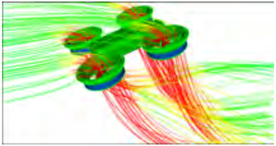
Overview of my contributions
State of the art





- Most of reduced order models considered are finite dimensional.
- But some natural phenomena have intrinsical delay behaviour, *e.g.*, transport equation.
- Idea : Consider time-delay reduced order models.

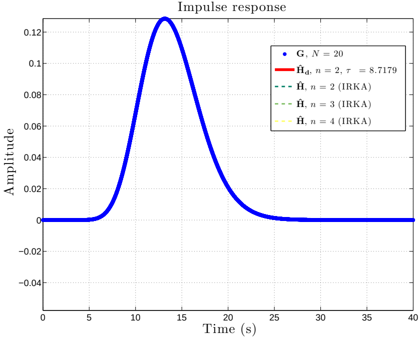
$$\boldsymbol{\Delta}(s) = e^{-\tau s}.$$



flow into quad-copter

- **Example 1:** Approximation of transport phenomena by time-delay structure.
- **Example 2:** Time-delay system stability charts estimation.
- **Example 3:** Hydroelectric EDF model (Rhin river).

Model approximation with time-delay structure
Example 1 (Transport equation)[1]



- Full order model has input-delay behavior.
- Finite dimensional reduced-order model not appropriate.
- Good input-delay approximation.

[1] Pontes Duff, I., Poussot-Vassal, C. and Seren, C. – "Optimal $\mathcal{H}_2$ model approximation based on multiple input/output delays systems." – **[Submitted to Automatica]**.

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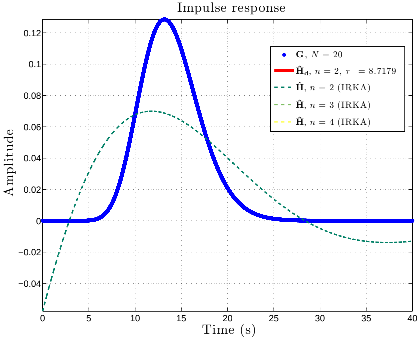

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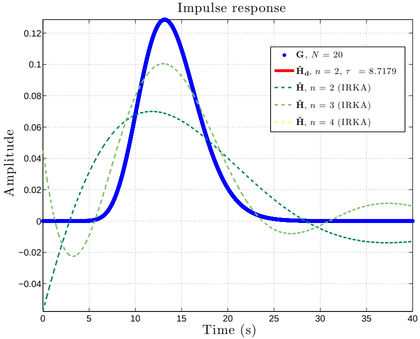

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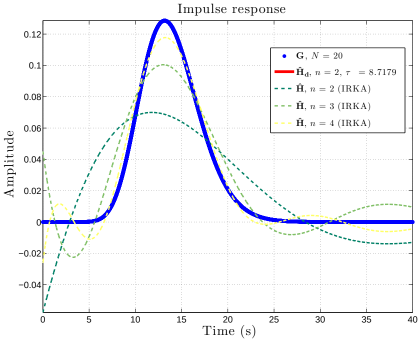

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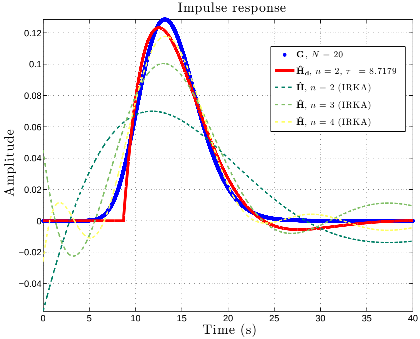

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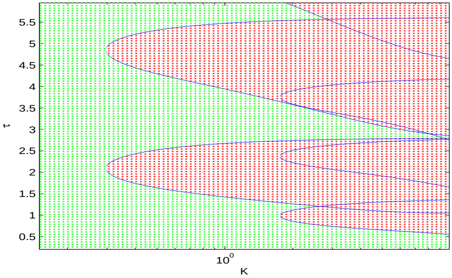
Stability estimation

Exploit model reduction techniques to analyse the stability of **TDS** w.r.t. parameters.

