Implications of turbulence interactions: A path toward addressing very high Reynolds number flows

Ye Zhou
University of California
Lawrence Livermore National Laboratory
Livermore, CA 94550, USA

ABSTRACT

The classical "turbulence problem" is narrowed down and redefined for scientific and engineering applications. From an application perspective, accurate computation of large-scale transport of the turbulent flows is needed. In this paper, a scaling analysis that allows for the large-scales of very high Reynolds number turbulent flows – to be handled by the available supercomputers is proposed. Current understanding of turbulence interactions of incompressible turbulence, which forms the foundation of our argument, is reviewed. Furthermore, the data redundancy in the inertial range is demonstrated. Two distinctive interactions, namely, the distance and near-grid interactions, are inspected for large-scale simulations,. The distant interactions in the subgrid scales in an inertial range can be effectively modelled by an eddy damping. The near-grid interactions must be carefully incorporated.

INTRODUCTION AND SUMMARY

In many problems in fundamental physics, we must simultaneously deal with uncertainty in the underlying equations of motion and with uncertainty in our ability to solve them. In turbulence, we have only the latter [1]. The turbulence problem is still referred to as the last unresolved classical physics problem [2].

The presence of strong nonlinear interactions make turbulence a truly multiple scale problem. The challenge in direct numerical simulations (DNS) of a very high Reynolds number flow is to account for all the scales, starting from the largest where the energy injection occurs to scales that are roughly two times the Kolmogorov dissipation wavenumber. The Reynolds number is around 10^8 for airplane wing and fuselage [2-3] and even higher for turbulent flows in space and astrophysical setting [4].

In such a computationally intensive field, we have witnessed an unprecedented advancement of the capabilities of the supercomputers (see the website www.top500.org for update). For grid generated turbulence or turbulent flows in a periodic box, brute-force DNS has already matched or surpassed experiments [5]. Pope [3] even suggested that we have entered an era of sufficient computer power.

At this juncture, what is the status of the last unresolved classical physics problem, given all these computing resources?

In this paper, "turbulence problem" is defined with a goal to compute the large-scales of complex turbulent flows at very high Reynolds numbers. A scheme is proposed so this goal can be achieved.

"TURBULENCE PROBLEM" REDEFINED

Most important properties of a high Reynolds number turbulent flow are determined by the transport dynamics of the large-scales[1]. Therefore, it makes sense to focus computing resources on capturing these scales accurately. From an application perspective, accurate and time-dependent, three-dimensional computations of the large-scales may be all that is needed. It is therefore necessary to chose the grid-size in a uniform fashion, using the boundary between the large-scales and inertial range.

In support of this argument, it will be illustrated that the self-similarity properties lead to the data redundancy; an advantage that should be fully exploited. The universality of the inertial range is indeed remarkable. The footprint of the flow type and its initial conditions, on the other hand, will persist, as indicated by the non-universal normalized energy dissipation rate [7][2].

Based on the above, this article claims (1) two distinct interactions have been identified; (2) a model that incorporates both interactions already exists; (3) a refined boundary for the inertial range can be located; (4) the self-similarity in the inertial range has been demonstrated.

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[1] We will not consider the problems of turbulence combustion where small scales are important [3] and other complex flows (such as the internal engine, see 2005 Annual Research Brief, Center of Turbulence Research, Stanford University [6])

[2] For discussions on normalized energy dissipation rate, see also references [8-10].

Finally, a scaling argument to scale the extremely high Reynolds number flows to high, but manageable Reynolds

number[3] in order to fit into the existing supercomputers is proposed.

FORMULATION OF A PHYSICAL PROBLEM

Filter classification and their spectral support

The objective of the filters is to separate the large-scales as faithfully as possible. Therefore, the filtering operation, which divides the flow into the subgrid and resolvable scales, should not adversely affect the large-scale properties.

Zhou *et al.* [12] pointed out that the resolvable scale interactions are affected when the filters with same spectral support are utilized. The so called "Type A category" filters include familiar Gaussian [13] and exponential filters [14]. The subgrid scale field, as well as the subgrid stresses, can be directly evaluated from the resolvable scale field.

However, in LES implementation, the maximum wavenumber is determined by the grid size. For wavenumber up the cutoff, k_C, all functions, the original, resolvable, and subgrid, are known. Nothing is known for $k > k_C$. Hence, the resulting subgrid stresses, though consistent, are only models of subgrid stress [12]. Similar propositions have been advanced [15-17].

