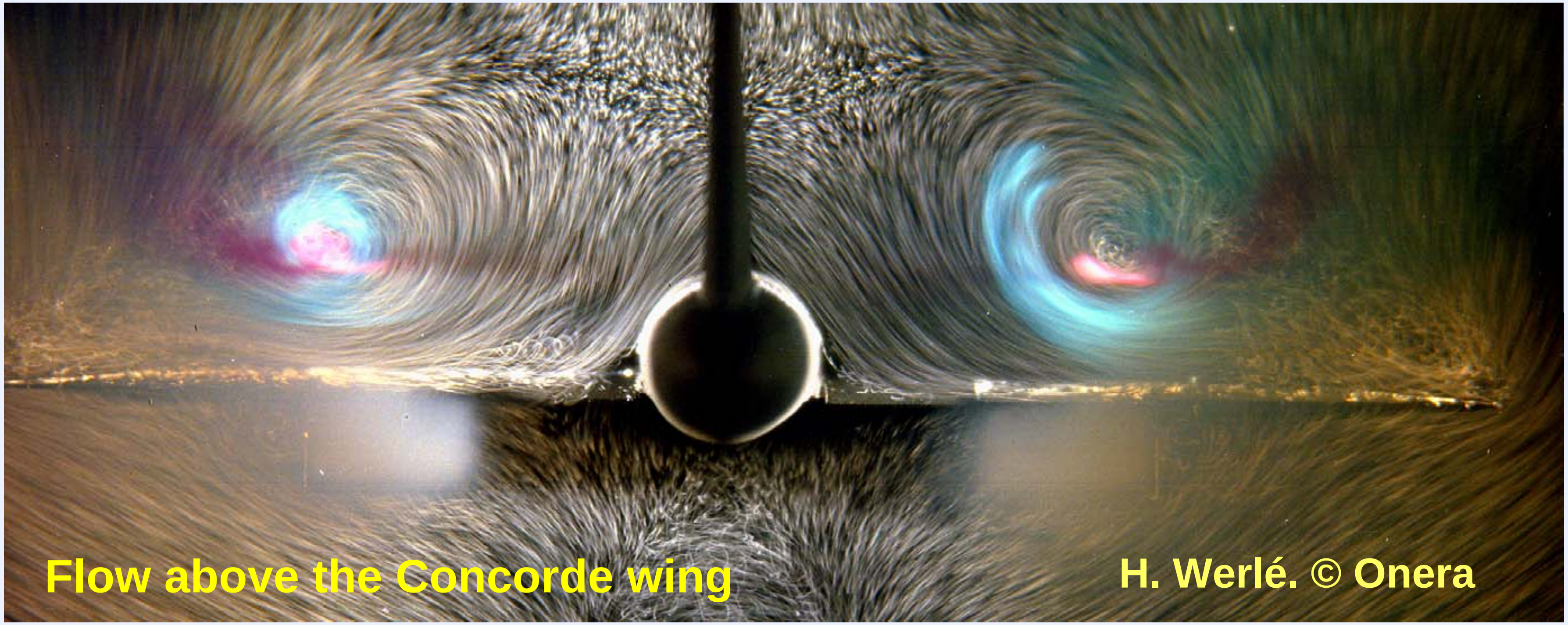
SEPARATION IN THREE-DIMENSIONAL FLOW: CRITICAL POINTS, SEPARATION LINES AND VORTICES



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Separation in three-dimensional steady flow

1 - The critical point theory and three-dimensional separation.

2 - The basic constitutive elements: separation lines of detachment and attachment, separation surfaces of detachment and attachment, vortex structures.

3 - Topology of some remarkable three-dimensional flows:

- ➔ vortex formation over a delta wing
- ➔ vortices of slender body
- ➔ vortex wake of a classical wing
- ➔ separation induced by an obstacle or a protuberance
- ➔ afterbody without and with propulsive jet
- ➔ vortex formation past a three-body automobile

4 - Two-dimensional separation revisited with 3D concepts or an apparently simple case.

Separation in three-dimensional steady flow

Part 1: THE CRITICAL POINT THEORY AND THREE-DIMENSIONAL SEPARATION

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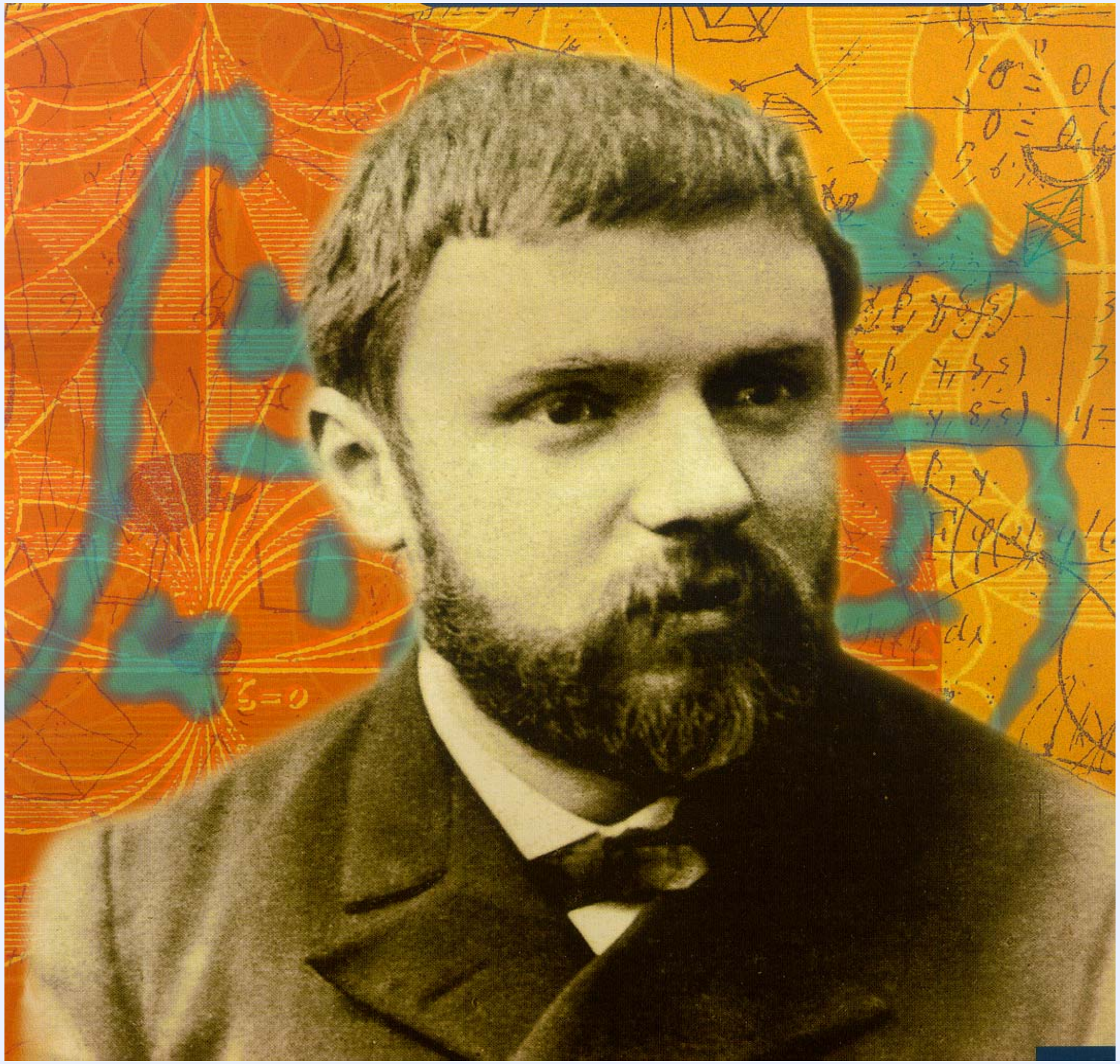
Towards a rational definition of 3D separation

Robert Legendre

Henri Werlé

The critical point theory and three-dimensional separation



Henri Poincaré, 1854 - 1912

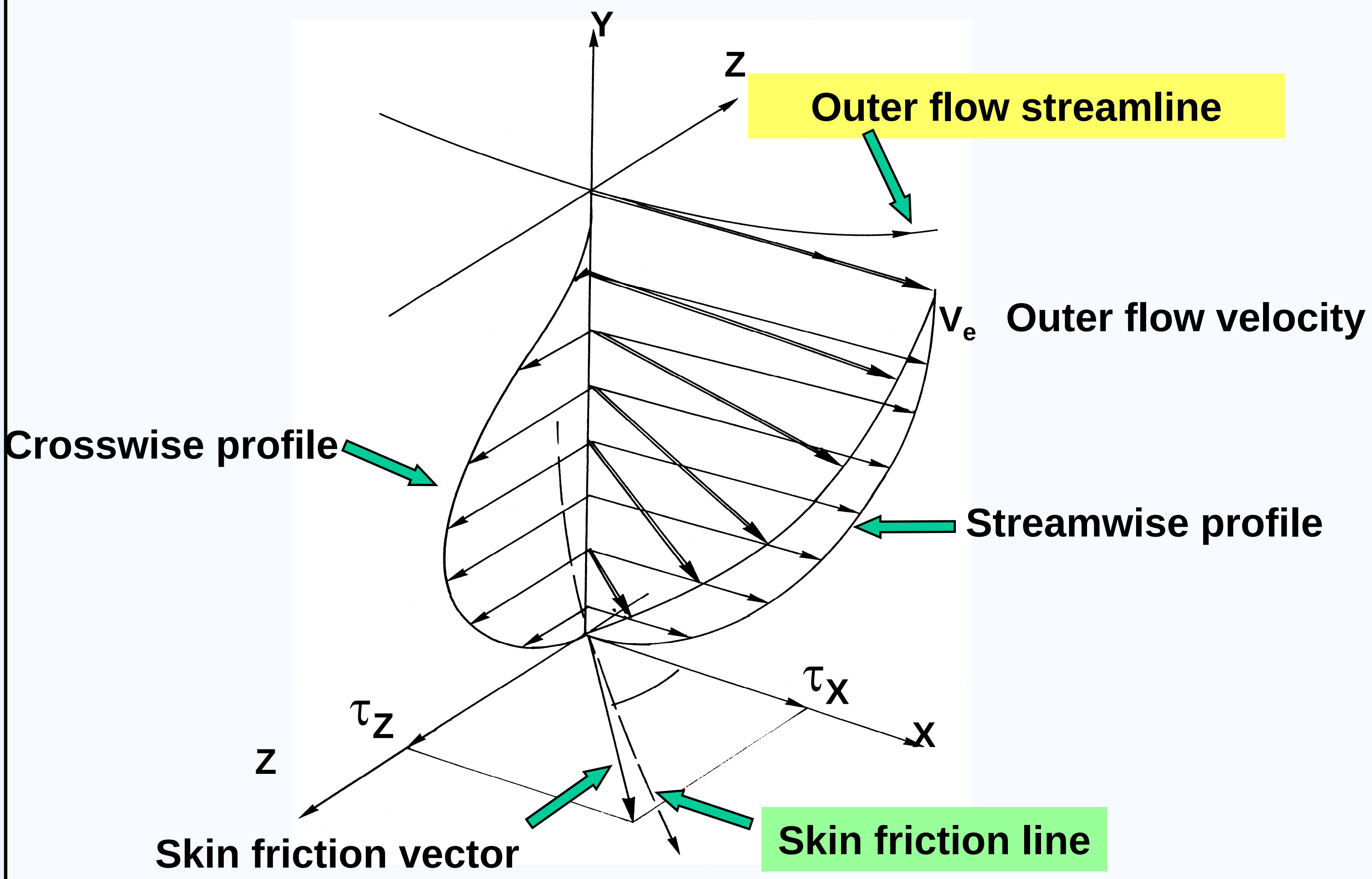
Some definitions or what we are about to speak

➔ **Three-dimensional separated flows are characterized by the existence of vortical structures defined as spatial concentration of vorticity resulting from boundary layer separation.**

