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Numerical Study of temperature transport in turbulent flows

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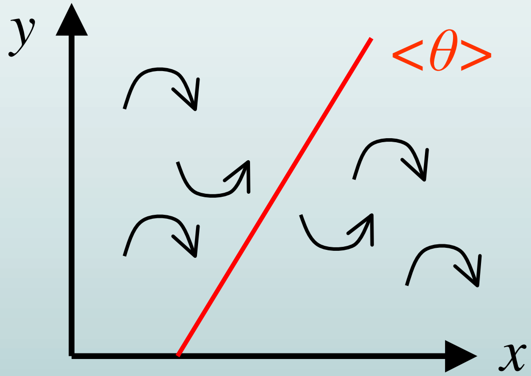
Context overview

- **Global context** : heat transport modelling in gas-particle turbulent flows.

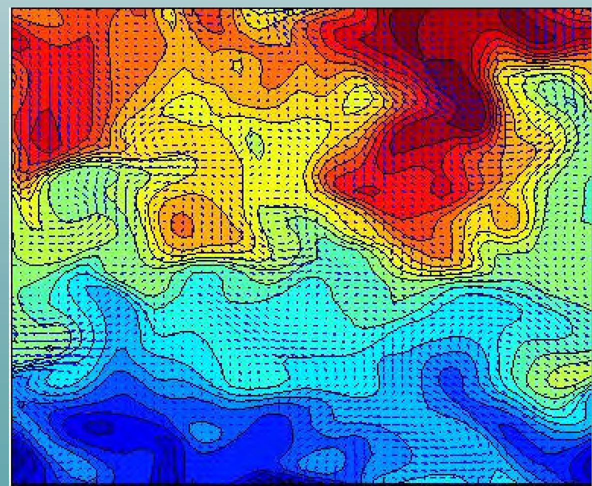
- **Objectives** :
  - Measurement of turbulent statistics along fluid element paths in homogenous isotropic forced dynamic turbulence with a linear temperature gradient ;
  - Evaluation of Lagrangian stochastic approach from measured Lagrangian correlation functions and moment transport equations.

Flow configuration

- DNS - 3D periodic box - 128^3 mesh ;
- Stationary Homogeneous Isotropic Turbulence (Eswaran and Pope, 1988) ;
- Fluid field submitted to a uniform mean temperature gradient $\gamma = \frac{\partial <\theta_f>}{\partial y}$ in y-direction.



Flow configuration



Snapshot of velocity and temperature fields

Configuration parameters

The Navier-Stokes and temperature fluctuations equations are solved.

$$\frac{\partial \theta_f}{\partial t} + u_{fj}\frac{\partial \theta_f}{\partial x_j} = -v_f\gamma + \kappa_f \frac{\partial^2 \theta_f}{\partial x_j \partial x_j}$$

Spatial resolution : second order finite-volume.

Time resolution :

