On the Interaction of Turbulent Shear Layers with Harmonic Perturbations

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OUTLINE

MOTIVATION AND OBJECTIVES

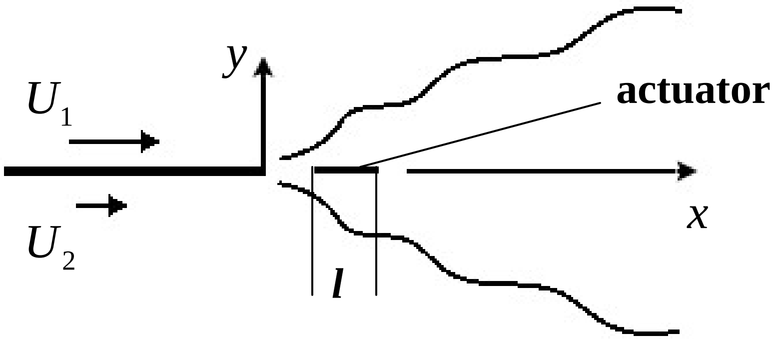
THEORETICAL MODEL

TURBULENT MIXING LAYER

- COMPARISON WITH EXPERIMENT
- MEAN-COHERENT FLOW INTERACTION
- COHERENT-RANDOM INTERACTION

CONCLUSIONS

Sketch of Setup



$$v = \frac{\omega A}{l}(x - x_1)\cos\omega t, \quad \omega = 2\pi f$$

$$U_m = 0.5(U_1 + U_2), \quad \lambda = \frac{U_1 - U_2}{U_1 + U_2}, \quad \theta = \int_{-\infty}^{+\infty} \frac{U - U_2}{U_1 - U_2}\left(1 - \frac{U - U_2}{U_1 - U_2}\right) dy$$

Parameters of interest:

$$U_1 = 13.5 \text{ m/s}, \quad U_2 = rU_1, \quad r = 0.3 - 0.6$$
$$l = 10 \text{ mm}, \quad A = 0.5 \text{ mm} - 3.5 \text{ mm}, \quad f = 30 \text{ Hz} - 60 \text{ Hz}$$

Amplitude and frequency effects on the momentum thickness

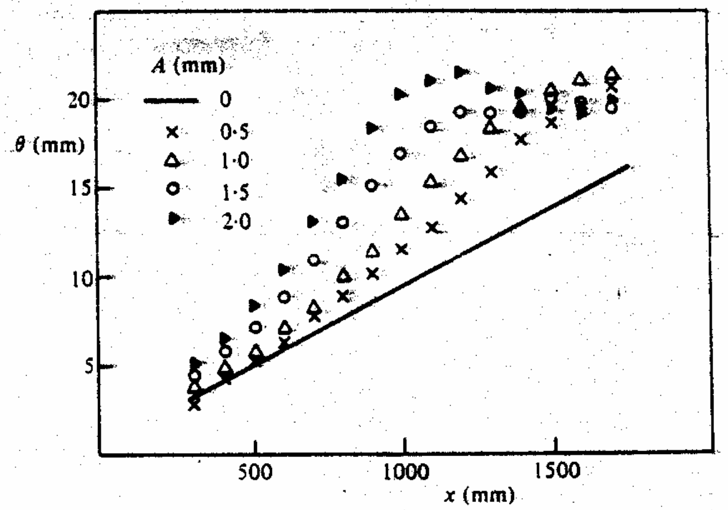


FIGURE 12. Variation of the momentum thickness at $r = 0{\cdot}6$, $f = 40$ Hz.

