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THE ROLE OF THE TURBULENT BACKGROUND IN THERMAL CONVECTION

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ABSTRACT

The turbulent background in thermal convection is characterized by low values of the thermal dissipation rate. To determine quantitatively the role of the turbulent background in thermal convection we evaluate the functions σ and τ (introduced in [1]) from the DNS and LES data generated for turbulent Rayleigh–Bénard convection in wide cylindrical containers of the aspect ratios $\Gamma = 5$ and 10 for the Rayleigh numbers $Ra = 10^5$, 10^6, 10^7, 10^8. It is shown that both the volume of the fluid which corresponds to the turbulent background and the contribution of the turbulent background to the volume averaged thermal dissipation rate increase with the Rayleigh number.

INTRODUCTION

One of the most interesting examples of thermal convection – turbulent Rayleigh-Bénard convection, i.e. the thermally driven fluid motion between a lower heated horizontal plate and an upper cooled plate – has been the subject of many fundamental investigations. For a review and literature on this classical problem we refer to Ahlers [2].

Most of the flow characteristics in Rayleigh-Bénard convection strongly depend on the Rayleigh and Prandtl numbers and the aspect ratio of the container. Studying these dependencies, Grossmann & Lohse [3], [4] analysed the thermal dissipation rate and suggested to investigate separately the turbulent background and the thermal boundary layers together with the plumes. Considering the thermal plumes as detached boundary layers, the authors split the volume averaged thermal dissipation rate into two contributions. The first is the thermal dissipation rate due to the plumes together with the smooth parts of the boundary layers and the second is the thermal dissipation rate of the turbulent background. According to the Grossmann–Lohse ansatz the turbulent background part of the thermal dissipation rate must dominate for very large Ra.

In [1] we developed a method to investigate the turbulent background quantitatively by introducing the functions τ and σ. Evaluating these functions in the numerical simulations one can estimate the contribution of the turbulent background to the volume averaged thermal dissipation rate and compute the volume of the fluid which corresponds to the turbulent background.

In the present work we apply this method to investigate the turbulent background in Rayleigh-Bénard convection in wide cylindrical containers of the aspect ratios $\Gamma = 5$ and 10 for the Rayleigh numbers $Ra = 10^5$, 10^6, 10^7, 10^8.

NUMERICAL EXPERIMENTS

The governing dimensionless equations for the Rayleigh-Bénard problem in Boussinesq approximation read

$$\mathbf{u}_t+\mathbf{u}\cdot\nabla\mathbf{u}+\nabla p=\left(\frac{Pr}{\Gamma^3 Ra}\right)^{1/2}\nabla^2\mathbf{u}+T\mathbf{z}, \quad (1)$$

$$\nabla\cdot\mathbf{u}=0, \quad (2)$$

$$T_t+\mathbf{u}\cdot\nabla T=\left(\frac{1}{\Gamma^3 Pr Ra}\right)^{1/2}\nabla^2 T \quad (3)$$

with $\mathbf{u}$ the velocity vector, T the temperature, $\mathbf{u}_t$ and T_t their time derivatives and p the pressure. $Ra=\alpha g H^3\Delta T/(\kappa\nu)$ denotes the Rayleigh number, $Pr=\nu/\kappa$ the Prandtl number, $\Gamma=D/H$ the aspect ratio with H the height and D the diameter of the cylindrical container. Further, α is the thermal expansion coefficient, g the gravitational acceleration, ΔT the temperature difference between the bottom and the top plates, ν the kinematic viscosity and κ the thermal diffusivity. The dimensionless temperature varies between $+0.5$ at the bottom plate and -0.5 at the top plate. An adiabatic lateral wall is prescribed by $\partial T/\partial r=0$. Finally, on the solid walls the velocity field vanishes according to impermeability and no-slip conditions.

To investigate the turbulent background in Rayleigh-Bénard convection we use the results of Direct Numerical Simulations (DNS) for $Ra=10^5$, 10^6, 10^7, $\Gamma=10$ and $Ra=10^6$, 10^7, $\Gamma=5$ and the results of Large-Eddy Simulations (LES) utilizing the tensor-diffusivity model by [5] with the top-hat filtering for $Ra=10^8$, $\Gamma=5$. In all simulations cylindrical containers filled with air, i.e. $Pr=0.7$, are considered. The fourth order accurate finite volume method developed for solving (1), (2), (3) in cylindrical coordinates (z,φ,r) on staggered non-equidistant grids and a computational mesh of (110, 512, 192) nodes clustered in the vicinity of the rigid walls are used in all simulations. For details of the method we refer to [6].

THERMAL DISSIPATION RATE ANALYSIS

Large values of the thermal dissipation rate

$$\epsilon_\theta=\Gamma^{-3/2}Ra^{-1/2}Pr^{-1/2}(\nabla T)^2$$

indicate the thermal plumes or the thermal boundary layers, while relatively small values of ϵ_θ stand for the turbulent background. This allows to investigate the turbulent background and its role in the heat transport.

