

# Hydrogen autoignition using Eulerian Stochastic Fields method with LES

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# Motivation

## Autoignition

Unsteady process  
Finite rate chemistry  
Turbulence–chemistry  
interaction

## Autoignition Model Requirements

LES or URANS  
**No** equilibrium, **No** fast chemistry  
approaches  
Models coupled with flow solver

# Contents

- LES Combustion Model
  - Sub-grid Probability density function (PDF)
- Eulerian Stochastic Fields method
- Results Hydrogen Autoignition
- Conclusions

# LES Equations

Filtered variable

$$\bar{f} = \int_V G(\mathbf{x} - \mathbf{x}'; \Delta) f(\mathbf{x}') dx'$$

Favre filtering

$$\bar{\rho} \tilde{f} = \overline{\rho f}$$

Continuity

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_j}{\partial x_j} = 0 \quad \frac{\partial \bar{\rho} \tilde{u}_i}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_j \tilde{u}_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) \right] - \frac{\partial \tau_{sgs}}{\partial x_j}$$

Momentum

Scalars (equal diffusivity)

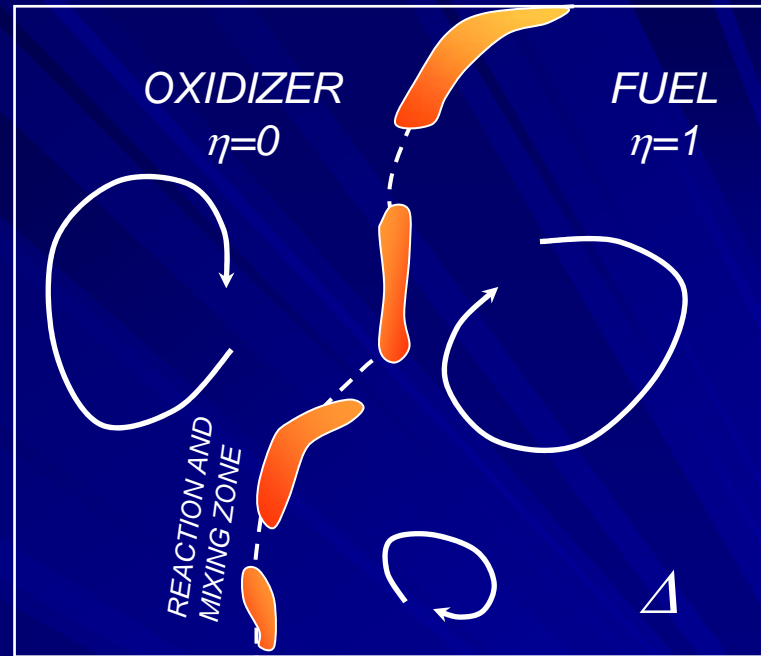
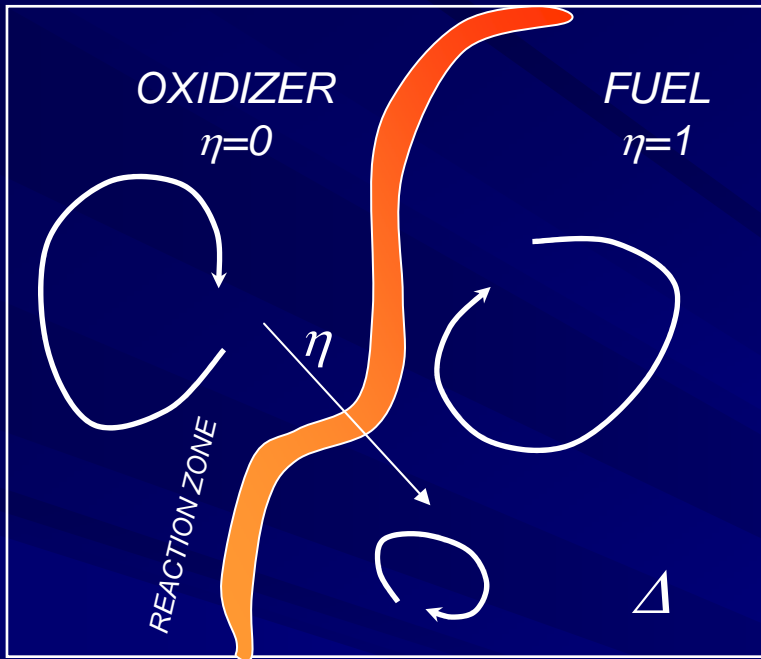
$$\frac{\partial \bar{\rho} \tilde{Y}_k}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_j \tilde{Y}_k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \bar{\rho} D \frac{\partial \tilde{Y}_k}{\partial x_j} \right] - \frac{\partial J_{sgs}^k}{\partial x_j} + \bar{\rho} \tilde{\omega}_k \quad ??? \quad \text{Combustion SGS phenomena}$$

SGS Closures (Smagorinsky)

