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Anisotropy of transport properties in magnetohydrodynamic turbulent flows

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ABSTRACT

Direct numerical simulations of a passive scalar in a turbulent conducting flow subject to an externally applied magnetic field are performed. The magnetohydrodynamics (MHD for short) equations are solved numerically, in parallel with an advection-diffusion equation for the passive scalar, using a spectral resolution of 256^3 *Fourier modes. The Magnetic Reynolds number is assumed to be lower than unity, so that the quasi-static approximation can be used. Turbulence statistics are computed to study the anisotropy induced by the magnetic field in the flow as well as in the scalar distribution.*

INTRODUCTION

Conducting fluids are encountered in many industrial and technological applications, from the fabrication of semi-conductors crystals to the design of efficient cooling blankets for future fusion reactors (tokamaks). For most laboratory flows, the magnetic Reynolds number $R_m = UL/\eta$ is very low (here η is the magnetic diffusivity while U and L represent characteristic velocity and length scales for the flow considered). In that case, an applied magnetic field can strongly affect the fluid's motion but there is practically no retroaction of the flow on the applied field. This regime of magnetohydrodynamics is governed by the so-called quasi-static approximation (or inductionless approximation) [1](see below). The main effects of an applied magnetic field on this kind of flows are the damping of turbulence through Joule dissipation and the creation of anisotropy which manifest itself by the elongation of flow structures in the direction of the magnetic field [2–5]. The purpose of this work is to analyze how those flow modifications affect the transport of passive scalar quantities which can be the temperature of the medium or a pollutant density depending on the application considered.

BASIC EQUATIONS

The evolution of the scalar in a flow is governed by an advection-diffusion partial differential equation :

$$\frac{\partial}{\partial t}\Theta(\mathbf{x},t) + (\mathbf{u}\cdot\nabla)\Theta(\mathbf{x},t) \;=\; \kappa\Delta\Theta(\mathbf{x},t) \qquad (1)$$

where Θ is the scalar quantity, **u** the velocity field and κ the diffusion coefficient. As is clear from the above equation, the transport is not directly affected by the applied magnetic field but is only modified through the damping and deformation of the velocity field. In order to obtain the evolution of Θ, Eq. (1) must be solved along the

Navier-Stokes equation for the velocity field $\mathbf{u}$:

$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla\right) \mathbf{u} = -\nabla(p/\rho) + \nu\Delta\mathbf{u} + \mathbf{f}_L + \mathbf{f} \quad (2)$$

where p is pressure, ρ the density, ν the viscosity, σ the electrical conductivity, $\mathbf{f}$ is a force density to sustain the turbulence and $\mathbf{f}_L$ is the Lorentz force. In the framework of the quasi-static approximation the latter depend only on the applied magnetic field (which, by convention, is directed along the z-direction) and of the velocity field (see for example [1]) ; A very important parameter of a flow governed by (2) is the so-called interaction parameter:

$$N = \sigma \frac{B_z^2 L}{\rho u}. \quad (3)$$

where u and L represent characteristic velocity and length scales. On dimensional grounds, the interaction parameter measures the relative strengths of the Lorentz force and non-linear terms in (2). If N is sufficiently large, the Lorentz force can drive the flow to a quasi two-dimensional state [6], where the flow structures are strongly elongated in the magnetic field direction.

NUMERICAL SIMULATIONS

The equations (1) and (2) are simultaneously solved in a three dimensional periodic box, using a pseudo-spectral code and a resolution of 256^3 Fourier modes. All the simulations consist in starting from a non-homogeneous random, zero-mean, distribution for the passive scalar Θ which decays freely in a turbulent forced velocity field. The initial spectrum of the scalar is supposed to be proportional to $k^{-5/3}$ and the Prandtl number ($Pr = \nu/\kappa$) is the same for every run and is equal to 1. The initial velocity field is chosen to be a developed turbulent field adequately resolved in the computational domain and is forced through injection of energy in the large scales of the flow. For each cases considered, we waited for a statistically stationary state to be reached after the magnetic field is switched on, before starting the decay of the scalar. In order to quantify the transport properties of the flow, several runs with different values of the interaction parameter N (0, 1 and 10) are considered. In this last case ($N = 10$) the flow is quasi two-dimensional.