No available scheme has been developed to restore the contamination caused by the Type A filter. It is therefore not appropriate for our purpose. The sharp cutoff filter, the "Type B category" which has distinctive spectral support, is recommended for LES applications.

Near-grid and distant interactions

Using a sharp cutoff filter, the subgrid of a given problem can be subdivided into two distinctive areas. The resolvable scale wavenumber is denoted **k** which satisfies a triad **k=p+q**. The subgrid region for *the near-grid interactions*, where one of the wavenumber is greater and the other is less than the cutoff wavenumber, k_C, is denoted as Δ_I. The subgrid region where both wavenumbers are greater than k_C (*the distant interaction region*), is denoted as Δ_{II}. Detailed studies of subgrid models in the energy transfer and momentum equations reveal the relationship between the eddy damping, backscatter and the Reynolds and cross stresses [18].

The near-grid and distant interactions plays distinctive roles in the energy transfer process [19]. The eddy viscosity $\nu^{>>}(k)$, resulting from the distant interactions, behaves in the same manner as the molecular viscosity. Therefore, an eddy viscosity model is acceptable. The eddy viscosity $\nu^{><}(k)$, resulting from the near-grid interactions, is responsible for the cusp-like behaviour of the spectral eddy viscosity first identified by Kraichnan [20]. See Fig. 1.

To confirm the importance of the near-grid interaction dynamics, the subgrid scale model should be accurately resolved [21]. A fictitious cutoff wavenumber is introduced in a DNS. Again, the results demonstrated that the near-grid interactions are critical for faithful computation of the large-scale evolutions.

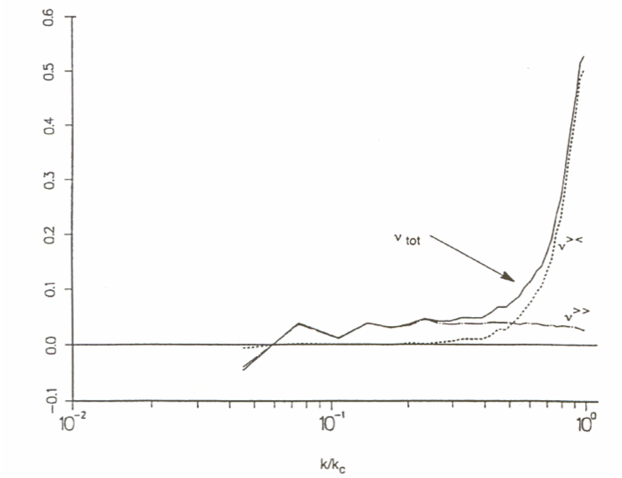

Fig. 1. *Spectral eddy viscosity and the individual contributions from both the near-grid and distant interactions (from ref. 19)*

Resolvable scale model equation

The first order of business is to derive a resolvable scale equation using the method of recursive renormalization (r-RG) group theory. This methodology was first proposed by Rose [22] for a model problem of passive scalar advection and was extended to Navier-Stokes equation [23-24]. Starting from the Kolmogorov dissipation wavenumber, the inertial range is divided into multiple shells, with their length as thin as possible. The first resolvable scale equation can be written symbolically as (*P* denotes the projection operator)

$$\partial u^{<} /\partial t + \nu_0 k^2 \; u^{<} = P\,[u^{<} u^{<} + 2\, u^{<} u^{>} + u^{>} u^{>}] \,. \qquad \text{-- (1)}$$

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[3] It is appropriate to compare the proposed method with that for LES and unsteady RANS. In LES, the grid-size is typically selected to capture about 80% of the energy [3]. As a result, LES is often restricted in dealing with turbulent flows at very high Reynolds numbers. The unsteady RANS models, on the other hand, typically rely on turbulence closure models to represent the un-resolvable scales (see for example, ref. [11]).

After removing the first subgrid shell, two types of subgrid interactions will make their distinctive contributions, and hence, they must be considered individually. First, the distant interactions will result in an enhanced eddy viscosity, ν_1 (k). Second, the near-grid interaction should be either computed directly, or approximated by an expression in the resolvable scale field. This process is repeated to remove the remaining subgrid scales shells.

