LAGRANGIAN TRACKING OF PARTICLES IN LARGE-EDDY SIMULATION WITH FRACTAL INTERPOLATION

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ABSTRACT

A closure model for Lagrangian tracking of particles, starting from LES flow data, is presented. The basic idea is to reconstruct the velocity field from the knowledge of its filtered values on a coarse grid, by means of fractal interpolation. Two different practical implementations of fractal interpolation are considered: in the first, the stretching parameters are assumed constant and a-priori assigned, while, in the second, they are computed by using DNS velocity fields. Validation is carried out by means of a-priori tests for turbulent channel flow. DNS data are filtered through either a cut-off or a top-hat filter; different filter widths are considered. Six different sets of particles are tracked, having characteristic relaxation times ranging over a huge interval. Particle statistics and concentration are computed, with and without fractal interpolation, and compared to those obtained starting from the DNS velocity fields.

INTRODUCTION

Direct Numerical Simulation (DNS) together with Lagrangian particle tracking has been quite extensively used to investigate and measure the mechanisms of particle transfer in a turbulent boundary layer. Clearly, DNS is limited to low Reynolds numbers, while the simulation of turbulent flows at higher Reynolds numbers can be tackled using Large-Eddy Simulation (LES). As for the fluid dynamic part, the closure problem of LES equations has been deeply investigated and several Sub-Grid Scale (SGS) models have been proposed and tested. Let us assume, for the moment, one-way coupling between the two phases (i.e. the fluid dynamics governing equations are unchanged). Since only the filtered fluid velocity, $\overline{\mathbf{u}}$, is available from LES, while the particle motion depends on the actual fluid velocity, a closure model should in principle be needed to reintroduce the SGS velocity fluctuations. However, this point has received little attention in the literature, especially if compared with the huge amount of work devoted to the closure problem for the fluid dynamic part. The work by Armenio et al. [1] on dispersion statistics of tracer particles in LES of turbulent channel flow indicated that SGS effects may be neglected for well resolved LES and several simulations in the literature were carried out without any SGS model for the particle motion equations. However, more recent a-priori and a-posteriori tests [2] [3] [4] [5] showed that LES is not able to accurately predict the statistics and the concentration of particles of finite inertia if SGS effects in the particle equations are ignored. In [2] and [4] a closure based on filter inversion or deconvolution was used. In [5] we proposed a closure model for the particle motion equations based on fractal interpolation, previously used by Scotti and Meneveau [6] to construct SGS models for the Navier-Stokes equations.

The aim of the present work is to provide further validation to this model through the Lagrangian tracking of 6 sets of particles, characterized by relaxation times spanning a huge interval, starting from filtered DNS or LES turbulent channel flow data. Furthermore, we improve upon the original formulation of the fractal interpolation by using an algorithm which allows the stretching parameters (free-parameters in the interpolation) to be computed from DNS data, without any a-priori knowledge of the fractal dimension of the velocity signal.

METHODOLOGY

Channel flow simulation

Particles are introduced in a pressure driven incompressible turbulent channel flow. We assume that particle number density and particle size are both small, and that there is no feedback of the particles onto the gas flow. The equations for the fluid phase are discretized through a pseudo-spectral method, using Fourier representations for the streamwise and spanwise directions and a Chebyshev representation for the wall-normal (non-homogeneous) direction. A two-level explicit Adams-Bashforth scheme for the nonlinear terms and an implicit Crank-Nicolson method for the viscous terms were employed for time advancement. In the present study, the shear Reynolds number is equal to 150 and the Reynolds number based on mean velocity and half duct width is $\simeq 2066$. The computational domain is $1885 \times 942 \times 300$ wall units in the streamwise, spanwise and normal directions; 128×128 Fourier modes in the homogeneous directions and 129 Chebyshev polynomia in the normal one are used for DNS. Statistics of the flow field match closely with those of other DNS published in the literature.