$$\tau_{sgs}^0 = -2\bar{\rho} \nu_{sgs} \tilde{S}_{ij}^0$$

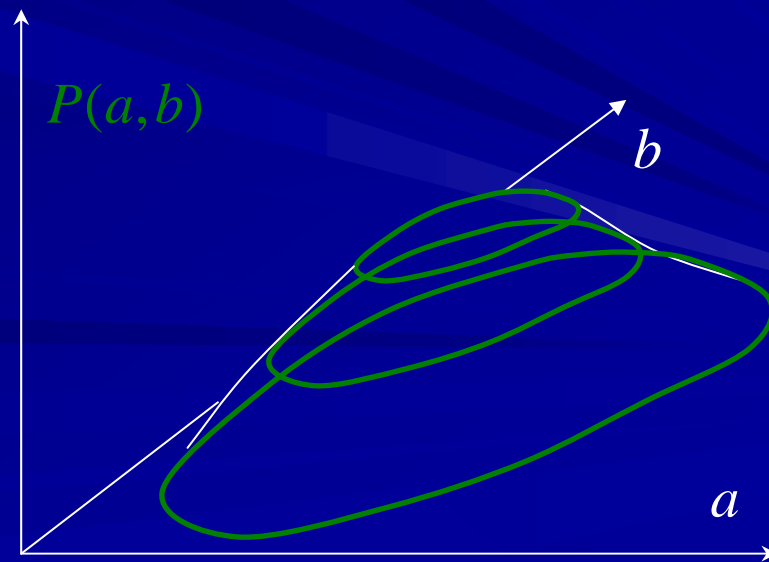
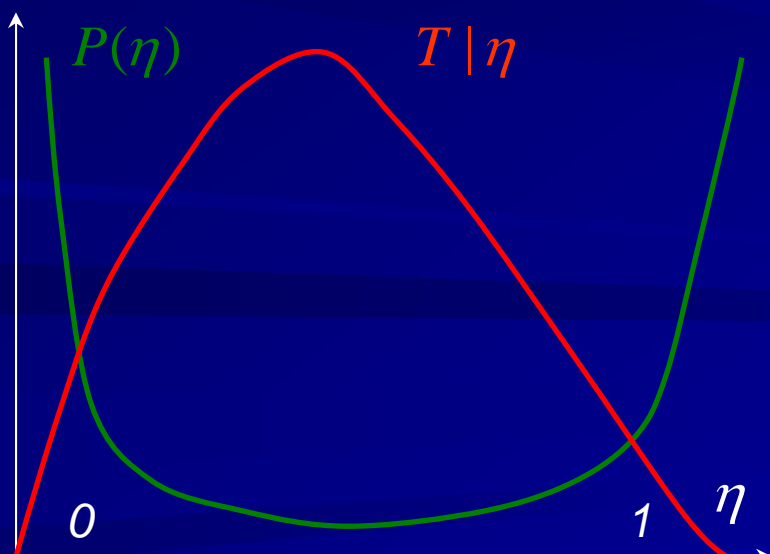
$$\nu_{sgs} = (C_s \Delta)^2 \|\tilde{S}_{ij}\|$$

$$J_{sgs}^k = \frac{\bar{\rho} \nu_{sgs}}{Sc} \frac{\partial \tilde{Y}_k}{\partial x_j}$$



Flamelets (Peters 80's)  
CMC (Klimenko, Bilger 90's)

Joint PDF (Pope 80's)



## Subgrid PDF or FDF (density weighted)

Probability of observing values of the scalar in  $\phi_\alpha \leq Y_\alpha \leq \phi_\alpha + d\phi_\alpha$

$$\bar{\rho} P(\underline{\phi}; \mathbf{x}, t) = \int_V \rho \prod_{k=1}^{N_s} \delta[\phi_k - Y_k] G(\mathbf{x} - \mathbf{x}'; \Delta) dx'$$

## FDF transport equation

$$\frac{\partial \bar{\rho} P}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_j P}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \bar{\rho} D_{eff} \frac{\partial P}{\partial x_j} \right] \quad \text{micromixing}$$

$$- \frac{\partial^2}{\partial \phi_\alpha \partial \phi_\beta} \left[ \bar{\rho} \left\langle D \frac{\partial Y_\alpha}{\partial x_j} \frac{\partial Y_\beta}{\partial x_j} \middle| \underline{Y} = \underline{\phi} \right\rangle P \right] - \frac{\partial}{\partial \phi_\alpha} [\bar{\rho} \omega_\alpha(\phi_\alpha) P]$$

## Linear Mean Square Estimation (LMSE) or interaction with the Mean (IEM)

$$- \frac{\partial^2}{\partial \phi_\alpha \partial \phi_\beta} \left[ \bar{\rho} \left\langle D \frac{\partial Y_\alpha}{\partial x_j} \frac{\partial Y_\beta}{\partial x_j} \middle| \underline{Y} = \underline{\phi} \right\rangle P \right] = \bar{\rho} \beta \frac{\partial}{\partial \phi_\alpha} \left[ (\phi_\alpha - \tilde{Y}_\alpha) P \right] \quad \text{SGS frequency}$$

$$\beta \approx C_D \frac{\nu + \nu_{sgs}}{\Delta^2}$$

# Eulerian Monte Carlo Field (Valiño 1998)

PDF represented as ensemble of **Stochastic Fields (SF)**

$$P(\underline{\phi}; \mathbf{x}, t) = \frac{1}{N_f} \sum_{n=1}^{N_f} \prod_{k=1}^{N_s} \delta[\phi_k - \zeta_k^n(\mathbf{x}, t)] \quad \tilde{Y}_k = \frac{1}{N_f} \sum_{n=1}^{N_f} \zeta_k^n$$

Transport equation of stochastic fields (SPDE)

$$\underbrace{\frac{\partial \bar{\rho} \zeta_k^n}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_j \zeta_k^n}{\partial x_j}}_{\text{Convection-Diffusion}} = \frac{\partial}{\partial x_j} \left( \bar{\rho} D_{eff} \frac{\partial \zeta_k^n}{\partial x_j} \right) + \omega_k(\zeta_k^n) + \underbrace{\left( \frac{2D_{eff}}{\Delta t^2} \right)^{1/2} \frac{\partial \zeta_k^n}{\partial x_j} dW_j^n}_{\text{Stochastic}} - \underbrace{\frac{\bar{\rho} \beta}{2} (\zeta_k^n - \tilde{Y}_k)}_{\text{IEM}}$$

Convection-Diffusion

Source term

Stochastic

IEM

# Summary SF model

## Pros

(vs. Lagrangian PDF)

Easy to implement  
Statistics easy to obtain  
No interpolation error

(vs. assumed PDF models)

More physics  
No assumptions chem-turbulence

## Cons

(vs. Lagrangian PDF)

Spatial discretization error

(vs. assumed PDF models)