The resulting recursion relation for these subgrid distant interactions lead to a fixed point-- the eddy viscosity. In the suggested model for the resolvable scale equation, the near-grid interactions is considered explicitly[4]

$$\partial u^{<} / \partial t + \nu(k)\, k^2\, u^{<} = P\, [u^{<} u^{<} + 2\, u^{<} u^{>}]. \qquad \text{--- (2)}$$

The wavenumber domain for the left hand side of (2) and the first term on the right hand side is $[0, k_C]$, while that for the second term is $[0, 2k_C]$.

PATH FOR RESOLVING A NARROWLY DEFINED TURBULENCE PROBLEM

Determination of the grid-size for LES

Now with a model for the resolvable scale Navier-Stokes equation in place, it is important to determine the grid size, k_C, uniformly for any given turbulent flow.

The traditional definition of the inertial range is the existence of a scale which is free from the large-scale forcing and small-scale viscous dissipation. A more precise definition can be introduced, where the upper and lower boundaries of the inertial range depend on the outer-scale (δ) and Reynolds number [26-28]:

Lower bound: $L_{\nu} \approx 50\, Re^{-3/4}\, \delta$, --- (3)

Upper bound: $L_{L\text{-}T} \approx 5\, Re^{-1/2}\, \delta$. --- (4)

The well known estimation that $Re > 10^4$ (Hinze, [29]), (or 100 when the Taylor microscale is used) is needed for an inertial range.

The upper bound of the inertial range in this model for resolvable scale equation is our grid size, which can be chosen as the grid-size in physical space or cutoff wavenumber in spectral space.

Selected benchmark flows

[4] In the analytical treatment [22-24], an approximated solution for the subgrid scale velocity field is substituted into the near-grid term in order to achieve the closure. Alternatively, many methods are available for estimating the velocity field in $[k_C, 2k_C]$ (for review, see [17] [25]).

Given the choice of the cutoff wavenumber[5], we are now in the position to estimate the computational requirement in a systematic fashion. The large-scales of two flows can be compared based on the same definition.

The resolution requirement for several benchmark flows are given in Table 1. As expected, even in our narrowly defined "turbulence problem", the demand for computationally reproducing the large-scales of these flows is beyond the current computational facilities (such as the Earth Simulator).

Flows **Re (R_λ)**	**k_c/k_δ**	**$2k_c/k_\delta$**	**k_ν/k_δ**
Airplane wing/fuselage [2-3] Re~108 (R_λ~25800)	2,000	4,000	20,000
Moscow wind tunnel [30] (R_λ~3180)	247	493	865
Tidal Channel [31] (R_λ~2000)	155	310	432
NASA/AMES Wind tunnel [32] (R_λ~1450)	113	226	266
Earth Simulator [10] Re~2.1×10^5 (R_λ~1201)	93	186	201

***Table 1**. Resolution requirements for benchmark flows. (k_δ is the outer-scale wavenumber.)*

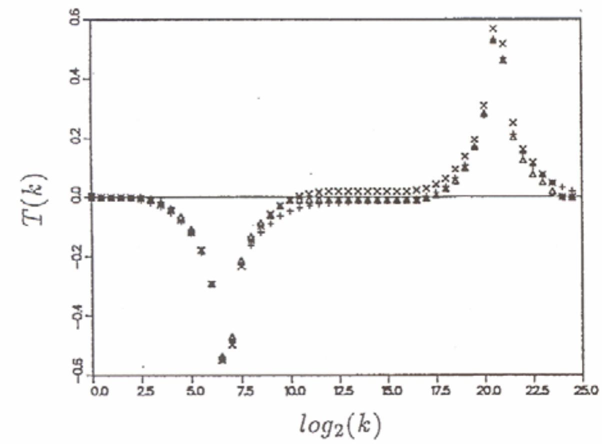

***Fig. 2** The energy transfer function of different grid sides for idealized Kolmogorov inertial range wavenumber. Rescaled to illustrate the "pipe without leak" analogy (from ref. 35)*

Data redundant in the inertial range

How can we compute the large-scale of these benchmark flows, or other higher Reynolds numbers found in astrophysical or geophysical flows? The answer lies in the universality of the inertial range, which we should exploit and utilize.

[5] The discussion can proceed in both physical and spectral spaces.

In the inertial range, the fractional energy flux for a given wavenumber scales against the scale disparity parameter with a -4/3 scaling law [33-35][6].