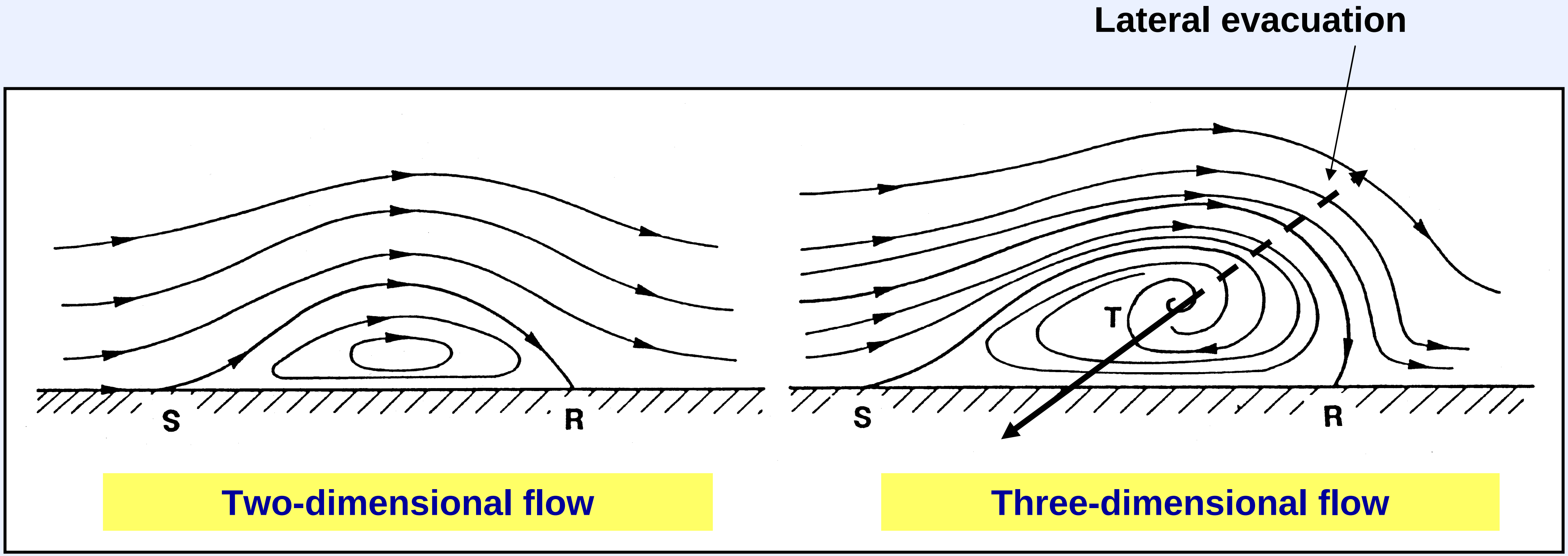
➔ **Once separation has occurred, vorticity tends to concentrate in the vicinity of surfaces – or sheets – whose rolling up forms vortices.**

➔ **In the reality, such surfaces – or sheets - defined as support of discontinuities (or singularities) do not exist. Such concepts belong to perfect fluid models. In the real world, vorticity is continuously distributed in space and occupies a certain volume in the vicinity of what we call separation or detachment surfaces.**

Structure of a three-dimensional boundary layer



Naive conceptions or separation



- In three-dimensional flows, velocity develops a transverse (crosswise) component allowing the flow to laterally escape.
- The separated flow is no longer trapped in a closed recirculation bubble, but can be evacuated along the transverse dimension.

Basic notions and definitions (1)

In a three-dimensional flow, skin friction is a vector.

➔ We call skin friction line a line tangent at each of its points to the local skin friction vector.

➔ Sometimes the concept of limit streamline is used: it is defined as the limit of a streamline when the distance y normal to the wall tends to zero.

➔ It can be shown that the limit direction of the velocity vector when y tends to zero is colinear with the skin friction vector (if the fluid is Newtonian).

➔ Skin friction lines and limit streamlines coincide.

Basic notions and definitions (2)

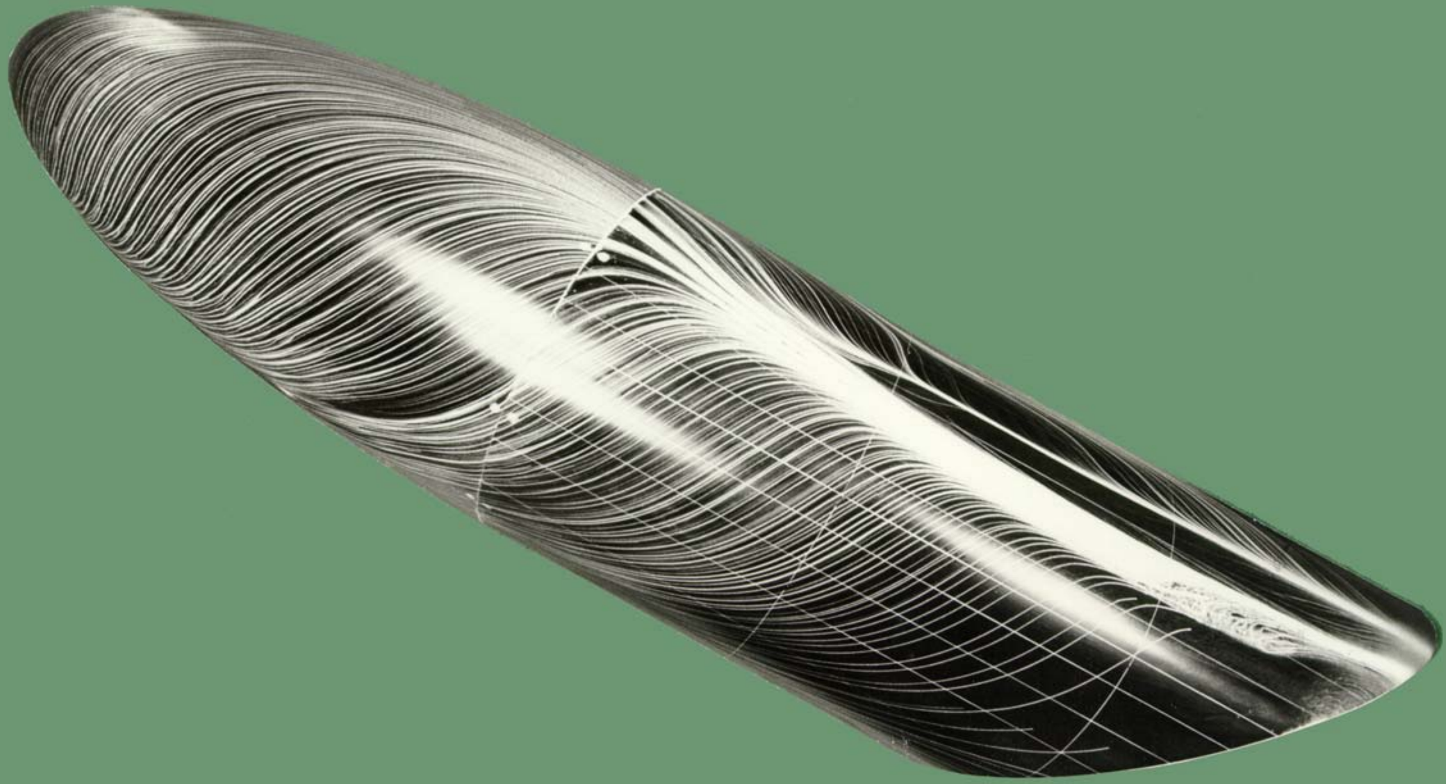
- ➔ Here we will use the physical concept of skin friction line rather than the limit streamline concept which results from a passage to the limit.

- ➔ The use of the wall streamline concept is still worst since there is no flow on a surface!

- ➔ The skin friction lines can be visualized by means of a viscous film deposited of the model surface: they have a real existence.

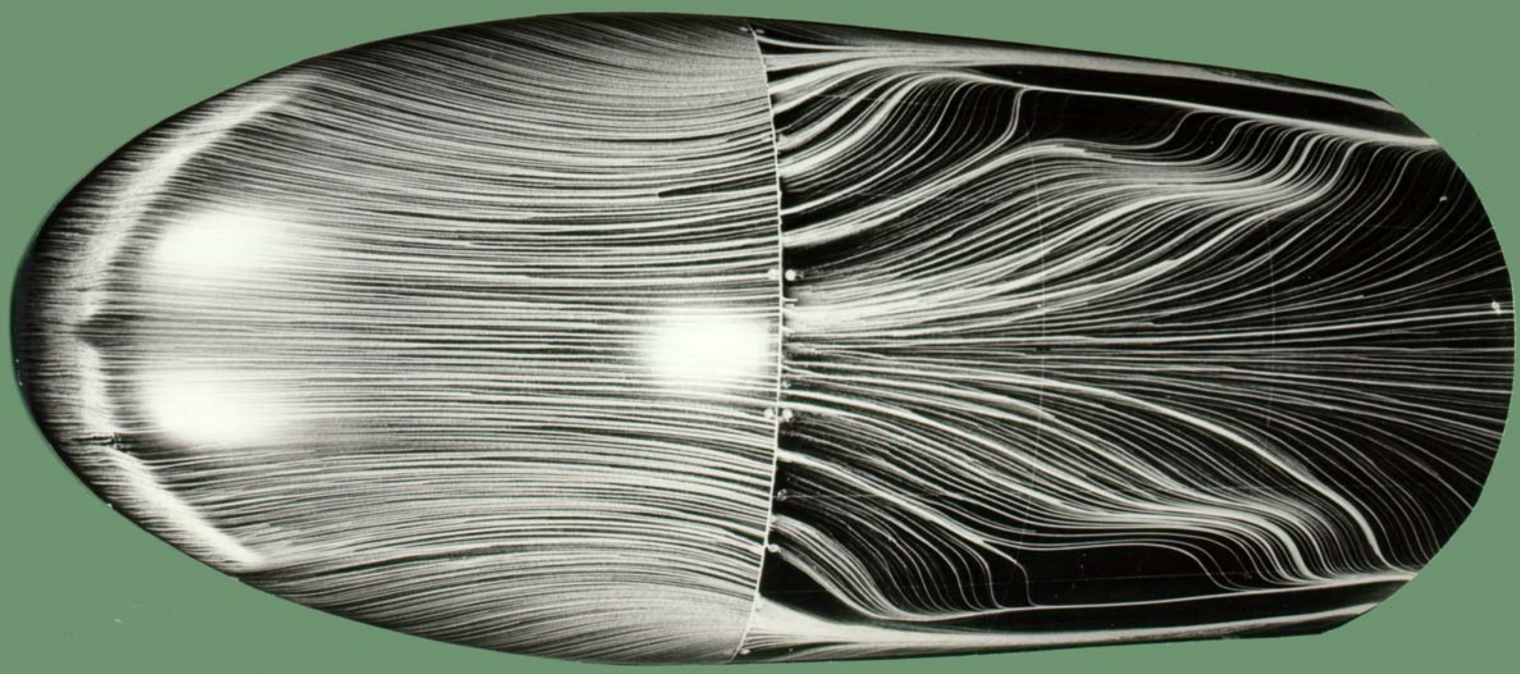
- ➔ The set of skin friction lines will be called the skin friction line surface pattern or more shortly skin friction line pattern or surface pattern.

Skin friction lines visualization on a blunted body



Wind tunnel S2Ch. © Onera

Skin friction lines visualization on a blunted body



Wind tunnel S2Ch. © Onera

The critical point theory (1)
(Poincaré, 1882 ; Legendre, 1956 ; Lighthill, 1963)

- The flow is assumed steady (not essential).

- We consider the two-dimensional space constituted by the surface of a three-dimensional body.

- We introduce the skin friction vector field on the body surface.

- We define the lines of force or trajectories of this field. Such lines are solutions of the differential system (x and z are two coordinates in the surface):

→ (1)

$$\boxed{\frac{dx}{\tau_x(x,z)} = \frac{dz}{\tau_z(x,z)}}$$

The critical point theory (2)

- In general, by one point on the body goes one and only one such trajectory called **a skin friction line.**

- This is not true at a point P_0 where the skin friction vector vanishes: then system (1) is singular.

- Such a point is called **singular or critical**, the solution of system (1) at a critical point leading to an eigenvalue problem.

