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LARGE-EDDY SIMULATION OF INCOMPRESSIBLE VISCOUS FLUID FLOW BY THE SPECTRAL ELEMENT METHOD

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ABSTRACT

We present the large-eddy simulation of the lid-driven cubic cavity flow by the spectral element method (SEM) using three subgrid-scale models. Two spectral filtering techniques suitable for these simulations have been implemented. Numerical results for Reynolds number Re $= 12.000$ *are showing very good agreement with the Direct Numerical Simulation (DNS) results and relatively good agreement with the experimental results found in the literature [5,7,8].*

INTRODUCTION

Spectral element methods have been mainly applied to the Direct Numerical Simulation (DNS) of fluid flow problems at low and moderate Reynolds (Re) numbers. Despite their high accuracy, spectral element methods are still far from reaching industrial applications that involve developed turbulence at Re values of the order of $10^6 - 10^7$. The reason for that dismal performance is that a resolved DNS including all scales from the large structures to Kolmogorov scales, needs a number of degrees of freedom (dof) that grows like $\mathrm{Re}^{9/4}$. Therefore with increasing Re, we have to increase the number of elements, E, and the degree, N, of the polynomial spaces. This places the computational load far out of the reach of present day computers.

Large-Eddy Simulation (LES) [9,11] represents an alternative to DNS insofar that it involves less dofs because the behaviour of the small scales are modeled. We will focus our attention on the Smagorinsky, dynamic and dynamic mixed models which will be evaluated by comparing LES and DNS numerical results on the three-dimensional cubic cavity flow.

MATHEMATICAL MODELLING

Large-scale quantities, designated by an "overbar", are obtained by a filtering process on the domain Ω. Assuming the filter commutes with differentiation and applying the filter to the Navier–Stokes equations in divergence form for the nonlinear term, one obtains the following relations:

$$\frac{\partial \overline{\mathbf{v}}}{\partial t} + \boldsymbol{\nabla} \cdot \overline{\mathbf{v}}\,\overline{\mathbf{v}} = -\boldsymbol{\nabla}\overline{p} + \nu \boldsymbol{\Delta}\overline{\mathbf{v}} - \boldsymbol{\nabla} \cdot \boldsymbol{\tau}, \qquad (1)$$

$$\boldsymbol{\nabla} \cdot \overline{\mathbf{v}} = 0. \qquad (2)$$

Here, $\overline{\mathbf{v}}$ is the filtered velocity, t denotes the time, $\overline{p}$ is the filtered pressure divided by the constant density, ν the kinematic viscosity. The symbols $\boldsymbol{\nabla}$ and $\boldsymbol{\Delta}$ represent the nabla and Laplacian operators, respectively. The SGS stress tensor $\boldsymbol{\tau}$ takes the small-scale effects into account and is given by

$$\boldsymbol{\tau} = \overline{\mathbf{v}\mathbf{v}} - \overline{\mathbf{v}}\,\overline{\mathbf{v}}. \tag{3}$$

Several large-eddy simulation models are considered to solve the filtered Navier–Stokes equations: the Smagorinsky model [12], the dynamic model of Germano *et al.* [3] and the dynamic mixed model [13].

NUMERICAL APPROXIMATION

Space discretization

The numerical approximation is obtained through a weak formulation of the Eqs. (1)–(2) discretized using the Lagrange–Legendre approximation. The reader is referred to the monograph by Deville, Fischer and Mund [2] for details. The velocity and pressure are expressed in the $P_N - P_{N-2}$ spaces where ${}_N$ is the set of polynomials of degree $\leq N$ in each space direction. This approximation avoids the presence of spurious pressure modes as it was proved by Maday and Patera [10]. The quadrature rules involved in the weak formulation define a Gauss–Lobatto–Legendre (GLL) grid for the velocity nodes and a Gauss–Legendre (GL) grid for the nodal pressures.