Cost  
Statistical error ,  $N_f$  low

Mueller-H<sub>2</sub>  
Mechanism

9 species  
19 reactions

Reaction Number		A mol cm s K	b	E cal mol <sup>-1</sup>
<b>H<sub>2</sub>/O<sub>2</sub> Chain Reactions</b>				
1	H + O <sub>2</sub> ↔ O + OH	1.9E+14	0.	16439
2	O + H <sub>2</sub> ↔ H + OH	0.508E+05	2.67	6290
3	H <sub>2</sub> + OH ↔ H <sub>2</sub> O + H	0.216E+09	1.51	3430
4	O + H <sub>2</sub> O ↔ OH + OH	2.97E+06	2.02	13400
<b>Dissociation/Recombination H<sub>2</sub>/O<sub>2</sub></b>				
5	H <sub>2</sub> + M ↔ H + H + M	4.577E+19	-1.4	104380
6	O + O + M ↔ O <sub>2</sub> + M	6.165E+15	-0.5	0
7	O + H + M ↔ OH + M	4.714E+18	-1	0
8	H + OH + M ↔ H <sub>2</sub> O + M	2.21E+22	-2	0
<b>Formation and consumption HO<sub>2</sub>/H<sub>2</sub>O<sub>2</sub></b>				
9	H + O <sub>2</sub> + M ↔ HO <sub>2</sub> + M	1.475E+12	0.	823
10	HO <sub>2</sub> + H ↔ H <sub>2</sub> + O <sub>2</sub>	1.66E+13	0.	295
11	HO <sub>2</sub> + H ↔ OH + OH	7.079E+13	0.	0
12	HO <sub>2</sub> + O ↔ O <sub>2</sub> + OH	0.325E+14	0.	-497
13	HO <sub>2</sub> + OH ↔ H <sub>2</sub> O + O <sub>2</sub>	2.89E+13	0.	11982
14	HO <sub>2</sub> + HO <sub>2</sub> ↔ H <sub>2</sub> O <sub>2</sub> + O <sub>2</sub>	4.2E+14	0.	-1629
15	H <sub>2</sub> O <sub>2</sub> + M ↔ OH + OH + M	2.951E+14	0.	48430
16	H <sub>2</sub> O <sub>2</sub> + H ↔ H <sub>2</sub> O + OH	0.241E+14	0.	3970
17	H <sub>2</sub> O <sub>2</sub> + H ↔ HO <sub>2</sub> + H <sub>2</sub>	0.482E+14	2.	7950
18	H <sub>2</sub> O <sub>2</sub> + O ↔ OH + HO <sub>2</sub>	9.55E+06	0.	3970
19	H <sub>2</sub> O <sub>2</sub> + OH ↔ HO <sub>2</sub> + H <sub>2</sub> O	5.8E+14	0.	0

# Test Case I (Markides)

$$T_{\text{air}} = 960\text{-}1000 \text{ K}$$
$$T_{\text{jet}} = 750 \text{ K}$$

$$U_{\text{jet}} = 26\text{-}120 \text{ m/s}$$
$$U_{\text{air}} = 26 \text{ m/s}$$

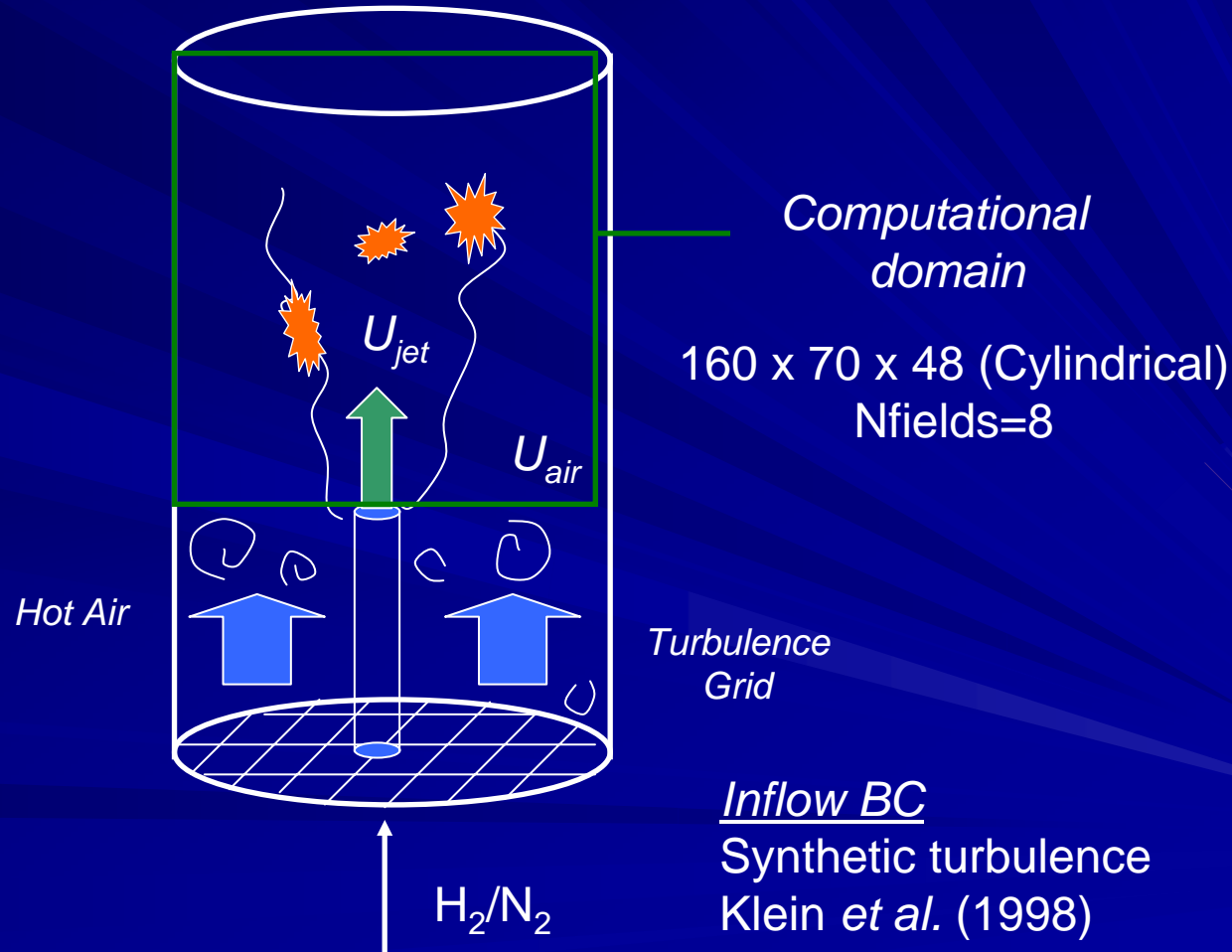
$$d_{\text{jet}} = 2.25 \text{ mm}$$
$$l_{\text{grid}} = 3 \text{ mm}$$

