On the Interaction of Turbulent Shear Layers with Harmonic Perturbations

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ABSTRACT

The problem of coherent perturbations in a turbulent shear layer is revisited for the purpose of developing a mathematical model based on the unsteady RANS equations. The ensemble-averaged flow parameters (velocity, pressure) are split into two parts: mean and coherent. The governing equations for these parts are derived, assuming eddy-viscosity equivalence for the random part of the flow, and solved by iterations to provide a coupled solution of the problem as a whole. Calculations agree well with experimental data in the upstream part of the layer, where the spreading rate grows rapidly and the mean-coherent flow interaction is the most important. In this region, the interaction changes the mean flow velocity distribution in such a manner that the neutral stability curve is shifted upstream relative to its position in the undisturbed layer and the perturbation intensity decreases further downstream. Experiments show that the coherent waves suppress the turbulent Reynolds-stress production downstream of this region, but the model fails to predict the layer spreading correctly due to an inadequate turbulence closure of the mean flow. In the case of a turbulent mixing-layer flow, we proposed a new closure, which takes into account this coherent-random interaction. Results of calculations with the improved closure correlate well with existing experimental data.

THE GOVERNING EQUATIONS

Because turbulent shear layers are sensitive to artificial periodic disturbances, they may be used for active flow control purposes. As it is known today, such a control has high potential for many applications (e.g., increase of maximum lift; drag reduction; improvement of aircraft performance, stability and control; etc.). The objective of this work is the developing of a mathematical model for understanding the physics of turbulent shear layers with weak oscillatory forcing.

For the purpose of analyzing the coherent perturbations in a statistically steady turbulent flow, we have derived a tool based on the RANS equations. The flow parameters (velocity, pressure) are split into the ensemble-averaged and the random components, and the ensemble-averaged part is represented as a sum of mean (time-averaged) and harmonics of the forcing frequency. For example, the velocity component u_k $(k = 1, 2, 3)$ is recast as follows:

$$\begin{aligned} &u_k = \bar{u}_k + u'_k, \quad \bar{u}_k = U_k + \tilde{u}_k \\ &\tilde{u}_k = \frac{1}{2}\sum_n \left(\tilde{u}_{k,n} e^{-in\omega t} + \tilde{u}^*_{k,n} e^{in\omega t} \right) \end{aligned} \tag{1}$$

where U_k is the mean flow velocity component, and the coherent part, $\tilde{u}_k$ is associated with the fundamental harmonic $\omega = 2\pi f$ of the oscillatory forcing. The asterisks in Eq. (1), and in what follows, stand for the complex conjugate. This representation is meaningful only when the coherent part is strong enough in comparison with the random component of the same frequency. Substitution of velocity components and pressure in the form of Eq. (1) into the Navier-Stokes equations and time-averaging leads to the RANS mean flow equations in tensor form (the index summation rule is imposed)

$$\begin{aligned} &\frac{\partial U_k}{\partial x_k} = 0 \\ &\frac{\partial U_k U_j}{\partial x_j} + \frac{\partial P}{\partial x_j} = \frac{\partial \tau_{kj}}{\partial x_j} + \frac{\partial \tilde{\tau}_{kj}}{\partial x_j} \end{aligned} \tag{2}$$

Here, the laminar viscosity is neglected, $\tau_{kj} = -\overline{u'_k u'_j}$ represents the mean part of the turbulent Reynolds stress (in the presence of the coherent perturbation), and $\tilde{\tau}_{kj}$ is the coherent Reynolds stress originating from the coherent part of Eq. (1)

$$\tilde{\tau}_{kj} = -\frac{1}{4}\sum_n \left(\tilde{u}_{k,n}\tilde{u}^*_{j,n} + \tilde{u}_{j,n}\tilde{u}^*_{k,n} \right) \tag{3}$$