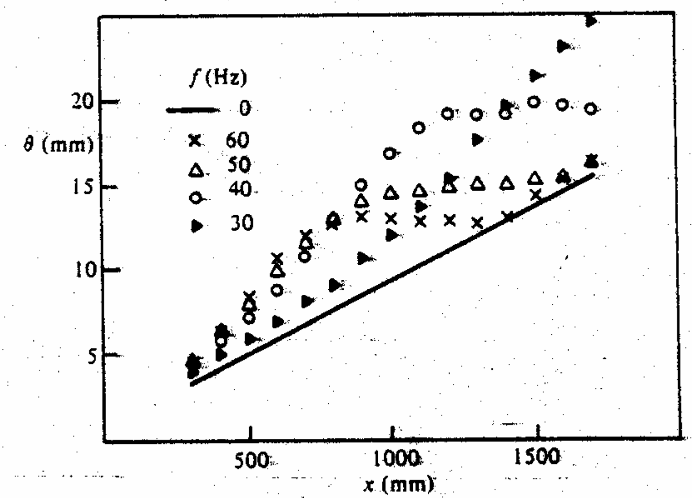


FIGURE 13. Variation of the momentum thickness for forced mixing layers at $r = 0{\cdot}6$, $A = 1{\cdot}5$ mm and 60 Hz > f > 30 Hz.

Oster & Wygnanski

1982

Representation of Flow Parameters

$$u_k = \overline{u}_k + u'_k, \quad \overline{u}_k = U_k + \tilde{u}_k$$

$$\tilde{u}_k = \frac{1}{2}\sum_n \left(\tilde{u}_{k,n} e^{-in\omega t} + \tilde{u}^*_{k,n} e^{in\omega t}\right), \quad \omega = 2\pi f$$

Mean Flow

Continuity equation

$$\partial U_k / \partial x_k = 0$$

Momentum equations

$$\frac{\partial U_k}{\partial t} + \frac{\partial U_k U_j}{\partial x_j} + \frac{\partial P}{\partial x_k} = \frac{\partial}{\partial x_j}\left(\nu \frac{\partial U_k}{\partial x_j} \right) + \frac{\partial \tau_{kj}}{\partial x_j} + \frac{\partial \tilde{\tau}_{kj}}{\partial x_j}$$

Reynolds stresses

$$\tau_{kj} = -\overline{u'_k u'_j}$$

$$\tilde{\tau}_{kj} = -\frac{1}{4}\sum_n \left(\tilde{u}_{k,n} \tilde{u}^*_{j,n} + \tilde{u}_{j,n} \tilde{u}^*_{k,n} \right)$$

Coherent Flow

Momentum equations for *n*-th mode

$$-in\omega\tilde{u}_{k,n} + U_j \frac{\partial \tilde{u}_{k,n}}{\partial x_j} + \tilde{u}_{j,n} \frac{\partial U_k}{\partial x_j} + \frac{\partial \tilde{p}_n}{\partial x_k} = \frac{\partial}{\partial x_j}\left(\nu \frac{\partial \tilde{u}_{k,n}}{\partial x_j} \right) + \frac{\partial r_{kj,n}}{\partial x_j} + \frac{\partial \tilde{\tau}_{kj,n}}{\partial x_j}$$

Nonlinear wave interaction

$$\tilde{\tau}_{kj,n} = -\frac{1}{2}\sum_m \left(\tilde{u}_{k,m+n}\tilde{u}^*_{j,m} + \tilde{u}_{j,m+n}\tilde{u}^*_{k,m} \right) - \frac{1}{2}\sum_m \tilde{u}_{k,m}\tilde{u}_{j,n-m}$$