$$Re_t = 90\text{-}160$$

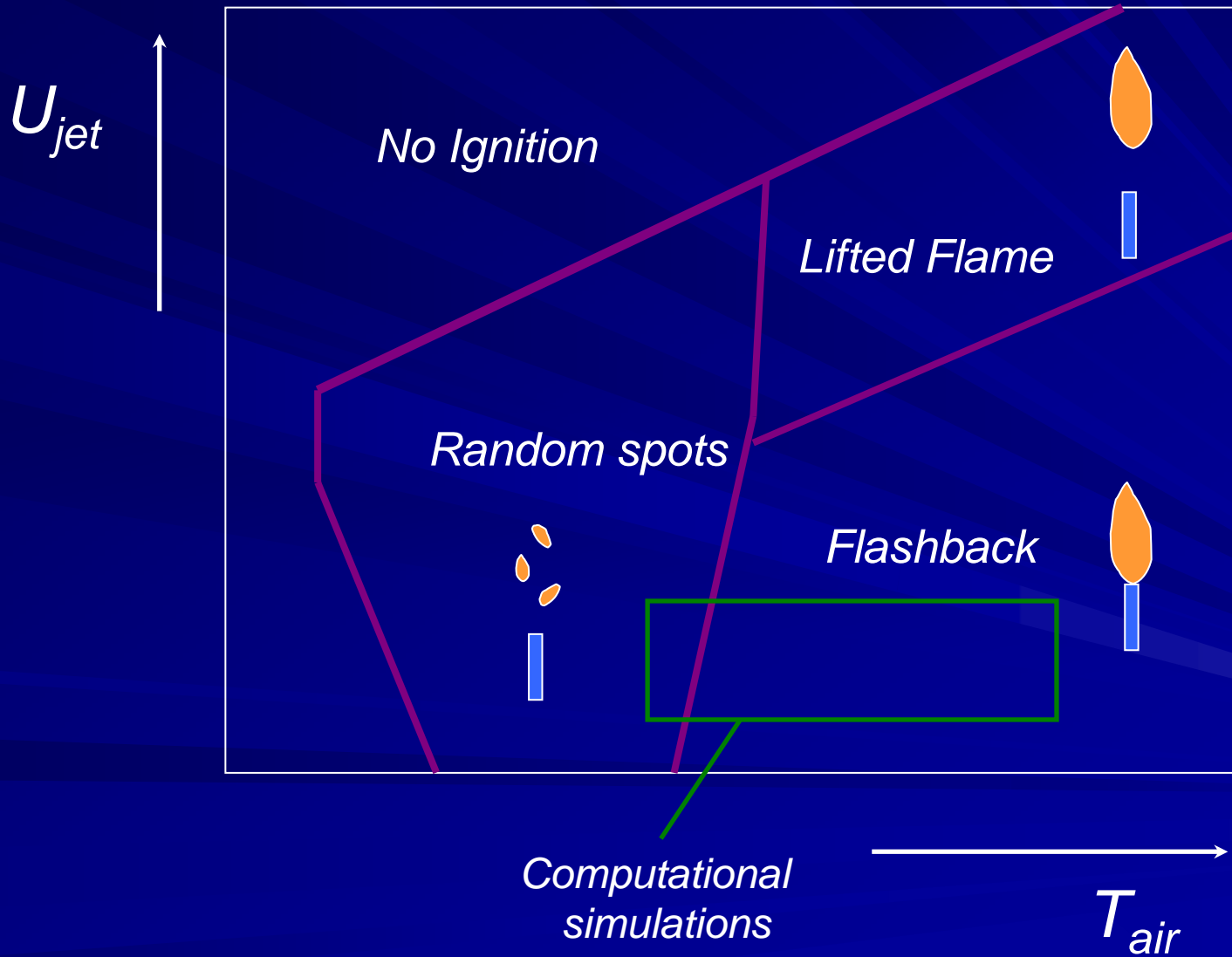
Jet

$$Y(\text{N}_2) = 0.87$$

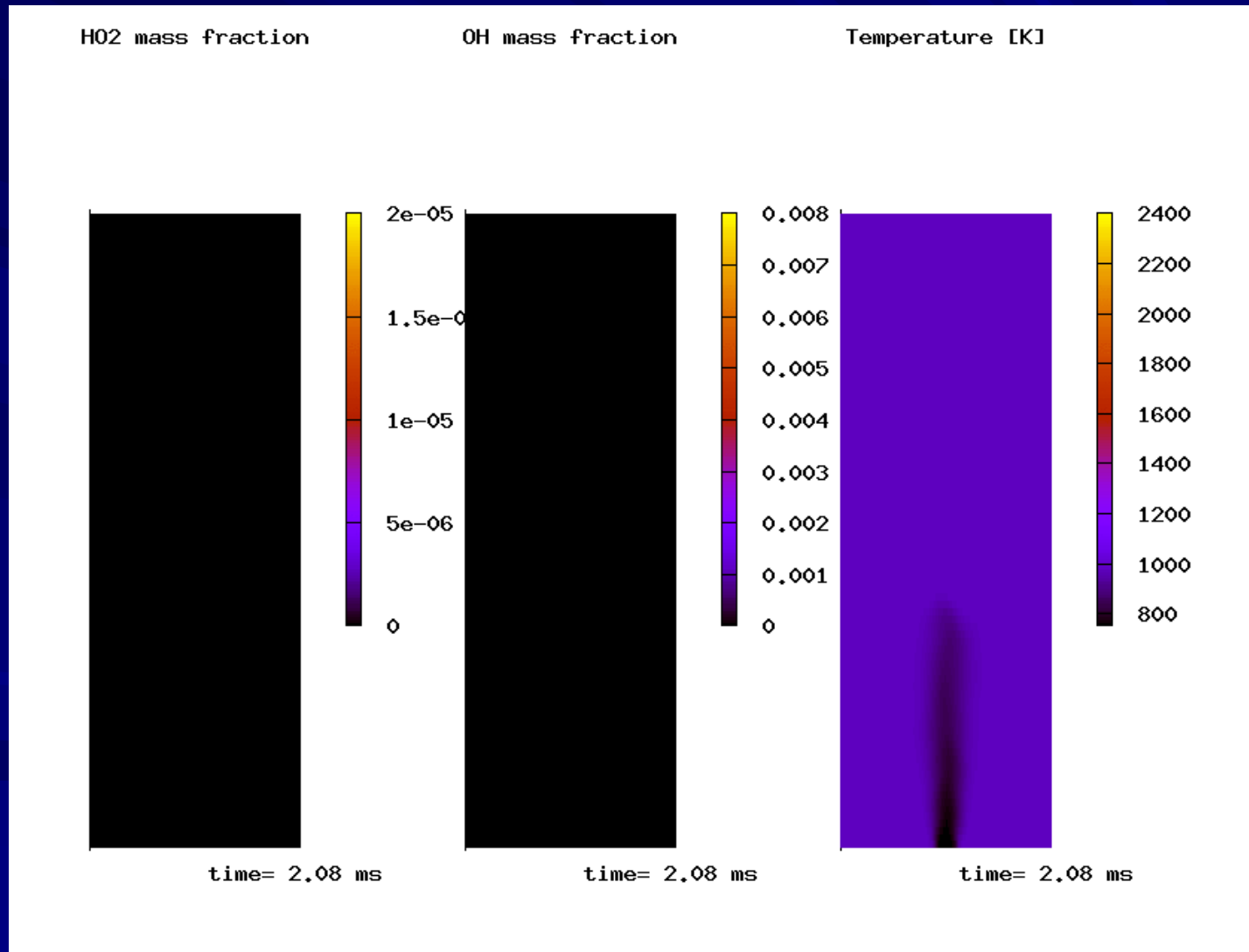
$$Y(\text{H}_2) = 0.13$$



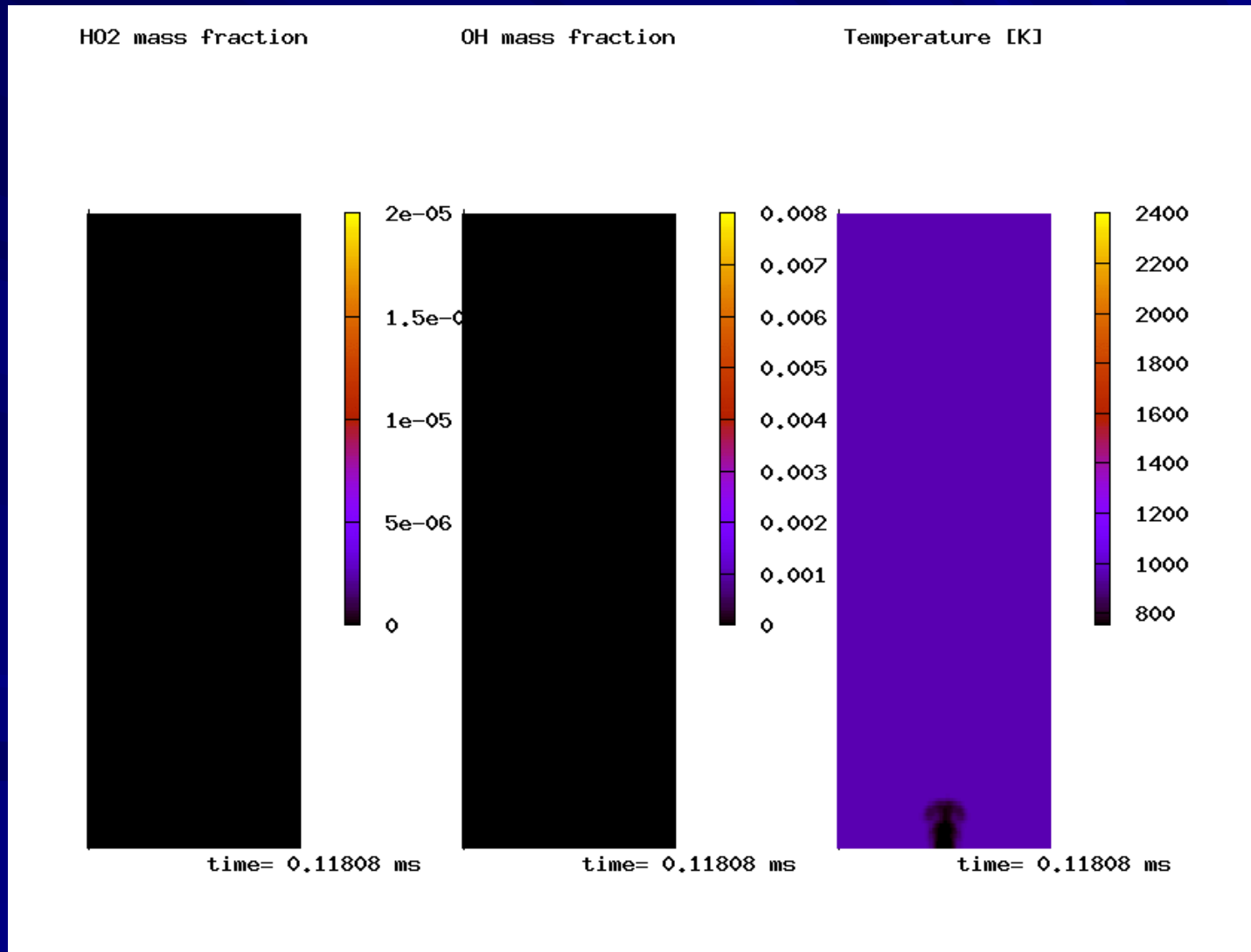