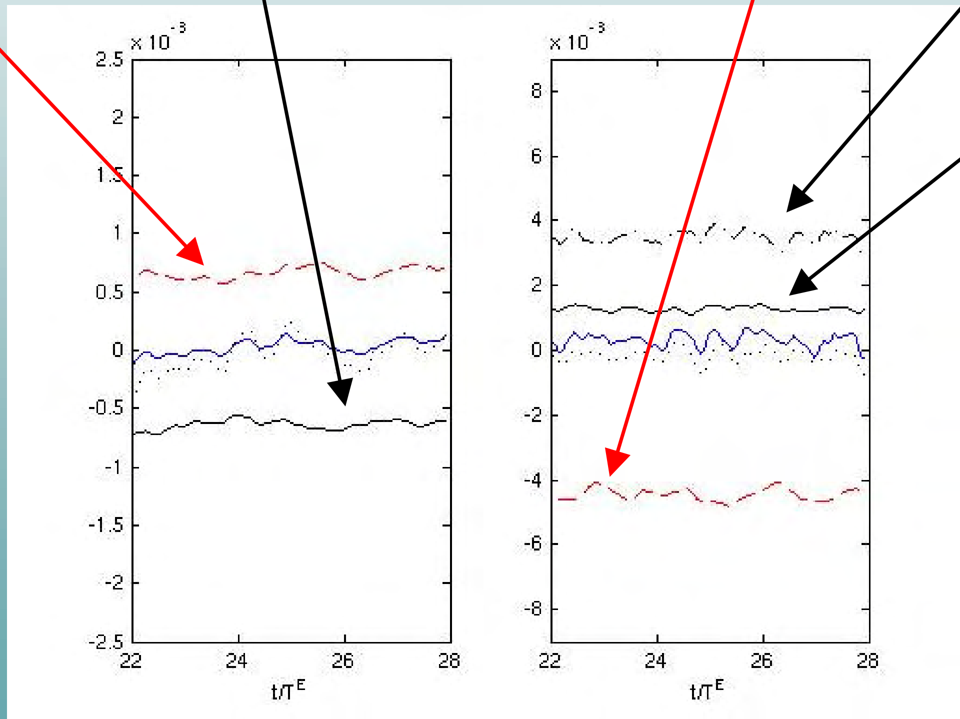
second order Runge Kutta method.

Grid size		128^3
Box size	L	0.128 m
Mean gradient temperature	γ	1
Reynolds number	Re_L	61
Prandtl Number	Pr	0.3/ 0.7/ 1
Integral scale	L_f	1.32 10^{-2} m
Kolmogorov microscale	η	6.51 10^{-4} m

Results : Stationary thermal field

$$\frac{1}{2}\frac{\partial\langle\theta_f^2\rangle}{\partial t}=-\langle v_f\theta_f\rangle\gamma-\kappa_f\left\langle\frac{\partial\theta_f}{\partial x_j}\frac{\partial\theta_f}{\partial x_j}\right\rangle$$

$$\frac{\partial<v_f\theta_f>}{\partial t}=-\langle v_f^2\rangle\gamma-\frac{1}{\rho_f}\left\langle\theta_f\frac{\partial P}{\partial x_2}\right\rangle-(\kappa_f+\nu_f)\left\langle\frac{\partial v_f}{\partial x_j}\frac{\partial\theta_f}{\partial x_j}\right\rangle$$

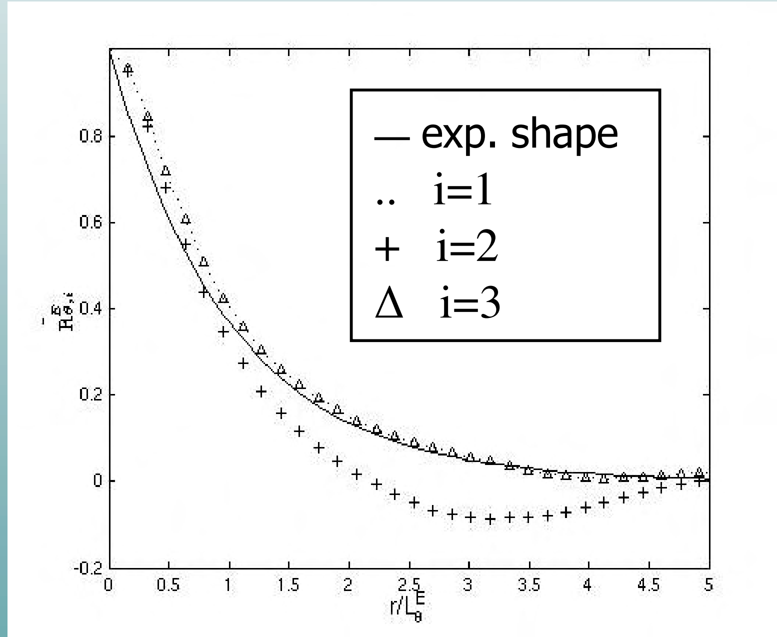


Balance of temperature variance and turbulent flux equations

Results : Thermal statistics (Pr=0.7)

$$R^E_{\theta,i}(r) = \langle \theta_f(x,t)\,\theta_f(x+re_i,t) \rangle \longrightarrow L^E_\theta = \int_0^\infty R^E_{\theta,1}(r)\,dr$$