For this purpose in [1] the functions $\tau(\xi)$ and $\sigma(\xi)$ were introduced and evaluated for $\Gamma=10$ and different Rayleigh numbers. The function $\tau(\xi)$ describes the percentage of the fluid volume, for which the thermal dissipation rate does not exceed $\xi\times 100\%$ of its maximum value, i.e. $\epsilon_{\theta,\max}=\max_V\epsilon_\theta$,

$$\tau(\xi)=\left\langle\vartheta(\xi\epsilon_{\theta,\max}-\epsilon_\theta)\right\rangle_V, \quad (4)$$

and $\sigma(\xi)$ describes the contribution to the volume averaged thermal dissipation rate from those parts of the domain, where ϵ_θ does not exceed $\xi\times 100\%$ of its maximum,

$$\sigma(\xi)=\frac{\left\langle\epsilon_\theta\vartheta(\xi\epsilon_{\theta,\max}-\epsilon_\theta)\right\rangle_V}{\left\langle\epsilon_\theta\right\rangle_V}. \quad (5)$$

Here ϑ is the Heaviside function,

$$\vartheta(x)=\begin{cases}1, & \text{if } x\geq 0,\\ 0, & \text{otherwise.}\end{cases} \quad (6)$$

In Fig. 1 and Fig. 2 the functions $\tau(\xi)$ and $\sigma(\xi)$ are plotted as they were evaluated in the simulations of turbulent Rayleigh-Bénard convection for different Ra and $\Gamma=10$ and 5, respectively. Since the turbulent background is indicated by lower values and the plumes together with the

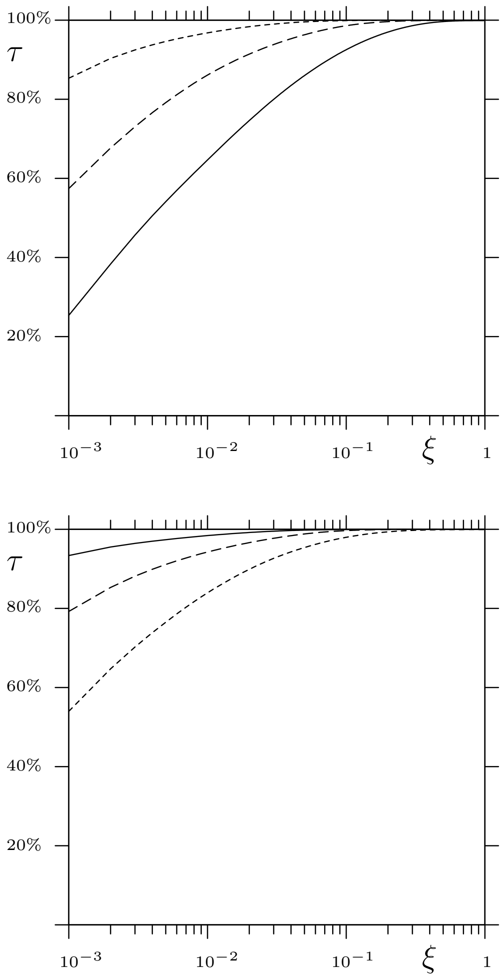

Fig. 1. Portion of the domain, where $\epsilon_\theta \leq \xi \epsilon_{\theta,\max}$, according (4) for $Pr = 0.7$ and (upper) $Ra = 10^5$ ——— , $Ra = 10^6$ – – – – , $Ra = 10^7$ - - - - - and $\Gamma = 10$; (lower) $Ra = 10^8$ ——— , $Ra = 10^7$ – – – – , $Ra = 10^6$ - - - - - and $\Gamma = 5$.