- In the vicinity of a critical point, the solution lines behaviour depends of the **nature (real or complex) and sign of the eigenvalues.**

The critical point theory (3)

- At point P_0 we simultaneously have:

$$\boxed{\tau_x(x,z) = 0 \quad , \quad \tau_z(x,z) = 0}$$

- Solution in the vicinity of P_0 is looked for via a first order Taylor series expansion:

$$\tau_x(x,z) = \left(\frac{\partial \tau_x}{\partial x}\right)_{P_0}(x - x_0) + \left(\frac{\partial \tau_x}{\partial z}\right)_{P_0}(z - z_0)$$

$$\tau_z(x,z) = \left(\frac{\partial \tau_z}{\partial x}\right)_{P_0}(x - x_0) + \left(\frac{\partial \tau_z}{\partial z}\right)_{P_0}(z - z_0)$$

The critical point theory (4)

For simplicity, the origin of system axis is placed at P_0 and index 0 is omitted.

$$\boxed{\frac{dx}{\frac{\partial \tau_x}{\partial x}x + \frac{\partial \tau_x}{\partial z}z} = \frac{dz}{\frac{\partial \tau_z}{\partial x}x + \frac{\partial \tau_z}{\partial z}z}}$$

The skin friction derivatives being assumed different from zero, then:

(2)

$$\frac{dx}{\frac{\partial \tau_x}{\partial x}x + \frac{\partial \tau_x}{\partial z}z} = \frac{dz}{\frac{\partial \tau_z}{\partial x}x + \frac{\partial \tau_z}{\partial z}z} =$$

$$\boxed{= \frac{\lambda dx + \mu dz}{\lambda\left(\frac{\partial \tau_x}{\partial x}x + \frac{\partial \tau_x}{\partial z}z\right) + \mu\left(\frac{\partial \tau_z}{\partial x}x + \frac{\partial \tau_z}{\partial z}z\right)}}$$

The critical point theory (5)

A solution is looked for by writing (2) in the following logarithmic form:

$$\boxed{\frac{\lambda dx + \mu dz}{\lambda\left(\dfrac{\partial \tau_x}{\partial x}x + \dfrac{\partial \tau_x}{\partial z}z\right) + \mu\left(\dfrac{\partial \tau_z}{\partial x}x + \dfrac{\partial \tau_z}{\partial z}z\right)} = \frac{d(\lambda x + \mu z)}{S(\lambda x + \mu z)}}$$

The solution is of the form:

$$\frac{d(\lambda x + \mu z)}{S(\lambda x + \mu z)} = \frac{df}{Sf} = -dt \quad \rightarrow \quad f = A\exp(-St)$$

$$\boxed{\lambda x + \mu z = A\exp[-St]}$$

t ⇨ integration dummy variable

problem ➡ find λ and μ

The critical point theory (6)

The previous form is possible if the following conditions are satisfied:

$$\lambda \frac{\partial \tau_x}{\partial \mathbf{x}} \mathbf{x} + \mu \frac{\partial \tau_z}{\partial \mathbf{x}} \mathbf{x} = \mathbf{S} \lambda \mathbf{x}$$

$$\lambda \frac{\partial \tau_x}{\partial \mathbf{z}} \mathbf{z} + \mu \frac{\partial \tau_z}{\partial \mathbf{z}} \mathbf{z} = \mathbf{S} \mu \mathbf{z}$$

First degree homogenous algebraic system for λ and μ:

$$\boxed{\begin{aligned} \left(\frac{\partial \tau_x}{\partial \mathbf{x}} - \mathbf{S} \right) \lambda + \frac{\partial \tau_z}{\partial \mathbf{x}} \mu &= \mathbf{0} \\ \frac{\partial \tau_x}{\partial \mathbf{z}} \lambda + \left(\frac{\partial \tau_z}{\partial \mathbf{z}} - \mathbf{S} \right) \mu &= \mathbf{0} \end{aligned}}$$

The critical point theory (7)

Trivial solution : $\lambda = 0$ et $\mu = 0$

Non trivial solution → **system determinant must be equal to zero:**

$$\begin{vmatrix} \dfrac{\partial \tau_x}{\partial x} - S & \dfrac{\partial \tau_z}{\partial x} \\ \dfrac{\partial \tau_x}{\partial z} & \dfrac{\partial \tau_z}{\partial z} - S \end{vmatrix} = 0$$

→ **Algebraic second degree equation for S**

$$S^2 - S\left(\frac{\partial \tau_x}{\partial x} + \frac{\partial \tau_z}{\partial z}\right) + \frac{\partial \tau_x}{\partial x}\frac{\partial \tau_z}{\partial z} - \frac{\partial \tau_x}{\partial z}\frac{\partial \tau_z}{\partial x} = 0$$

The critical point theory (8)

Jacobian matrix

$$\Rightarrow \quad \boxed{\mathbf{F} = \begin{vmatrix} \dfrac{\partial \tau_x}{\partial x} & \dfrac{\partial \tau_x}{\partial z} \\ \dfrac{\partial \tau_z}{\partial x} & \dfrac{\partial \tau_z}{\partial z} \end{vmatrix}}$$

p = - trace of F et q = determinant of F

$$\mathbf{p} = -\left(\frac{\partial \tau_x}{\partial x} + \frac{\partial \tau_z}{\partial z} \right) \qquad \mathbf{q} = \left(\frac{\partial \tau_x}{\partial x} \frac{\partial \tau_z}{\partial z} - \frac{\partial \tau_z}{\partial x} \frac{\partial \tau_x}{\partial z} \right)$$

p and q depend only of the skin friction derivatives at the critical point

Eigenvalue equation $\Rightarrow$ $\boxed{\mathbf{S}^2 + \mathbf{p}\,\mathbf{S} + \mathbf{q} = 0}$

The critical point theory (9)

★ The solutions of the previous equation (eigenvalue of matrix F) are written:

$$S_{1,2} = \frac{-p \pm \sqrt{p^2 - 4q}}{2}$$

★ The nature – real or complex – of the eigenvalues depends of the sign of the discriminant:

$$\boxed{D = p^2 - 4q}$$

The critical point theory (10)

eigenvalue S_1 $\rightarrow$ λ_1, μ_1 eigenvector (1)

eigenvalue S_2 $\rightarrow$ λ_2, μ_2 eigenvector (2)

$\lambda_1, \mu_1, \lambda_2, \mu_2$ depend of $\dfrac{\partial \tau_x}{\partial x}, \dfrac{\partial \tau_x}{\partial z}, \dfrac{\partial \tau_z}{\partial z}, \dfrac{\partial \tau_z}{\partial x}$

Values of λ and μ

$$\lambda_1 = -\left(\frac{\partial \tau_z}{\partial x}\right)_{P_0} \qquad \mu_1 = \left(\frac{\partial \tau_x}{\partial x}\right)_{P_0} - S_1$$

$$\lambda_2 = \left(\frac{\partial \tau_z}{\partial z}\right)_{P_0} - S_2 \qquad \mu_2 = -\left(\frac{\partial \tau_x}{\partial z}\right)_{P_0}$$

$$\lambda_1 x + \mu_1 z = A_1 \exp\left[-S_1 t\right]$$

$$\lambda_2 x + \mu_2 z = A_2 \exp\left[-S_2 t\right]$$

A_1, A_2 are determined by matching conditions with the surrounding field

The critical point theory (11)

The solution in the vicinity of a critical point is of the form:

$$x(t) = \frac{A_1\mu_2 \exp\left(-S_1 t\right) - A_2\mu_1 \exp\left(-S_2 t\right)}{\lambda_1\mu_2 - \lambda_2\mu_1}$$

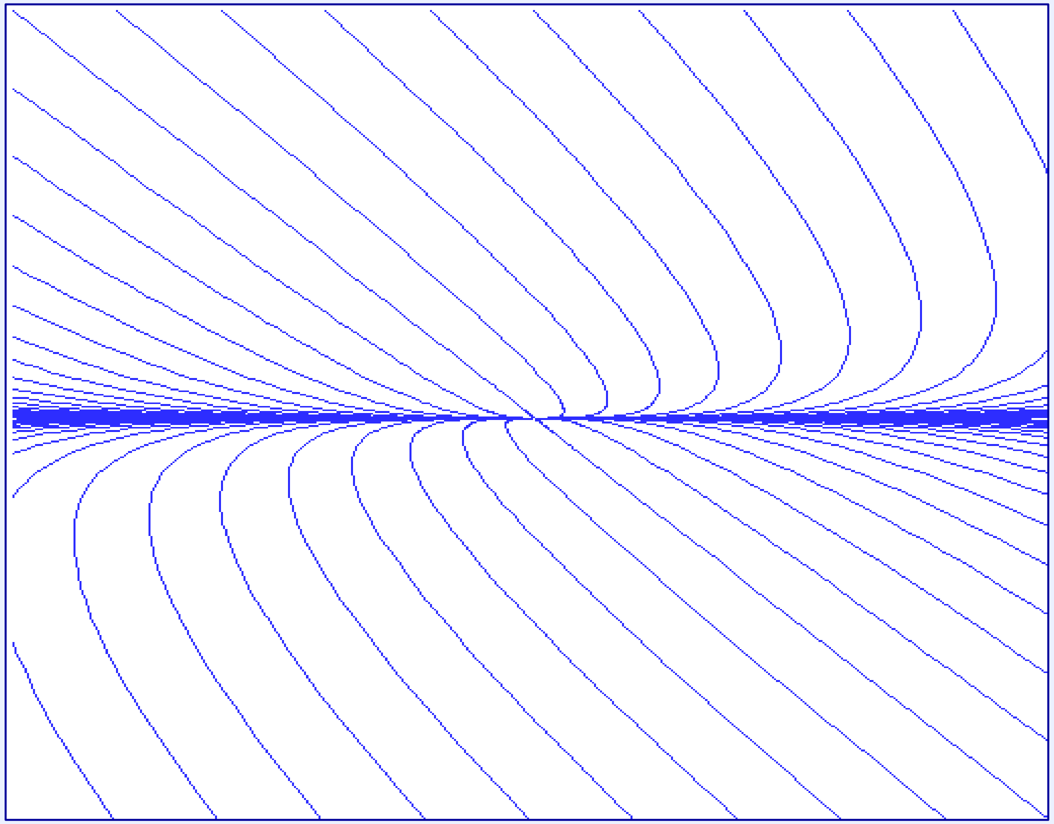
$$z(t) = -\frac{A_1\lambda_2 \exp\left(-S_1 t\right) - A_2\lambda_1 \exp\left(-S_2 t\right)}{\lambda_1\mu_2 - \lambda_2\mu_1}$$

The critical point theory (12)

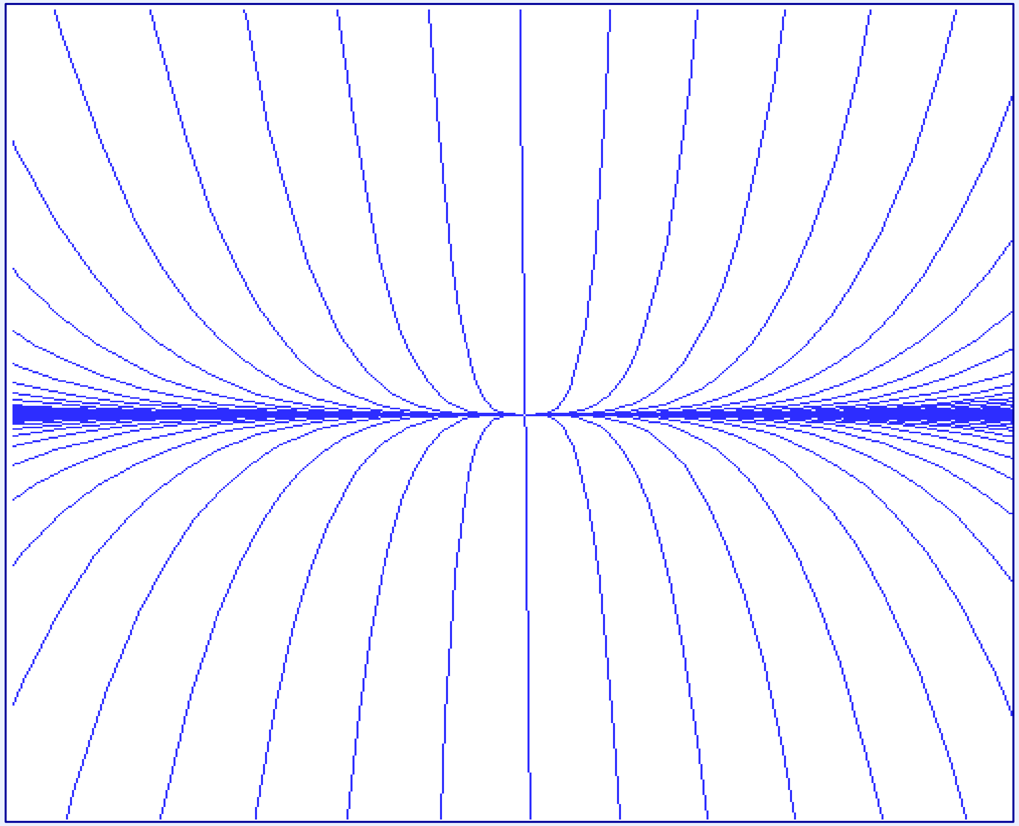
The different critical points:

