

# THE DECAY LAW OF GRID TURBULENCE IN A ROTATING TANK

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## ABSTRACT

*The energy decay of grid-generated turbulence in a rotating tank is experimentally investigated by means of particle image velocimetry. For times smaller than the Ekman timescale, a range of approximate self-similar decay is found, in the form  $u^2(t) \propto t^{-n}$ , with the exponent  $n$  decreasing from 2 to values close to 1 as the rotation rate is increased. This observation is interpreted in the frame of a phenomenological model based on the exponent of the energy spectrum, in which both the effects of the rotation and the confinement are taken into account.*

## INTRODUCTION

Assuming self-similarity, the decay of high Reynolds number homogeneous and isotropic turbulence is usually described by a power law [1–4],

$$u^2 \propto (t - t^*)^{-n}, \quad (1)$$

where  $u$  is the velocity variance,  $n$  is the decay exponent and  $t^*$  is a virtual origin. The value of the exponent  $n$  depends on whether the size of the energy-containing eddies is free to grow ( $n = 6/5$ ) or is bounded by the experiment size ( $n = 2$ ), with a possible changeover between these two regimes [4].

In the presence of rotation, in addition to the turnover time  $\ell/u$  (where  $\ell$  is the size of the energy-containing eddies), two other timescales are present in the problem, which have opposite effects on the turbulence decay: the rotation timescale,  $\Omega^{-1}$ , and, for bounded systems, the

Ekman timescale,  $t_E = h(\nu\Omega)^{-1/2}$  (where  $h$  is the size of the experiment along the rotation axis). The ratio of the turbulent time scale and the rotation rate is the Rossby number,  $Ro = u/2\Omega\ell$ . While the primary effect of the rotation is to reduce the energy dissipation [5], the effect of the Ekman friction is to accelerate the decay at large times, shortening the temporal range for the turbulent decay even at large Reynolds numbers. Based on the assumption that the energy transfers time scale is governed by  $\Omega^{-1}$ , Squires et al. [6] have proposed a self-similar decay with an exponent half of that without rotation,  $u^2 \propto t^{-3/5}$ , in the limit of zero Rossby number.

In experiments, where both confinement and finite Rossby numbers are to be considered, the situation is more complex. In small experiments in which  $h \simeq O((\nu/\Omega)^{1/2})$  (e.g. Ibbetson & Tritton [7]), the inhibition of the energy decay is hidden by the extra dissipation in the Ekman layers, and is not observed. Larger experiments (e.g. Jacquin et al. [5]), in which a significant range  $\Omega^{-1} \ll t \ll t_E$  exists, have indeed confirmed

the expected inhibition of the energy decay, but without further characterizing the decay law at large time, due to the limited extent of the experiment. The aim of the present experiment is to investigate the combined effects of the background rotation and the confinement on the decay law of energy in a rotating tank, starting from an approximately homogeneous and isotropic state.

## EFFECTS OF THE ROTATION AND THE CONFINEMENT ON THE DECAY LAW

### Decay without rotation

In the absence of rotation, the decay law of an unbounded homogeneous turbulence can be derived assuming a two-range model for the energy spectrum [1,2]: A “permanent” part  $E(k) \simeq Ak^2$  for small  $k$ , followed by the Kolmogorov spectrum  $E(k) \simeq C\epsilon^{2/3}k^{-5/3}$  at larger  $k$ . Solving for  $\epsilon = -d(u^2)/dt$ , where  $u^2 = \int_0^\infty E(k)dk$  is (twice) the total kinetic energy, yields  $u^2 \propto (t + \tilde{t})^{-6/5}$ , i.e. a decay exponent  $n = 6/5$ . In this expression, the crossover time  $\tilde{t} > 0$  does not necessarily correspond to the virtual origin  $t^*$  introduced in Eq. (1), even though their order of magnitude should be both given by the timescale of the initial large eddies. During the decay, the energy-containing scale  $k_e(t)^{-1}$  (where  $k_e$  is the the crossover between the two spectral ranges) grows without bound. In a real experiment, a bounding size  $L$  is present, defining a minimum wavenumber  $k_0 \simeq L^{-1}$  towards which  $k_e(t)$  saturates after a given time  $t_s$ . Setting  $E(k) = 0$  for  $k < k_0$  to account for the confinement yields, for  $t \gg t_s$ , a sharper decay, with an exponent  $n = 2$  (Skrbek & Stalp [4]) (see Table 1).