Thermal Integral scale	L^E_θ	$1.35\ 10^{-2}\ m$
Dynamic Eulerian timescale	T^E	0.20
Dynamic Lagrangian timescale	T^L	0.16
Thermal Eulerian timescale	T^E_θ	$0.22\ s$
Thermal Lagrangian timescale	T^L_θ	$0.30\ s$



Spatial Eulerian autocorrelation temperature functions $R^E_{\theta,i}$

Results :Lagrangian statistics

Interpolation of Eulerian predictions on 100 000 fluid element trajectories

→ Shape Functions Method (Maxey, 1989)

Lagrangian temperature autocorrelation functions

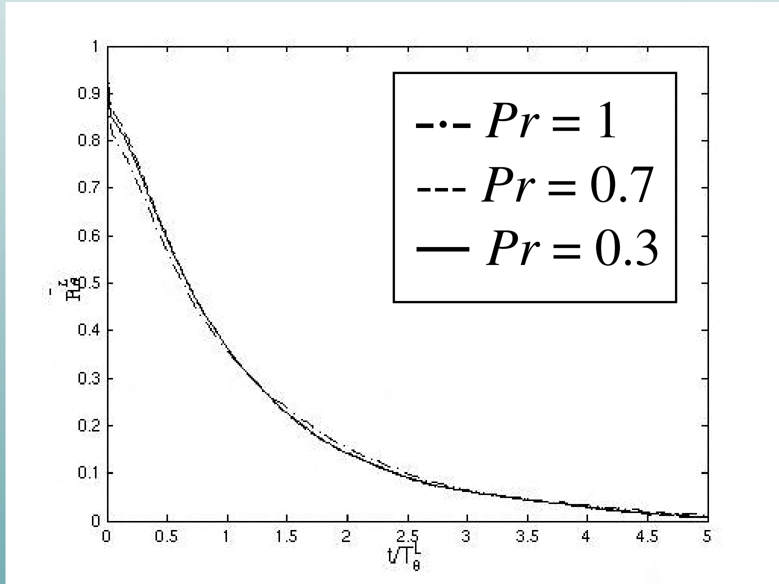
$$R_{\theta}^{L}(\tau)=\left\langle \theta_f\left(x(t),t\right)\theta_f\left(x(t+\tau),t+\tau\right)\right\rangle$$

Lagrangian cross-correlation functions

$$R_{ab}^{L}(\tau)=\left\langle a_f\left(x(t),t\right)b_f\left(x(t+\tau),t+\tau\right)\right\rangle$$