In fact, the universality in the inertial range can be demonstrated by computing the triadic energy transfer functions (for definition, see, for example, Domaradzki and Rogallo [37]). In Zhou [33-34], these triadic interactions are selected such that they satisfy the self-similarity scaling laws of Kraichnan [38]. The reconstructed energy transfer function, T(k), based on calculations for several grid sizes, has been shown to differ only in its range (or extent) in the spectral domain.

With a given energy input, this ideal Kolmogorov inertial range is essentially a pipe without leak. The different length of the pipe only reflects the different resolutions (or in other words, different Reynolds number) of the flows.

Based on this understanding, one can rescale these energy transfer function (Fig. 2) without affecting the large-scale. This is the most clear evidence of data redundancy in the inertial range.

Arbitrary Reynolds number flows

What would be the minimum resolution requirement (minimum model) for a faithful model calculation of the large-scale of a flow? The answer is that the near-grid interactions must be in the inertial range. The condition for this requirement can be found by demanding that the upper wavenumber of the near-grid scale, $2k_c$, be equal to the lower boundary of the inertial range, k_v (inner viscous scale). The Reynolds number is about 1.6×10^5 when this condition is met. The highest wavenumber for the calculation is $2k_c/k_\delta \equiv k_v/k_\delta = 160$ (and $k_c/k_\delta = 80$). This resolution requirement is achievable (see, for example, the Earth Simulator data in Table 1).

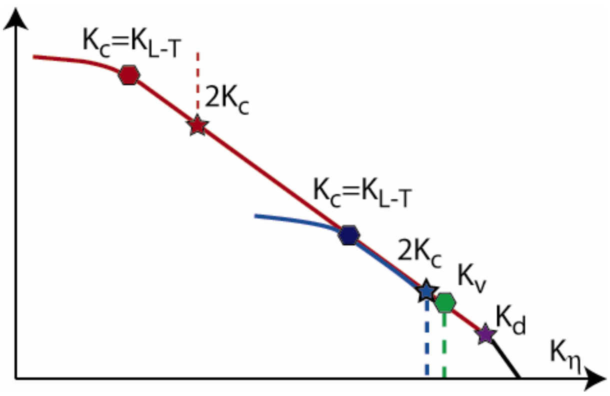

***Fig. 3:** Illustration of how an arbitrary high Reynolds number flows (in red) can be scaled down to a more manageable one (shown in blue), but still capture the physical aspects of the large-scale.*

For flows with arbitrary high Reynolds number, the resolvable scale momentum can be scaled to the minimum model. This claim is based on two arguments. First, the eddy viscosity, when it only includes the distant interactions, can be expressed analytically to account for any length scale in the inertial range of a given flow. Furthermore, the subgrid velocity fields for computing the near-grid interactions are in the inertial range for the "minimum model" and the extremely high Reynolds number flows. The near-grid interactions in both cases consist of the same physical aspects used in the large-scale computation. See Fig. 3.

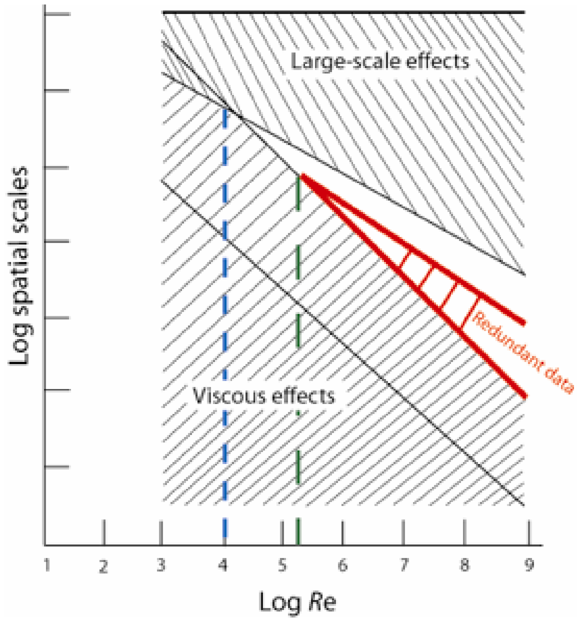

***Fig. 4.** Another illustration of how an arbitrarily high Reynolds number flows can be scaled down to a manageable one (shown in green), but still captures the physics of the large-scale. Based on a figure in Dimotakis with added marks in colour.*

Fig. 4 provides another way to illustrate this argument using the physical length scales variation with the Reynolds number. The Reynolds numbers where an inertial range first occurs and the "minimum" are marked blue and green, respectively. The shaded area represents the redundant data of the inertial range.

