

# DNS of a Turbulent Channel Flow with Streamwise Rotation at Different Reynolds Numbers

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## ABSTRACT

*In this work a turbulent channel flow rotating about the streamwise direction is presented. The theory is based on the investigations of [3] employing the symmetry group theory. It was found that a cross flow in the spanwise direction is induced. Statistical evaluations have shown that all six components of the Reynolds stress tensor are non-zero. A series of direct numerical simulations (DNS) has been conducted at rotation number  $Ro=20$  for different Reynolds numbers. In this paper the results of the DNS are presented and discussed.*

## INTRODUCTION

Rotating turbulent flows play more a major role in engineering applications such as in gas turbine blade passages, pumps and rotating heat exchangers to name only a few. In these cases secondary flows are induced caused by centrifugal or Coriolis forces. Investigations of [3] using symmetry theory showed that there is a new turbulent scaling law related to the turbulent channel flow rotating about the mean flow direction. Figure 1 depicts the flow geometry.

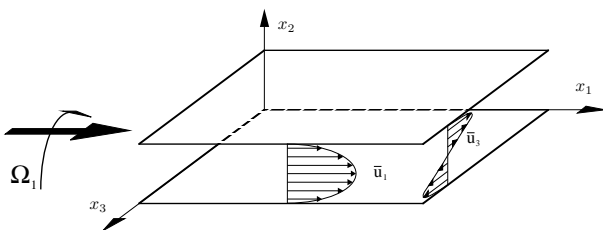


Fig. 1. Sketch of the flow geometry of a turbulent channel flow rotating about the mean flow direction

The flow has several common features with the classical rotating channel flow rotating about the spanwise direction [1] but also has some different characteristics. The induction of a mean velocity in  $x_3$ -direction [4] is the most obvious difference compared to the classical case. This cross flow

can be deduced by investigating the mean momentum equation and the Reynolds stress transport equation. Statistical evaluations have shown that all six components of the Reynolds stress tensor are non-zero.

Both the predicted cross flow and the non-zero components of the Reynolds stress tensor could be verified in a DNS at Reynolds number  $Re=180$  for different rotation rates [5]. In this paper the results of a DNS at rotation rate  $Ro=20$  at three different Reynolds numbers ( $Re=180, 270$  and  $560$ ) are presented and discussed. The main objective of the present paper is to analyze these effects at different Reynolds numbers.

## DIRECT NUMERICAL SIMULATION

### Numerical Method and Computations

The numerical technique which was chosen is a standard spectral method with Fourier decomposition in streamwise and spanwise direction as well as Chebyshev decomposition in wall-normal direction. The numerical code for channel flow was developed at KTH/Stockholm [2]. Additional features such as the streamwise rotation and statistics were added during the project. All flow quantities are non-dimensionalized by  $h/2$  and  $u_{cl}$  where  $h$  is the channel width and  $u_{cl}$

is the center line velocity of the flow field. The boundary conditions are non-slip at  $x_2 = \pm 1$  and periodic in  $x_1$ - and  $x_3$ -direction. For all computations the pressure-gradient is kept constant. Further details on the numerical scheme may be obtained from [2].

After the simulations were finalized all flow quantities were normalized on the friction velocity  $u_\tau$ . Hence the Reynolds number is defined by

$$Re_\tau = \frac{hu_\tau}{2\nu} \quad (1)$$

and the rotation number as

$$Ro = \frac{\Omega h}{u_\tau}. \quad (2)$$

Three computations at rotation number  $Ro=20$  for Reynolds numbers  $Re = 180$ ,  $Re = 270$  and  $Re = 560$  have been conducted. The size of the domain used in the  $x_1$ ,  $x_2$ , and  $x_3$  directions are respectively  $8\pi$ , 2 and  $4\pi$  for all computations. The used grid resolution at Reynolds number  $Re = 180$  is  $256 \times 129 \times 128$ , at  $Re = 270$   $256 \times 193 \times 256$  and at  $Re = 560$   $512 \times 257 \times 256$ . All computations were run for  $10000 \frac{h/2}{u_{cl}}$  time units and the statistics accumulation was performed for the last 5000 time units.