### Decay with rotation

A very crude way to account for the effect of rotation is to modify the scaling of the high-

wavenumber part of the spectrum, but without including the anisotropy. Assuming that  $E(k)$  depends on  $\epsilon$ ,  $\Omega$  and  $k$ , dimensional analysis yields

$$E(k) = C_p \Omega^{(3p-5)/2} \epsilon(t)^{(3-p)/2} k^{-p}. \quad (2)$$

where the exponent  $p$  can take any value between 1 and 3, and  $C_p$  is a non dimensional constant. For  $\Omega = 0$ , the Kolmogorov exponent  $p = 5/3$  is recovered. For  $\epsilon = 0$ , the Kraichnan spectrum in the enstrophy cascade regime ( $p = 3$ ) is recovered. Finally, the intermediate case  $p = 2$  is the spectrum proposed by Zhou [9], on the assumption of an energy transfer timescale given by  $\Omega^{-1}$ . Solving for  $\epsilon = -d(u^2)/dt$ , and assuming that the spectral exponent  $p$  in (2) remains constant during the decay, the decay exponent for unbounded rotating turbulence is found,

$$n = \frac{3}{5} \left( \frac{3-p}{p-1} \right). \quad (3)$$

The value  $n = 6/5$  is recovered for the Kolmogorov spectrum  $p = 5/3$ , and the limiting case  $p = 3$  of the enstrophy cascade regime yields  $n = 0$  (i.e. conservation of energy). The exponent  $p = 2$  of Ref. [9] yields  $n = 3/5$ , which is twice smaller than the exponent without rotation, as first noticed by Squires et al. [6] on dimensional grounds.

*Effects of the rotation and confinement* — If the confinement is now taken into account, the same procedure as above may be used, by setting  $E(k) = 0$  for the low wavenumber range, which gives the decay exponent

$$n = \frac{3-p}{p-1}. \quad (4)$$

As for the non rotating case, the exponent with confinement is larger by a factor of 5/3 than that without confinement. One now obtains  $n = 2$  for  $p = 5/3$  and  $n = 1$  for  $p = 2$ , with again the

factor 2 between the zero rotation and the fast rotation regimes.

Table 1

Predictions for the decay exponent  $n$ .

	not confined	confined
no rotation	6/5 (Ref. [1])	2 (Ref. [4])
rotation	3/5 (Ref. [6])	1

## EXPERIMENTAL RESULTS

In order to characterize the effects of the background rotation and the confinement, a series of experiments of decaying grid-generated turbulence have been carried out in a rotating tank. The experimental set-up, described in Morize et al. [10], consists of a water filled square tank,  $L = 35$  cm in side, mounted on a rotating turntable, whose angular velocity  $\Omega$  has been varied between 0.13 and 4.34 rad s<sup>-1</sup>. Turbulence is generated by rapidly towing a square grid of mesh  $M = 39$  mm at a mean velocity  $V_g = 0.65$  m s<sup>-1</sup> from the bottom to the top of the tank. The grid Reynolds number is  $Re_g = MV_g/\nu = 2.5 \times 10^4$ , a value larger than that of most conventional wind-tunnel experiments, and the grid Rossby number  $Ro_g = V_g/2\Omega M$  is kept sufficiently large, in the range 2 – 65, in order to minimize the effect of the rotation on the turbulence production mechanism in the near wake of the grid. Instantaneous velocity fields in the horizontal plane at mid-height of the tank are measured by particle image velocimetry (PIV), using a corotating high resolution camera. Ensemble averages over 50 realizations of the decay are performed to achieve statistical convergence.

The energy decay is shown for three rotation rates in Fig. 1. For low rotation rate ( $\Omega = 0.12$  rad s<sup>-1</sup>), after a period of approximately constant energy, a well defined power law with a decay exponent