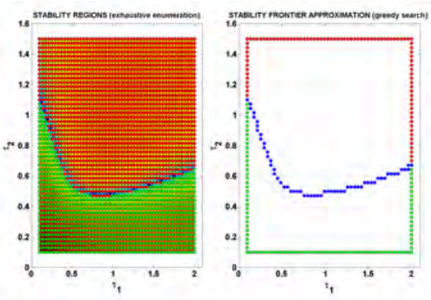
Application to Robotics:



2 Pontes Duff, I., Vuillemin, P., Poussot-Vassal, C., Briat, C. and Seren, C. — "Approximation of stability regions for large-scale time-delay systems using model reduction techniques." — In Proceedings of the 2015 ECC.

Perpectives:

Implement research boundary algorithm using evolutionary methods (PR GENETIC)



GENETIC

a GENeric Evolutionary computation Toolbox for Identification and Control

Rhin flow system

Example 3 [1] (EDF collaboration)

⇝ **PDE** St-Venant fluid model

- Infinite dimensional linear parametric model;
- Modeling = relationship between outflow q_s and inflow q_e at any given nominal flow q_0 s.t.:

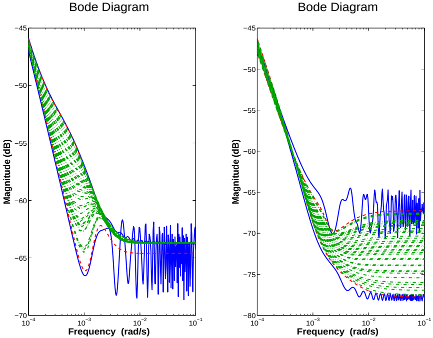
$$Z(q_0, s) = [G_e(q_0, s) \quad - G_s(q_0, s)] \cdot \begin{bmatrix} q_e(s) \\ q_s(s) \end{bmatrix}$$

G_e and G_s are rational functions of hyperbolic.

[1] Dalmas, V., Robert, G., Poussot-Vassal, C., Pontes Duff, I. and Seren, C. – "Parameter dependent irrational and infinite dimensional modelling and approximation of an open-channel dynamics." – **[Accepted to the 15th European Control Conference, 2016.]**

Rhin flow system
Example 3 [1] (EDF collaboration)

⇝ **PDE** St-Venant fluid model



⇒ Result: finite dimensional parametric reduced model

[1] Dalmas, V., Robert, G., Poussot-Vassal, C., Pontes Duff, I. and Seren, C. – "Parameter dependent irrational and infinite dimensional modelling and approximation of an open-channel dynamics." – **[Accepted to the 15th European Control Conference, 2016.]**

Main contributions

- Model approximation for reduced order modeling, time-delay structures: theoretical and algorithmic contributions.
- Methodological solutions for **TDS** stability charts estimation.
- Application to several representative industrial cases.
- Scientific collaborations:
  1. S. Gugercin/C. Beattie (Virginia Tech - 3 months stay).
  2. C. Briat (ETH-Zürich).
  3. G. Robert/V. Dalmas (EDF).

Conclusions and perspectives
Third (and last) year future works

Virginia **T**ech (**VT**) campus

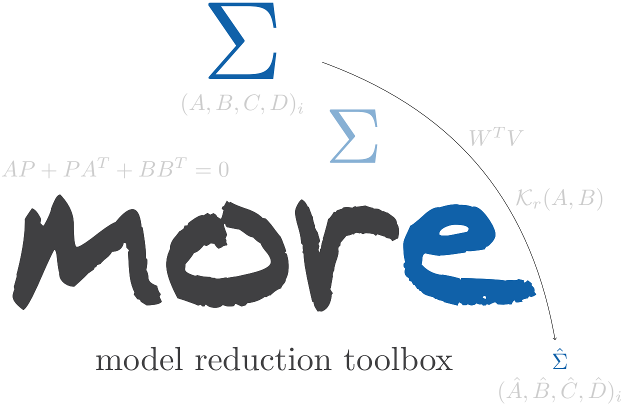
Onera - Toulouse

- Methodologies extension
- Methods will be available in the MOdel REduction toolbox.

more
model reduction toolbox

- Thesis defence planned in December 2016.
- New PhD position opened on model approximation.