# Bulk behaviour



# Animation (U=26 m/s T=960 K)



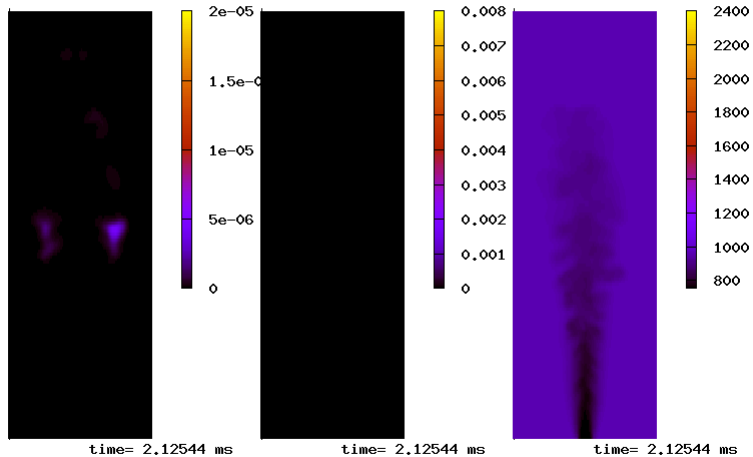
# Animation ( $U=70$ m/s $T=960$ K)



H<sub>2</sub>O mass fraction

OH mass fraction

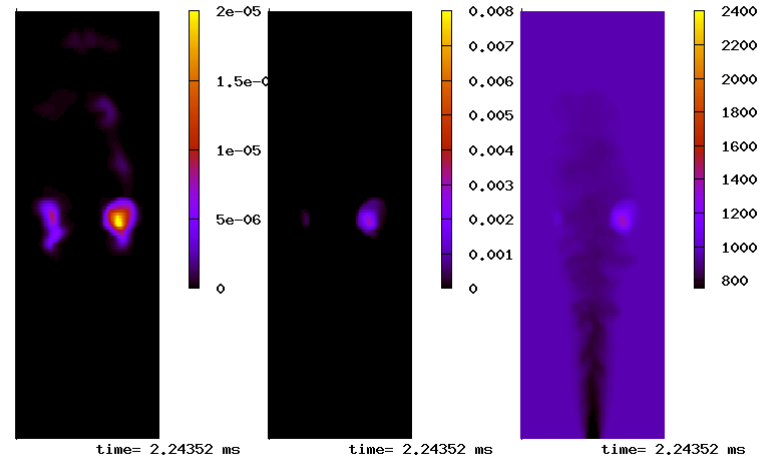
Temperature [K]