- S_1, S_2 real and of same sign ⟶ node
- S_1, S_2 real and such that $S1 = S2$ ⟶ isotropic node
- S_1, S_2 real with opposite signs ⟶ saddle point
- S_1, S_2 complex conjugate ⟶ focus
- S_1, S_2 imaginary ⟶ degenerate focus or centre

Node type critical point



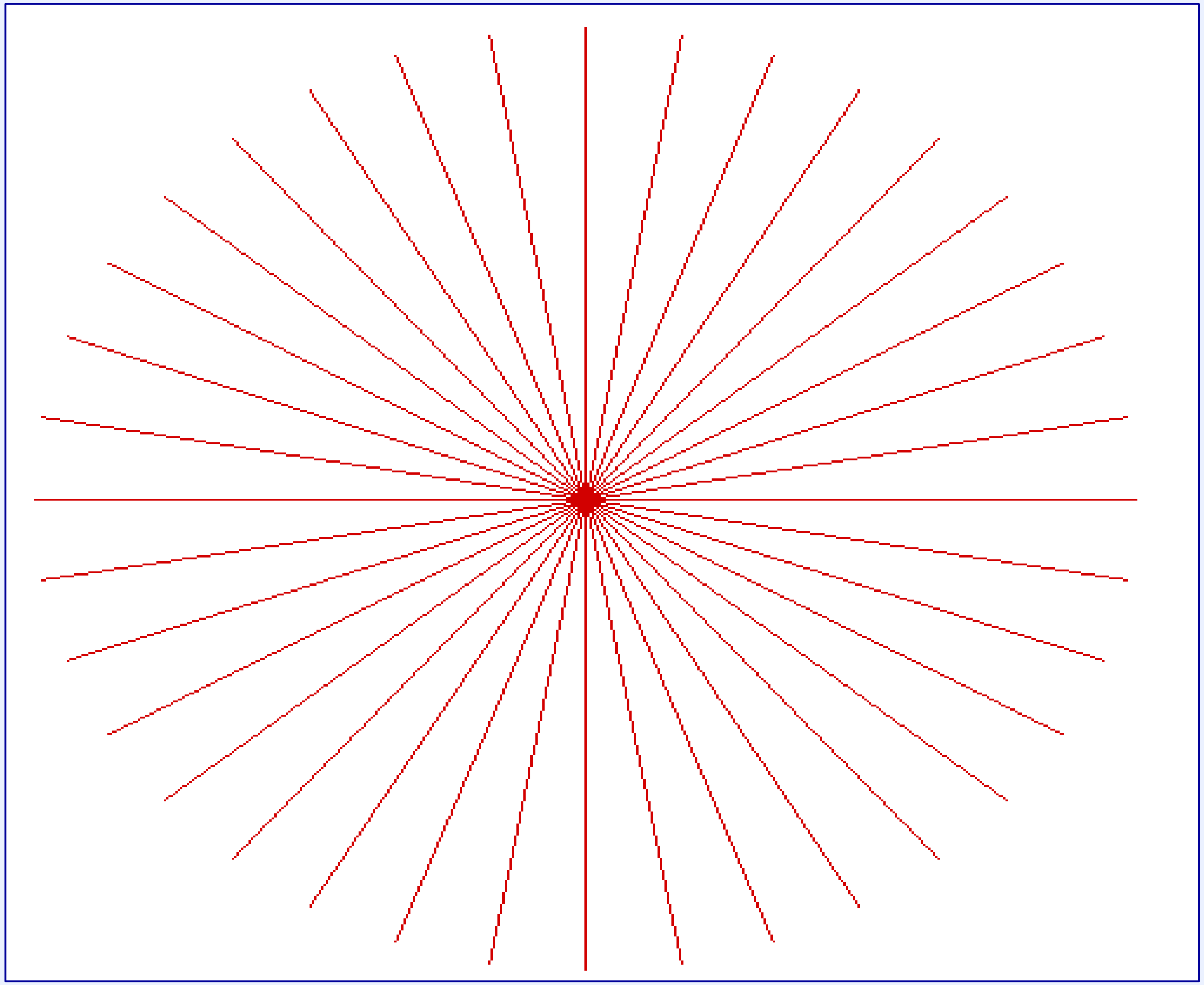
$\lambda_1 = -1 \quad , \quad \lambda_2 = 0$