Since the grid-scale is determined by the outer-scale and the Reynolds number of the problem, any high Reynolds number flow can be easily scaled down to a very high, but computationally achievable Reynolds number flow.

While not necessary, the computed large-scales from the "minimum model" could be scaled up,[7] if so desired during the post-processing operation.

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[6] This -4/3 scaling has been confirmed by Gotoh and Watanabe [36]

[7] Following Ryutov *et al.* [39]

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BIBLIOGRAPHY

[1] Nelkin, M., "*Universality and scaling in fully developed turbulence*," Adv. in Phys., Vol. 43, pp. 143-181, 1994.

[2] Moin, P., and Kim, J., "*Tracking turbulence with supercomputers*," Scientific American, Vol. 276, pp. 62-68, 1997.

[3] Pope, S.B., "*Ten questions concerning the large-eddy simulation of turbulent flows*," New J. Phys., Vol. 6, 35, pp. 1-24, 2004.

[4] Zhou, Y., Matthaeus, W.H., and Dmitruk, P., "*Magnetohydrodynamic turbulence and time scales in astrophysical and space plasma*," Rev. Mod. Phys., Vol. 76, pp. 1015-1035, 2004.

[5] Jiménez, J., "*Computing high-Reynolds-number turbulence: will simulation ever replace experiments*?" J. of Turbulence, Vol. 4, 022, pp.1-14, 2003.

[6] Annual Research Brief – 2005, Center for Turbulence Research, Stanford University and NASA Ames Research Center, Stanford, CA

[7] Burattini, P., Lavole, P., and Antonia, R.A., "*On the normalized energy dissipation rate*," Phys. Fluids, Vol. 17, 098103, pp.1-4, 2005.

[8] Sreenivasan, K.R., "*On the scaling of the turbulence energy dissipation rate,"* Phys. Fluids, Vol. 27, pp. 1048-1051, 1984; Sreenivasan, K.R., "*An update on the energy dissipation rate in isotropic turbulence,"* Phys. Fluids, Vol. 10, pp. 528-529, 1998.

[9] Pearson, B.R., Krogstad, P.-A., and van de Water, W., "*Measurements of the turbulent energy dissipation rate,"* Phys. Fluids, Vol. 14, pp. 1288-1290, 2002.

[10] Kaneda, Y., Ishihara, T., Yokokawa, M., Itakura, K., and Uno, A., "*Energy dissipation rate and energy spectrum in high resolution direct numerical simulations of turbulence in a periodic box*," Phys. Fluids, Vol. 15, L21-24, 2003.

[11] Johansen, S.T., Wu, J., and Shyy, W., "*Filter-based unsteady RANS computation,*" Int. J. Heat and Fluid Flow, Vol. 25, pp. 10-21, 2004.

[12] Zhou, Y., Hossain, M., and Vahala, G., "*A critical look at the use of filters in large eddy simulation*," Phys. Lett. A, Vol. 139, pp. 330-332, 1989.

[13] Leonard, A., "*Energy cascade in large-eddy simulations of turbulent fluid flows*," Adv. Geophy. S., 18A, pp. 237–248, 1974.

[14] Germano, M., "*Differential filters for the large eddy simulation of turbulent flows,* Phys. Fluids, Vol. 29, pp. 1755-1757, 1986.

[15] Langford, J.A., and Moser, R.D., "*Optimal LES formulations for isotropic turbulence*," J. Fluid Mech., Vol. 398, pp. 321-346, 1999.

[16] Domaradzki, J.A., and Loh, K.-C., "*The subgrid-scale estimation model in the physical space representation,"* Phys. Fluids, Vol. 11, pp. 2330-2342, 1999.

[17] Domaradzki, J.A., and Adams, N.A., "*Direct modelling of subgrid scales of turbulence in large eddy simulations*," J. of Turbulence, Vol. 3, 024, pp. 1-19, 2002.

[18] Zhou, Y., "*Eddy damping, backscatter, and subgrid stresses in subgrid modeling of turbulence,* " Phys. Rev. A, Vol. 43, pp 7049-7052, 1991.