Particle tracking

Particles are injected into the flow at concentration low enough for particle-particle interaction due to their inertial force to be negligible and particles are assumed pointwise, rigid, spherical and obeying the following vectorial Lagrangian equation of motion:

$$\frac{d\mathbf{v}}{dt} = \frac{1 + 0.15 Re_p^{0.687}}{\tau_P} (\mathbf{u} - \mathbf{v}) \qquad (1)$$

in which $\mathbf{v}$ is the particle velocity vector, $\mathbf{u}$ is the fluid velocity vector at particle location, Re_p is the particle Reynolds number, τ_P is the particle relaxation time. In the present simulations, 10^5 flyash particles, characterized by a particle to fluid density ratio equal to 769.23, have been released at randomly chosen locations within the computational box. The Lagrangian particle tracking code integrates Eq. (1) by an explicit method, using the channel flow code to supply the fluid velocity field at each time step. The initial velocities of the particles were set equal to the interpolated fluid velocities at each particle location. Eq. (1) does not include wall effects: in our calculations, we simply considered that a particle is elastically reflected away from the wall when its center is at a distance from the boundary lower than half of the particle diameter. The fluid forces acting on each single particle are calculated with a Lagrange interpolation of order three. Six different sets of particles have been tracked, characterized by different Stokes numbers (adimensionalized relaxation times), viz. 0.2, 1, 5, 10, 25 and 125. The time step for the numerical simulation of the channel flow is equal to half the relaxation time of the smallest particles.

Fractal interpolation

The aim of fractal interpolation is to reconstruct the velocity field $\mathbf{u}(\mathbf{x}_i, t)$ from the knowledge of its filtered value $\overline{\mathbf{u}}$. This is done by iteratively applying in each direction an affine mapping procedure to the filtered field. In this way, starting from a coarse grid on which $\overline{\mathbf{u}}$ is defined, a signal can be reconstructed on a given finer grid (we refer to [6] and [5] for more details). The characteristics of the reconstructed signal depend on two stretching parameters. It can be shown [6] that they are related to the fractal dimension of the signal. In [6] they are considered constant in time and space and the adopted values are obtained from experimental velocity signals of homogeneous and isotropic turbulence; in particular, these parameters are set to $d_1 = 2^{-1/3}$ and $d_2 = -2^{-1/3}$, corresponding to a fractal dimension of the velocity signal of 1.7. As a first approach, the same constant values have been used in the present work. Furthermore, we also apply an algorithm [7] which allows the stretching parameters to be locally computed only using the discrete values on a fine grid (DNS data), without the a-priori knowledge of the fractal dimension of the velocity signal. However, the locally computed values show significant fluctuations which reflect the instantaneous fluctuations of the DNS velocity fields from which they have been derived. Thus, to use these local values in the fractal interpolation procedure, a 3D distribution of the stretching parameters should be computed and stored for each velocity component at each time step. This renders the whole procedure much more complex, but, most of all, this is impossible to be carried out in actual LES simulations for which the instantaneous DNS fields are obviously not available. To overcome this problem, we adopted the following strategy: from the locally computed values of the stretching parameters we compute the fractal dimension (for each velocity component and at each time step) for fractal interpolation in the x direction, by considering the x distribution of the variable at each discrete y and z locations as a single signal. Then, this is averaged over the homogeneous spanwise direction y and in time. The averaged fractal dimension for fractal interpolation in the y direction is computed analogously. The output of this computation is the averaged fractal dimension of each velocity component and for each horizontal plane (constant z), for reconstruction in the x and y directions respectively. As an example Fig. 1 shows the averaged fractal dimension of the normal velocity component *signal* in the x and y directions as a function of the distance from the wall (in wall units). Then, the stretching parameters for each velocity component and for each direction (x and y) are assumed constant in time and over the horizontal planes. These constant values can be easily derived from the previously computed fractal dimension, using the same relationship as in [6]. The computed values are shown in Fig. 2. Note that in all cases the new stretching parameters show a noticeable variation in the wall-normal direction and they are significantly lower than the value experimentally obtained for homogeneous turbulence.

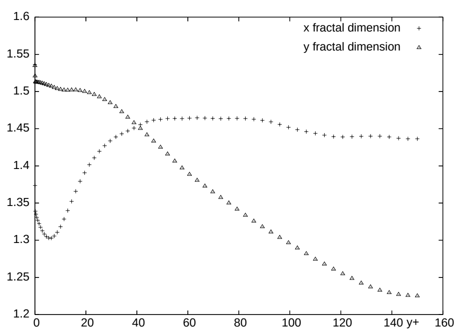
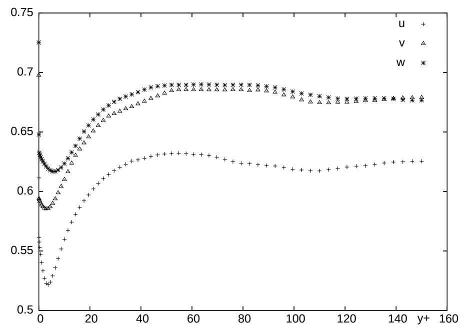


Fig. 1. Normal profiles of the averaged fractal dimension of the normal velocity component *signal* in the x and y directions.

