Strongly anisotropic turbulence statistical theory & DNS

Turbulence and Interactions **IGESA**

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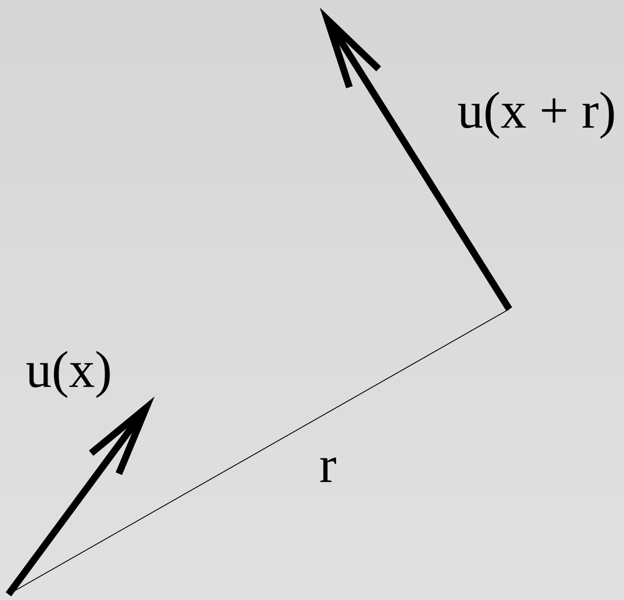
Outline

- ‘Directional’ and ‘polarization’ anisotropy
- A strategy for HAT (Homogeneous Anisotropic Turbulence)
  -) Rapid/slow decomposition suggested by linear theory (so-called RDT).
  -) To transfer the QNM machinery to slow variables
- Rotating turbulence. True Wave-Turbulence ?
- Turbulence in a stably stratified fluid. Waves *and* turbulence.
- Discussion of some issues in other cases (pdf text): *e. g.* Pure plane shear (next talk on Thursday !)
- Concluding remarks. Cross-fertilization
  -) Zonal eddy damping, -) Vortex and Wave ‘linear modes’,
  -) Perspectives towards inhomogeneous flows: WKB RDT, ... etc.

Anisotropic description

- ANISOTROPY/ inhomogeneity/ Intermitency
- structure functions or correlations, two-point : $R_{ij}(\boldsymbol{r}) = \langle u_i(\boldsymbol{x}) u_j(\boldsymbol{x} + \boldsymbol{r}) \rangle$



-) Single-point: componentality only

-) Two-point: directional anisotropy

- Low dimension parameterization, SO(3) symmetry group (Arad *et al.*,PRE,1999)

Anisotropic description. 3D Fourier space

- Anisotropic scalar (e. g. spherical harmonics)

$$\frac{1}{2}R_{ii}(\boldsymbol{r}) \quad \rightarrow \quad \frac{1}{2}\hat{R}_{ii}(\boldsymbol{k}) = e(\boldsymbol{k})$$

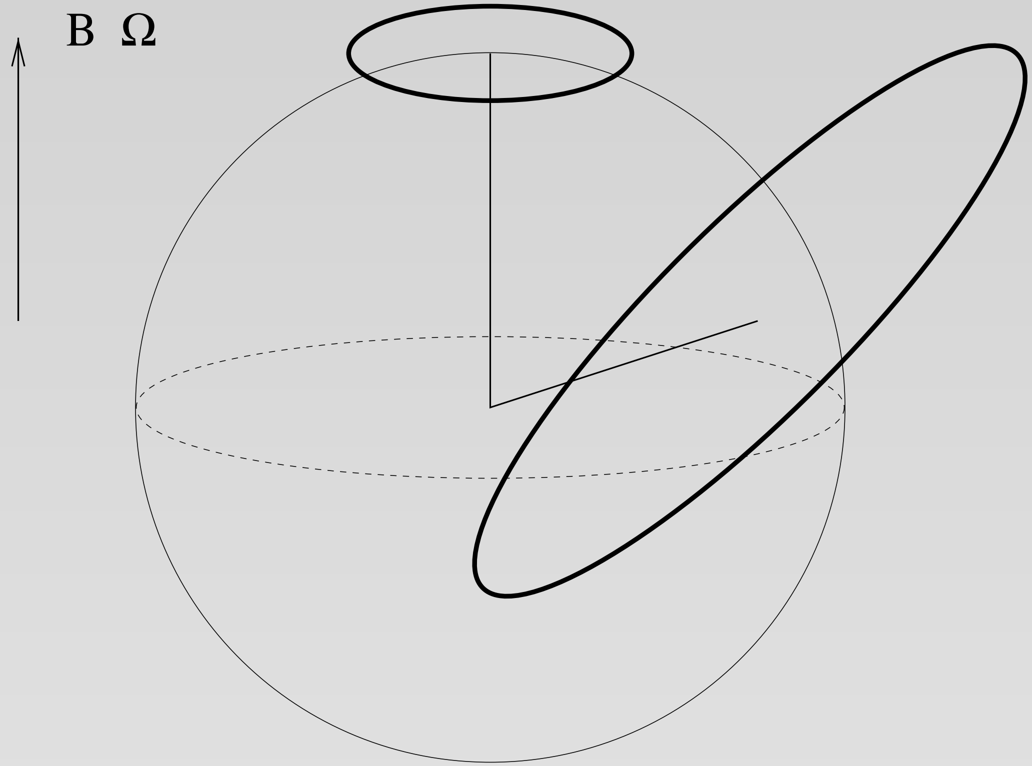
$$\sum r_n^m(r)Y_n^m(\theta_r, \phi_r) \quad \rightarrow \quad \sum \varphi_n^m(k)Y_n^m(\theta_k, \phi_k)$$

