

# Interaction between a deformable buoyant bubble and a homogeneous isotropic turbulence

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## ABSTRACT

*This communication reports on results of a 3D Direct Numerical Simulation (DNS) of a strongly deformable bubble in isotropic turbulent flows. The complex interaction between interface and turbulence is fully resolved. This two-way coupling phenomenon is found to be of great importance for the flow dynamic. An explicit filtering of the simulation has been employed to evaluate the order of magnitude of the specific subgrid contributions in the Large Eddy Simulation (LES) modelling case. Closure models are proposed.*

## INTRODUCTION

Two-phase flows abound in nature and in engineering applications. In most cases, both phases are turbulent (with high Reynolds numbers) and complex turbulence/interface interaction takes place. Consequently, DNS calculations have to entail a number of degrees of freedom proportional to the third power of the Reynolds number to correctly describe the flow behaviour. This extremely hard constraint makes it impossible to use DNS for industrial applications. In order to successfully carry out industrial simulations their numerical cost has to be reduced. In one-phase flow, LES allows a drastic reduction of the number of nodes in the space discretization.

Until now, most numerical calculations for multi-phase turbulent flows use RANS (Reynolds Averaged Navier-Stokes) or LES modelling in order to add a diffusion-like term. However, very few theoretical or experimental justifications have been proposed. Because local experimental measurements of complex turbulence interface interac-

tion are very difficult, the only tool that is able to provide information seems to be numerical experiments. In the context of DNS of two-phase flow, most of the literature is dedicated to two types of studies:

- On the one hand, calculations focus on deformable interfaces but no really on developed turbulence. For instance, Bunner and Tryggvason (2003) study the effect of bubble deformation on the properties of bubbly flows [1]. In their simulations, vortical structures are only produced by the wake of bubbles. The so-called pseudo-turbulence induced by a bubble swarm does not have the same properties than a really developed turbulence whose the energy spectrum has an inertial zone.
- On the other hand, the turbulence is fully developed but the interfaces are not physically deformed. So, complex interactions between fluid velocity fluctuations and interface deformations can not exist. In a large amount of works in this category, the size of the particles is smaller than the Kolmogorov length scale and, even when the particle's diame-

ter is much bigger than this scale, inclusions are supposed non-deformable. For example, Merle *et al.* (2004) simulates the dynamics of a clean spherical bubble fixed in a turbulent pipe flow [2].

In the present contribution, we investigate a DNS of the motion of a strongly deformable bubble in a Homogeneous Isotropic Turbulence (HIT) in order to provide new information on the complex turbulence interface interaction. The *Interfaces and Sub-grid Scales* (ISS) is our proposal of a two-phase equivalent for the one-phase LES concept. In this method, the geometry of interfaces is fully resolved. The challenge of ISS is to integrate into subgrid models the unresolved scales of the two-way coupling phenomena between interfaces and turbulence. In this work, we generalize, with the same methodology, the results of our previous *a priori* tests for ISS modelling [3] wherein we only make 2D DNS, so we do not verify if the proposed closures are efficient when vortex stretching mechanism occurs. Firstly, we present our numerical method. Then, we describe the DNS realised. Finally, we evaluate specific subgrid contributions in the ISS modelling case and propose closure models.

## NUMERICAL METHOD

In each phase, the flow is incompressible and isothermal. The mathematical formalism we use is the so-called one-fluid formulation where each one-fluid variable,  $\phi$ , is defined by  $\phi = \sum_k \chi_k \phi_k$  with  $\chi_k$  the phase indicator ( $\chi_k = 1$  in phase  $k$  and 0 elsewhere),  $u$  the velocity,  $t$  the time,  $p$  the pressure,  $\rho$  the density,  $\mu$  the dynamic viscosity,  $n = n_g$  the normal to the interface (from gas to liquid),  $\kappa$  the curvature of interface and  $\delta_\sigma$  the Dirac function indicating interface:

$$\nabla \cdot u = 0 \quad (1a)$$

$$\rho \frac{\partial u}{\partial t} + \rho \nabla \cdot (u \otimes u) = -\nabla p + \rho g + \sigma \kappa n \delta_\sigma + \nabla \cdot (\mu (\nabla u + \nabla^T u)) \quad (1b)$$