### Main Profiles and Reynolds Stress Tensor

In figures 2 and 3 the mean velocity profiles for three different Reynolds numbers are visualized. The predicted cross flow could also be verified in the DNS. Both the streamwise and spanwise profiles increase with increasing Reynolds number. From Reynolds number  $Re=180$  to  $Re=270$  rather weakly but for  $Re=560$  the increase is clearly visible.

From the variety of statistical one-point quantities, the Reynolds stress tensor is shown in figures 4 and 5 at rotation number  $Ro=20$  for two Reynolds numbers  $Re=180$  and  $Re=560$ . The stresses for  $Re=270$  (not shown here) are very similar to these of  $Re=180$ . As predicted from the lie group analysis [3] the DNS shows also that all six components of the Reynolds stress tensor are non-zero and all statistical curves are symmetric or skew-symmetric about the centerline.

The normal and shear stresses clearly increase with increasing Reynolds number.

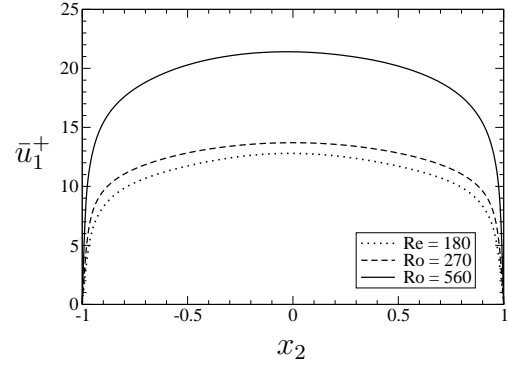


Fig. 2. Streamwise mean velocity profiles at  $Ro=20$  and different Reynolds numbers.

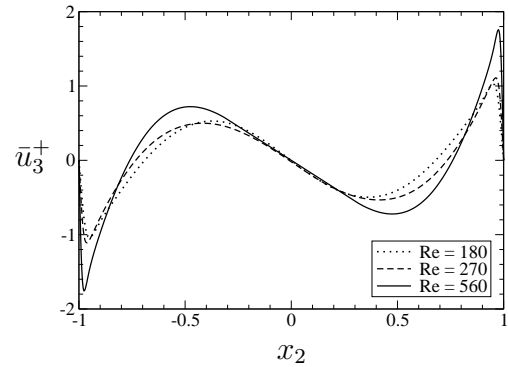


Fig. 3. Streamwise mean velocity profiles at  $Ro=20$  and different Reynolds numbers.

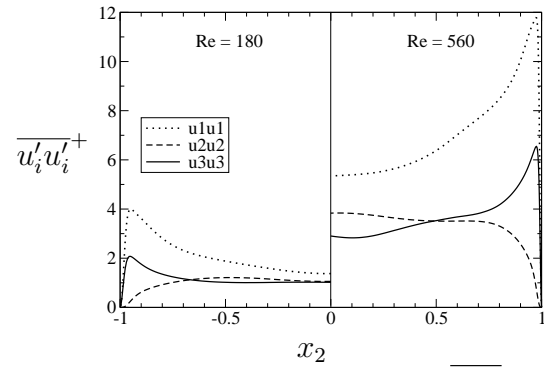


Fig. 4. Reynolds normal stresses  $\overline{u'_i u'_i}$ .

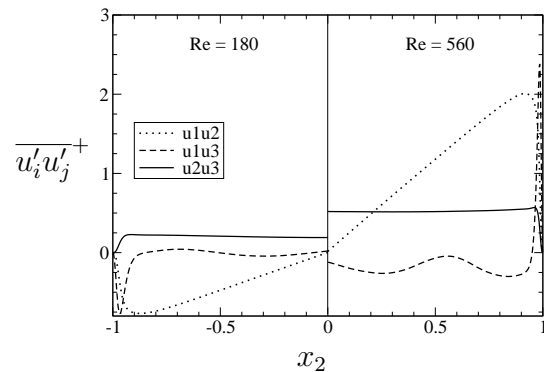


Fig. 5. Reynolds shear stresses  $\overline{u'_i u'_j}$ .