RESULTS

Velocity field

The evolution of the kinetic energy $E_K = < u_i u_i > /2$ of the flow is shown in figure 1. The moment $t = t^*$ correspond to the time of application of the magnetic field, and we can clearly see the additional dissipation due to the Joule effect. In this figure, as well as in the subsequent ones, time has been non dimensionalised using the eddy turn over time calculated before the application of the magnetic field, $\tau_u = L(t^*)/u(t^*)$. The strange behaviour in the $N = 10$ case is

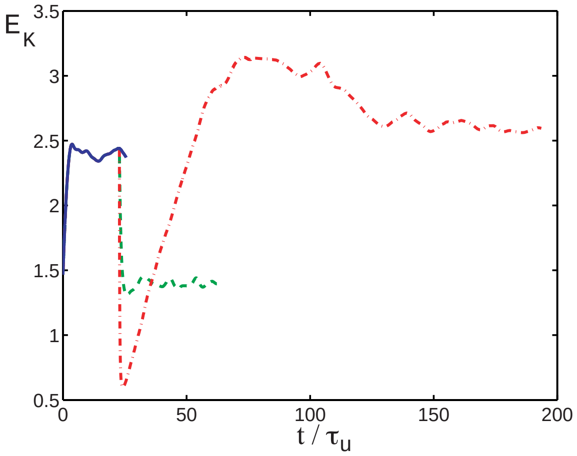

Fig. 1. Kinetic Energy evolution. (–) : $N = 0$, (- -) : $N = 1$, (-.) : $N = 10$.

explained by the modifications in the energy cascade and in the velocity field spectra, induced by the magnetic field [3], which allow to accumulate more energy in the flow subject to the strongest field.

The velocity field spectra are plotted in figure 2, again for the three cases considered. With increasing magnetic field, the Kolmogorov scale η become larger, confirming the fact that the magnetic field has a damping effect on the turbulence intensities. The largest scales on the contrary carry more energy than in the hydrodynamic case, reflecting the phenomenon already observed in fig.1.

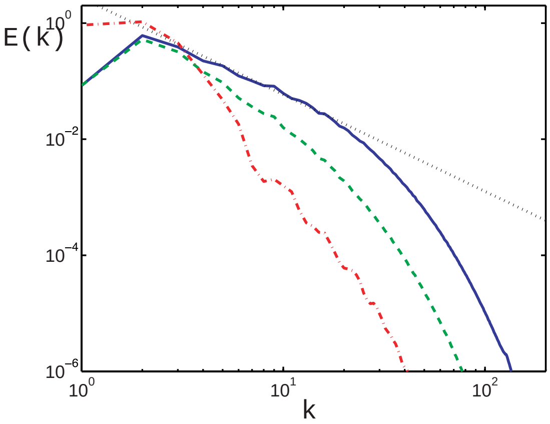

Fig. 2. Energy spectra. (–) : $N = 0$, (- -) : $N = 1$, (-.) : $N = 10$, (...) : $k^{-5/3}$.

Scalar Energy decay

As for the velocity field, we define the *energy* for the scalar field as the mean-square of its fluctuations, $E_\Theta = \frac{1}{2} < \Theta^2 >$. Figure 3 shows the evolution of E_Θ with time. The decay is clearly slower with increasing magnetic field. We might first think that this is due to the less effective convective transport when the magnetic damping is stronger, but this is not the case. Indeed, the relative kinetic energy levels are as follow (see Fig.1), $E_K(N = 1) < E_K(N = 0) < E_K(N = 10)$. In the specific problem we considered, the largest energy carrying scales plays a less important role in the process of scalar decay because there is no net scalar flux. The most part of the scalar dissipation is carried out by the small scales, which are more energetic with decreasing magnetic field.