Dissipation term

$$r_{kj} = \overline{u'_k u'_j} - < u'_k u'_j >$$

Turbulence Closure

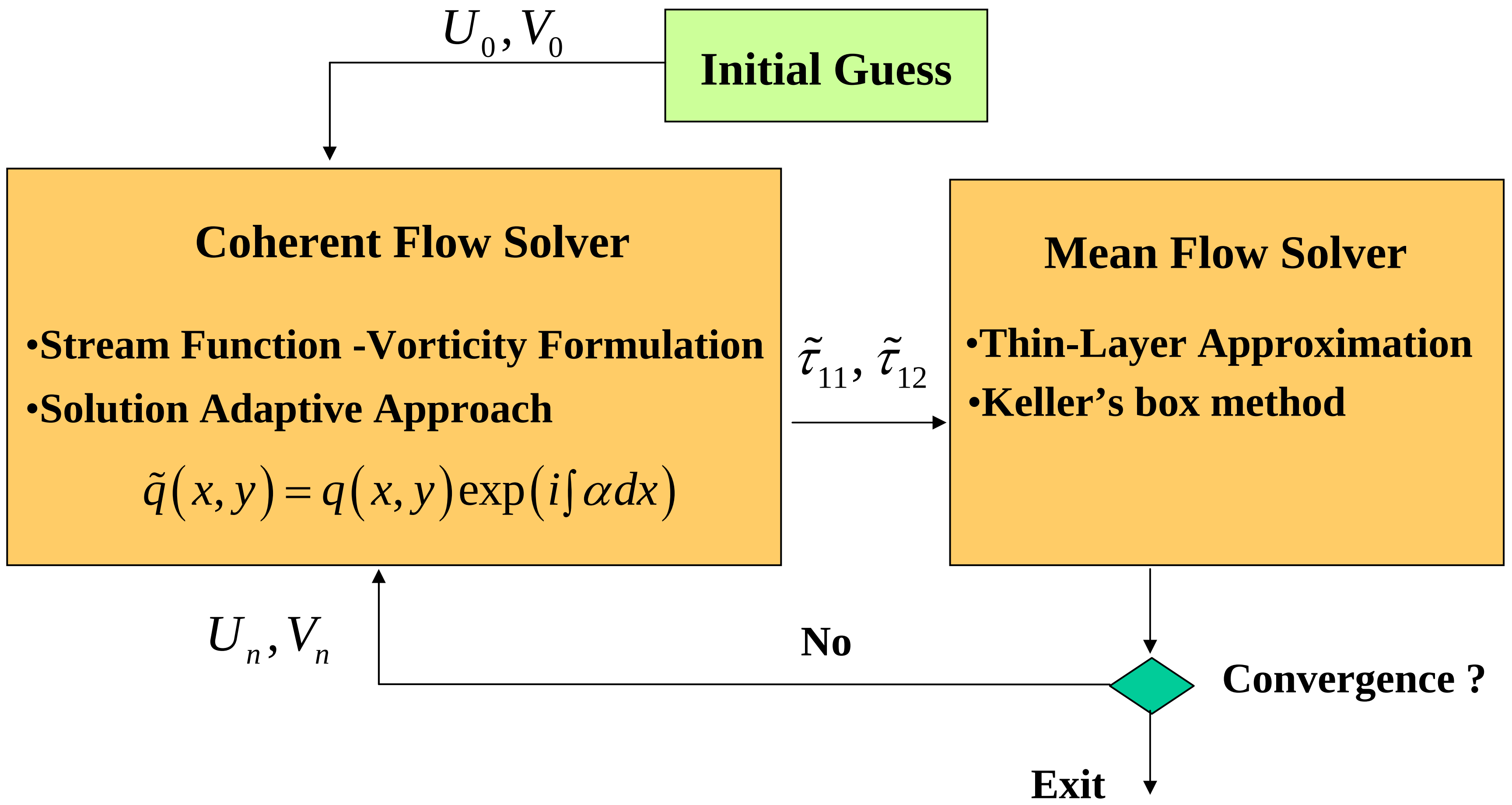
Newtonian eddy-viscosity model

$$\tau_{kj} = 2\nu_T S_{kj}, \quad r_{kj,n} = 2\tilde{\nu}_T \tilde{s}_{kj,n}, \qquad s = \frac{1}{2}\left(\frac{\partial u_k}{\partial x_j} + \frac{\partial u_j}{\partial x_k}\right)$$