### Isosurface of the wall-normal velocity field

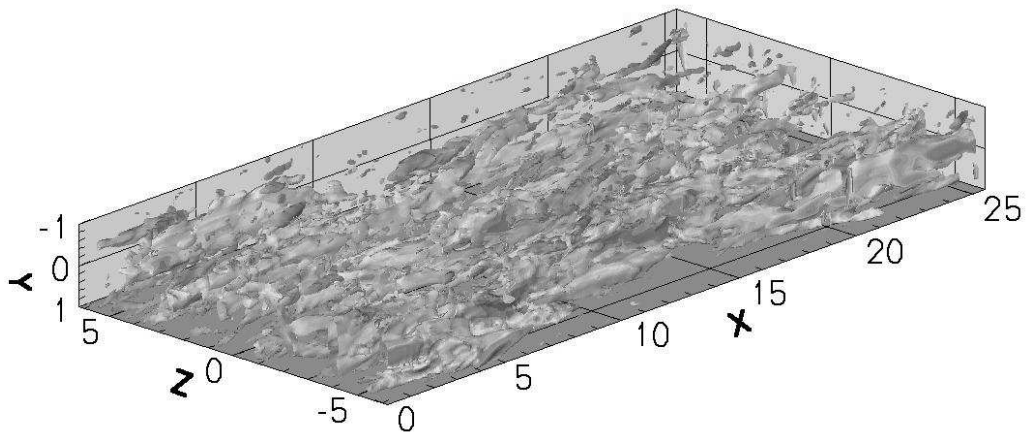


Fig. 6. Isosurface of the wall-normal velocity field at  $Ro=20$  and  $Re=180$ .

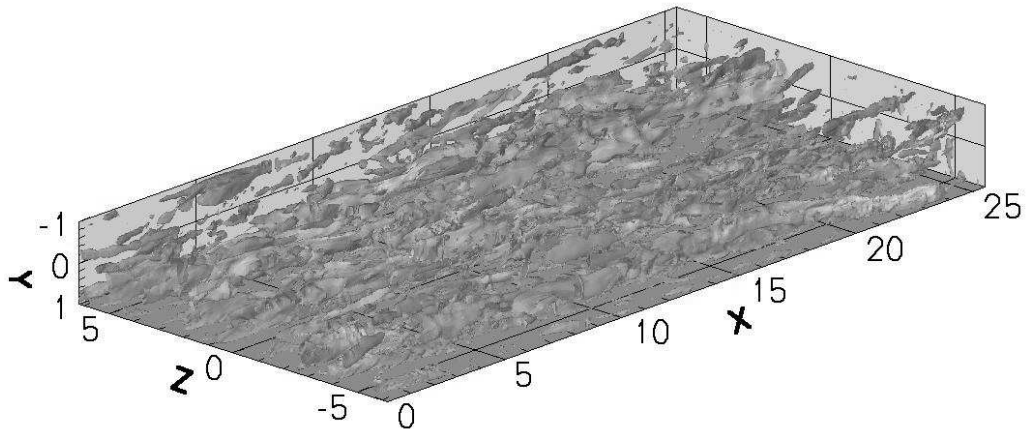


Fig. 7. Isosurface of the wall-normal velocity field at  $Ro=20$  and  $Re=270$ .