Substituting velocity and pressure in the form of Eq. (1) into the Navier-Stokes equations, phase-averaging and subtraction of Eq. (2) results in the RANS equations for amplitude of the *n*-th harmonic

$$\begin{aligned} &\frac{\partial \tilde{u}_{k,n}}{\partial x_k} = 0 \\ &-in\omega\tilde{u}_{k,n} + U_j \frac{\partial \tilde{u}_{k,n}}{\partial x_j} + \tilde{u}_{j,n}\frac{\partial U_k}{\partial x_j} + \frac{\partial \tilde{p}_n}{\partial x_k} = \frac{\partial r_{kj,n}}{\partial x_j} + \frac{\partial \tilde{\tau}_{kj,n}}{\partial x_j} \end{aligned} \tag{4}$$

Here $r_{kj,n}$ stands for the *n*-th harmonic amplitude of the turbulent Reynolds stress, $r_{kj} = \overline{u'_k u'_j} - < u'_k u'_k >$ caused by the interaction of the coherent signal and the background turbulence, bar and <> denote time-averaging and phase-averaging, respectively, and $\tilde{\tau}_{kj,n}$ is associated with the nonlinear interaction of harmonics which are determined by Eq. (1)

$$\tilde{\tau}_{kj,n} = -\frac{1}{2}\sum_{m}\left(\tilde{u}_{k,m+n}\tilde{u}^{*}_{j,m} + \tilde{u}_{j,m+n}\tilde{u}^{*}_{k,m}\right) - \frac{1}{2}\sum_{m}\tilde{u}_{k,m}\tilde{u}_{j,n-m} \quad (5)$$

Now, one has to adopt a closure approximation for τ_{kj} and $r_{kj,n}$ in Eqs. (2) and (4), respectively. In the general case, different closure approximations are possible for these two terms. However, in the case of thin shear layers, we take the Newtonian eddy model for both the mean flow field and for the coherent constituent (Reynolds & Hussain [1]). The latter means that

$$\tau_{kj} = 2\nu_T S_{kj}, \ \ r_{kj,n} = 2\tilde{\nu}_T \tilde{s}_{kj,n} \quad (6)$$

where S_{kj}, $\tilde{s}_{kj}$ are the strain rate tensors of the mean and the coherent parts of the flow, respectively. Actually, only τ_{12} is used in the two-dimensional mean flow problem, therefore, this approximation causes minimal errors.

In the case of a two-dimensional, incompressible thin shear layer along axis x, Eq. (2) of the mean flow is used in the boundary-layer approximation. The correspondent finite-difference equations are solved for the known coherent Reynolds stress $\tilde{\tau}_{kj}$ by Keller's box marching method. Velocity components $\tilde{u}_i$ for evaluation of $\tilde{\tau}_{kj}$ by Eq. (3) are obtained from the solution of the coherent problem Eq. (4).

In the past, this problem was solved using Linear Stability Theory (LST) under the assumption of a quasi-parallel shear flow. However, this approach suffers from the uncertainty of the amplitude normalization, which impairs the coherent stress evaluation. The uncertainty can be removed by the method of multiple scales under the assumption that the one scale is significantly different from the other. However, this method is cumbersome because its realization is based on calculation of eigenfunctions of the direct and the adjoint problems for spatially growing perturbations, and on evaluation of integrals associated with the inner product. In addition, parameters corresponding to the problem of interest led to relatively low Reynolds numbers, indicating that the nonparallel-flow effects might be very significant, and that more terms have to be taken into account in the method of multiple scales [2]. Because a self-consistent problem assumes that a wave propagates in the mean flow determined by the Reynolds stresses of the wave itself, it does not enable solutions at moderate excitation level, when the mean-coherent flow interaction starts to play an appreciable role. This limitation can be avoided if the flow is simulated by RANS calculation with a proper turbulence model. However, straightforward computation of the unsteady RANS equations requires the resolution of the different scales associated with the base flow and the perturbation. The latter makes this approach very time-consuming.