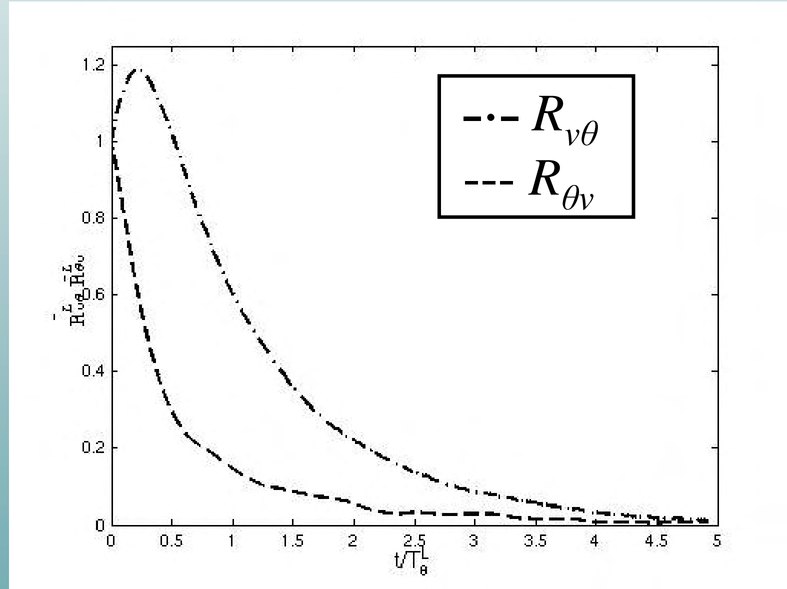
Results :Lagrangian statistics

Normalized Lagrangian temperature autocorrelations



The effect of Laminar Prandtl number is weak

Normalized Lagrangian velocity-temperature cross-correlations



$$R_{v\theta}(\tau) \neq R_{\theta v}(-\tau)$$

Lagrangian stochastic model

Pdf modelling requires closure model for Lagrangian fluid quantities

The Langevin model approach for both velocity and temperature

$$\begin{cases} dv_f^* = -\dfrac{1}{\tau_v} v_f^* dt + B_v dW_v \\ d\theta_f^* = -v_f^* \gamma dt - \dfrac{1}{\tau_\theta} \theta_f^* dt + B_\theta dW_\theta \end{cases}$$

dW_v, dW_θ independent Wiener process and $<\theta_f^{*2}> = <\theta_f^2>$ $<\theta_f^* v_f^*> = <\theta_f v_f>$

Lagrangian autocorrelations and cross-correlations derived from model

Lagrangian stochastic model

$$R^L_{\theta v} = <\theta_f v_f> e^{-\frac{t}{\tau_v}}$$

$$R^L_{v\theta} = <\theta_f v_f> e^{-\frac{t}{\tau_\theta}} + \xi_2 \left(e^{-\frac{t}{\tau_v}} - e^{-\frac{t}{\tau_\theta}} \right)$$

$$R^L_v = <v_f^2> e^{-\frac{t}{\tau_v}}$$

$$R^L_\theta = <\theta_f^2> e^{-\frac{t}{\tau_\theta}} + \xi_1 \left(e^{-\frac{t}{\tau_v}} - e^{-\frac{t}{\tau_\theta}} \right)$$

$$\begin{cases} \xi_1 = \gamma \left\langle v_f \theta_f \right\rangle \dfrac{\tau_\theta \tau_v}{\tau_\theta - \tau_v} \\ \xi_2 = \gamma \left\langle v_f^2 \right\rangle \dfrac{\tau_\theta \tau_v}{\tau_\theta - \tau_v} \end{cases}$$

By integrating Lagrangian autocorrelations :

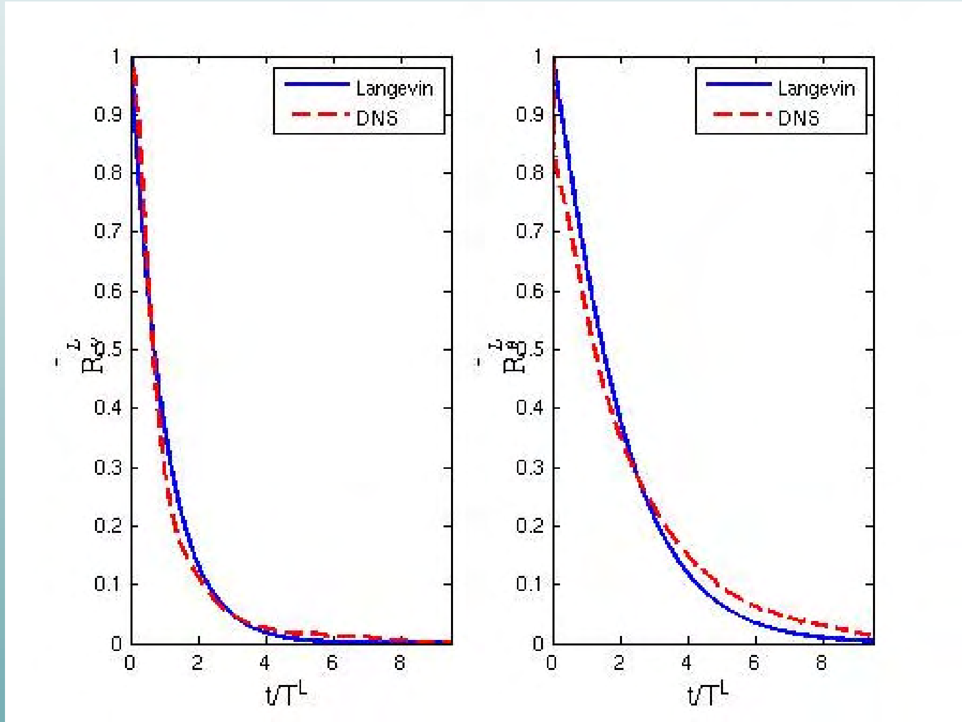
$$\tau_v = T^L \qquad \tau_\theta = A T^L_\theta$$