$\lambda_1 = 100 \quad , \quad \lambda_2 = 0$

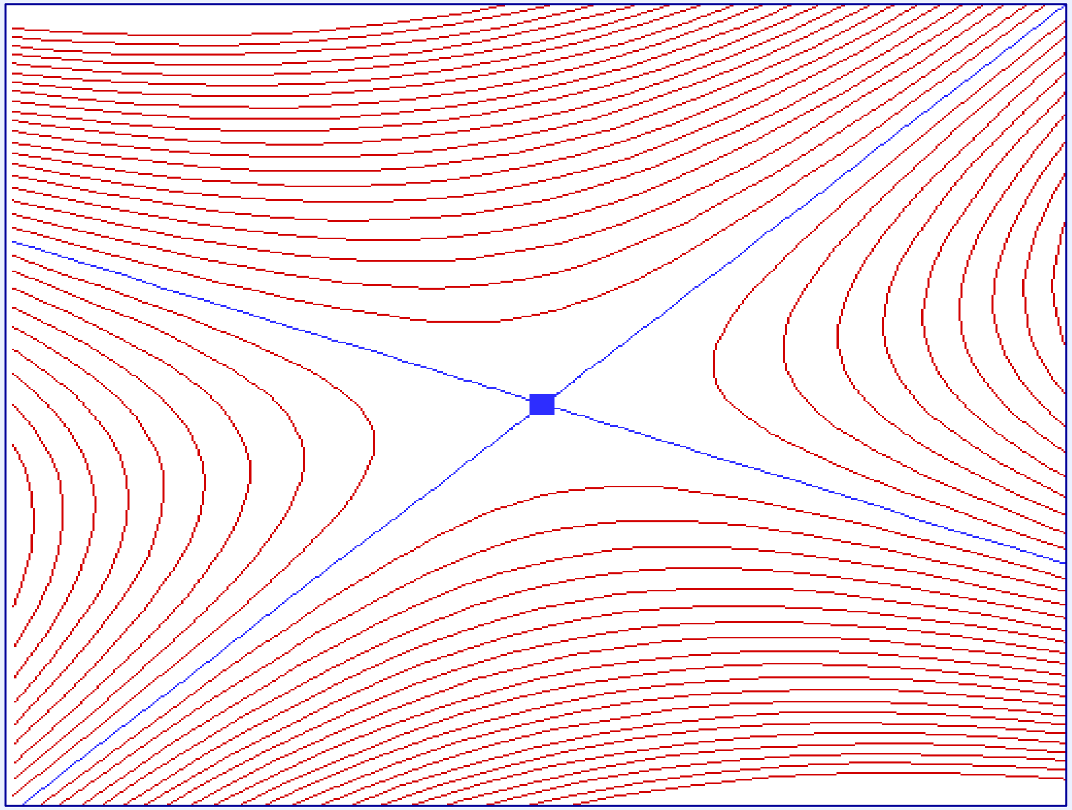
$S_1 = 1 \quad , \quad S_2 = 4$

Isotropic node type critical point

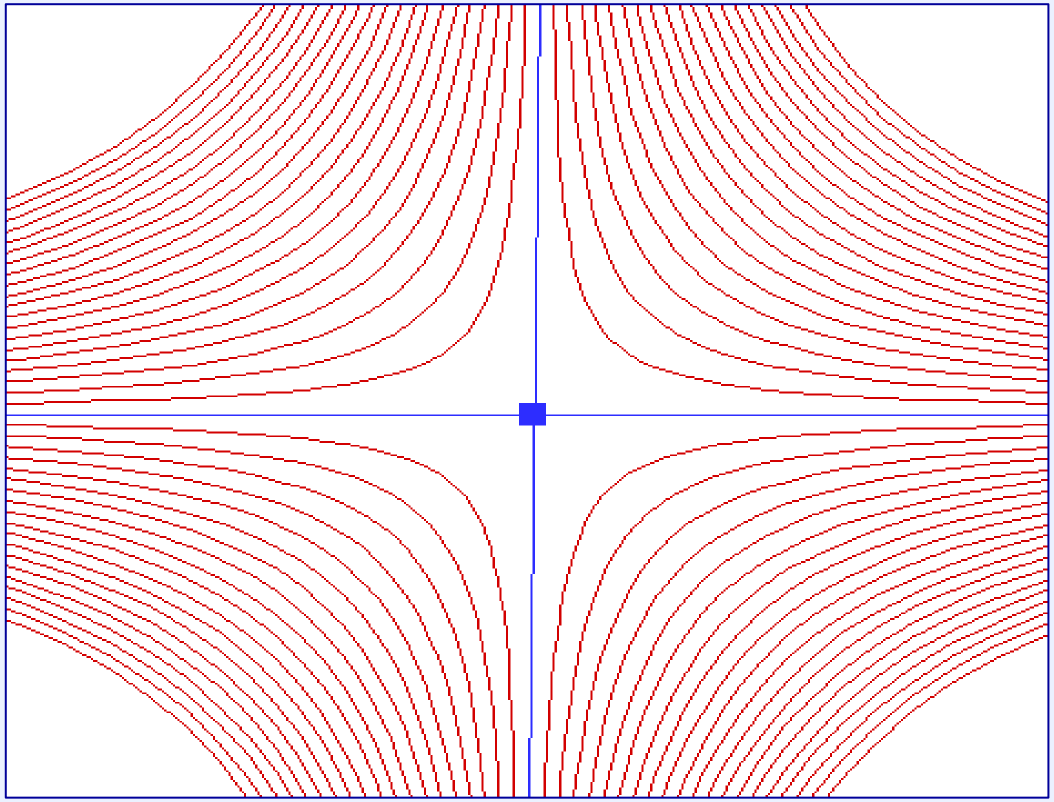


$S_1 = S_2$

Saddle type critical point



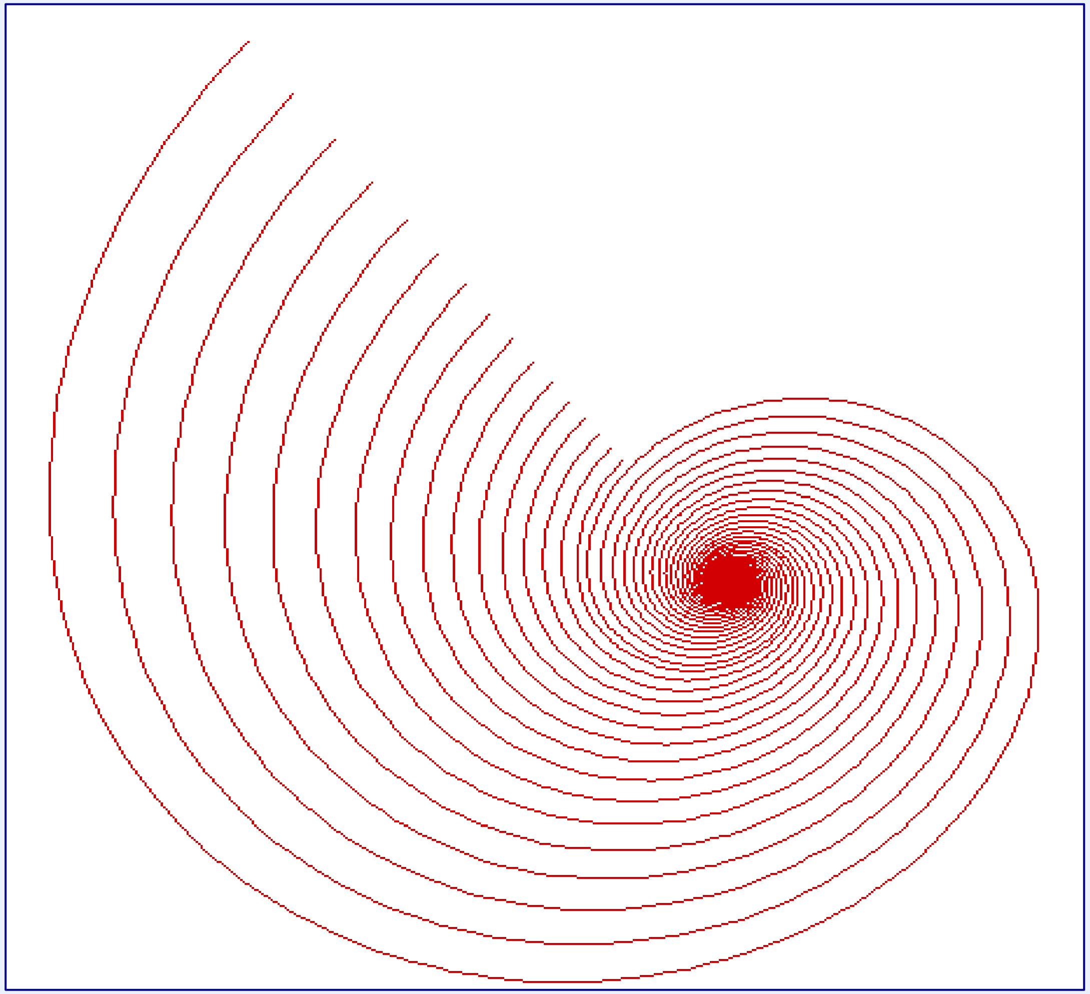
$\lambda_1 = 1 \quad , \quad \lambda_2 = -1$