The main idea of the Parabolized Stability Equations (PSE) approach [3] is to separate the "fast" and the "slow" scales in the partial differential equation by representing functions as

$$\tilde{q}(x,y) = q(x,y)\exp\left(i\int \alpha dx\right) \quad (7)$$

where $\alpha(x)$ is the complex perturbation wave number, and $q(x,y)$ is the amplitude function. Because q and α are considered as slow functions of the streamwise coordinate, the latter allows dropping of the highest streamwise derivatives with respect to the coordinate x in the viscous terms. The phase function $\alpha(x)$ is iterated at every step of the marching procedure to adjust the solution to auxiliary problem of α definition. The PSE approach provides an efficient tool because the equations include all the terms responsible for the non-parallel flow effects, and they allow a straightforward analysis of the non-linear effects. However, because the equations still possess an elliptic character via the pressure perturbation term in the momentum equation, the marching step or another regularization parameter cannot be chosen to be arbitrarily small, in order to avoid numerical instabilities. Another source of error can be a transitional region near the inflow boundary, where variations of q and α are not small simultaneously, and the PSE approximation fails.

A solution adaptive approach of Guo *et al.* [4] adopts transformation (7) without further simplification of the amplitude equations. This procedure grants the advantage of scale separation, i.e., the equations for the harmonic perturbations are solved on the same grid as the mean flow equations (a significant reduction of computing time), but there is no obstacle associated with above shortcomings of the PSE method and the corresponding loss of accuracy.

We use both, the PSE and the solution adaptive approach, to solve the coherent flow problem. In the case of turbulent boundary layer, the PSE method is used with primitive variables [5]. The far wake and mixing layer flows are calculated using the solution adaptive approach in the stream function-vorticity formulation [6, 7] or the velocity-vorticity formulation in the case of three-dimensional coherent flow [8]. The coherent wave Eq. (4) is solved with the obtained mean flowfield. After an iteration, the coherent stress is updated and the calculation is repeated. Convergence is usually achieved after a few iterations.

THE MEAN-COHERENT INTERACTION

Most of the experimental data belong to turbulent mixing layers with velocities U_1 and U_2 , harmonically excited by an oscillating flap at the end of the splitter plate. We shall consider this type of flow for validating our model by comparing its results with experiment. For mixing layers, we neglect molecular viscosity effects, whereas the eddy viscosity is modeled within the scope of a Prandtl-type algebraic formula

$$\nu_T(x) = 4\chi\theta(x)\left(U_1 - U_2\right) \quad (8)$$

Here, θ is the momentum thickness, and χ is an experimental constant determined by particular properties of the setup. The mean field of unforced mixing layer under consideration is self-similar:

$$U = U_m \left[1 + \lambda U_0(\eta)\right], \quad \eta = y/\Delta_0 x, \quad \Delta_0^2 = \nu_{T0}/U_m x \qquad (9)$$

Here, $U_m = (U_1 + U_2)/2$, $\lambda = (U_1 - U_2)/(U_1 + U_2)$, the momentum thickness $\theta_0 = p\lambda x$, and $\nu_{T0}(x)$, are linear functions of x, and p is an empirical constant replacing χ for calculating Δ_0; taken from experiments of Ref. [9], $p = 0.036$.

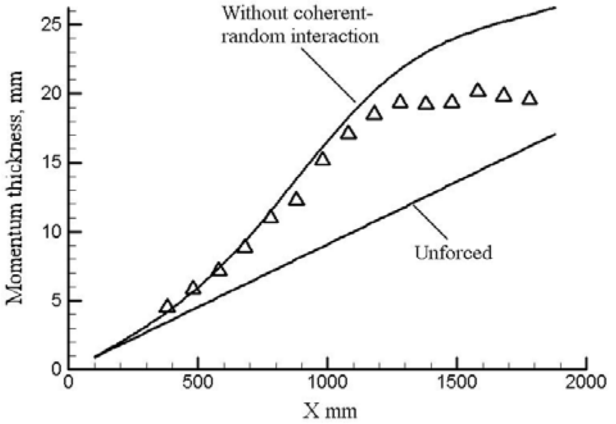

Figure 1: *Variation of the momentum thickness: solid lines – results of calculation, symbols – experiment [9].*

Reynolds & Hussain [1] used triple decomposition to derive the dynamic equations for coherent flow, and found that action of the random field can be considered as a result of the oscillation of the background Reynolds stress due to the passage of the organized disturbances. They also proposed to approximate this stress by the Newtonian eddy model given by Eq. (6). Comparison with experimental data of Ref. [10] and [11] at small flap amplitudes indicates a good potential of this approach for shear-layer flows if $\tilde{\nu}_T = \nu_T$. We use this equality in the following.