$$A = \left(1 - \gamma \frac{\left\langle v_f \theta_f \right\rangle}{<\theta_f^2>} T^L \right)^{-1}$$

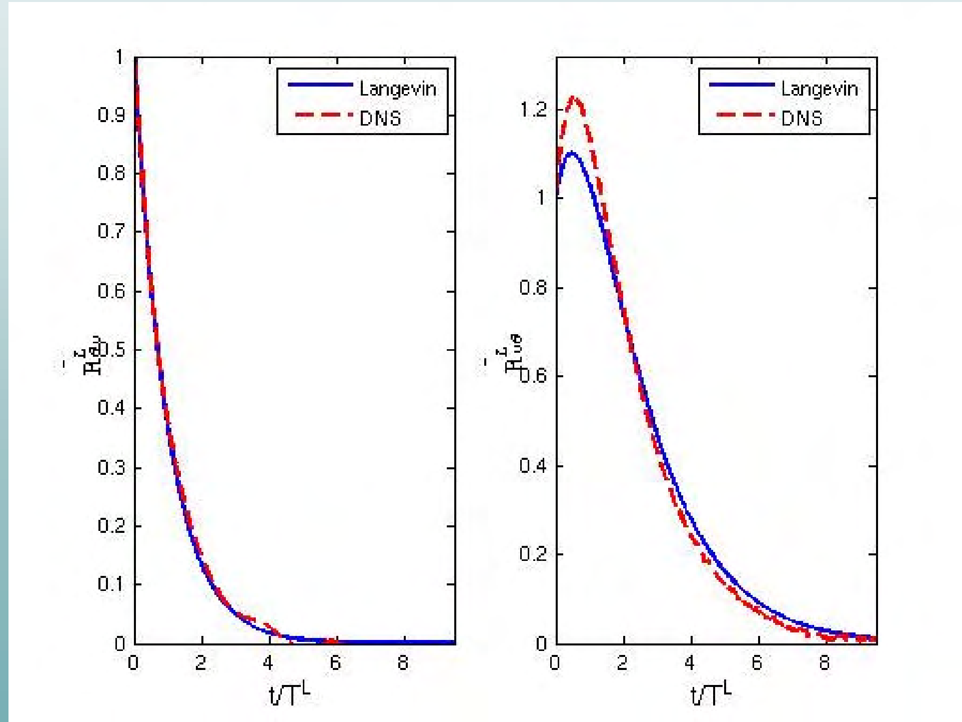
A ~ 0.9 (Pr = 0.7)