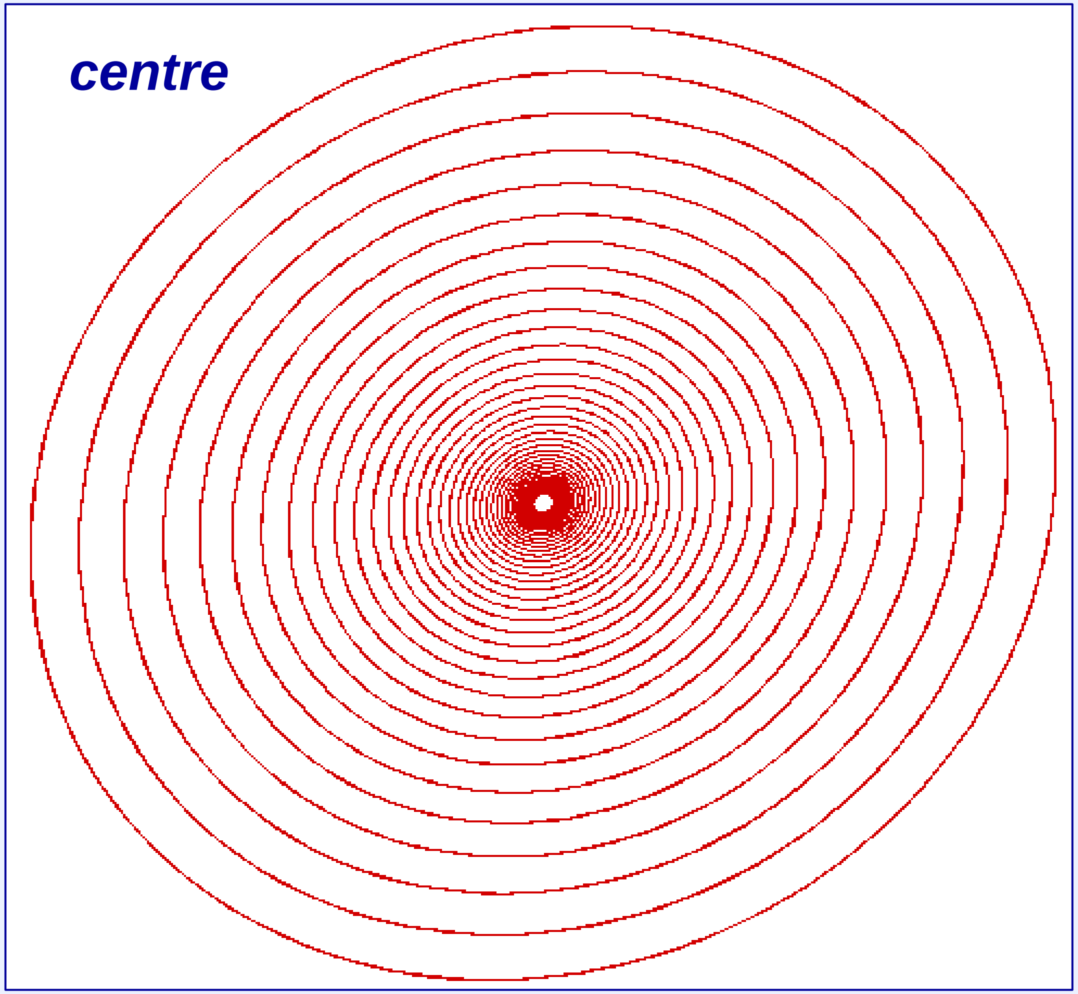


$\lambda_1 = 0 \quad , \quad \lambda_2 = -100$

$S_1 = -1 \quad , \quad S_2 = 1$

Focus type critical point

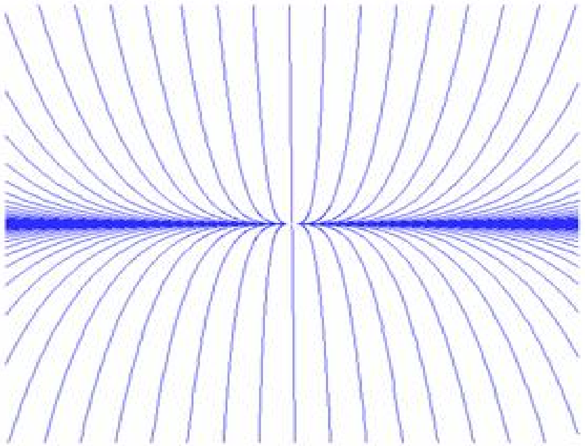




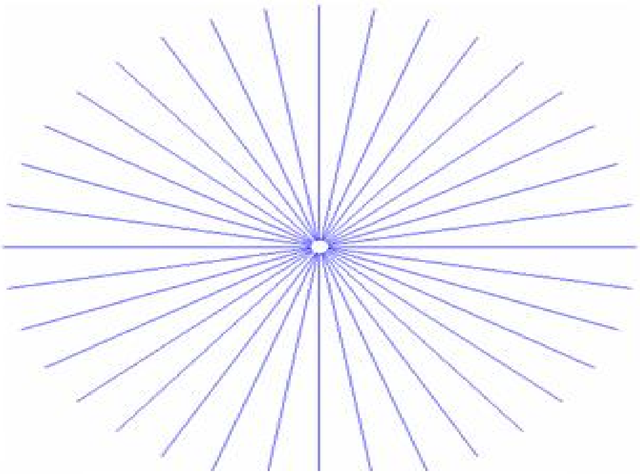
$$S_{1,2} = 1 \pm i \times 4$$

$$S_{1,2} = \pm i \times 4$$

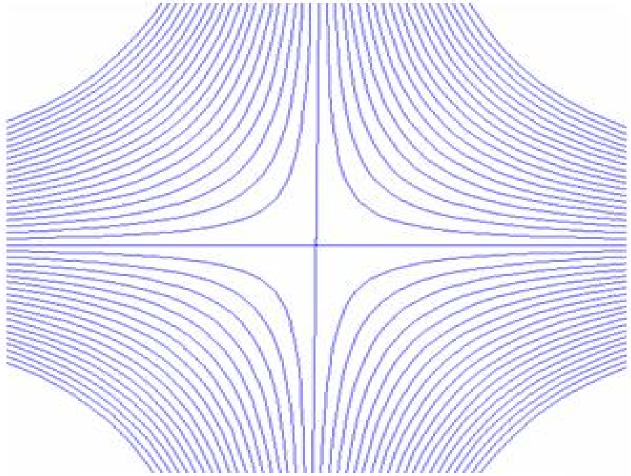
The different critical points



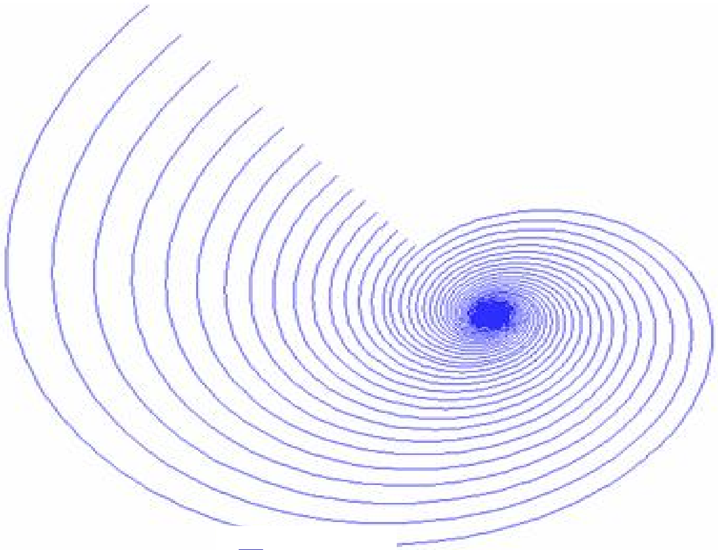
Node



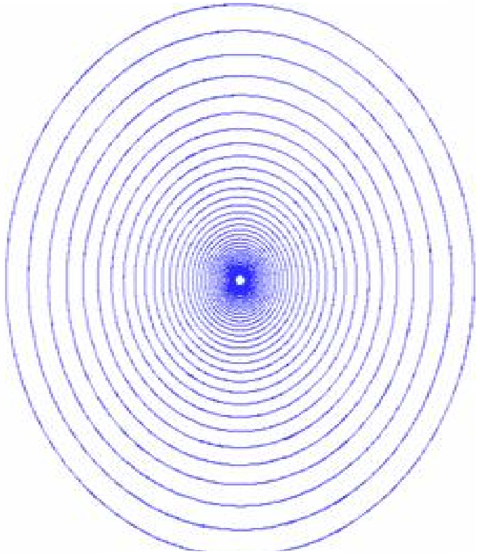
Isotropic node



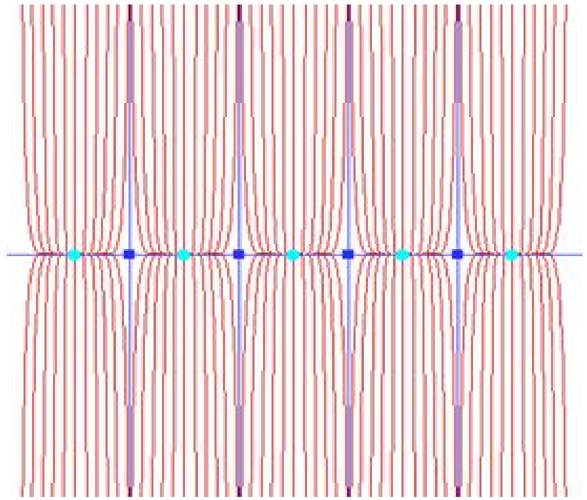
Saddle point



Focus

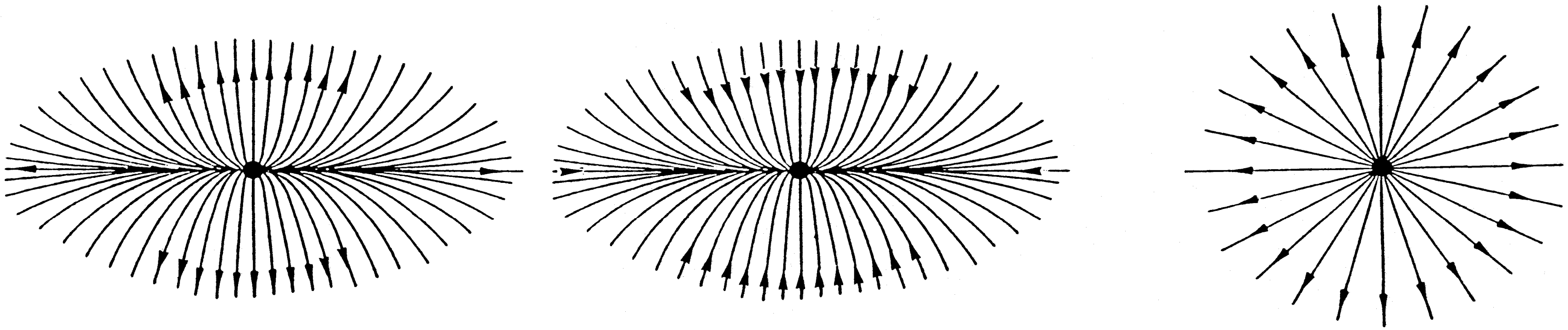


Centre

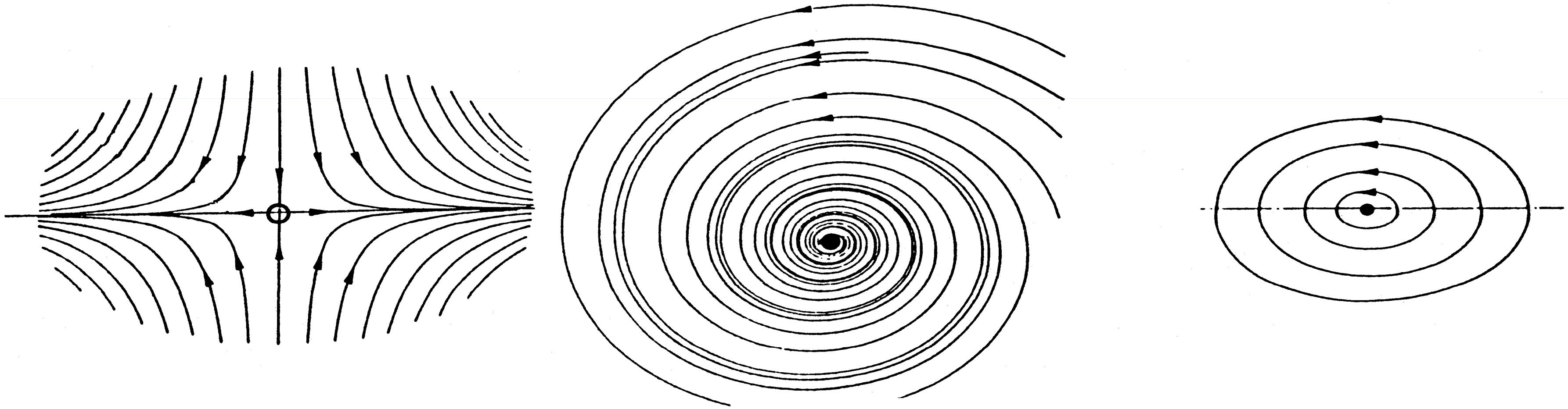


Node-saddle pt. combination

The main critical points



Attachment node — Detachment node — Isotropic attachment node

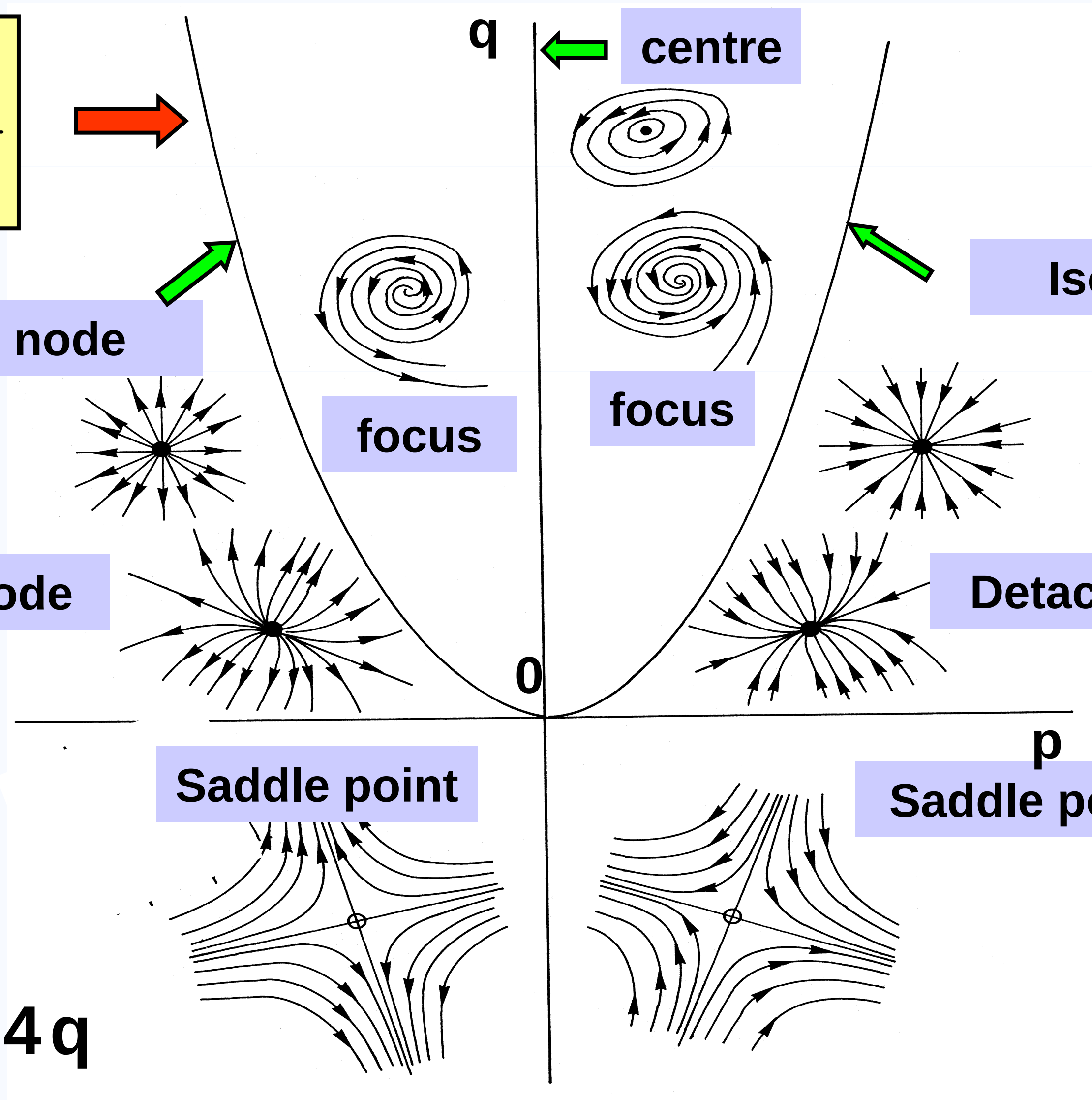


Saddle point — Focus — Centre

The sense of displacement along the trajectories gives a physical meaning to the solution behaviour in the vicinity of the critical points

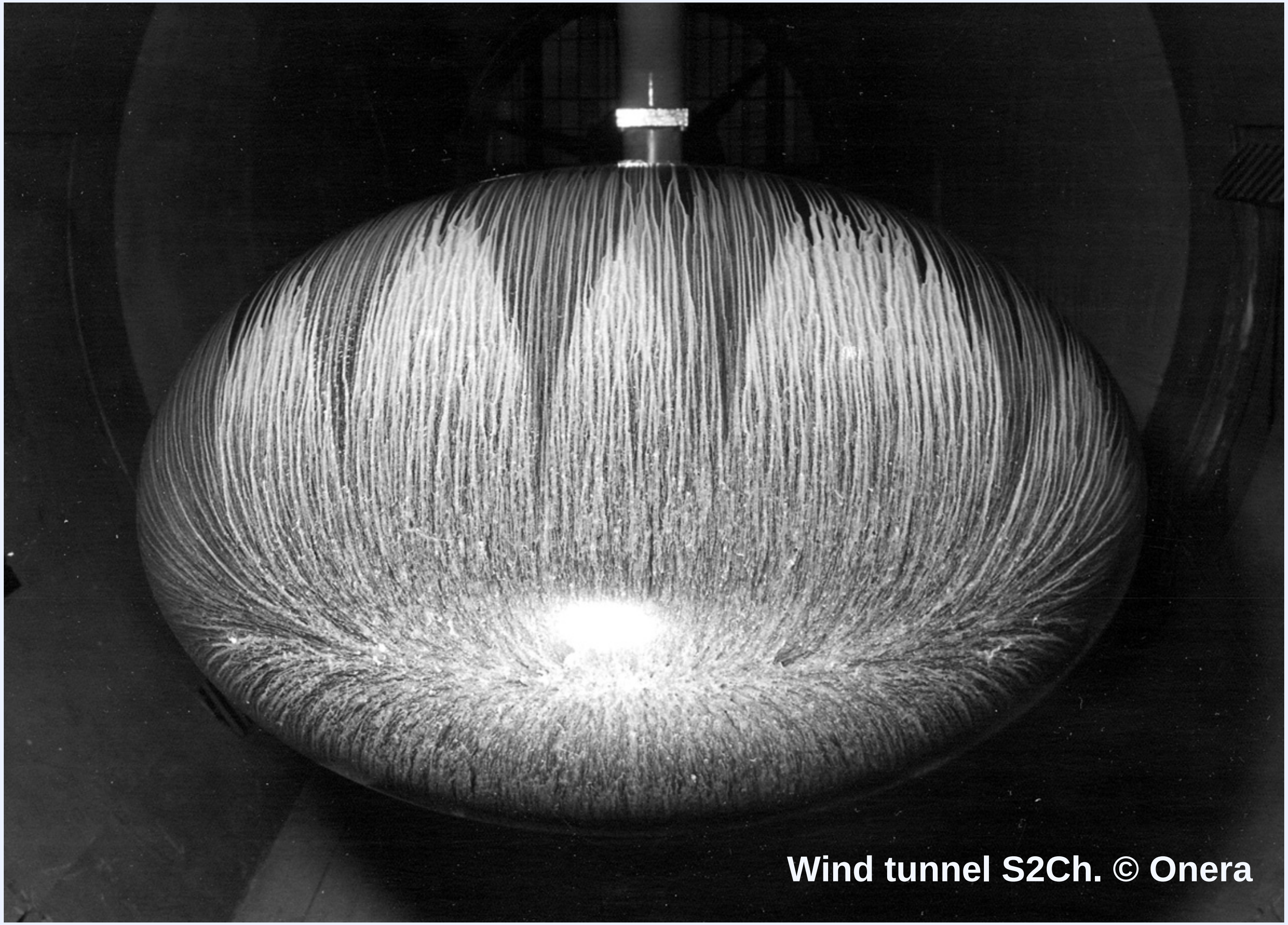
Critical point classification in the plane [p,q]