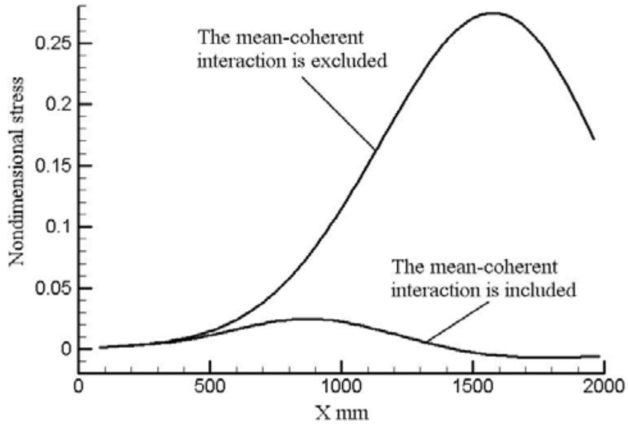

Figure 2: *Variation of the coherent Reynolds stress, $\tilde{\tau}_{12}(x,0)$ for $f = 40$ Hz, $A = 1.5$ mm.*

Comparison of the momentum thickness from the present calculations with the data of Ref. [9] is shown in Fig. 1 for the mixing layer with $U_1 = 13.5$ m/s, $U_2 = 8.1$ m/s, excited by a flap oscillation with amplitude $A = 1.5$ mm and frequency $f = 40$ Hz. Excellent agreement obtains upstream of the region, where momentum thickness saturates, but fails downstream, showing that the mathematical model is not uniformly good in the entire flow region. Similar results were also obtained in the case of a turbulent wake with harmonic perturbations [6]. As in the case of the mixing layer, the model correlates well with experimental data of Ref. [11] in the upstream part of the wake; however, the agreement becomes poor downstream due to the turbulence closure limitation, which ignores the changes of turbulence caused by the perturbations.

The bell shape of the coherent Reynolds stress distribution in cross sections of a turbulent mixing layer has a maximum near the centerline. Variation of the maximums is shown in Fig. 2 as results of calculations with, and without, the mean-coherent interaction. In the fore part of the layer, where the stress is small, both curves are close one to the other. Downstream, the interaction distorts the mean-flow velocity profiles in such a manner that the stress decreases. A mechanism of this behavior can be described in the scope of the LST approximation. Figure 3 presents two neutral stability curves, $f = f_s(x)$, calculated for initial self-similar and distorted velocity profiles. When propagating downstream in a turbulent mixing layer, the coherent signal of frequency f amplifies if $f < f_s(x)$ and decays otherwise. The position of two neutral points in Fig. 3, corresponding to $f = 40$ Hz, correlates well with data of Fig. 2. This demonstrates that the LST approximation is valid if it is applied to a distorted flowfield.

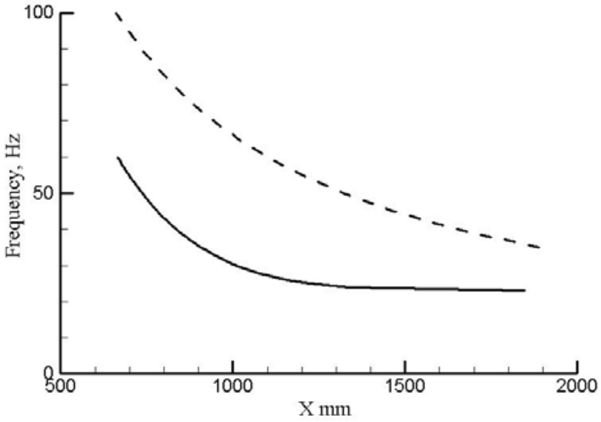

Figure 3: *Neutral stability curves $f = f_s(x)$, dashed line – self-similar velocity profile, solid line – velocity profile at $x = 700$ mm of the flow for $f = 40$ Hz, $A = 1.5$ mm.*

The velocity distortion in our example is less than 8%, however, it causes a considerable upstream shift of the neutral stability curve. This effect can be explained by asymptotic theory which connects the neutral stability point position with the distribution of the cross-flow vorticity derivative, $\partial^2 U/\partial\eta^2$. Distortion of the latter is a lot more than U and concentrated near the mixing layer centerline. This property allowed us to develop an asymptotic pattern of the flow [7].