Avoiding a ‘schizophrenic’ viewpoint ! (Cambon & Teissèdre 1985, CRAS Paris)

- A trace-deviator decomposition restricted to solenoidal space

$$\hat{R}_{ij} = \underbrace{\underbrace{U(k)P_{ij}}_{\text{isotropic}} + \underbrace{\mathcal{E}(\boldsymbol{k})P_{ij}}_{\text{directional}}}_{eP} + \underbrace{\Re\left(Z(\boldsymbol{k})N_iN_j\right)}_{\text{polarization}}.$$

(Cambon & Jacquin, JFM, 1989), $P_{ij} = \delta_{ij} - \frac{k_i k_i}{k^2}$, $\boldsymbol{N}$ ‘helical mode’. Helicity ?



Rotating turbulence, MHD simplified case

Weakly anisotropic description

- 3D description $\boldsymbol{k}$: $\hat{R}_{ij}(\boldsymbol{k}) = \underbrace{U(k)P_{ij}}_{\text{isotropic}} + \underbrace{\mathcal{E}(\boldsymbol{k})P_{ij}}_{\text{directional}} + \underbrace{\Re\left(Z(\boldsymbol{k})N_iN_j\right)}_{\text{polarization}}$

- Averaging over $\boldsymbol{k}/k$: $\varphi_{ij}(k) = 2E(k)\left(\frac{\delta_{ij}}{3} + \underbrace{H^{(e)}_{ij}(k) + H^{(e)}_{ij}(k)}_{H_{ij}}\right)$

- Averaging over k: $\overline{u_iu_j} = q^2\left(\frac{\delta_{ij}}{3} + \underbrace{b^{(e)}_{ij} + b^{(z)}_{ij}}_{b_{ij}}\right)$

  (with Kassinos *et al.*, JFM, 2000, as a byproduct)

Reconstructing $\hat{\mathbf{R}}(\boldsymbol{k})$

$$e(\boldsymbol{k}) = \underbrace{\frac{E(k)}{4\pi k^2}}_{U} \left(1 - 15 H^{(e)}_{mn}(k) \frac{k_m k_n}{k^2}\right), \quad Z(\boldsymbol{k}) = 5 \frac{E}{4\pi k^2} H^{z}_{mn}(k) N^*_m N^*_n$$

$$\hat{R}^{(pol)}(\boldsymbol{k}) = 5U(k) \left(P_{im} P_{jn} H^{(z)}_{nm} - \frac{1}{2} P_{ij} H^{(z)}_{mn}(k) P_{mn}\right)$$

(Cambon & Rubinstein 2006, generalizing and gathering several weakly anisotropic models, *including the one by Kaneda & Yoshida*). Full consistency with spherical harmonics second degree expansions.

Rotating turbulence

- Wave-turbulence with a 'singular' zero-frequency mode

$$\boldsymbol{u}(\boldsymbol{x},t) = \sum e^{\imath \boldsymbol{k}\cdot\boldsymbol{x}} \left(a_{+1}\boldsymbol{N} e^{\imath\sigma_k t} + a_{-1}\boldsymbol{N}^* e^{-\imath\sigma_k t} \right)$$

-) A new set of 'slow' variables : $a_{\pm 1}(\boldsymbol{k},t)$

-) Anisotropic dispersion frequency of inertial waves: $\sigma_k = 2\boldsymbol{\Omega}\cdot\frac{\boldsymbol{k}}{k}$

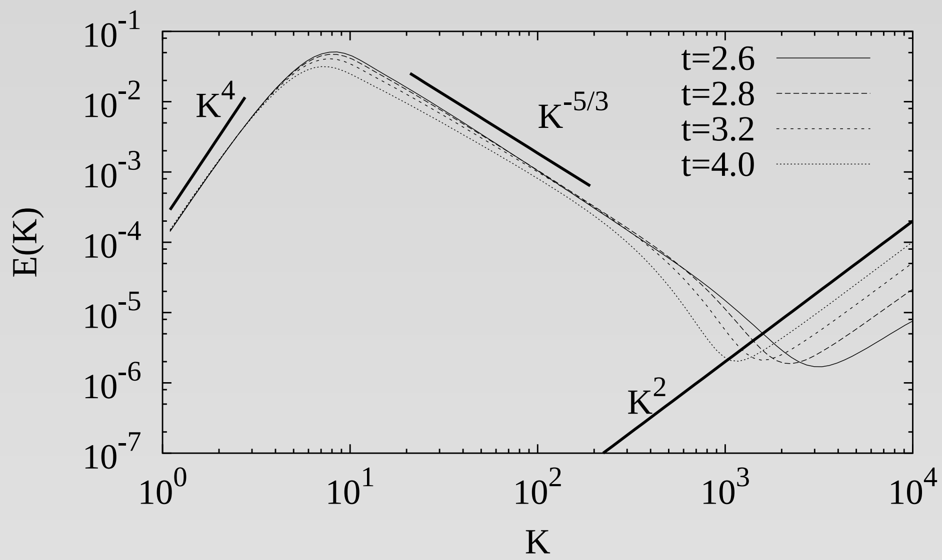
- Rewriting governing equations in terms of $a_{\pm 1}$:

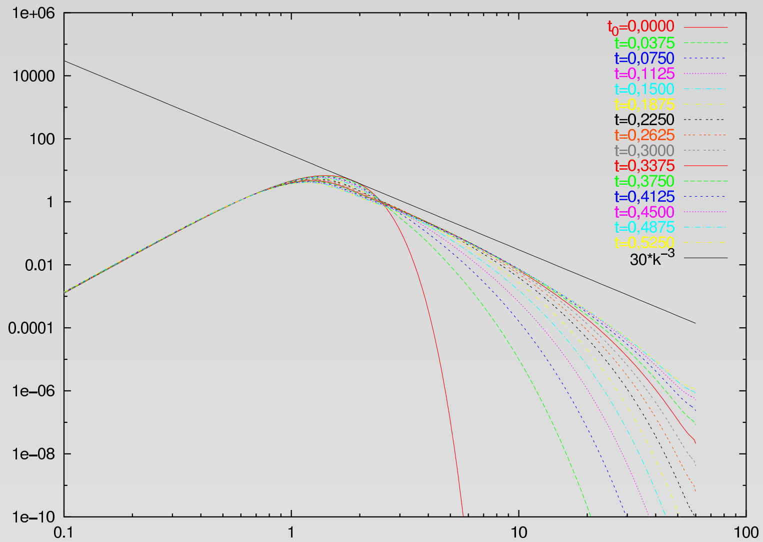
-) triadic resonance condition $\sigma_k \pm \sigma_p \pm \sigma_q = 0$

- Rewriting any spectral closure (e.g. Lin) : $\frac{\partial e}{\partial t} + 2\nu k^2 e = T^{(e)}(\boldsymbol{k},t)$

Results. Statistical theory

- From classical EDQNM (isotropic, no rotation, Wouter & Bertoglio, 20006) ...

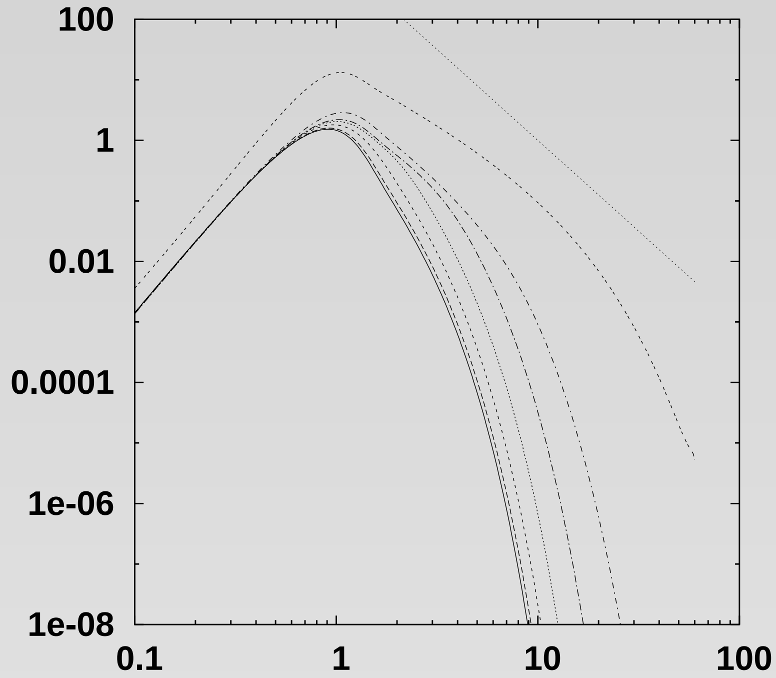




- ... to EDQNM3 $\longrightarrow$ (A) QNM energy equation (Bellet *et al.*, JFM, 2006)