Time discretization

As the LES viscosity is not invariant, we modify the standard time integration scheme in such a way that this space varying viscosity be handled explicitly as this was done e.g. in [1, 4]. Let us define the effective viscosity as

$$\nu_{\text{eff}} = \nu + \nu_T = \nu_{\text{cst}} + (\nu_{\text{eff}} - \nu_{\text{cst}}) \tag{4}$$

where ν_{cst} is the sum of ν and the spatial average of ν_T over the computational domain, being by construction constant in space but not in time. The time scheme is a second-order backward differentiation formula for the linear viscous and pressure terms while the nonlinear term is treated by a second-order extrapolation method [2]. The implicit part is solved by a generalized block LU decomposition, using a standard fractional-step method with pressure correction which may be preconditioned by various algorithms.

FILTERING

As spectral elements offer high accuracy for the flow at hand, we construct the filters using two spectral techniques. The first one is a nodal filter acting in physical space on the nodal velocity components (and pressure) to stabilize the computations. The second method is designed as a modal filter and is carried out element-wise in spectral space and corresponds to the convolution kernel of the LES filtering.

THE LID-DRIVEN CUBICAL CAVITY PROBLEM

The lid-driven cavity presents although the geometry is simple, complex physical phenomena. As in this case, we have no homogeneous direction, the presence of side walls confining the full flow modifies the flow patterns and the route to turbulence.

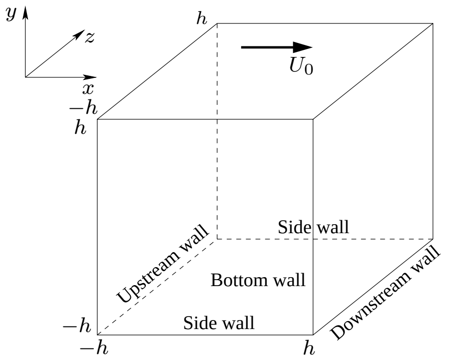

Fig. 1. Sketch of the geometry of the lid-driven cubical cavity.

Figure 1 shows the cubical cavity. The flow motion is induced by the top lid that moves in the x-direction with a constant unit velocity $U_0 = 1$. The Reynolds number is consequently $\mathrm{Re} = U_0 2h/\nu$. We will essentially address the case of the flow at $\mathrm{Re} = 12.000$. The kinetic energy is provided to the flow by the shear stress at the top lid through viscous diffusion. The amplitude of the Reynolds stress below the lid is negligible indicating that the flow under the lid is mainly laminar but transient. The momentum transfer from the lid induces a region of strong pressure in the upper corner of the downstream wall as the flow, mainly horizontal prior the corner, has to change direction and moves vertically downwards. This sharp turn dissipates energy in that region.

In order to resolve the boundary layers along the lid and the downstream wall, the spectral elements are unevenly distributed. The spatial discretization has $E_x = E_y = E_z = 8$ elements in the three space directions with $N_x = N_y = N_z = 8$ polynomial degree. The spectral element calculation has two times less points per space direction than the DNS of Leriche-Gavrilakis [8] who employed a 129^3 Chebyshev discretization. No-slip conditions are applied to the other walls. The time step is chosen as $\Delta t = 10^{-3}$ and the complete simulation comprises 387.000 iterations leading to a total effective simulation time of 387 time units (h/U_0) for the dynamic and dynamic mixed models only. Partial simulations comprising 33.000 iterations have been performed for the Smagorinsky model and for the under-resolved DNS. The reference results are the DNS data of Leriche [6] and the experimental ones from Koseff and Street [5], corresponding to 1.000 and 145.5 time units respectively. In the cavity flow, the average is obtained by time averaging. One-dimensional lines in the mid-plane $z = 0$ are presented in Fig. 2–5, with the following legend: Experimental data by Koseff *et al.* (crosses), DNS by Leriche (solid line), LES dynamic model (dashed line) and LES dynamic mixed model (dotted line).