As it was showed by Lifshitz & Degani [8], the mean-velocity distortion caused by oblique harmonic perturbations of a turbulent mixing layer can also change significantly the wave growth rate.

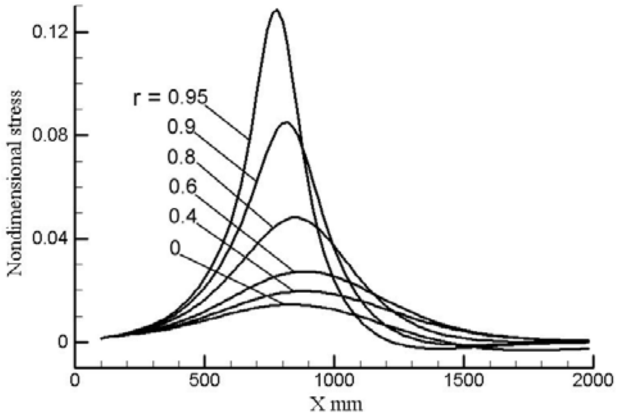

Figure 4: *Variation of the coherent Reynolds stress, $\tilde{\tau}_{12}(x,0)$ for several wavenumbers ratio, $r=\beta/\alpha$, in a turbulent mixing layer with $p=0.046$ forced by oblique harmonic perturbations for $f=40$ Hz and $A=1.5$ mm.*

The oblique wave introduces an additional parameter, $r=\beta/\alpha$, which is the ratio of spanwise wavenumber, β, to streamwise wavenumber, α. Calculations in Ref. [8] show that the coherent Reynolds stress variation weakly depends on r if the mean-coherent interaction is neglected. Results obtained when the interaction is taken into account are presented in Fig. 4. They demonstrate that $\max\tilde{\tau}_{12}(x,0)$ for $r=0.95$ exceeds more than tenfold its value for $r=0$.

THE COHERENT-RANDOM INTERACTION

Equation (8) of the Prandtl (algebraic) turbulence viscosity ignores the changes of turbulence caused by the organized waves and must be improved to reflect the variation of momentum thickness, which is well observed in experiments. In harmonically perturbed shear layer, the RHS of equation of turbulent kinetic energy consists of two extra terms [1, Eq. (3.2c)]

$$\tilde{\mathcal{P}} = -\overline{<u_i'u_j'>\tilde{s}_{ij}} - \overline{\tilde{u}_j\frac{\partial}{\partial x_j}<\frac{1}{2}u_i'u_i'>} \qquad (10)$$

According to the Newtonian closure used in our model, this term is transformed to

$$\tilde{\mathcal{P}} = 2\nu_T\left(\overline{\tilde{s}_{ij}\tilde{s}_{ij}}\right) \qquad (11)$$

which represents the extra production of turbulent kinetic energy due to the coherent-random interaction.

Taking a simplified form of Eq. (11) for estimating the mixing-layer length scale, one can improve Eq. (8) and to derive the following relation for turbulence viscosity parameter, $b=\nu_t/\nu_0$, which is needed for calculation of the mean and the coherent parts of the flow

$$\frac{\nu_{T0}}{\nu_T} = \frac{\theta_0}{\theta(x)} + C_t\frac{\nu_T}{\nu_{T0}}\frac{fx}{U_m^3\Delta_0}\int_{-\infty}^{\infty}\left|\frac{\partial\tilde{u}_1}{\partial\eta}\right|^2 d\eta \qquad (12)$$

where C_t is an empirical constant. We take $C_t=0.036$ by adjusting the calculation according to the experimental data of Ref. [9] for the flow shown in Fig. 1.