Angle-dependent spectrum

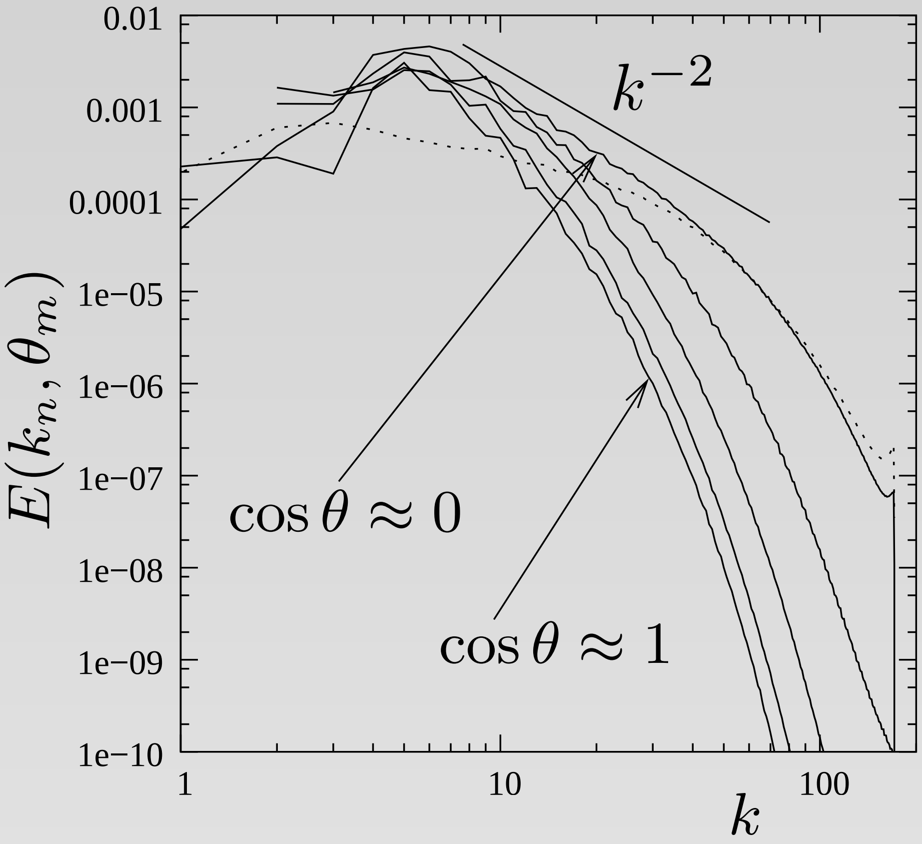
- Isotropy breaking by spectral transfer $T^{(e)}(\boldsymbol{k})$: directional anisotropy:

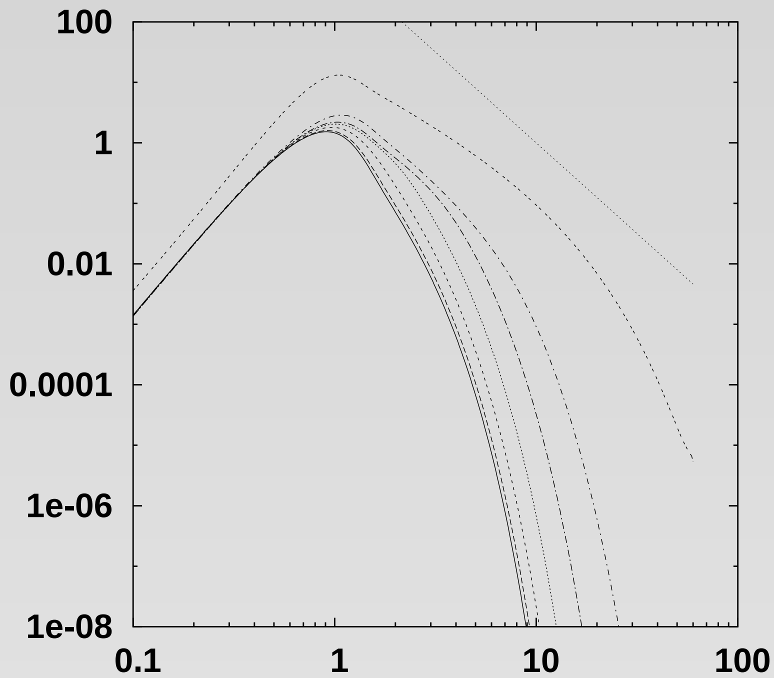


$$4\pi k^2 e(\boldsymbol{k}, t_f) = 4\pi k^2 e(k, \underbrace{\cos\theta}_{k_{\parallel}/k}, t_f)$$

