



DNS of Multiphase Flows

Studies of Bubbly Channel Flows by Direct Numerical Simulations

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Conservation of Momentum

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \nabla \cdot \mathbf{u} \mathbf{u} = -\nabla p + \mathbf{f} + \nabla \cdot \mu (\nabla \mathbf{u} + \nabla^T \mathbf{u}) + \int_F \sigma \kappa \mathbf{n} \delta(\mathbf{x} - \mathbf{x}_f) da$$

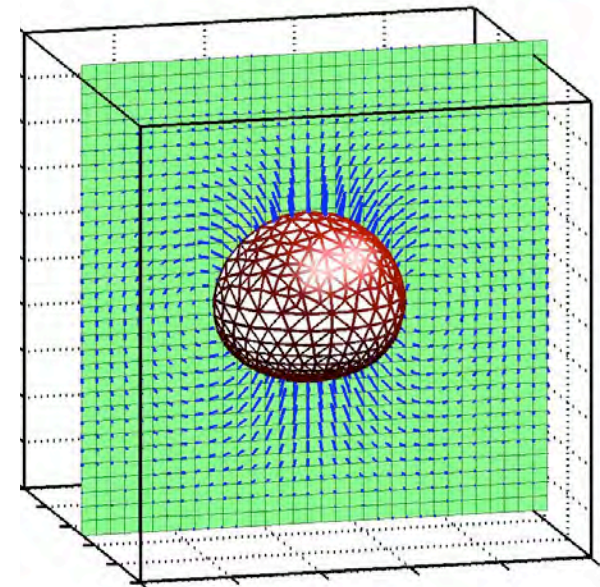
Singular interface term

Conservation of Mass

$$\nabla \cdot \mathbf{u} = 0 \quad \text{Incompressible flow}$$

Equation of State:

$$\frac{D\rho}{Dt} = 0; \quad \frac{D\mu}{Dt} = 0$$



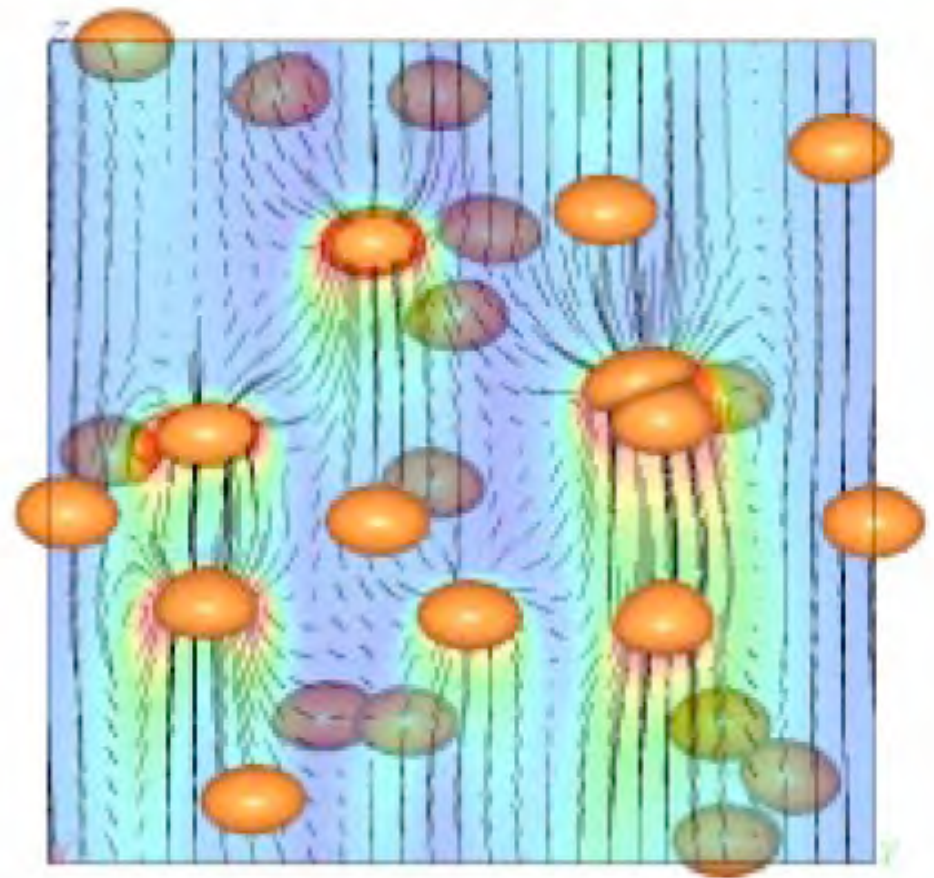
The conservation equations are solved on a regular fixed grid and the front is tracked by connected marker points