H<sub>2</sub>O mass fraction

OH mass fraction

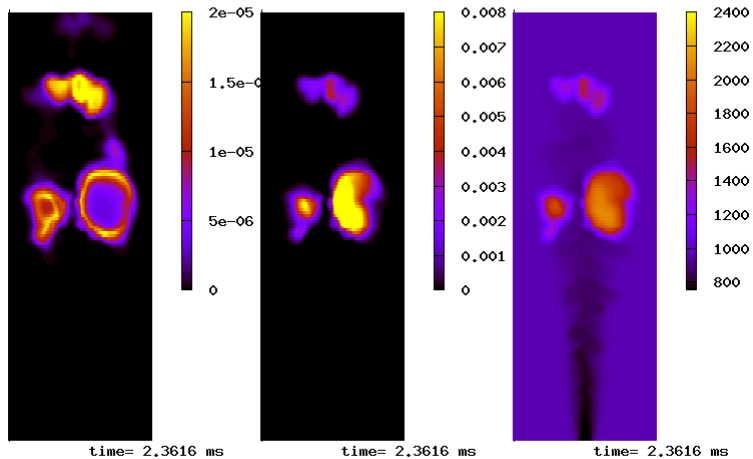
Temperature [K]



H<sub>2</sub>O mass fraction

OH mass fraction

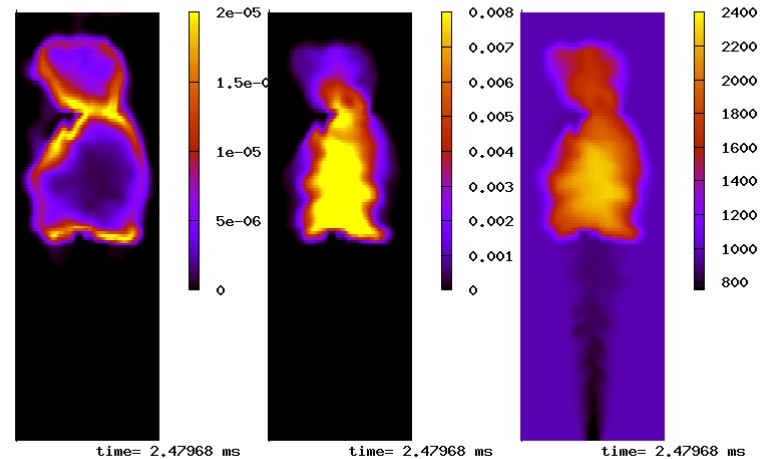
Temperature [K]



H<sub>2</sub>O mass fraction

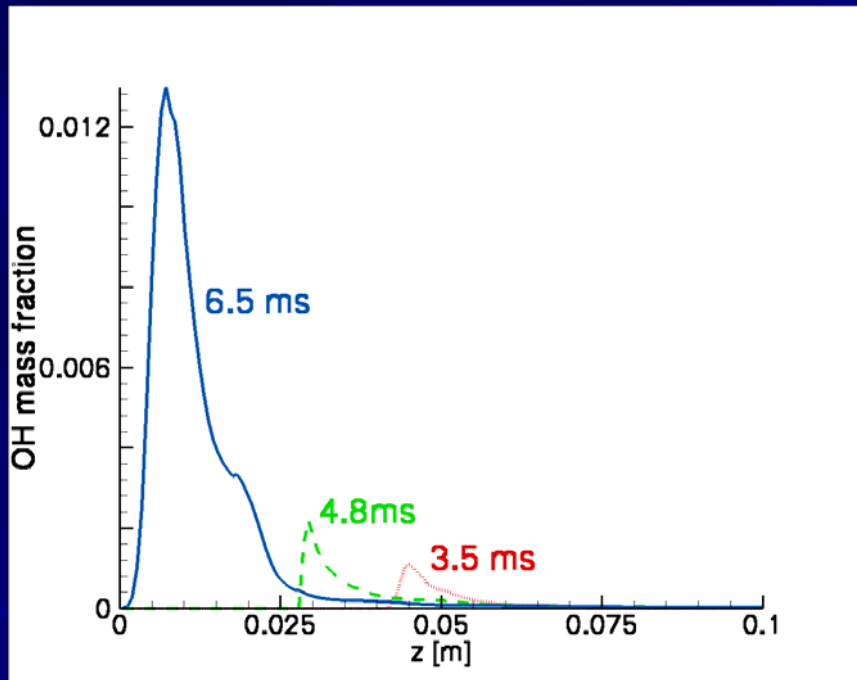
OH mass fraction

Temperature [K]