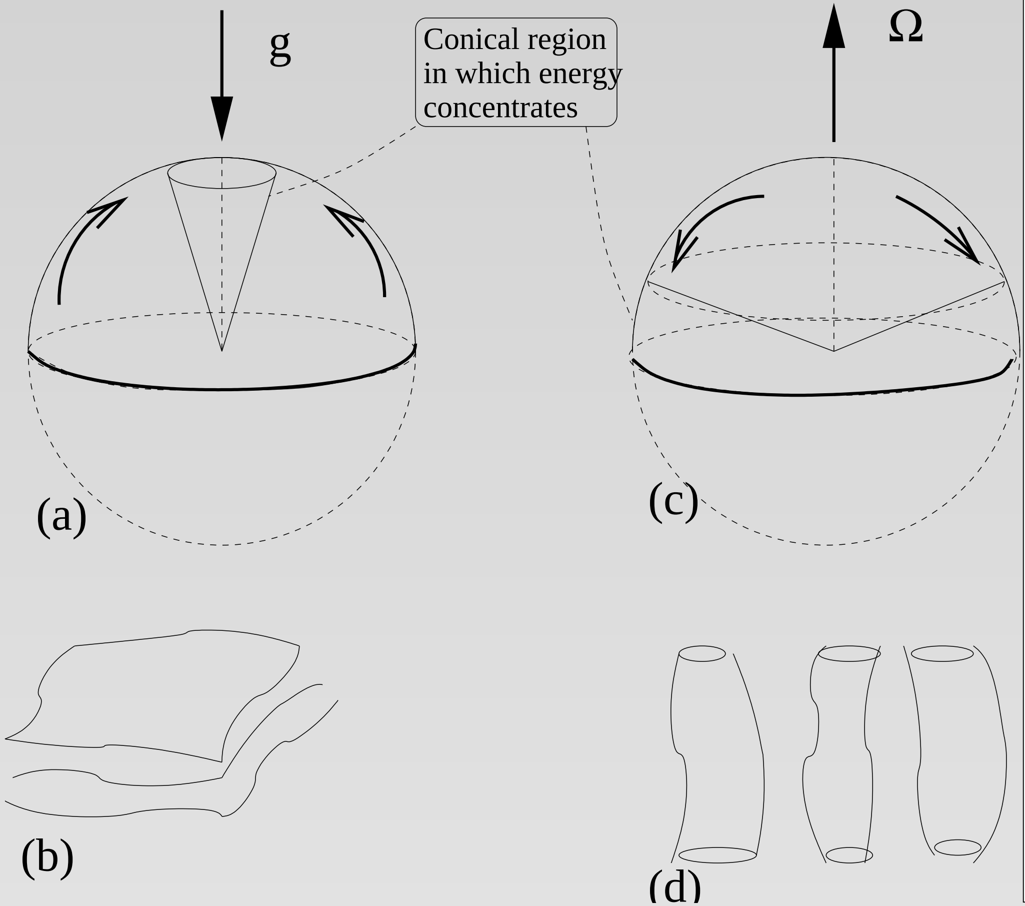
- Spherical averaging $\longrightarrow E(k, t_f)$, prefactor $E \sim \frac{\Omega}{t} k^{-3}$, not 2D !