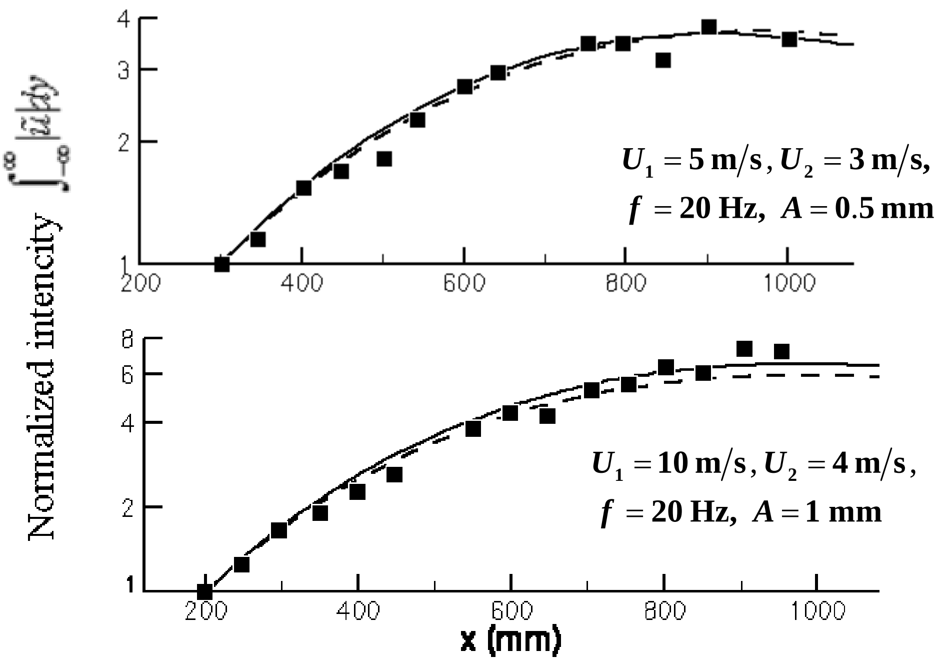
Prandtl viscosity for a turbulent mixing layer

$$\tau_{xy} = \nu_T(x)\frac{\partial U}{\partial y}, \quad \nu_T = 4\chi\theta(x)(U_1 - U_2), \quad \tilde{\nu}_T = \nu_T$$

Flow Chart of the Solver



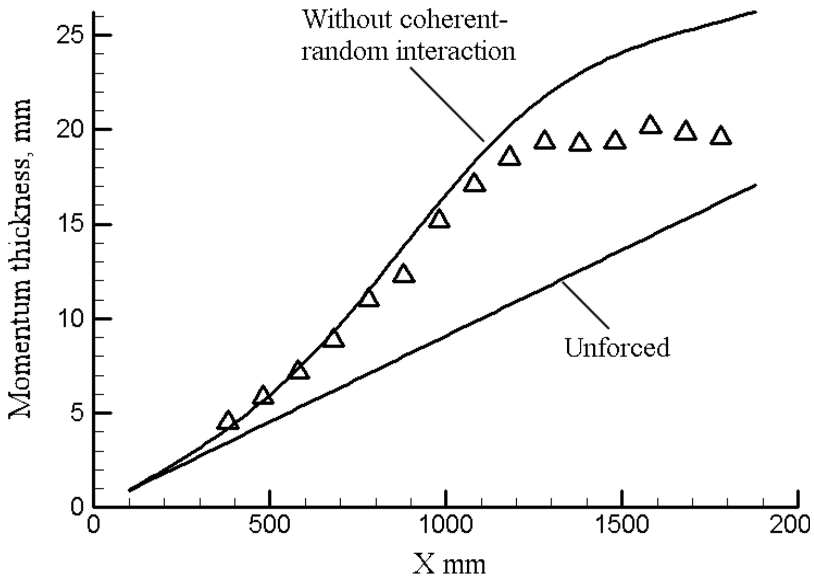
Experimental Data of Gaster *et al.* (1985)