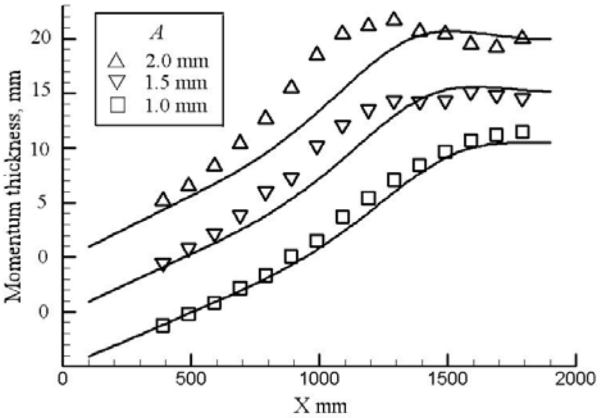

Figure 5: *Variation of the momentum thickness for mixing layer with $U_2/U_1=0.6$ forced by $f=40$ Hz.*

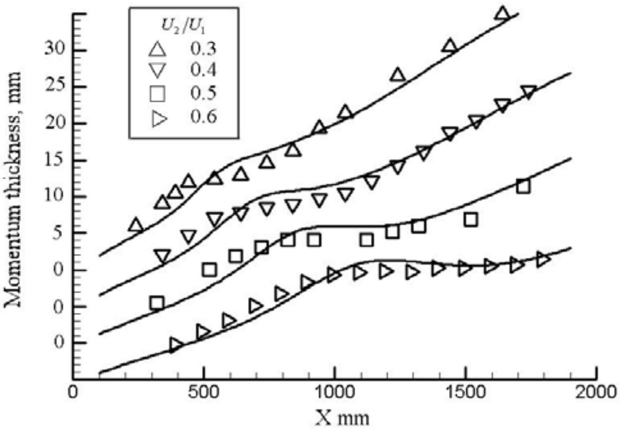

Figure 6: *Variation of the momentum thickness for forced mixing layer for $f=50$ Hz and $A=1.5$ mm.*

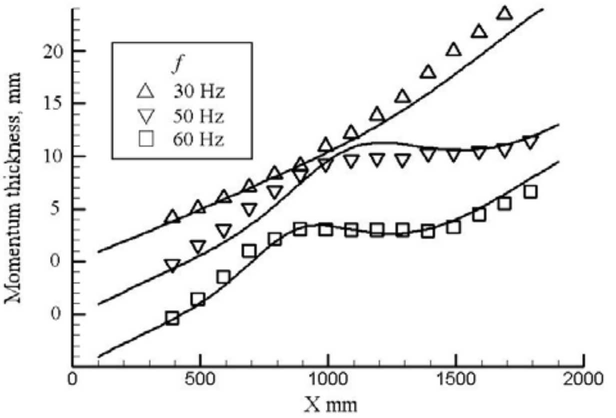

Figure 7: *Variation of the momentum thickness for mixing layer with $U_2/U_1=0.6$ forced by $A=1.5$ mm.*

Then, the calculations were carried out for flows with following parameters: $U_1=13.5$ m/s, $U_2/U_1=0.6$, 0.5, 0.4, and 0.3, frequencies, $f=30$, 40, 50, and 60 Hz, and amplitudes, $A=1$, 1.5, and 2 mm, as reported in Ref. [9].

The results were compared with the experimental data. The comparison presented in Figs. 5-7 shows satisfactory matching with the maximum disagreement in Fig. 5 for strong forcing with flap amplitude, $A = 2\ \text{mm}$. The agreement is better in Figs. 6-7 where the data for flap amplitude $A = 1.5\ \text{mm}$.

CONCLUDING REMARKS

We applied the mean-field approach to develop a mathematical model of harmonically perturbed turbulent shear layers. The model marks out two regions of the shear layer. In the first region, the organized wave increases the momentum thickness of the layer and distorts the mean flowfield, causing early attenuation of the wave. In the second region, the reduction of the turbulent viscosity, stemming from the coherent-random interaction, becomes dominant. These results contradict existing views which consider the impact of waves on the mean flow through the coherent Reynolds stress as the main interaction. However, the current results may explain the long-range action of oscillations on the flowfield as seen in experiments with delayed boundary layer separation.

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