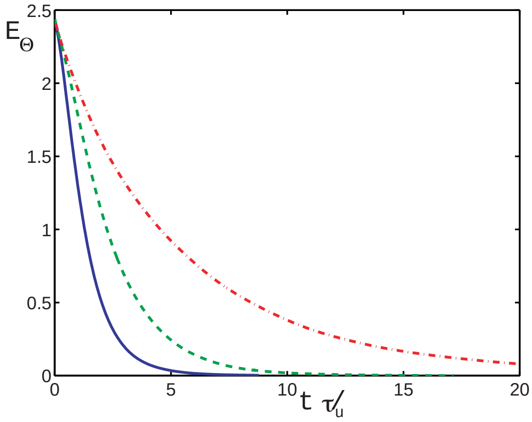

Fig. 3. Scalar energy decay. (–) : $N = 0$, (- -) : $N = 1$, (-.) : $N = 10$.

Anisotropy

Figures 4(a) and 4(b) show the partition of the kinetic energy between the different components of the velocity in the $N = 1$ and $N = 10$ cases. It clearly demonstrate the anisotropy of the flow with increasing interaction parameter. In order to study the anisotropy level of the scalar, we computed in the three cases an anisotropy coefficient defined by

$$\alpha(t) = \frac{< (\partial_\perp \Theta)^2 >}{< (\partial_\parallel \Theta)^2 >} \tag{4}$$

In figure 5 we can see the evolution of α with time. It shows that the transport is indeed anisotropic, the gradients in the direction parallel being almost equal to twice the gradients in the direction parallel in the case $N = 10$. From this figure we can also conclude that the anisotropy level seem to stabilize to a statistically constant value.

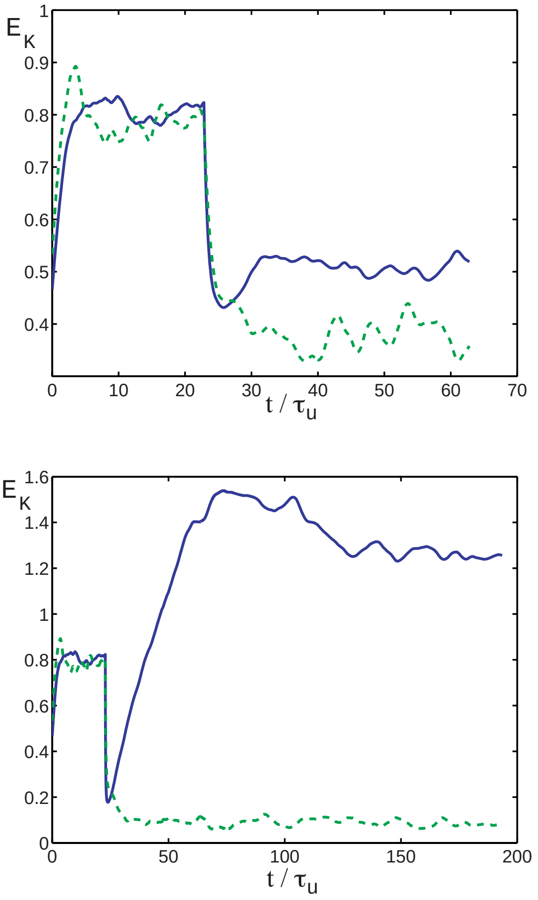

Fig. 4. Kinetic energy partition. (–) : $< u_\perp^2 >= (< u_x^2 > + < u_y^2 >)/2$, (- -) : $< u_\parallel^2 >= u_z^2$ for $N = 1$(a) and $N = 10$ (b).

Scalar Spectra

Although the turbulence cannot be considered to be isotropic anymore, it is still axisymmetric. Then the three dimensional scalar energy spectrum can be defined, in the Fourier space, as a function of the wave number, k, and the angle, γ, between the wave vector **k** and the magnetic field, by the relation:

$$E_\Theta(t) = \int_0^\pi \int_0^\infty E_\Theta(k,\gamma,t)dkd\gamma \tag{5}$$

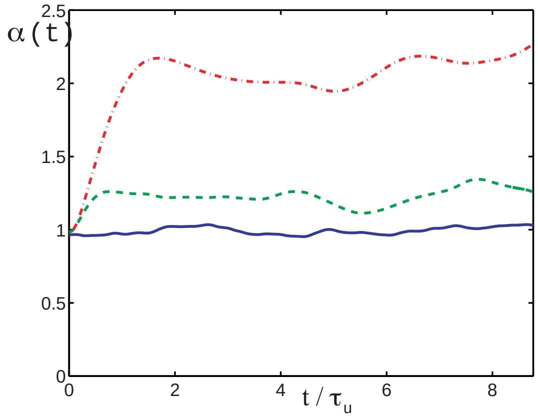

Fig. 5. Anisotropy coefficient. (–) : $N = 0$, (- -) : $N = 1$, (-.) : $N = 10$.