[19] Zhou, Y., and Vahala, G., "*Reformulation of recursive-renormalization-group based subgrid modeling of turbulence*," Phys. Rev. E, Vol. 47, pp. 2503-2519, 1993.

[20] Kraichnan, R.H., "*Eddy viscosity in two and three dimensions*," J. Atmos. Sci., Vol. 33, pp. 1521-1536, 1976.

[21] Dubois, T., Jauberteau, F., and Zhou, Y., "*Influences of subgrid scale dynamics on resolvable scale statistics in large-eddy simulations,"* Physica D, Vol. 100, pp. 390-406, 1997.

[22] Rose, H.A., "*Eddy diffusivity, eddy noise, and subgrid-scale modelling*," J. Fluid Mech., Vol. 81, pp. 719-734, 1977.

[23] Zhou, Y., Vahala, G., and Hossain, M., "*Renormalization-group theory for the eddy viscosity in subgrid modeling*," Phys. Rev. A, Vol. 37, pp. 2590-2598, 1988.

[24] Zhou, Y., Vahala, G., and Hossain, M., "*Renormalization eddy viscosity and Kolmogorov's constant in forced Navier-Stokes turbulence*," Phys. Rev. A., Vol. 40, pp. 5865-5874, 1989.

[25] Sagaut, P., *Large Eddy Simulation for Incompressible Flow* (Springer, Berlin, 2004).

[26] Dimotakis, P.E., "*The mixing transition in turbulent flows"*, J. Fluid Mech., Vol. 409, pp. 69-98, 2000.

[27] Zhou, Y., Robey, H.F., and Buckingham, A.C., "*Onset of turbulence in accelerated high-Reynolds-number flow*," Phys. Rev. E, Vol. 67, 056305, pp. 1-11, 2003a.

[28] Zhou, Y., Remington, B.A., Robey, H.F., *et al.,* "*Progress in understanding turbulent mixing induced by Rayleigh-Taylor and Richtmyer-Meshkov instability*," Phys. Plasma, Vol. 10, pp. 1883-1896, 2003b.

[29] Hinze, J.O., *Turbulence,* 2nd Edn, (McGraw-Hill, New York, 1975).

[30] Praskovsky, A.A., Gledzer, E. B., Karyakin, M. Yu., and Zhou, Y., "*The sweeping decorrelation hypothesis and energy-inertial scale interaction in high Reynolds number flows*," J. Fluid Mech., Vol. 248, pp. 493-511, 1993

[31] Grant, H.L., Stewart, R.W., and Moilliet, A., "*Turbulence spectra from a tidal channel,*" J. Fluid Mech., Vol. 12, pp. 241-268 (1962)

[32] Saddoughi S.G., and Veeravalli S.V., "*Local isotropy in turbulent boundary layers at high Reynolds number,*" J. Fluid Mech. Vol. 268, pp. 333–372, 1994.

[33] Zhou, Y., "*Degree of locality of energy transfer in the inertial range,*" Phys. Fluids A, Vol. 5, pp. 1092-1094, 1993a.

[34] Zhou, Y., "*Interacting scales and energy transfer in isotropic turbulence,*" Phys. Fluids A, Vol. 5, pp. 2511-2524, 1993b.

[35] Zhou, Y., and Speziale, C.G., "*Advances in the fundamental aspects of turbulence, Energy transfer, interacting scales, and self-preservation in isotropic decay,*" ASME Appl. Mech. Rev, Vol. 51, pp. 267-301, 1998.

[36] Gotoh, T., and Watanabe, T., "*Statistics of transfer fluxes of the kinetic energy and scalar variance,*" J. of Turbulence, Vol. 6, 33, pp. 1-18, 2005.

[37] Domaradzki, J.A., and Rogallo, R.S., "*Local energy transfer and non-local interactions in homogeneous, isotropic turbulence,*" Phys. Fluids A, Vol. 2, pp.413-426, 1990.

[38] Kraichnan, R.H., "*Inertial range transfer in two- and three-dimensional turbulence,*" J. Fluid Mech., Vol. 47, pp. 525-535, 1971.

[39] Ryutov, D., Drake, R.P., Kane, J., Liang, E., Remington, B.A., and Wood-Vasey, W.M., "*Similarity criteria for the laboratory simulation of supernova hydrodynamics,*" The Astrophys. J., Vol. 518, pp. 821-832, 1999.