We use a "sharp interface" version of the Front-Tracking approach, which does not resort to usual explicit smoothing function of the interfaces. Since the interfaces are not smeared, the method can accurately capture the turbulent transfer between the two phases. It was assessed on many application tests comparing with analytical solutions or experimental data [4]. This original method benefits from the VOF method for the calculation of the indicator function. Nevertheless, the interfaces are explicitly described using a Lagrangian mesh, moving on an Eulerian mesh for the flux calculation as for the classical Front-Tracking method. Each elementary surface (respectively segment) describing the interface in 3D (respectively 2D) is designed with 3 (respectively 2) points. The displacement of these points follows the relation:

$$\frac{\partial \chi_k}{\partial t} = u \cdot n_k \delta_\sigma \quad (1c)$$

This system is approximated by explicit finite volumes of second order in space (the convection operator is centred) and third order in time (we use a Runge-Kutta scheme).

## DNS OF A DEFORMABLE BUBBLE IN HIT

For DNS of a deformable bubble in HIT, two computations are necessary. The first one,  $s_1$ , is a single-phase flow, which corresponds to the classical homogeneous isotropic turbulence in a three dimensional periodic box. The second one,  $s_2$ , is a two-phase simulation. At first, each simulation reaches separately a statistically stationary state. Then, we use one boundary of  $s_1$  as an inlet condition of imposed velocity for  $s_2$  (see fig. 1). The mesh size of  $s_1$  and  $s_2$  is 128x128x128 allocated to 8 processors. We have paid a particular attention to the task of providing realistic inlet condition. Indeed, in  $s_1$ , kinetic energy is rescaled

at every time step and a uniform translation motion is added in order to perform the simulation in the bubble's frame of reference. Our method to force turbulence corresponds to a linear forcing method proposed by Lundgren (see [5]). Our results are in agreement with those of Rosales *et al.* [5]. In particular, we find the Kolmogorov  $k^{-5/3}$  slope and the most energetic structures (that correspond also to the beginning of the inertial zone) have the size of the domain of simulation (fig. 2). Furthermore, the good decay (until the Kolmogorov length scale) of our spectra proves that  $s_1$  is fully resolved. In  $s_2$ , we simulate the interaction of the buoyant bubble with the isotropic turbulent flow. The size of our bubble is in the inertial zone of the kinetic energy spectrum (see fig. 2). Most relevant dimensionless numbers of the simulation whose the present contribution uses the results are given in Tab. 1 ( $\eta$  is the Kolmogorov length scale,  $D_b$  the bubble diameter,  $T_b$  the relaxation time of the bubble,  $L$  and  $T$  the time and space integral scales). However, we make a parametric study on the value of surface tension. We find a critical Weber number for bubble breakup of about 2 and the breakup process is in good agreement with the mechanisms identified by the study of F. Risso and J. Fabre [6]. Finally, we realize a mesh convergence for the laminar case of a simple rising bubble. We demonstrate that a 96x96x96 mesh is enough to capture the terminal velocity of the bubble.

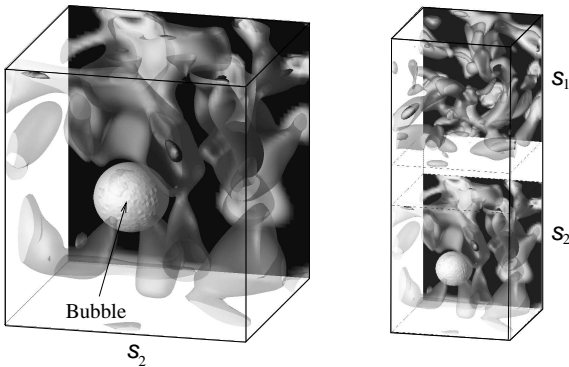


Fig. 1. Visualization of bubble and Q-isosurfaces

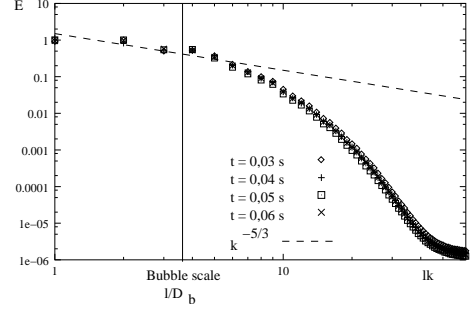


Fig. 2. Kinetic energy spectrum ( $l$  domain length)