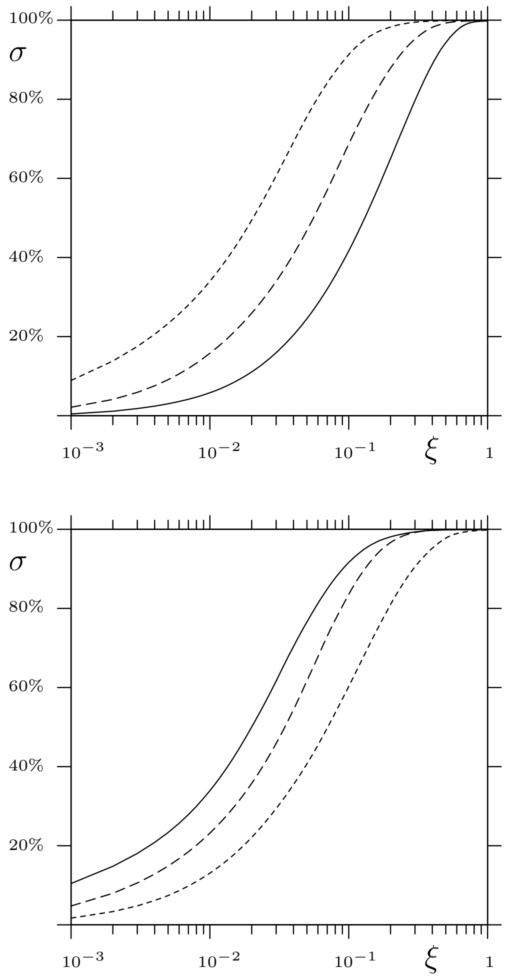

Fig. 2. Contribution to the volume averaged thermal dissipation rate from the parts of the domain, where $\epsilon_\theta \leq \xi \epsilon_{\theta,\max}$, according (5) for $Pr = 0.7$ and (upper) $Ra = 10^5$ ——— , $Ra = 10^6$ – – – – , $Ra = 10^7$ - - - - - and $\Gamma = 10$; (lower) $Ra = 10^8$ ——— , $Ra = 10^7$ – – – – , $Ra = 10^6$ - - - - - and $\Gamma = 5$.

boundary layers by higher values of the thermal dissipation rate, there exists a certain value of ξ, which separates these two regions. Furthermore, Fig. 1 and Fig. 2 reveal that the values of $\tau(\xi)$ and $\sigma(\xi)$ obtained for a certain Ra are always higher than the corresponding values for a lower Ra. Thus, the results presented above show that both the portion of the whole domain, which corresponds to the turbulent background, and the background contribution to the volume averaged thermal dissipation rate increase with Ra. This is true at least for wide containers of the aspect ratios 10 and 5 and Ra from 10^5 to 10^8 and supports the conjecture by Grossmann & Lohse [3], [4] about the leading role of the turbulent background in thermal convection for very large Ra.

Considering the functions

$$\tau_s(\xi) = \left\langle \vartheta(\xi \epsilon_{\theta,\max} - \epsilon_\theta) \right\rangle_{S_z} \tag{7}$$

and

$$\sigma_s(\xi) = \frac{\left\langle \epsilon_\theta \vartheta(\xi \epsilon_{\theta,\max} - \epsilon_\theta) \right\rangle_{S_z}}{\left\langle \epsilon_\theta \right\rangle_{S_z}} \tag{8}$$

with the Heaviside function ϑ (6), which determine the portion of the horizontal cross-section

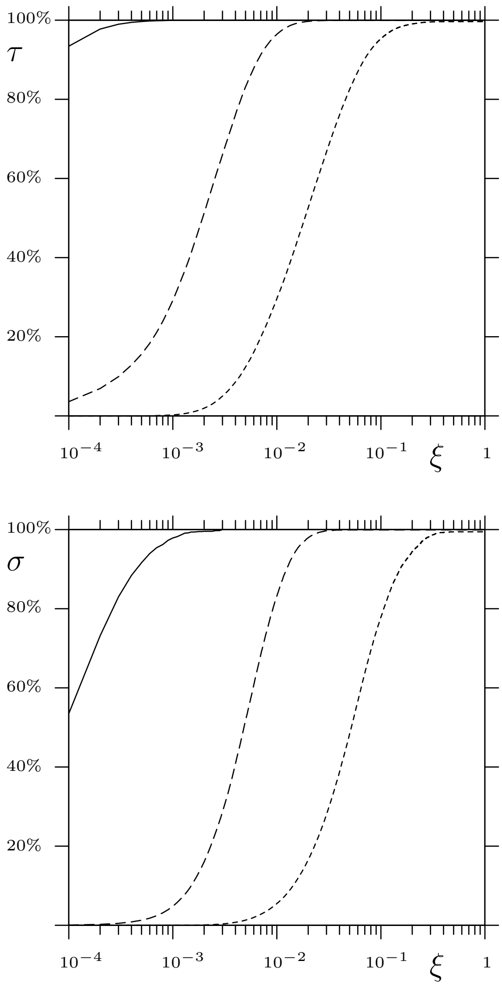

Fig. 3. (upper) Percentage of the horizontal cross–section, where $\epsilon_\theta \le \xi\epsilon_{\theta,\max}$ and (lower) contribution to the area-averaged thermal dissipation rate from the horizontal cross-section, where $\epsilon_\theta \le \xi\epsilon_{\theta,\max}$, for $Ra = 10^8$, $\Gamma = 5$ and $z = 0.5H$ (———), $z = H/(2Nu)$ (– – – –) and $z = 10^{-3}H$ (- - - - -).

S_z and the contribution to the area averaged thermal dissipation rate from those parts of S_z, where $\epsilon_\theta \le \xi\epsilon_{\theta,\max}$, for different z (see Fig. 3 lower and upper, respectively) one concludes that near the horizontal plates the contribution of the turbulent background which corresponds to small values of the thermal dissipation rate is negligible, while in the bulk large values of the thermal dissipation rate are hardly ever reached.

Further, the contribution of the horizontal boundary layers (without plumes) to the volume averaged thermal dissipation rate increases with the Rayleigh number (see, for example, [7]). As shown above, the contribution of the turbulent background also increases. It means that starting with a certain Ra the role of the thermal plumes in thermal convection decreases with increasing Rayleigh number.

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