AQNM and DNS





512^3 DNS by Liechtenstein *et al.*, JOT, 2005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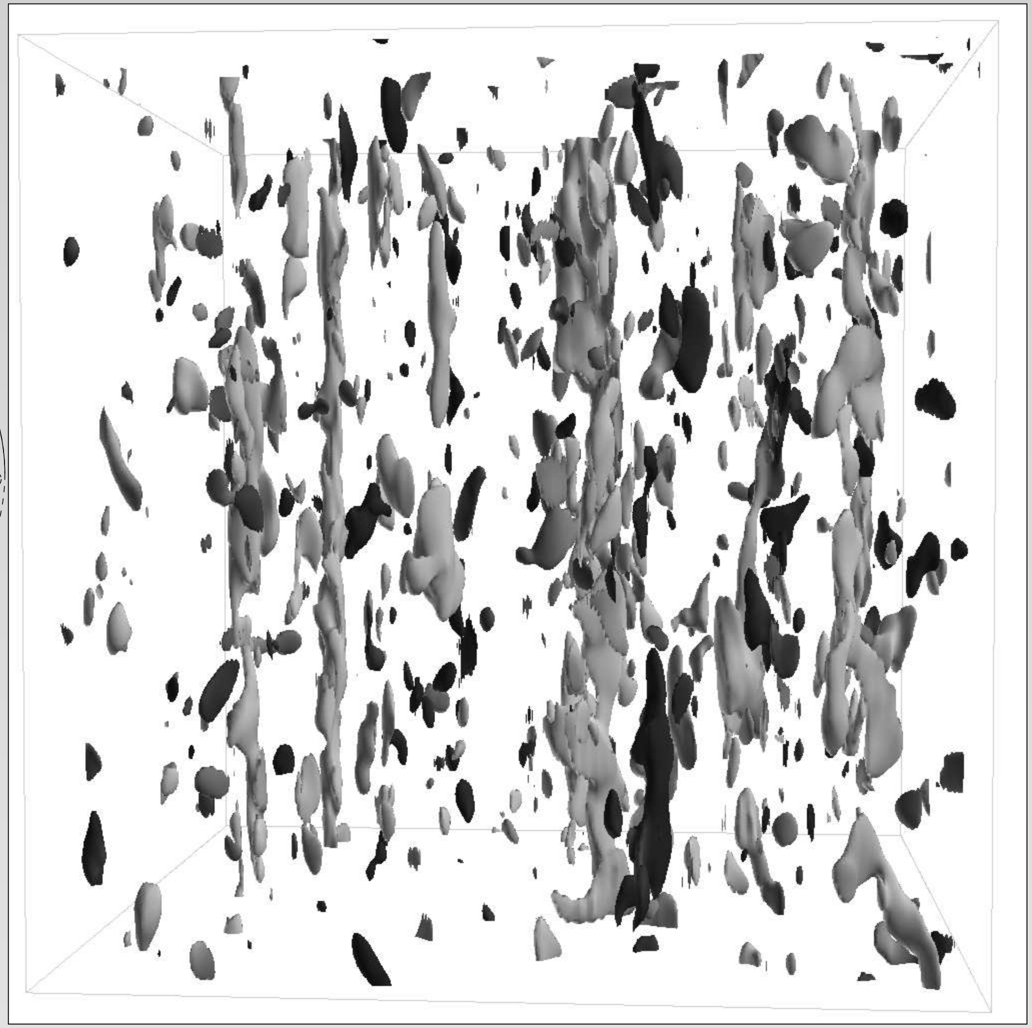
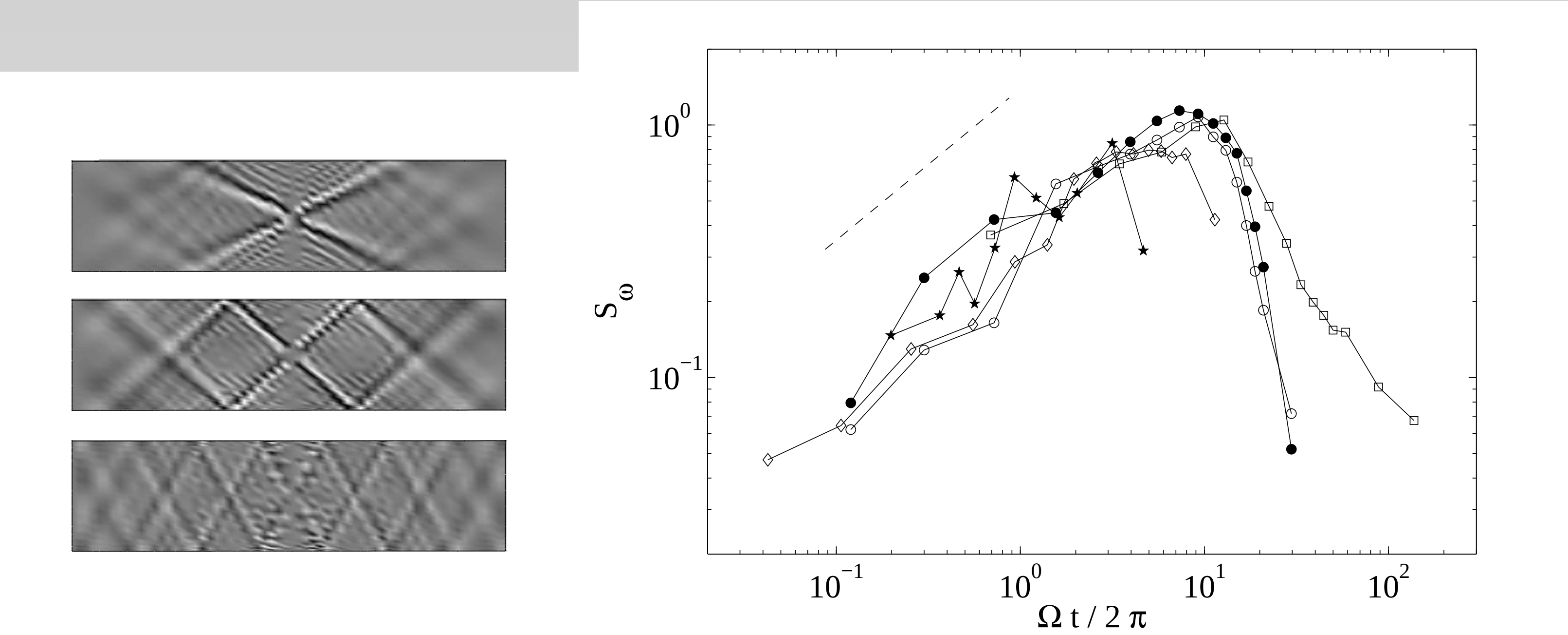
Conical region
in which energy
concentrates
Ω
(a)
(c)
(b)
(d)



Perspectives and open issues

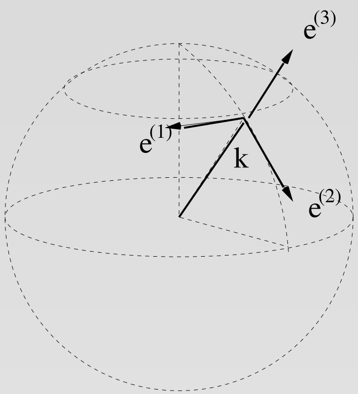


- Dynamics of the slow mode ? AQNM, EDQNM3, classical Wave-Turbulence (Galtier 2003, also Waleffe 1993) and under-resolved DNS/LES
- EDQNM3 and AQNM poorly exploited: much more statistics must be extracted
- including triple vorticity correlations for cyclonic/ anticyclonic vorticity asymmetry