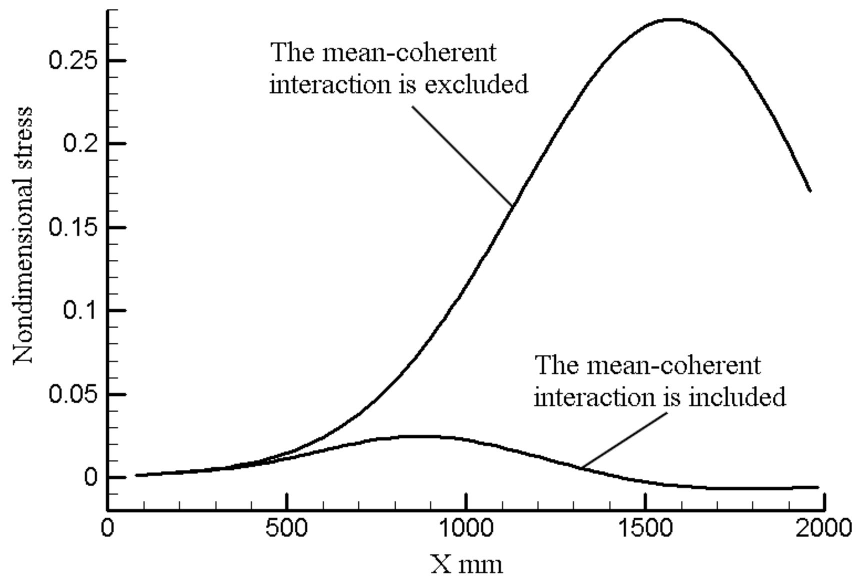
Solid lines – with the mean-coherent interaction;
Dashed lines – without the mean-coherent interaction

Experimental Data of Oster & Wygnanski (1982)