The spectrum $E_\Theta(k,t) = \int_0^\pi E_\Theta(k,\gamma,t)d\gamma$ is shown at figure 6 for one specific instant situated in the beginning of the decay process; the very initial spectra is also shown for comparison. The behaviour is quite different in the three cases. In the hydrodynamic case (continuous curve) the slope of the convective range is lower than the $-5/3$ law predicted by the KOC phenomenology. Of course, at low Reynolds number, the observation of a scaling region is not necessarily easy and can be closer to -1.3 ([7]). For the two flows submitted to an external magnetic field, the evolution of the spectra show that the largest scales transfer their energy to the small scales at a lower rate with increasing magnetic field. The scalar energy cascade seem to be modified in the same way as the kinetic energy cascade : a smaller amount of scalar energy is allowed to travel from the large to the small scales and the Batchelor length scale, η_Θ become larger than in the hydrodynamic case.

The angular dependence of the spectrum is studied as follow. First we define the angular spectra as $E_\Theta(\gamma,t) = \int_0^\infty E_\Theta(k,\gamma,t)dk$, and then we plot the evolution of the evolution of the quantity

$$\beta(t) = \frac{E_\Theta(\gamma=0,t)}{E_\Theta(\gamma=\pi/2,t)} \tag{6}$$

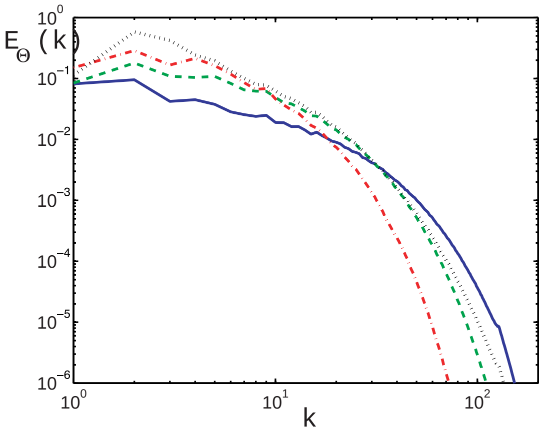

Fig. 6. Scalar spectra. (–) : $N = 0$, (- -) : $N = 1$, (-.) : $N = 10.$, (...) : Initial spectrum.

with time. Again it seems that starting from an isotropic state, the scalar distribution evolve quickly to reach an asymptotic level of anisotropy, in which the scalar energy is dominated by contributions in the vicinity of $\gamma = \pi/2$.

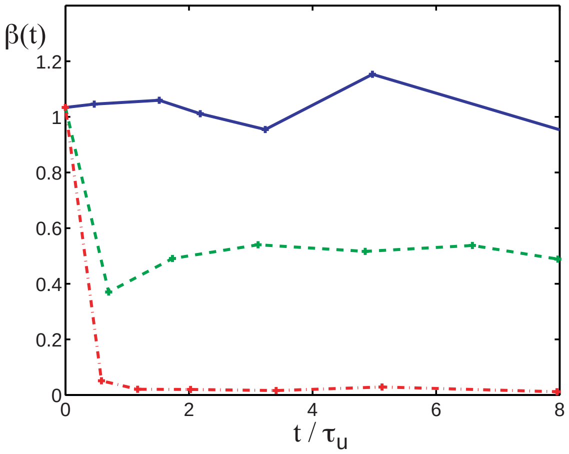

Fig. 7. $\beta(t)$. (–) : $N = 0$, (- -) : $N = 1$, (-.) : $N = 10$.

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