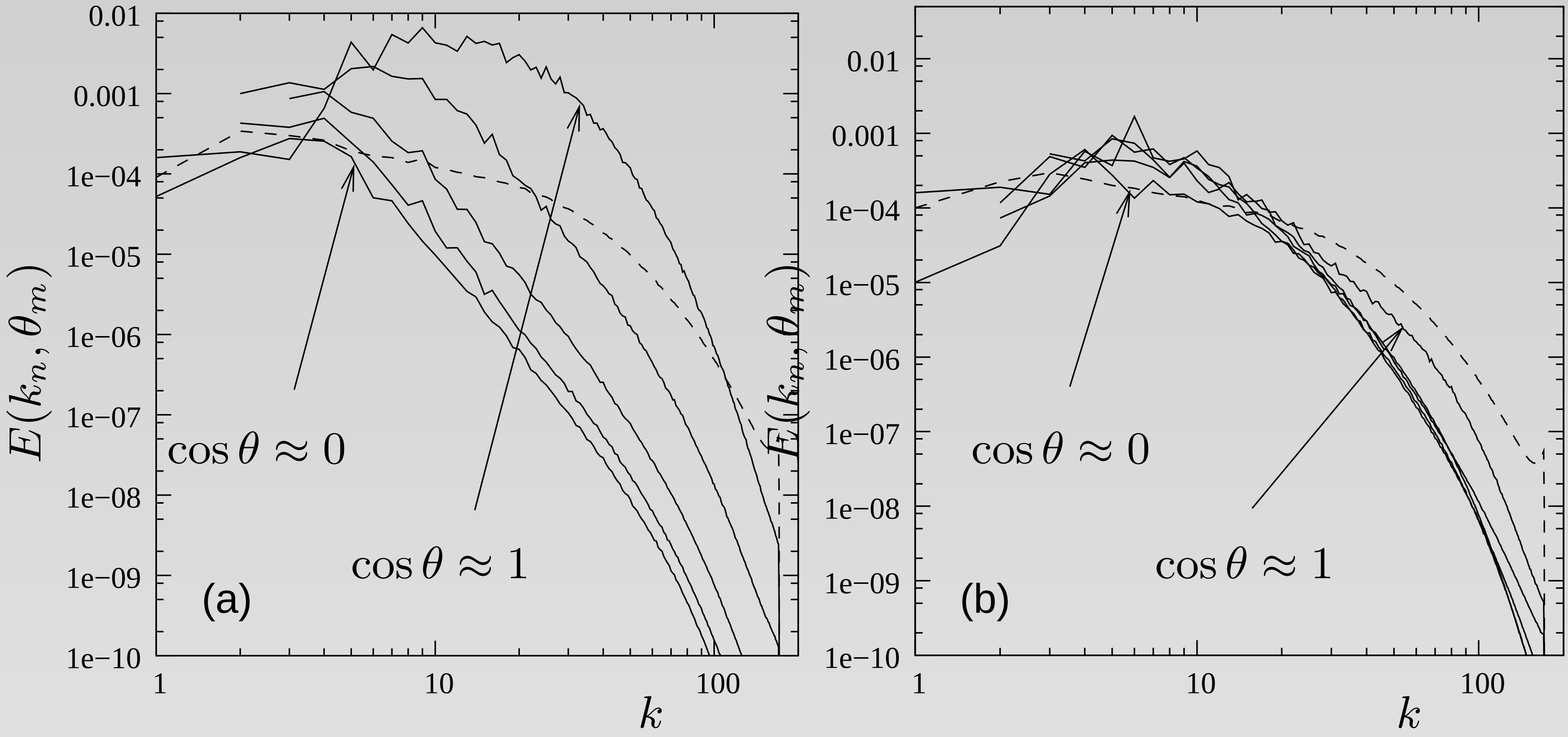
Stably-stratified turbulence

- A new nonpropagating mode : (vertical) vortex, toroidal, QG, linearized PV ...