Spreading Rate

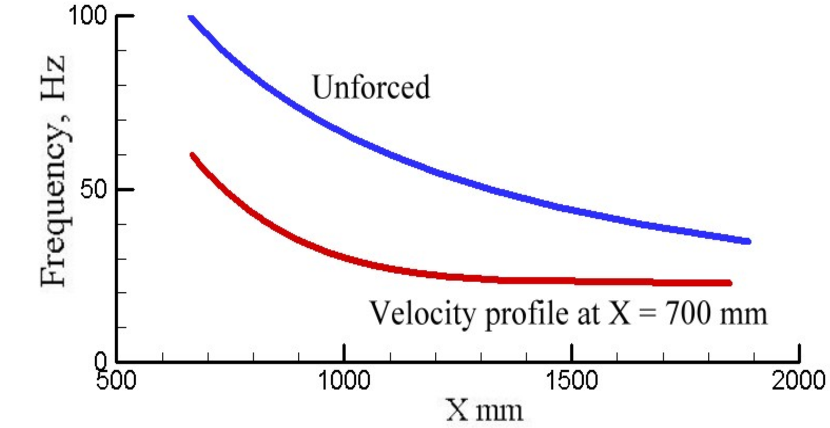


Coherent Reynolds Stress at the Centerline

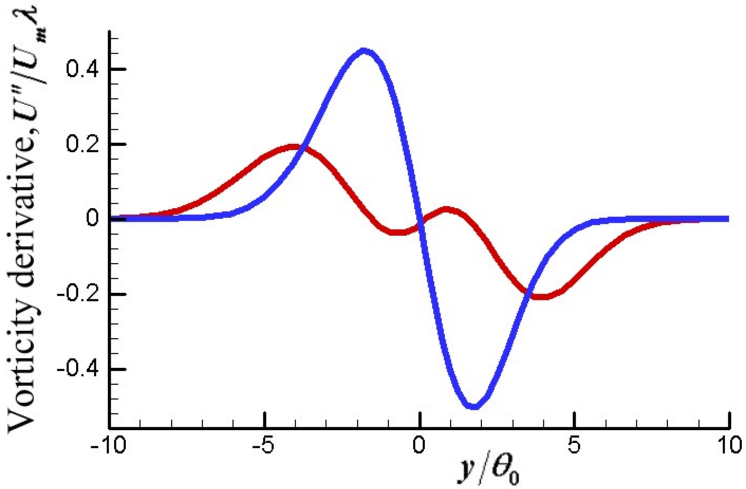


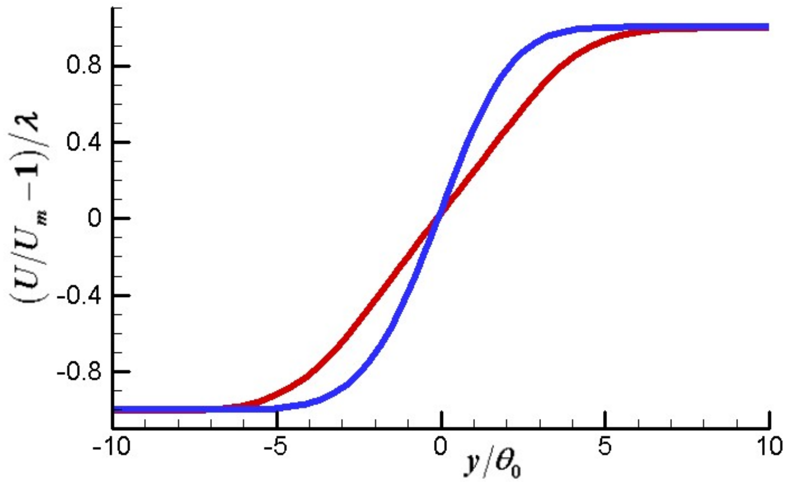
$U_1 = 13.5$ m/s, $U_2 = 8.1$ m/s, $f = 40$ Hz, $A = 1.5$ mm , $d\theta_0/dx = 0.036\lambda$

Neutral Stability Curves



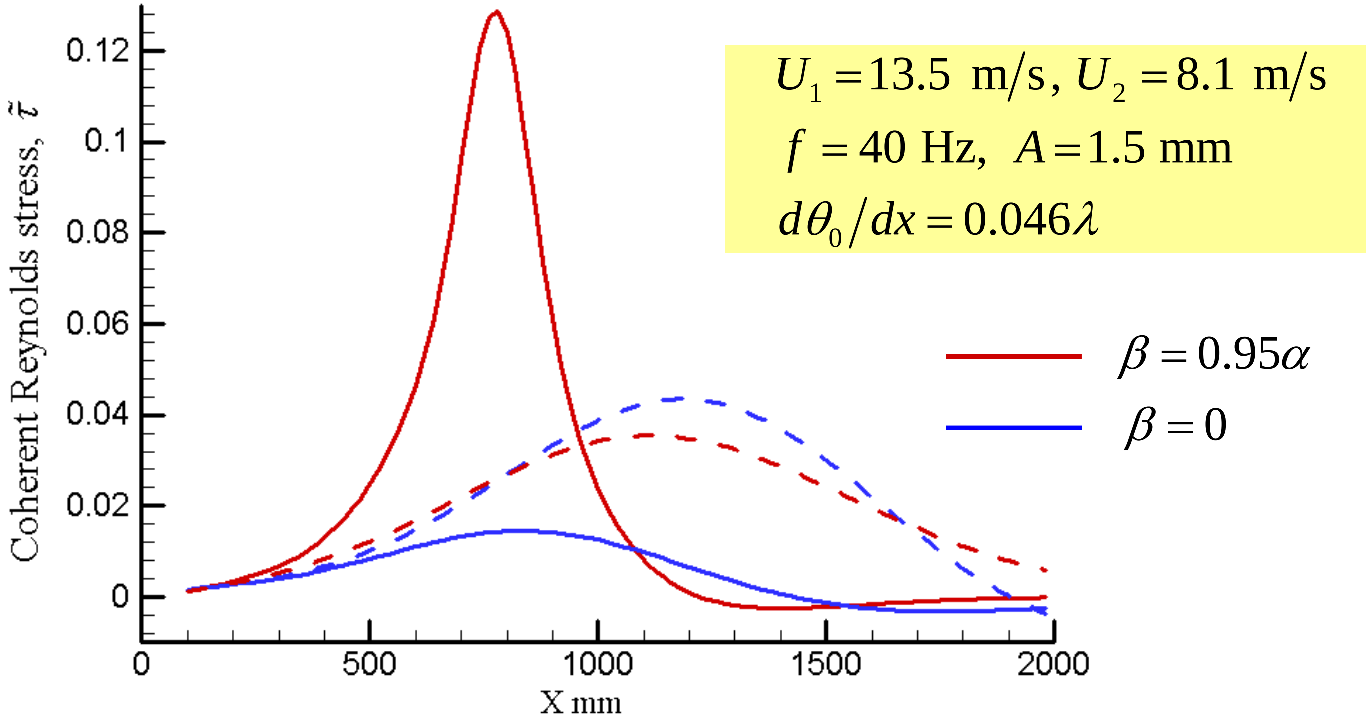
$$\left(U-\omega/\alpha\right)\left(\psi''-\alpha^2\psi\right)-U''\psi=0$$