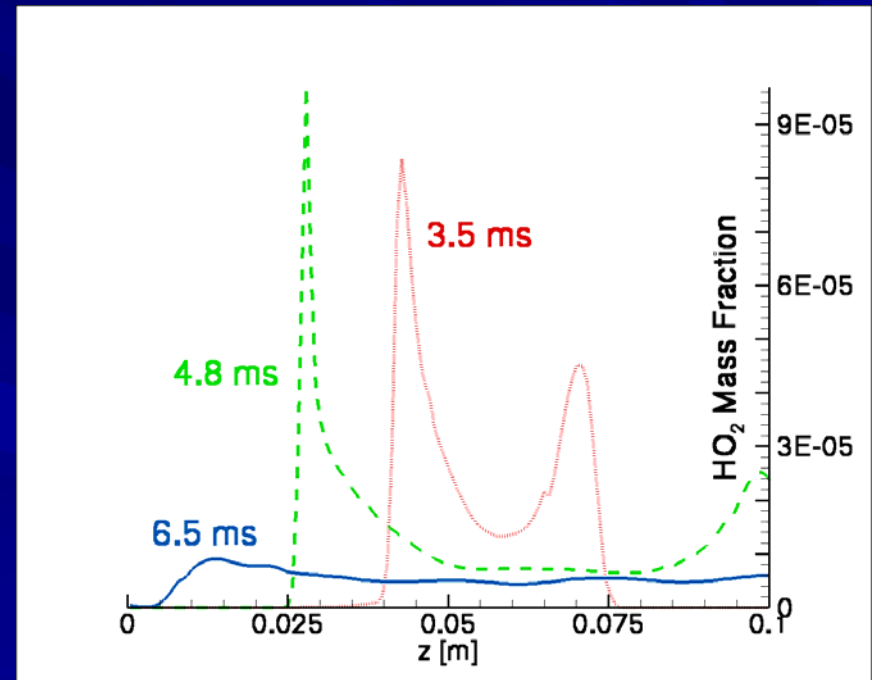


# Results I

$U_{jet} = 26 \text{ m/s}$   $T = 960 \text{ K}$

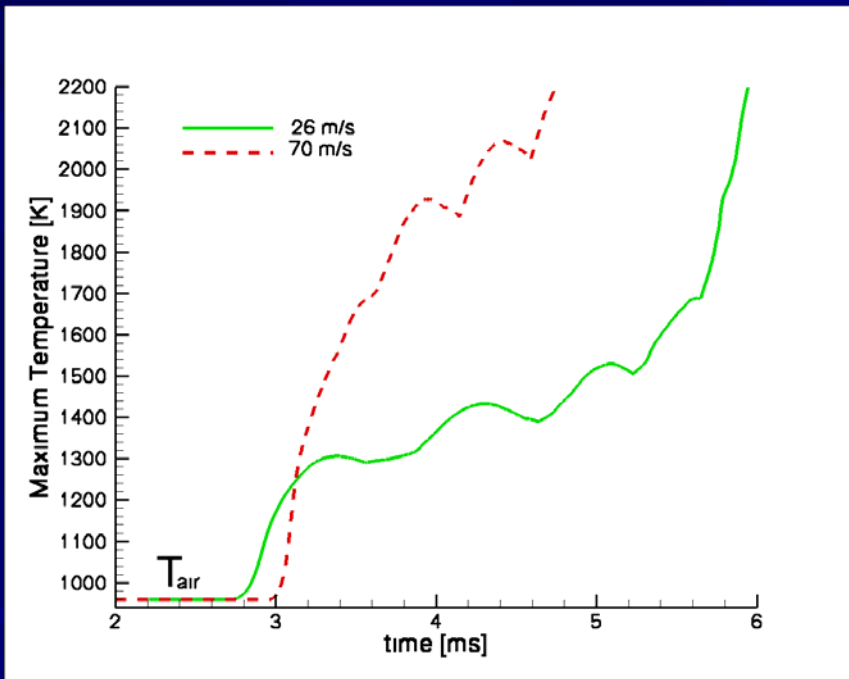


OH

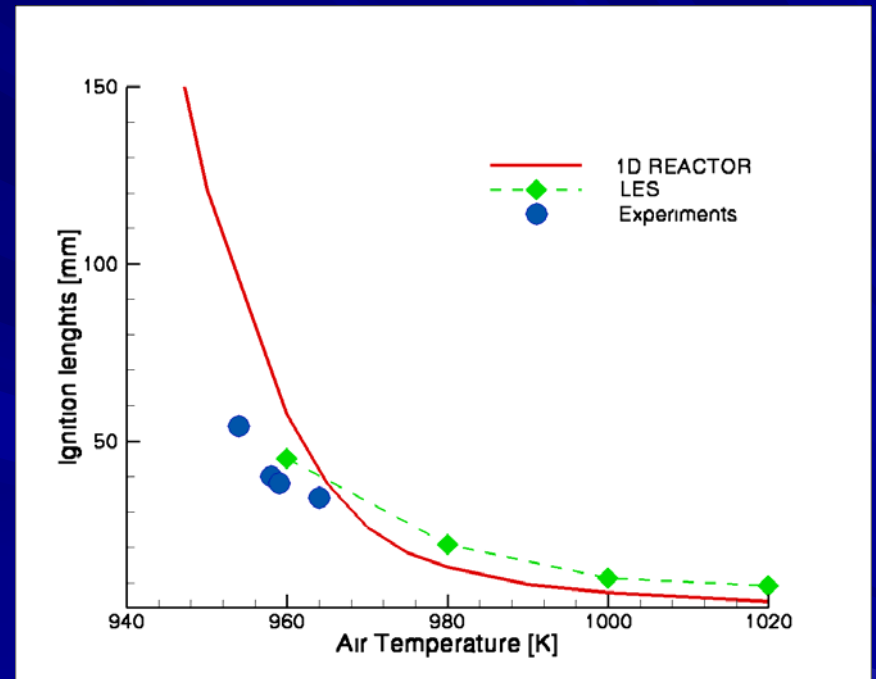


HO<sub>2</sub>

# Results II



Maximum T



Ignition Length

# Test-Case II (Cabra flame)

$$V_{\text{jet}} = 107 \text{ m/s}$$

$$V_{\text{air}} = 3.5 \text{ m/s}$$

$$T_{\text{jet}} = 305 \text{ K}$$

$$T_{\text{air}} = 1045 \text{ K}$$

$$Re = 23650$$

$$D = 4.57 \text{ mm}$$

Air

$$X(\text{O}_2) = 0.147$$

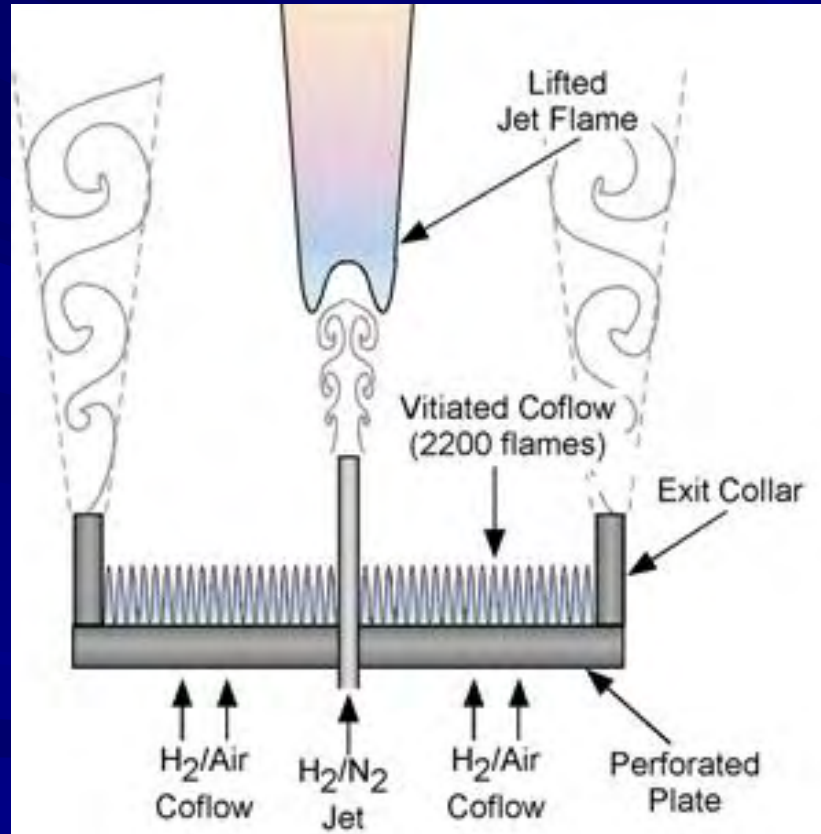
$$X(\text{N}_2) = 0.7532$$

$$X(\text{H}_2\text{O}) = 0.0989$$

Jet

$$X(\text{N}_2) = 0.75$$

$$X(\text{H}_2) = 0.25$$



Grid (cylindrical)  
*coarse*  
160 × 80 × 48

PDF  
8-16 fields

CPU Cost  
(*coarse 8 fields*)  
PDF 960 CPUh

Mixture Fraction