$$q = \frac{p^2}{4}$$



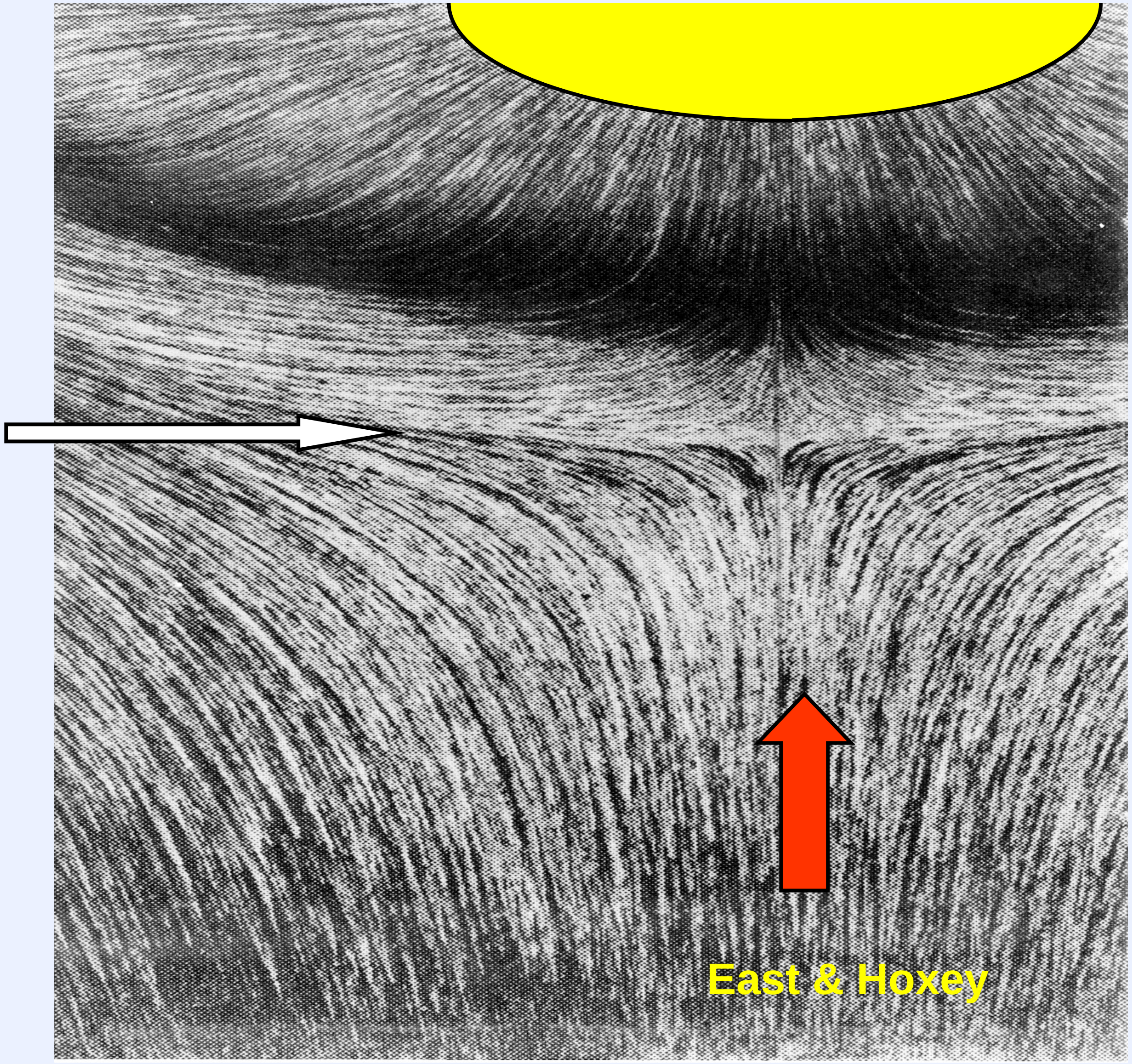
$$D = p^2 - 4q$$

Attachment node at the nose of a blunt body



Attachment node at the nose of a blunt body

Saddle point



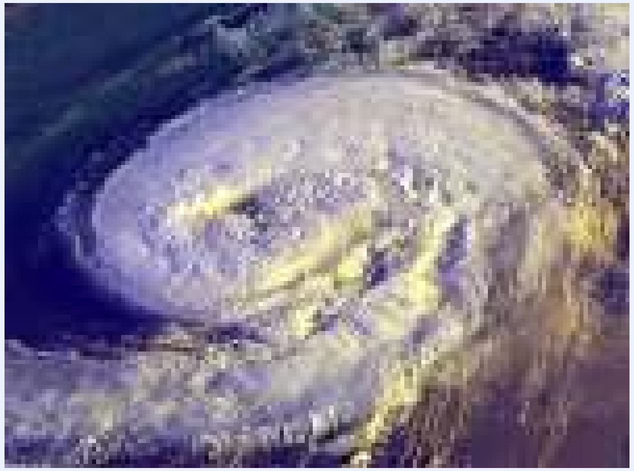
Separation in front of an obstacle

Spectacular foci

Hurricanes

Hugo

Mitch



Bonnie

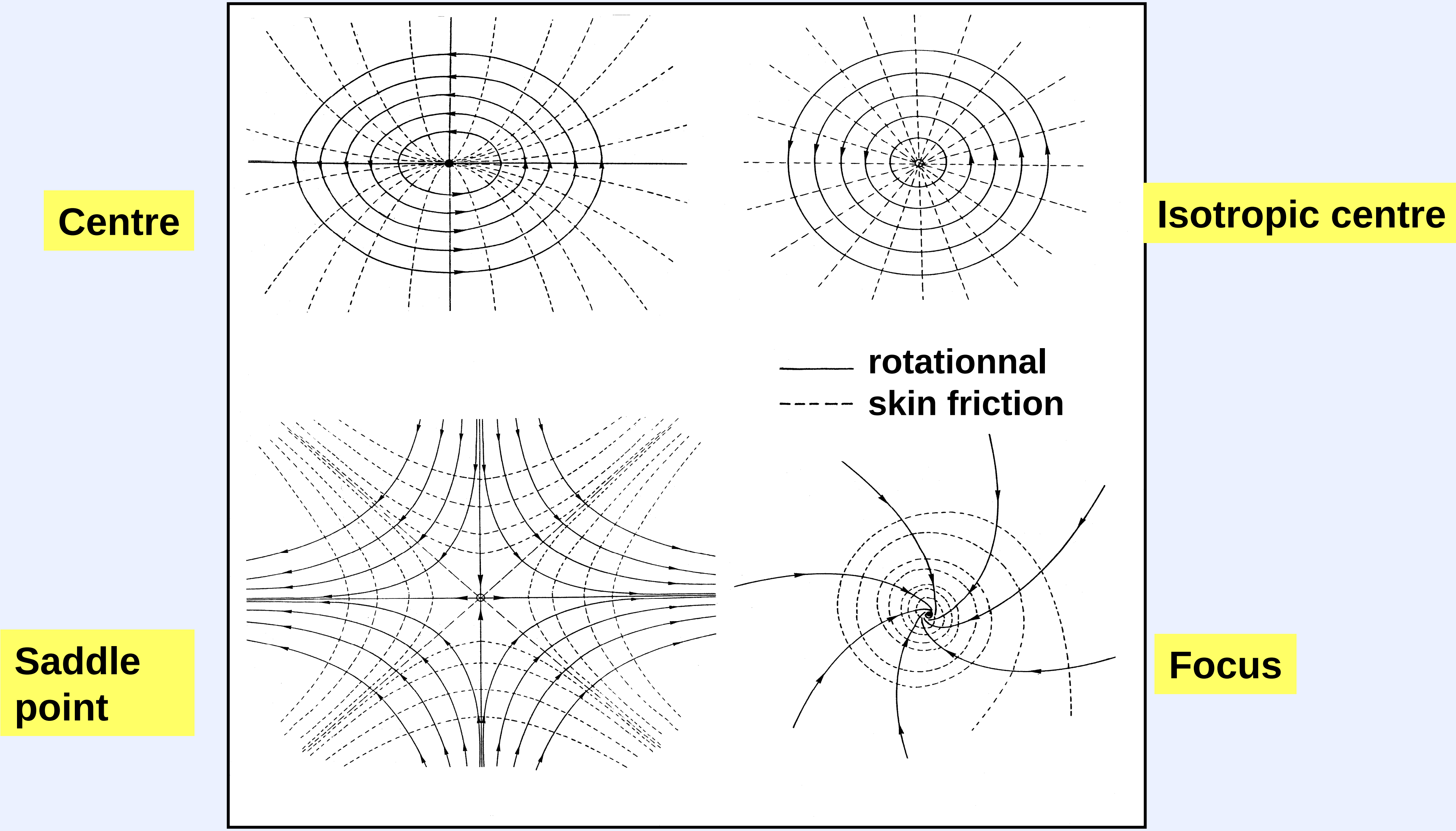
Luis

Critical points for the rotational at the wall

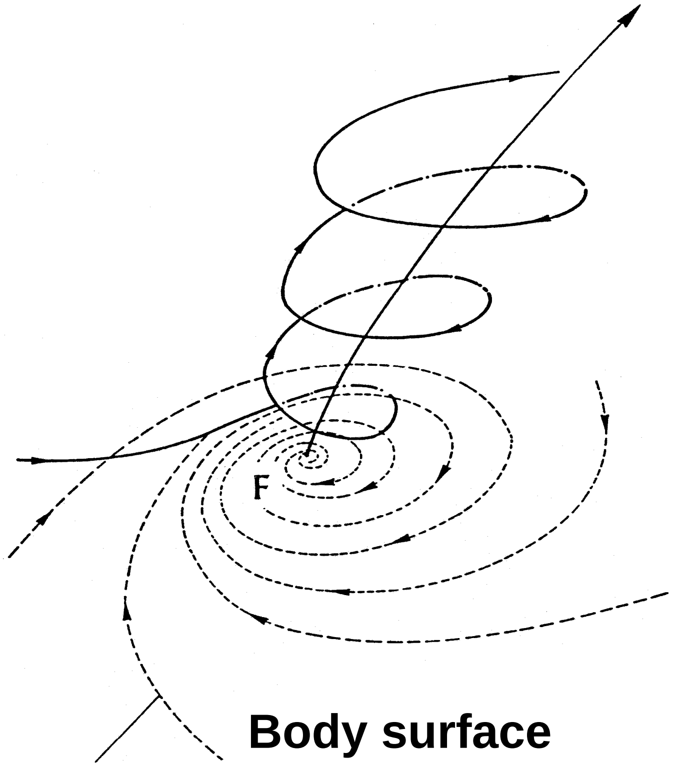
Well known results:

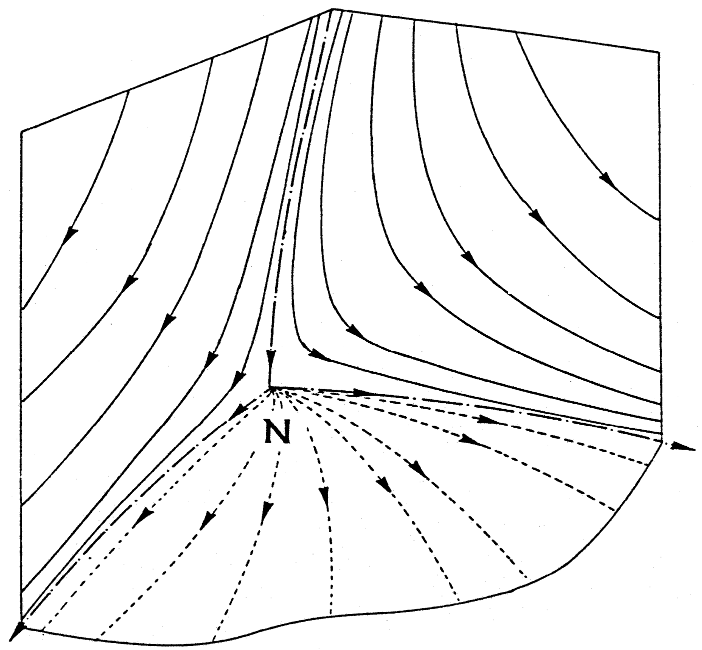
- ➔ **On a surface, the rotational vector $\vec{\Omega}$ is tangent to the surface.**
- ➔ **At a point on a surface, the skin friction vector $\vec{\tau}$ and the rotational vector $\vec{\Omega}$ are orthogonal.**
- ➔ **The skin friction lines and the rotational trajectories are two families of orthogonal curves.**
- ➔ **The critical points of the skin friction field are also critical points for the rotational field (in general of different nature).**