The mean velocity distortion at X = 700 mm

Oblique-Wave Excitation $\tilde{u}_k = \bar{u}_k(x_1, x_2)e^{i(\alpha x_1 + \beta x_3 - \omega t)}$



Solid lines – with the mean-coherent interaction;
Dashed lines – without the mean-coherent interaction.

The Coherent-Random Interaction

Turbulent kinetic energy equation

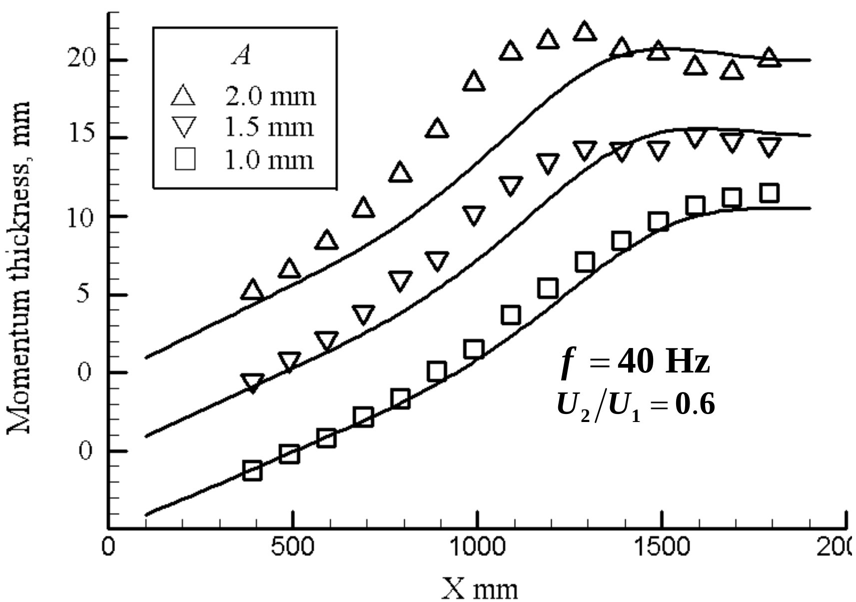
$$\frac{Dk}{Dt} = \mathrm{P} - \varepsilon + \mathrm{D}$$