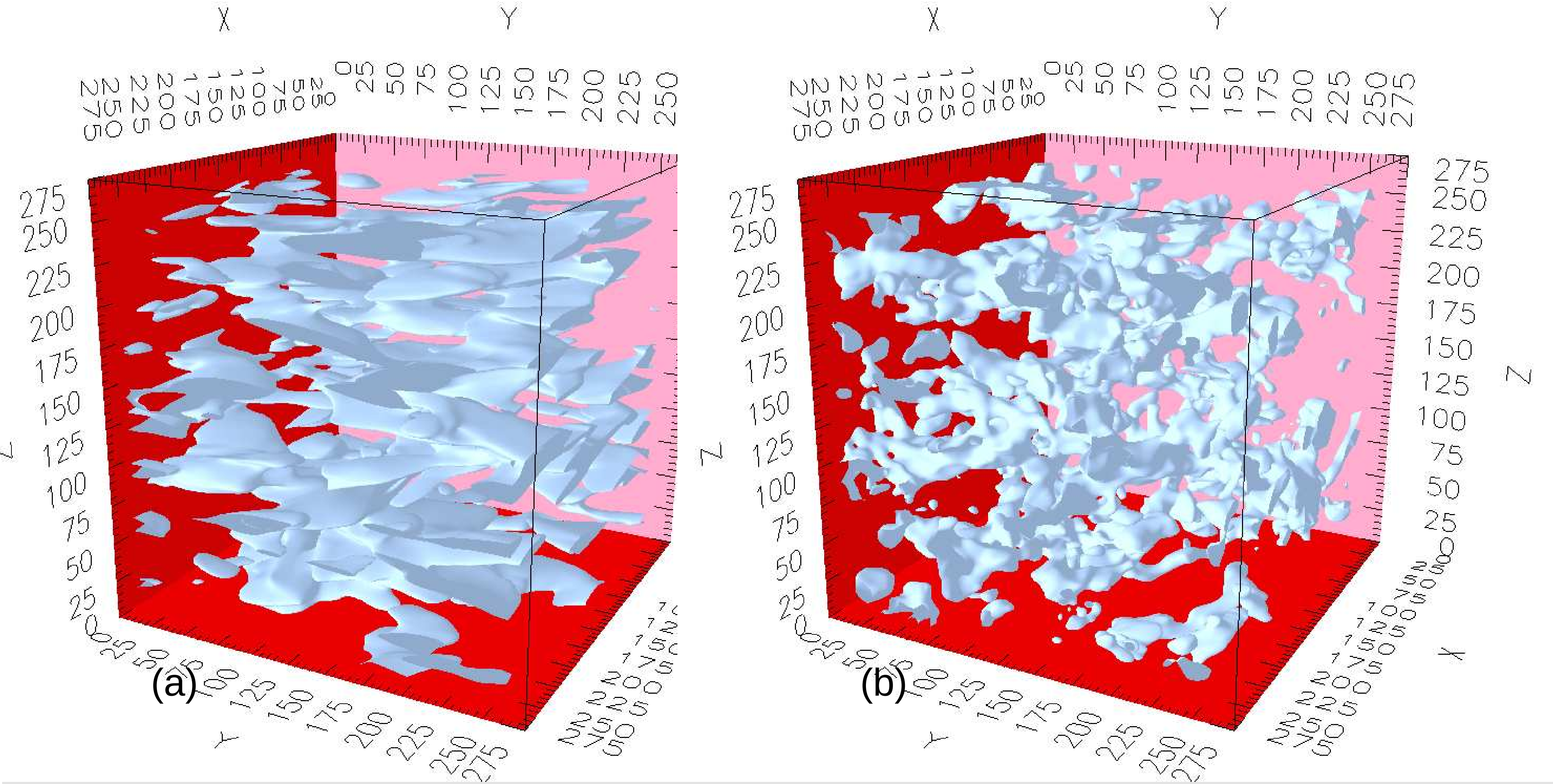
$$\boldsymbol{v}(\boldsymbol{x},t) = \sum e^{\imath \boldsymbol{k}\cdot\boldsymbol{x}} \left(\underbrace{a_0 \boldsymbol{N}^{(0)}}_{vortex} + a_{+1}\boldsymbol{N}^{(1)} e^{\imath\sigma_k t} + a_{-1}\boldsymbol{N}^{(-1)} e^{-\imath\sigma_k t} \right)$$



- $\boldsymbol{v} = (\boldsymbol{u}, b)$, gravity waves with $\sigma_k = N\frac{k_\perp}{k}$



Angle-dependent toroidal and poloidal modes (Liechtenstein, 2006)


(a)
(b)
X
Y
Z

Perspectives and open issues

- Motion in a stratified fluid is not quasi-2D but *ANTI-2D* !
- Toroidal turbulence: statistical approach to horizontal layering (vs. zig-zag instability, Billand & Chomaz, Lindborg ...)
- Very good EDQNM2-DNS comparisons (Godeferd & Cambon 1994, Godeferd & Staquet 2003): improvements ? EDQNM3, ED, very high Reynolds
- Rotation + stratification (DNS by Liechtenstein *et al.*), revisiting a QG model

RDT without waves: pure plane shear

- Triangular coupling, simplified in the Craya-Herring frame, but no diagonal form for the Green's function. Two modes anyway.

$$u^{(\alpha)}(\boldsymbol{k}(t),t) = g_{\alpha\beta}(\boldsymbol{k},t,t_0)\underbrace{u^{(\beta)}(\boldsymbol{k}(t_0),t_0)}_{\text{new slow ?}} + \int_{t_0}^{t} g_{\alpha\beta}(\boldsymbol{k},t,t')f^{(\beta)}(\boldsymbol{k}(t'),t')dt$$

- $\boldsymbol{k}$-space distortion: $k_i = F^{-1}_{ji}(t)K_j \quad k_1 = K_1, k_2 = K_2 - K_1 St, k_3 = K_3,$
  $\boldsymbol{x}$-space distortion: $x_i = F_{ij}(t)X_j \quad x_1 = X_1 + StX_2, x_2 = X_2, x_3 = X_3$
- Non uniform convergence at large St for $k_1 = 0$ and $k_1 \neq 0$
- See you on Thursday ? issues for compressible shear !

Conclusion, perspectives and open issues

- Very important anisotropy, even if poorly reflected by classical descriptors, *directional, dimensionality* 3D, 2D, 1D.
- A relevant strategy : RDT $\subset$ WT $\subset$ (ED) QNM3 $\subset$ LRA.
- A zonal ED, WT (or AQNM) is a natural limit for EDQNM3, issues in the absence of waves ?
- RDT with space distortion, exponential growth ?
- WKB RDT towards inhomogeneous flows: nonlinearity ?