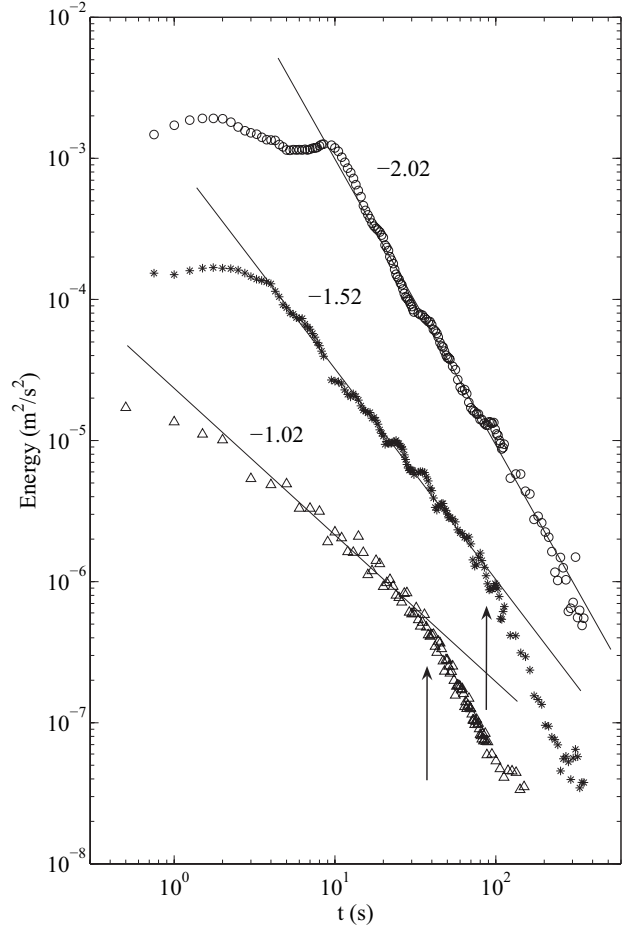


Fig. 1. Energy decay for (○)  $\Omega = 0.12$  rad/s, (☆)  $\Omega = 0.40$  rad/s, and (△)  $\Omega = 1.50$  rad/s (2nd and 3rd curves divided by 10 and 100 for visibility). The full arrows indicate the Ekman time.

$n \simeq 2.02 \pm 0.10$  is found. As the rotation rate is increased, the friction from the Ekman layers arises earlier (full arrows in Fig. 1), shortening the range of power law. Although the quality of the power law is poorer for large rotation rates, a significant power law range is found, with an exponent that decreases continuously from  $n \simeq 2$  to values close to 1, as shown in Fig. 2. However it is not clear from this figure whether a saturation towards  $n = 1$  is present or not.

Although the assumption of a constant spectral exponent during the decay is not realistic [10], the trend for  $n$  is found to be consistent with the model presented above, when confinement

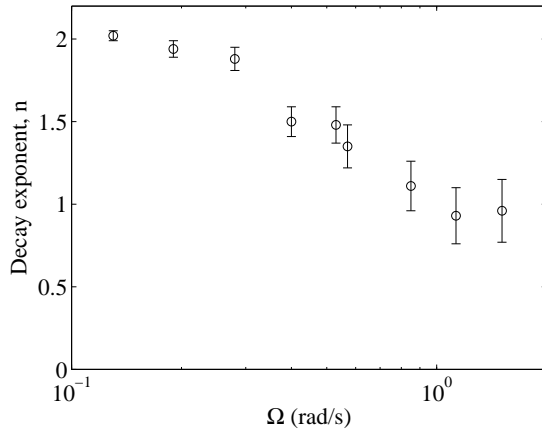


Fig. 2. Semi-logarithmic plot of the decay exponent  $n$  as a function of the rotation rates  $\Omega$ .

effects are included (see Table 1). The fact that the confinement plays an important role from the early time of the decay originates from the rapid growth of the vertical integral scale through the propagation of inertial waves, which quickly saturates to the tank height  $h$  [11]. In this aspect, decaying rotating turbulence in confined geometry strongly differs from that in a wind tunnel with a rotating honeycomb [5], which may have lateral confinement effects but no confinement along the rotation axis. In spite of this difference, it is remarkable that the factor 2 between the exponents at high and low rotation rates (Squires et al. [6]) is recovered here.

## CONCLUSION

To summarize, a clear evidence of the reduction of the energy decay by the rotation has been observed for times smaller than the Ekman timescale. The most important result is that, in addition to the dissipation in the Ekman layers, the confinement plays a central role in the decay law of rotating turbulence. By making the growth of the integral scale along the rotation axis to quickly saturate to the experiment size even at very low rotation rate, the confinement leads to a sharper decay than for unbounded tur-

bulence, although compatible with the reduced dissipation induced by the rotation.

When the range between the rotation-induced saturation time of the vertical lengthscale and the Ekman timescale is large enough, a significant self-similar energy decay is observed, characterized by a decay exponent  $n$  decreasing from 2 to values close to 1 as the rotation rate is increased. These exponents are found to be in qualitative agreement with a phenomenological model based on the exponent of the energy spectrum, in which both the effects of the rotation and the confinement are taken into account. These observations may be of primary importance for the modelling of turbulence in rotating containers, as the confinement is shown to significantly influence the energy decay even for very weak rotation rate and large experiment size.

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