$Re_b$	$We$	$Mo$	$Bo$	$\frac{\eta}{D_b}$	$\frac{L}{D_b}$	$\frac{T}{T_b}$
25.8	0.2	$2.2 \cdot 10^{-5}$	2.1	0.06	1.4	0.5

Table 1

Dimensionless numbers

## TOWARDS ISS MODELLING

One interest of such simulation is to provide information for ISS. The ISS (like LES) corresponds to a low-pass filtering in frequency of the fields [7]. We define the space filtering operation  $\overline{\cdot}$  and the surface filtering operation  $\overline{\cdot}^s$ :

$$\overline{\phi}(x) = \frac{1}{V} \int_{V_x} \phi(y) dy \quad (2a)$$

$$\overline{\phi}^s(x) = \begin{cases} \frac{\int_{V_x} \phi(y) \delta_\sigma(y) dy}{\int_{V_x} \delta_\sigma(y) dy} & \text{if } \int_{V_x} \delta_\sigma(y) dy \neq 0 \\ 0 & \text{otherwise} \end{cases} \quad (2b)$$

Applying the spatial filter,  $\overline{\cdot}$ , to the one-fluid equations leads to the following system where the interface is smoothed (*i.e.*, its width is the same than the low-pass frequency filtering operator)

$$\nabla \cdot \overline{u} = 0 \quad (3a)$$

$$\overline{\rho} \frac{\partial \overline{u}}{\partial t} + \overline{\rho} \nabla \cdot (\overline{u} \otimes \overline{u}) = -\nabla \overline{p} \quad (3b)$$

$$+ \nabla \cdot (\overline{\mu} (\nabla \overline{u} + \nabla^T \overline{u}))$$

$$+ \overline{\rho} g + \sigma \overline{\kappa}^s \overline{n}^s \overline{\delta}_\sigma$$

$$+ \tau_1 + \tau_2 + \tau_3 + \tau_4$$

$$\frac{\partial \overline{\chi}_k}{\partial t} = \overline{u} \cdot \overline{n}_k^s \overline{\delta}_\sigma + \tau_5 \quad (3c)$$

with

$$\tau_1 = \overline{\rho} \frac{\partial \overline{u}}{\partial t} - \overline{\rho \frac{\partial u}{\partial t}} \quad (3d)$$

$$\tau_2 = \overline{\rho} \nabla \cdot (\overline{u} \otimes \overline{u}) - \overline{\rho \nabla \cdot (u \otimes u)} \quad (3e)$$

$$\tau_3 = \nabla \cdot \left( \overline{\mu (\nabla u + \nabla^T u)} - \overline{\mu (\nabla \overline{u} + \nabla^T \overline{u})} \right) \quad (3f)$$

$$\tau_4 = \overline{\sigma \kappa n \delta_\sigma} - \overline{\sigma \kappa^s \overline{n^s} \delta_\sigma} \quad (3g)$$

$$\tau_5 = \overline{u \cdot n_k \delta_\sigma} - \overline{u \cdot \overline{n_k^s} \delta_\sigma} \quad (3h)$$

$\tau_1$ ,  $\tau_2$ ,  $\tau_3$  and  $\tau_4$  are four specific tensors of the momentum conservation equation associated respectively to acceleration, advection, viscous effects and interfacial forces;  $\tau_5$  is the specific subgrid term of the mass conservation equation. All these tensors appear due to the filtering and averaging operations [3]. They must be modelled to close the ISS description of two-phase flows. The first interest of *a priori* tests is to sort out the different subgrid scale terms of the two-phase flow equations according to their relative quantitative impact in the under-resolved case. Subgrid scale terms associated to interfacial forces (respectively viscous effects) are 10000 (respectively 10) times smaller than those associated to inertia and can be neglected in first approximation (as we demonstrated previously for 2D flows [3]).

To close terms associated to inertia, we firstly examine the eddy-viscosity assumption for  $\tau_2$ :

$$\tau_2 : \nabla u = \mu_t (\nabla u + \nabla^T u) : \nabla u \quad (4)$$

As we see in [3], it appears that classical closure models based on this assumption are inadequate to mimicking the negative part of the *equivalent*  $\mu_t$  due to a dispersive phenomenon of great importance near the interfaces. That is the reason why we propose to use a structural modelling. When the structural modelling is limited to the Leonard term, this is equivalent to a classical “scale similarity hypothesis” [7].