Thanks for your attention, any questions ?



➠ website link: http://w3.onera.fr/more/

Workshops:

- **2nd Workshop on Delay Systems, October 2013 (CNRS-LAAS, Toulouse):**
  "Model reduction for norm approximation."
- **3rd Workshop Delay Systems, October 2014 (GIPSA-Lab, Grenoble):**
  "Model reduction of time-delay systems and stability charts."

Congresses/Seminars:

- **GT MOSAR, November 2104 (ONERA, Toulouse):**
  "Model reduction of infinite dimensional systems."
- **Matrix Computation Seminar, October 2015 (Virginia Tech, USA):**
  "$\mathcal{H}_2$ model approximation, interpolation and time-delay systems."
- **SIAM Student Chapter, November 2015 (VirginiaTech, USA):**
  "$\mathcal{H}_2$ model approximation, stability charts and time-delay systems."
- **7th European Congress of Mathematics, July 2016, (TU Berlin):**
  "$\mathcal{H}_2$ model approximation for time-delay reduced order systems." [invited]

Book chapter:

- Pontes Duff, I., Vuillemin, P., Poussot-Vassal, C., Briat, C. and Seren, C.
  Model reduction for norm approximation: an application to large-scale time-delay systems.
  [To Appear] in Springer Series: Advances in Dynamics and Delays.

Conference papers:

- Pontes Duff, I., Vuillemin, P., Poussot-Vassal, C., Briat, C. and Seren, C.
  Stability and performance analysis of a large-scale aircraft anti-vibration control subject to delays using model reduction techniques.
  [Accepted] in the Proceedings of the 2015 EuroGNC Conference.
- Pontes Duff, I., Vuillemin, P., Poussot-Vassal, C., Briat, C. and Seren, C.
  Approximation of stability regions for large-scale time-delay systems using model reduction techniques.
  [Accepted] in the Proceedings of the 14^{th} European Control Conference, 2015.
- Pontes Duff, I., Poussot-Vassal, C. and Seren, C.
  Realization independent time-delay optimal interpolation framework.
  [Accepted] at the 54^{th} IEEE Conference on Decision and Control, 2015.

Academic outputs
Accepeted/Submitted papers

Journal papers:

- Pontes Duff, I., Poussot-Vassal, C. and Seren, C.
  Optimal $\mathcal{H}_2$ model approximation based on multiple input/output delays systems.
  [Submitted] to Automatica journal, 2015.

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- Dalmas, V., Robert, G., Poussot-Vassal, C., Pontes Duff, I. and Seren, C.
  Parameter dependent irrational and infinite dimensional modelling and approximation of an open-channel dynamics.
  [Accepted] to the 15^{th} European Control Conference, 2016.
- Pontes Duff, I., Gugercin, S., Beattie, C., Poussot-Vassal, C. and Seren, C.
  $\mathcal{H}_2$-optimality conditions for reduced time-delay systems of dimension one.
  [Accepted] to the 13^{th} IFAC Workshop on Time Delay Systems, 2016.

Journal papers:

- Pontes Duff, I., Poussot-Vassal, C. and Seren, C.
  Model reduction and stability charts of time-delay systems.
  [On going work] $\rightsquigarrow$ European Journal of Control (?)

- Pontes Duff, I., Gugercin, S., Beattie, C., Poussot-Vassal, C. and Seren, C.
  $\mathcal{H}_2$-optimality conditions for structured reduced order models.
  [On going work] $\rightsquigarrow$ SIAM Journals on matrix analysis and applications (?).

Technical Report:

- Pontes Duff, I., Gugercin, S. and Beattie, C.
  Stability and model reduction of family of TDS models.
  [On going work] $\rightsquigarrow$ Event not identified.