(a)



(b)

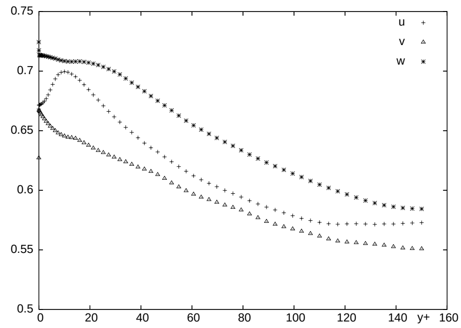


Fig. 2. Normal profiles of the averaged stretching parameters for the reconstruction of the velocity components in the x (a) and y (b) directions.

RESULTS

In the a-priori tests the DNS velocity fields are filtered to different resolutions in the streamwise and the spanwise directions only. In the normal direction the data are not filtered, since often in LES the normal resolution is DNS like. Either a cut-off or a top-hat filter is applied in the wave number space. Starting from these filtered fields, the motion equation (1) is integrated for the previously described set of particles, with and without the fractal interpolation procedure. When fractal interpolation is used the velocity field is reconstructed up to the DNS resolution. Particle statistics and concentration are computed and compared with those obtained starting from the DNS velocity fields. The results of a-priori tests carried out without fractal interpolation show that, except for the largest considered particles ($St = 125$), filtering has a significant effect on the particle velocity fluctuations and on the particle concentration, especially near the wall, also at resolutions commonly used in LES. The particle set characterized by $St = 25$ appears to be the most sensitive to filtering. As an example, Fig. 3 shows the instantaneous concentration of this set of particles as a function of the wall normal coordinate (in wall units), obtained using the velocity fields from DNS and those filtered at a resolution of 32×32 Fourier modes through a cut-off filter, without fractal interpolation. As already observed in [2], it can be seen that filtering, without any closure in the particle equation of motion, leads to a large underestimate of the turbophoresis phenomenon, i.e. the tendency of particle to cluster near the wall. The results obtained with

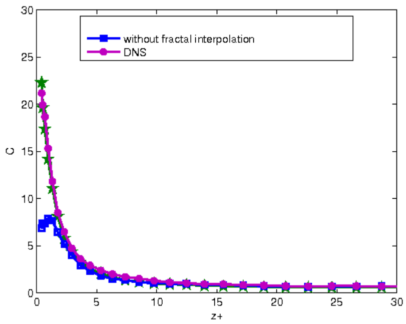


Fig. 3. Instantaneous concentration profiles for $St = 25$ particles, obtained starting from DNS and filtered DNS without fractal interpolation.

fractal interpolation, by using the constant values of the stretching parameters of $\pm 2^{-1/3}$, show an improved agreement with DNS for both particle statistics and concentrations (see, e.g. , [5]). However, in our opinion, the previous constant values of the stretching parameters are not well suited for channel flow, since they were obtained from homogeneous and isotropic turbulence velocity signals. A preliminary analysis, carried out by simply changing the values of the stretching parameters, showed that the results obtained for small particles are significantly sensitive to the adopted values. Thus, we repeated the same a-priori tests with fractal interpolation by using the new stretching parameters computed following the previously described procedure. Preliminary results are shown in Fig. 4, in which the profiles of the r.m.s of the wall normal velocity component obtained for particles having $St = 25$ starting from fluid velocities filtered on a $16 \times 16 \times 129$ grid are compared with those obtained from the DNS flow fields. It is evident that with the new values of the stretching parameters the particle velocity fluctuations are signifcantly underestimated, at least for this very coarse grid resolution. The analysis of the reasons of this behavior will be the object of further investigation.

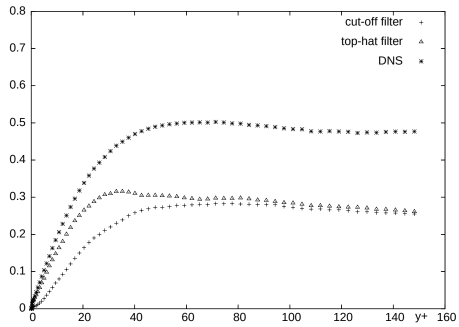


Fig. 4. Normal profiles of the r.m.s of the wall normal component of the particle velocity ($St = 25$) obtained with fractal interpolation with a $16 \times 16 \times 129$ grid resolution.

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