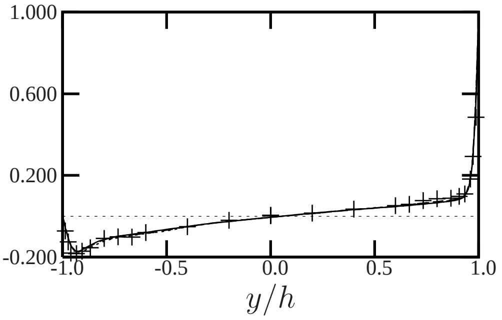

Fig. 2. $\langle U \rangle$ on the centerline $x = 0$ in the mid–plane $z = 0$, Experimental data by Koseff *et al.* (crosses), DNS by Leriche (solid), LES dynamic model (dashed) and dynamic mixed model (dotted)

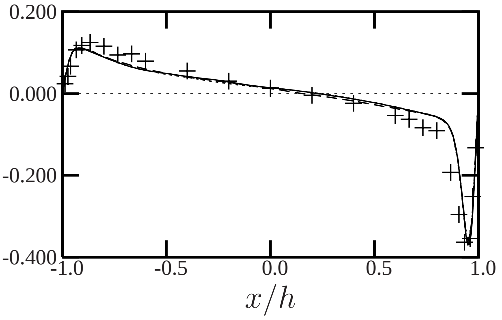

Fig. 3. $\langle V \rangle$ on the centerline $y = 0$ in the mid-plane $z = 0$

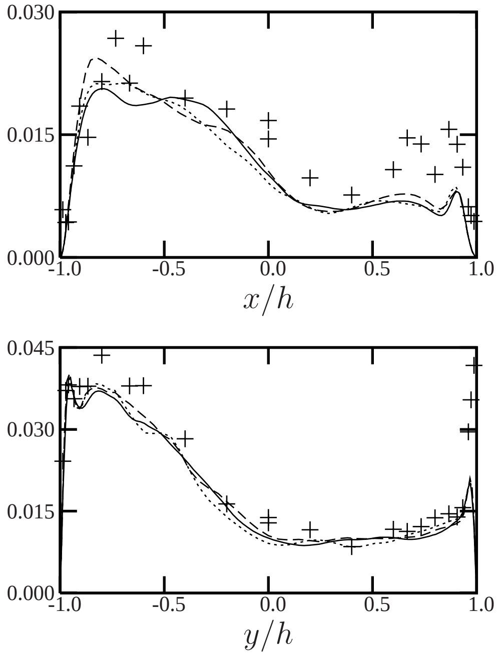

Fig. 4. $\sqrt{\langle u^2 \rangle}$ on the centerline $y = 0$ in the mid–plane $z = 0$ (top), on the centerline $x = 0$ in the mid-plane $z = 0$ (bottom)

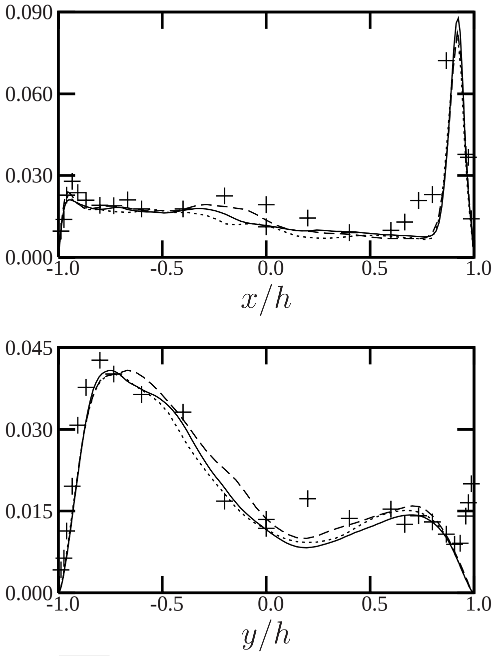

Fig. 5. $\sqrt{\langle v^2 \rangle}$ on the centerline $y = 0$ in the mid–plane $z = 0$ (top), on the centerline $x = 0$ in the mid-plane $z = 0$ (bottom)

CONCLUSIONS

Large-eddy simulations of the three-dimensional lid-driven cubical cavity flow using the spectral element method has been presented. The treatment of the subgrid-scales relies on three different eddy viscosity models: Smagorinsky model, dynamic model and a dynamic mixed model. These LES have been carried out on a parallel architecture with a relatively coarse grid and numerical results appeared to be extremely close to the DNS results available in the literature and relatively close to the experimental results. Moreover the results of the under-resolved DNS on the coarse grid (not shown here) are far from providing any insight into the physics of the problem.

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