The Leonard and Germano [8] decomposition splits any  $\psi$  or  $\phi$  variables into averaged and spatially fluctuating components, and formally decomposes a subgrid term into three parts:

$$\phi = \overline{\phi} + \phi' \quad (5a)$$

$$\overline{\phi \psi} - \overline{\phi} \overline{\psi} = L + C + R \quad (5b)$$

$$L = \overline{\overline{\phi} \overline{\psi}} - \overline{\phi} \overline{\psi} \quad (5c)$$

$$C = \overline{\overline{\phi} \psi'} - \overline{\phi} \overline{\psi'} + \overline{\phi' \overline{\psi}} - \overline{\phi'} \overline{\psi} \quad (5d)$$

$$R = \overline{\phi' \psi'} - \overline{\phi'} \overline{\psi'} \quad (5e)$$

$L$  is usually referred as the Leonard term (this term does not need to be modelled),  $C$  is the crossed term (it involves products of averaged and fluctuating quantities),  $R$  is equivalent to the Reynolds stress tensor. This structural modelling is turned into a mixed model (scale similarity model and eddy-viscosity assumption) with the choice of the following closures:

$$L = \overline{\overline{\phi} \overline{\psi}} - \overline{\phi} \overline{\psi} \quad (6a)$$

$$C + R = \text{classical diffusion model} \quad (6b)$$

This scale similarity model, reinterpreted near the discontinuity, has been applied to the subgrid terms coming from  $\tau_1$  and  $\tau_2$ :

$$\tau_1^{mod} = \overline{\rho} \frac{\partial \overline{u}}{\partial t} - \overline{\rho \frac{\partial u}{\partial t}} \quad (7a)$$

$$\tau_2^{mod} = \overline{\rho} \nabla \cdot (\overline{u} \otimes \overline{u}) - \overline{\rho \nabla \cdot (u \otimes u)} \quad (7b)$$

It is important noting that contrary to equation (6a),  $\rho$  is filtered only one time in equations (7). This is coherent with the fact that ISS modelling assumes that interfaces are fully resolved and tracked in a DNS-like approach. Furthermore, we do not add the diffusion-like term (*i.e.*, we model  $C + R$  by zero) because it can be neglected for *a priori* tests (however, a diffusion-like term is certainly required for real LES in order to stabilize simulations). Our first results indicate that it is a good way to tackle with the dominant term. Figure 3 shows the results for the models given by equations (7) with a filter three times bigger than the DNS mesh. The two preponderant subgrid terms are clearly well modelled. So, the results of previous 2D studies [3] are generalized to a really turbulent case.

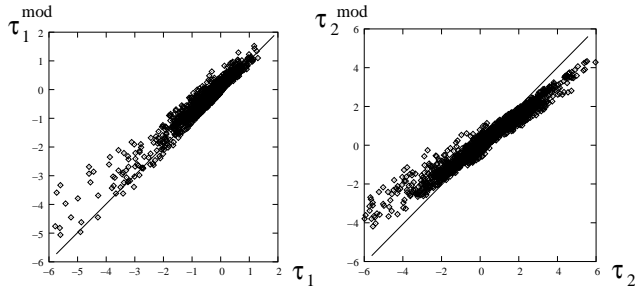


Fig. 3. Correlation: proposed model and real DNS contribution

## CONCLUSIONS AND PERSPECTIVES

An original and reliable DNS of the interaction between a deformable buoyant bubble and a homogeneous isotropic turbulence is realized. This type of numerical experiments is able to provide new information about the complex two-way coupling phenomenon.

This study relying on the isothermal momentum transfer through an interface in really turbulent flows completes the initial contributions on 2D flows:

- a consistent mathematical formalism for the system of two-phase one-fluid equations,
- the space filtered system of two-phase one-fluid equations (LES-like),
- the hierarchy of the order of magnitude of the subgrid LES terms,
- models to close the most important of these terms,
- validation of the models on *a priori* tests.

In another contribution [9], we demonstrate that our proposed closures are also efficient for heat transfer. However, the proposed closures suppose that interfaces are sharp and not smooth. Current works consist in finding the equivalent system for a sharp interface to the system (3). This step is necessary for using both the proposed closures and our numerical method. Further works concern implementation and *a posteriori* tests of the presented models.

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