Lagrangian stochastic model

R_v^L R_θ^L



$R_{\theta v}^L$ $R_{v\theta}^L$



Comparison of Lagrangian velocity and temperature autocorrelation

Comparison of Lagrangian velocity-temperature cross-correlation

Lagrangian stochastic model

Validation of the Langevin model from the moment transport equation

⟶ Derived from the fluid pdf Boltzman equation

$$\frac{\partial f_f}{\partial t}+\frac{\partial}{\partial x_j}\left(c_{f,i} f_f\right)=-\frac{\partial}{\partial c_{f,i}}\left[<\frac{du_f}{dt}\middle|\mathbf{c}_f,\xi_f> f_f\right]-\frac{\partial}{\partial \xi_f}\left[<\frac{d\theta_f}{dt}\middle|\mathbf{c}_f,\xi_f> f_f\right]$$

Where $f_f\left(\mathbf{c}_f,\xi_f;x,t\right)d\mathbf{c}_f d\xi_f$

The probable number of fluid particles with a velocity u_f in [$\mathbf{c}_f$, $\mathbf{c}_f$+d $\mathbf{c}_f$] and a temperature θ_f in [ξ_f, ξ_f d ξ_f].

$\frac{du_f}{dt}$ and $\frac{d\theta_f}{dt}$ are modeled with the velocity-temperature Langevin model

Moments equations

By integrating the pdf transport equation, moment equations are written,

$$\frac{\partial\left\langle \theta_f{}^2 \right\rangle}{\partial t} = -2\left\langle v_f\ \theta_f \right\rangle \gamma - \frac{2}{\tau_\theta}\left\langle \theta_f{}^2 \right\rangle + B_\theta^2$$

$$\frac{\partial\left\langle v_f \theta_f' \right\rangle}{\partial t} = -\left\langle v_f^2 \right\rangle \gamma - \left(\frac{1}{T^L} + \frac{1}{\tau_\theta} \right) \left\langle v_f \theta_f' \right\rangle$$

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$$\boxed{\frac{2\left\langle \theta_f^{\;2}\right\rangle}{\tau_\theta} - B_\theta^2 = 2\varepsilon_\theta = 2\kappa_f\left\langle \frac{\partial\theta_f}{\partial x_j}\frac{\partial\theta_f}{\partial x_j}\right\rangle}$$

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- In the considered case the modeled turbulent flux equation leads to a gradient closure model :

$$\left\langle v_f\theta_f\right\rangle=-\left(\frac{1}{T^L}+\frac{1}{\tau_\theta}\right)^{-1}\left\langle v_f^2\right\rangle\gamma$$

Thermal dispersion Coefficient (Pr=0.7)

Assuming a gradient approximation :

$$\langle v_f \theta_f \rangle = -D_\theta^t \gamma$$

The thermal dispersion coefficient can be measured from the DNS results :

$$D_\theta^t = -\frac{\langle v_f \theta_f \rangle}{\gamma}$$

In the frame of moment approximation in the pdf approach, the thermal dispersion coefficient is :

$$D_\theta^t = \frac{T^L \tau_\theta}{T^L + \tau_\theta} \langle v_f^2 \rangle$$

Re_L	$T^L \langle v_f^2 \rangle$	$D^t_{\theta DNS}$ (m^2/s)	$D^t_{\theta Stoc}$ Stochastic model
39	$5.1\ 10^{-4}$	$5.87\ 10^{-4}$	$4.29\ 10^{-4}$
61	$6.8\ 10^{-4}$	$6.02\ 10^{-4}$	$4.48\ 10^{-4}$

Conclusion

- Direct Numerical Simulations are used to characterize velocity and temperature Lagrangian statistics in non-isothermal turbulent flows.

- The velocity scalar stochastic model is evaluated from Lagrangian auto-correlation functions.

- Additional validation via moment approach is carried out by comparing the thermal dispersion coefficient measured by DNS and the one obtained by the turbulent flux moment equation.

Outlook

- Testing of alternative stochastic model in order to improve the turbulent flux prediction
- Tracking of discrete solid particles suspended in homogeneous isotropic forced dynamic turbulence with a linear temperature gradient