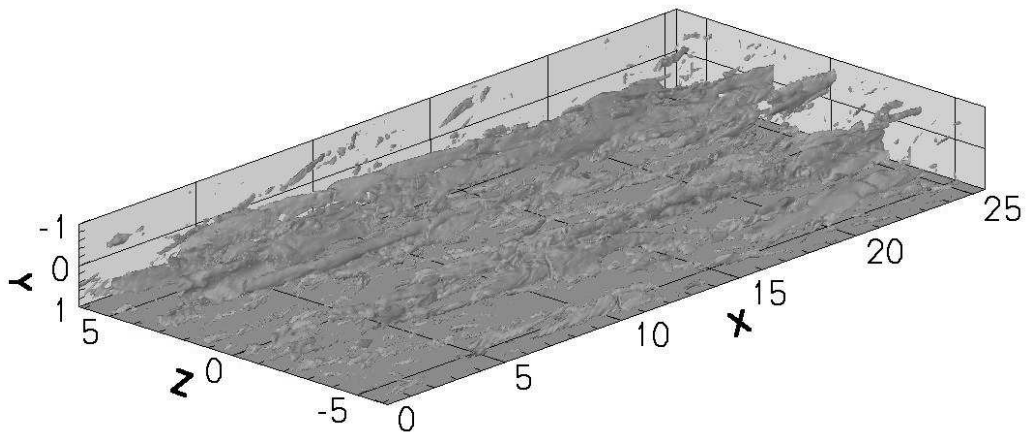


Fig. 8. Isosurface of the wall-normal velocity field at  $Ro=20$  and  $Re=560$ .

In figures 6 - 8 isosurfaces of the wall-normal velocity field are visualized at an instantaneous time unit  $t = 10000 \frac{h/2}{u_{cl}}$  for different Reynolds numbers. Apparently with increasing Reynolds number the formed turbulent structures become longer and thinner.

### xz-slices of the streamwise vorticity field

The figures 9 and 10 show the xz-slices for  $y=0.9$  (near the wall) of the streamwise vorticity field at an instantaneous time unit  $t = 10000 \frac{h/2}{u_{cl}}$  for two different Reynolds numbers. Because of the limited space only the first half of the channel is shown. The plot at  $Re=270$  looks very similar to this at  $Re=180$ , therefore it is not shown here. For increasing Reynolds numbers the number of vortices increases, but the structures become smaller. It is important to mention that caused by the rotation the structures are diverted from the streamwise direction.

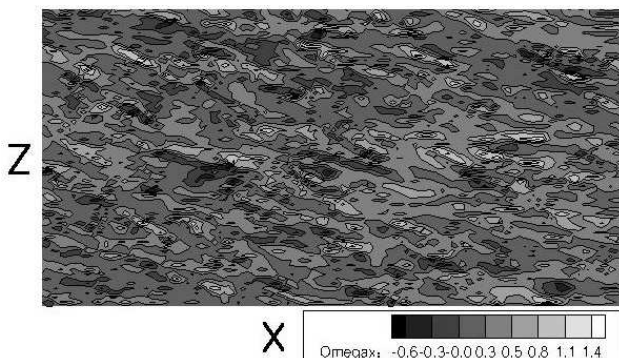


Fig. 9. xz-slice ( $y=0.9$ , near the wall) of the streamwise vorticity field at  $Ro=20$  and  $Re=180$ .

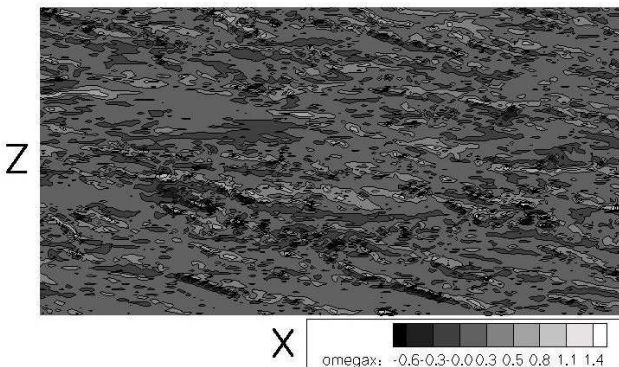


Fig. 10. xz-slice ( $y=0.9$ , near the wall) of the streamwise vorticity field at  $Ro=20$  and  $Re=560$ .

## CONCLUSIONS

With the DNS the induced phenomena of a cross flow in spanwise direction has been computed for different Reynolds numbers. It is to mention that the mean velocity in both streamwise and spanwise direction increases for a representative rotation number  $Ro=20$  at different Reynolds numbers. Furthermore, it is shown that all components of the Reynolds stress tensor are non-zero and that all statistical curves are symmetric or skew-symmetric about the centerline. In the isosurfaces of the velocity field it is shown that elongated turbulent structures are formed. For higher Reynolds numbers the structures are much longer and thinner. In the vorticity field the structures are diverted from the streamwise direction because of the rotation. This effect should be also analyzed for different rotation rates. Future research is under current investigation.

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