OH mass fraction

Temperature [K]



time= 0.0662 ms



time= 0.0662 ms

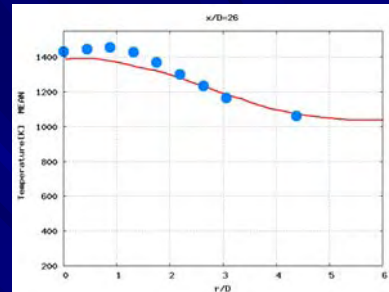
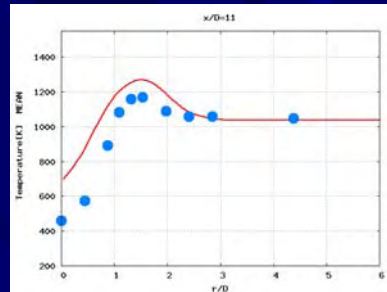
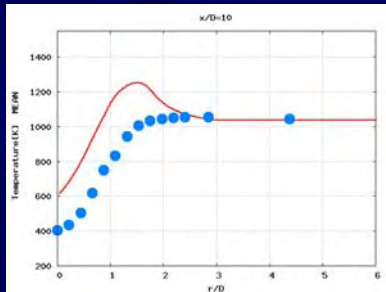


time= 0.0662 ms

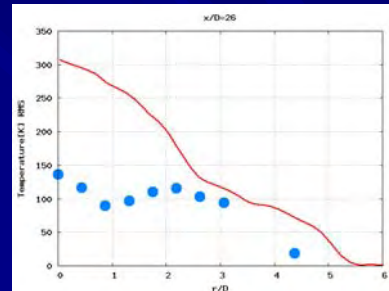
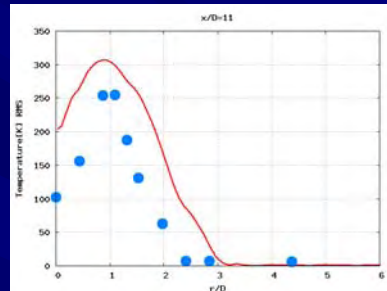
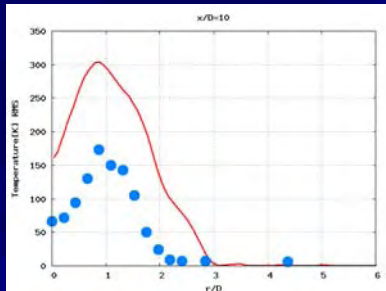
LES-PDF

T [K]

MEAN

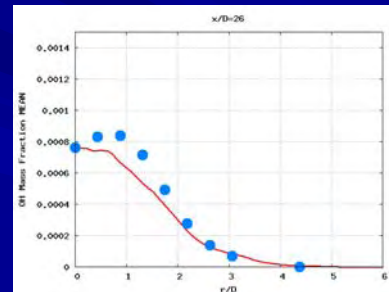
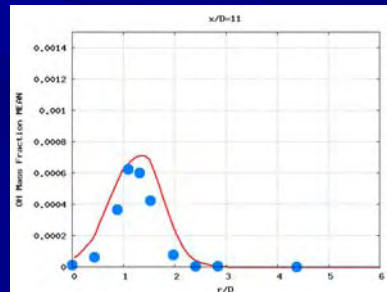
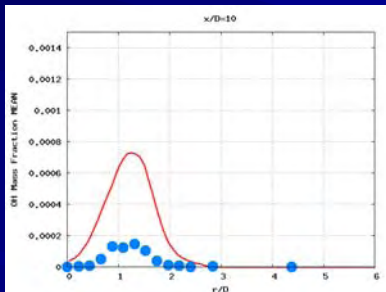


RMS

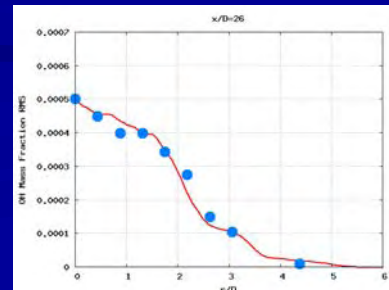
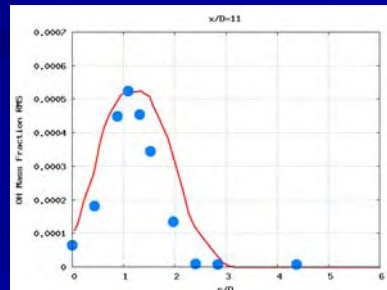
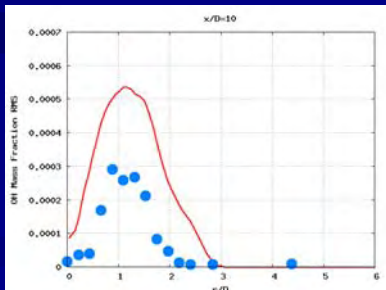


OH

MEAN



RMS



# Conclusions

- Model capable, “acceptable” results
- Random spot regime elusive  
*Increase Number of fields*
- *Very sensitive to air Temperature errors.*  
*Few experimental data*