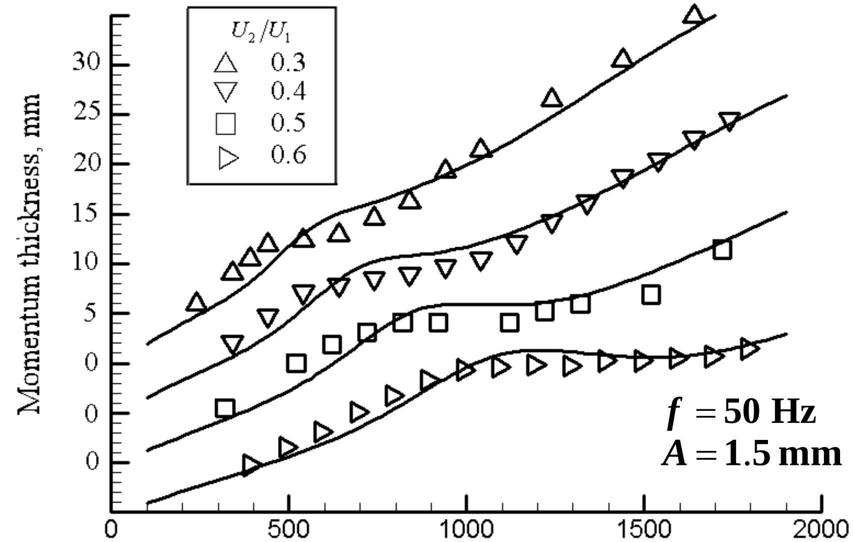
$$\mathrm{P} = -\overline{\left(u_i' u_j'\right)} \frac{\partial U_i}{\partial x_j} - \overline{\left\langle u_i' u_j' \right\rangle \frac{\partial \tilde{u}_i}{\partial x_j}} - \overline{\tilde{u}_j \frac{\partial}{\partial x_j} \left\langle \frac{u_i' u_i'}{2} \right\rangle} = -\overline{\left(u_i' u_j'\right)} \frac{\partial U_i}{\partial x_j} + 2\nu_T \overline{\left(\tilde{s}_{ij} \tilde{s}_{ij}\right)}$$

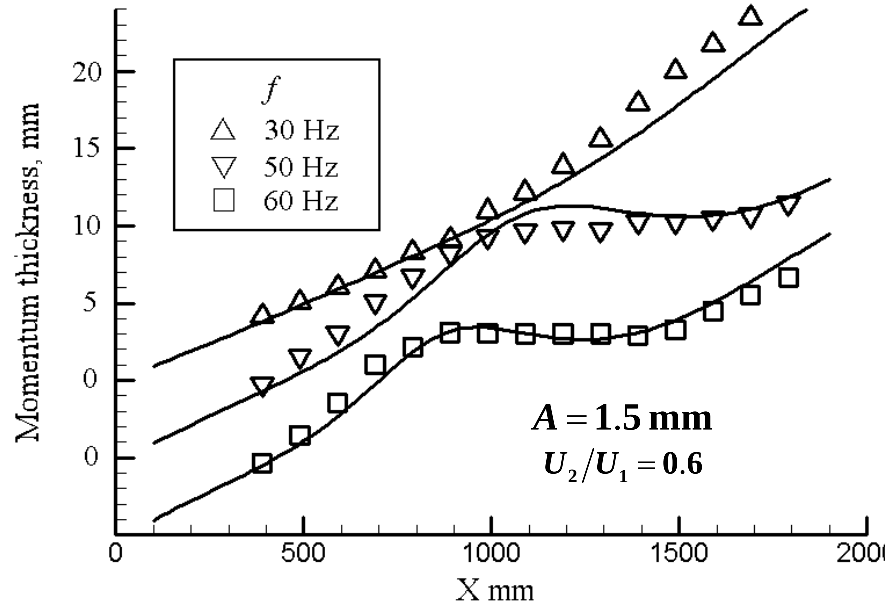
Modified turbulent viscosity

$$\frac{\nu_{T0}}{\nu_T} = \frac{\theta_0}{\theta(x)} + C_t \frac{\nu_T}{\nu_{T0}} \frac{fx}{U_m^3 \Delta_0} \int_{-\infty}^{\infty} \left| \frac{\partial \tilde{u}_1}{\partial \eta} \right|^2 d\eta$$

Comparison with Experiment of Oster & Wygnanski







Conclusions

Our mathematical model reveals two processes following the propagation of a finite amplitude harmonic wave in a turbulent shear layer.

The first one is the mean-field distortion caused by the coherent Reynolds stresses. The distortion strongly influences the wave itself making the normal wave more stable and the oblique wave less stable in comparison with the wave of infinitesimal amplitude. In any case, the wave attenuates rapidly after arriving at the neutral stability curve.

The second process is connected with extra production terms in the turbulence kinetic energy equation caused by interaction between the random and coherent velocity fields. Being dominant downstream, this interaction changes the turbulent viscosity and may explain the long-range action of the oscillations on the flowfield.