Critical points for the rotational at the wall



Three-dimensional nodes and foci



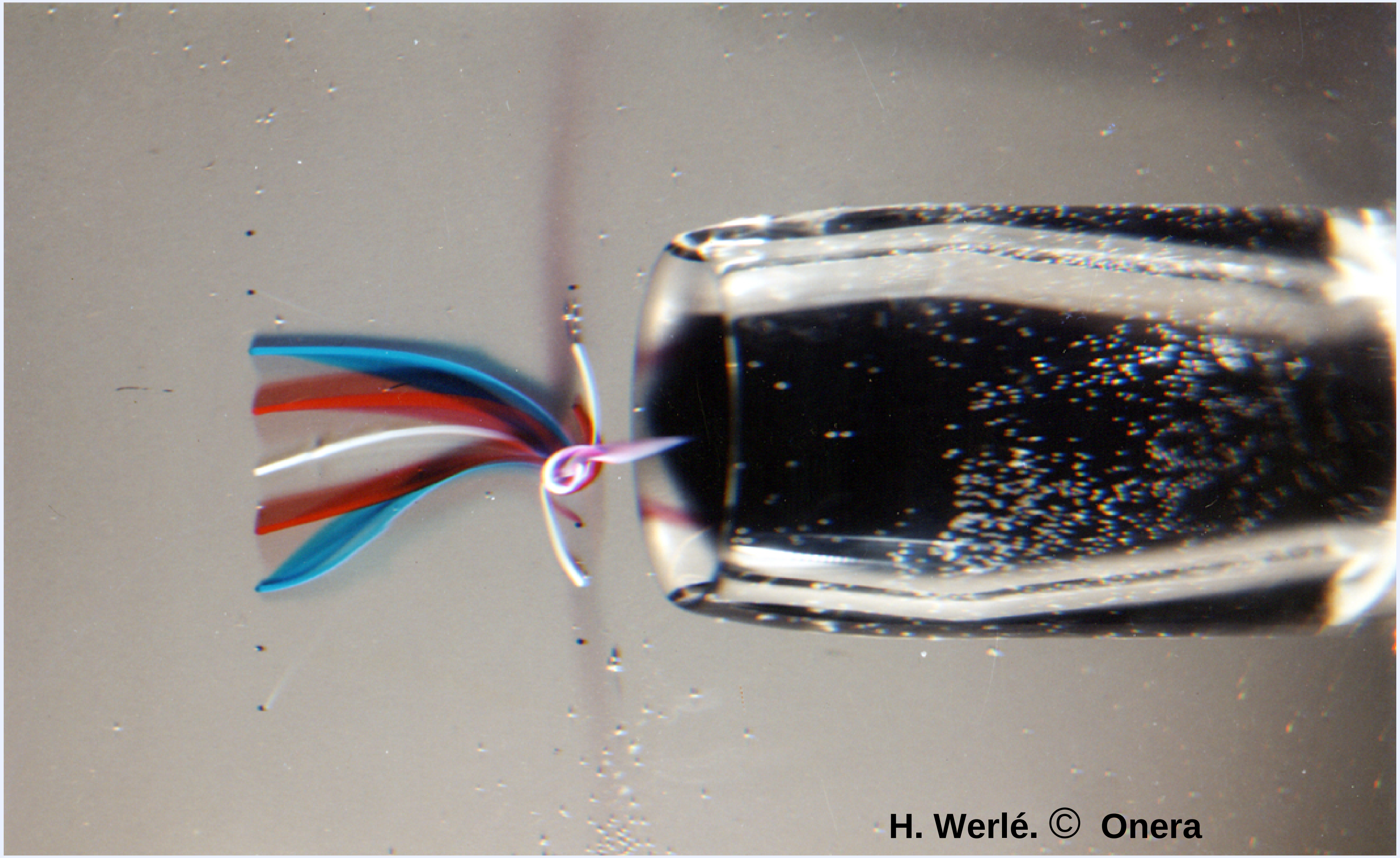


Focus

Node

Skin friction line

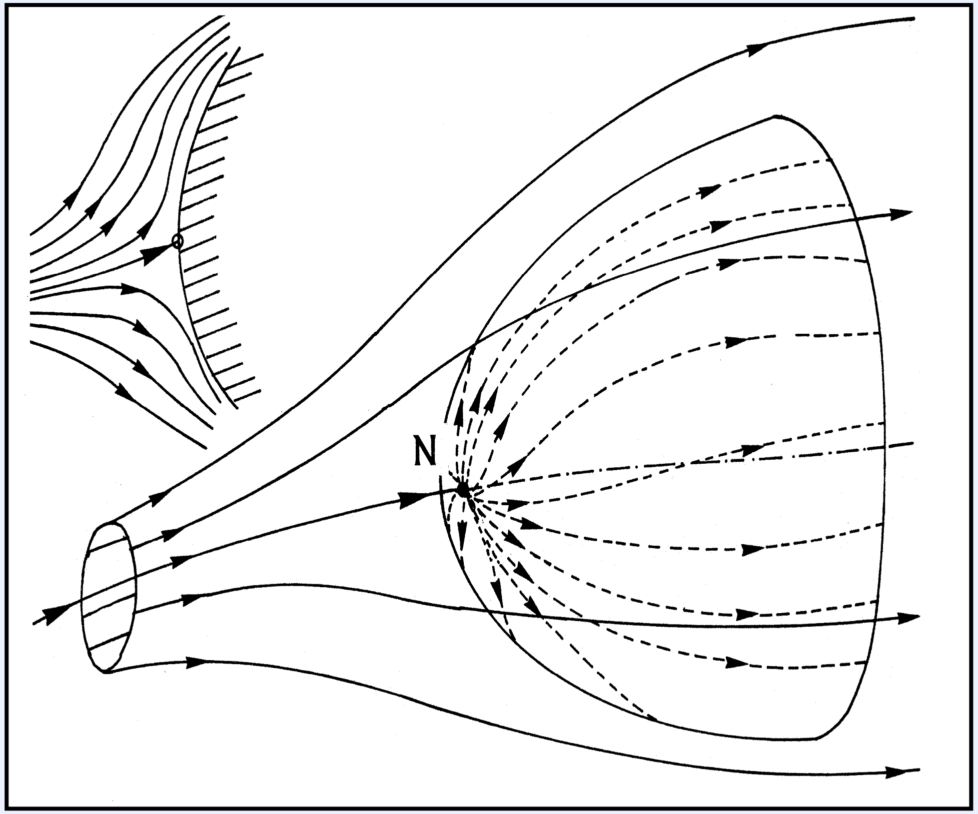
Vortex swallowed by an air intake



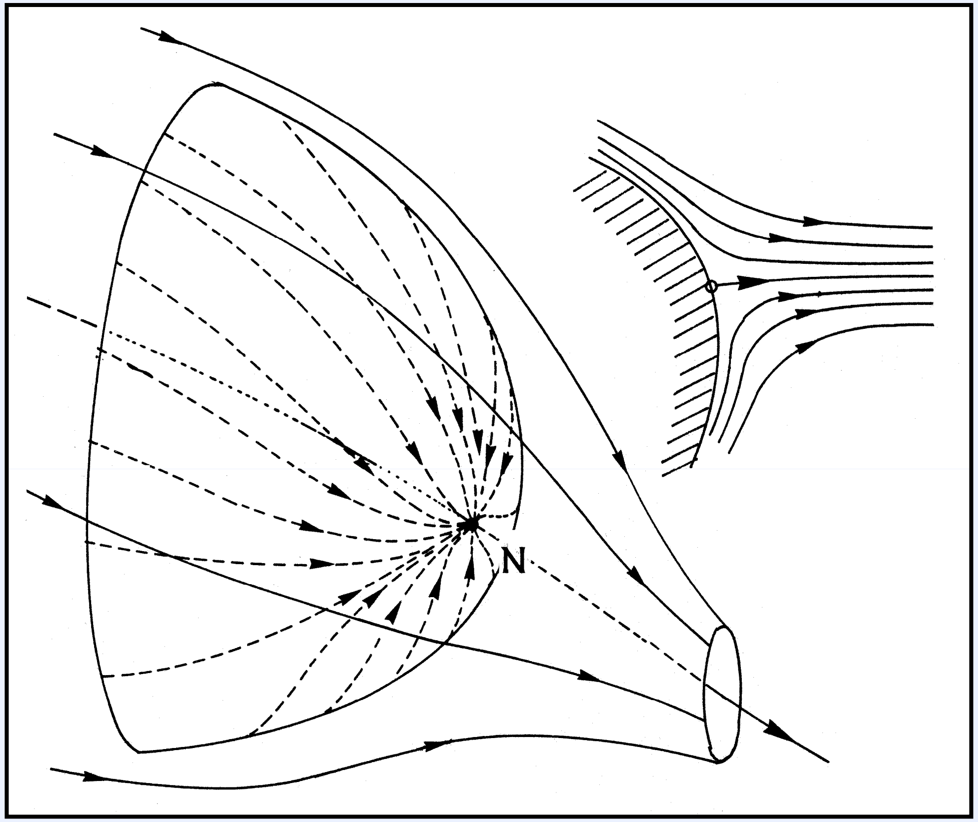
The air intake suction effect induces the formation on a vortex springing from the ground. Here the vortex is swallowed by the air intake.

Three-dimensional nodes and foci

Attachment point

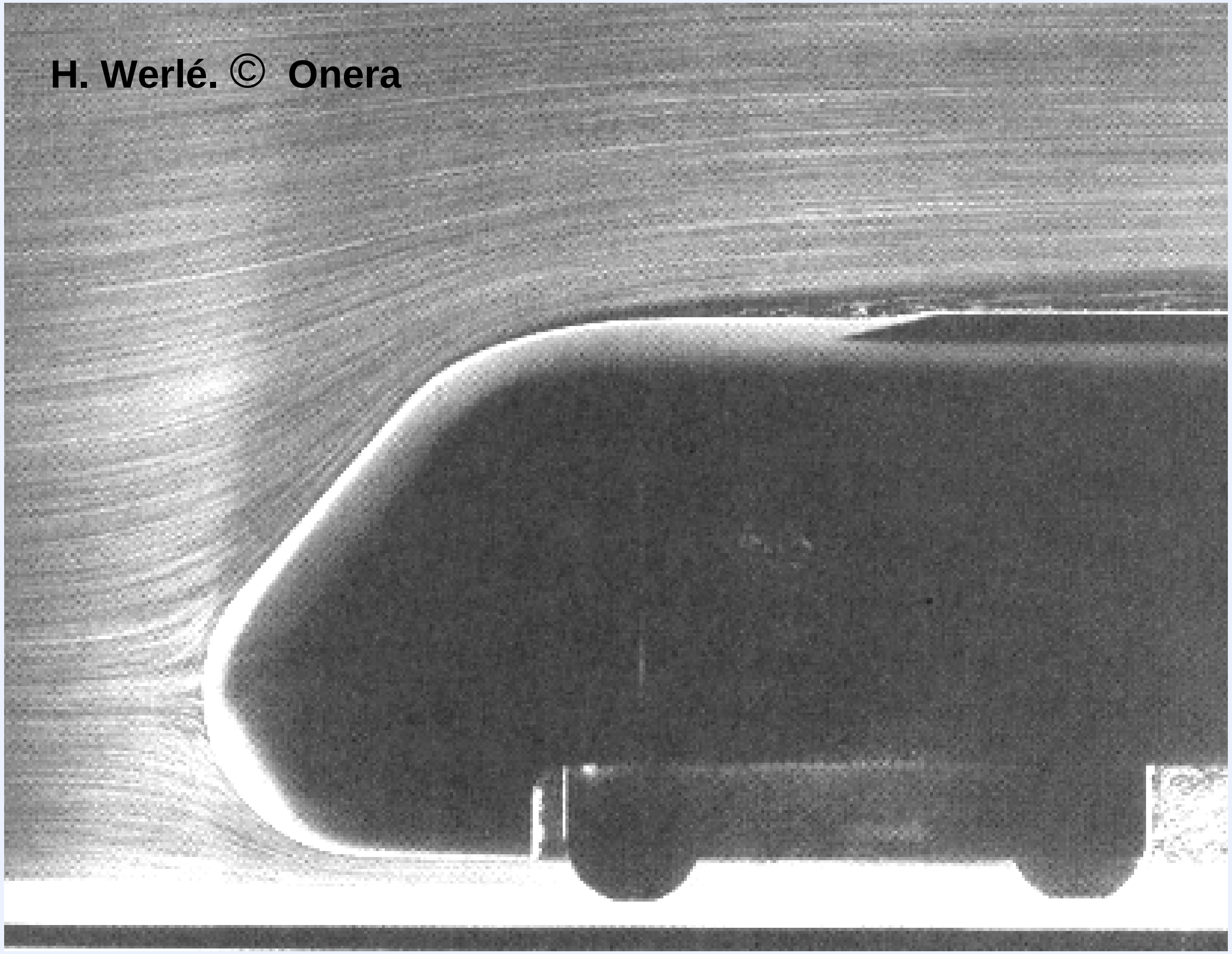


Detachment point



To a node on the surface is associated a saddle point in the contiguous flow

Three-dimensional nodes and foci



Attachment node on the front part of a high speed train