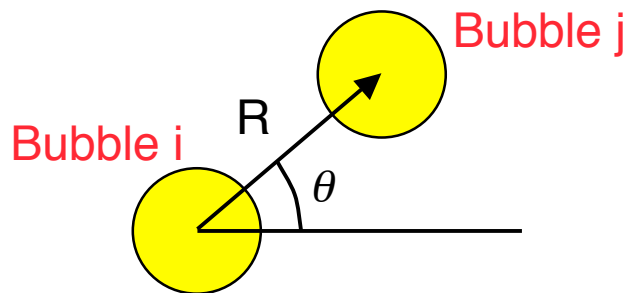
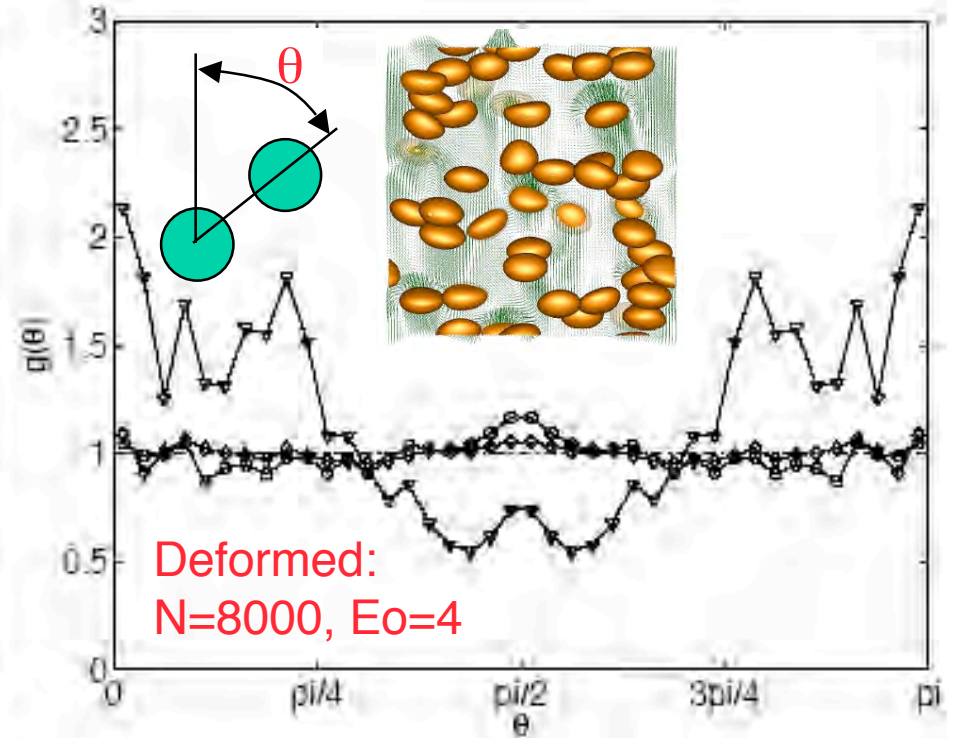
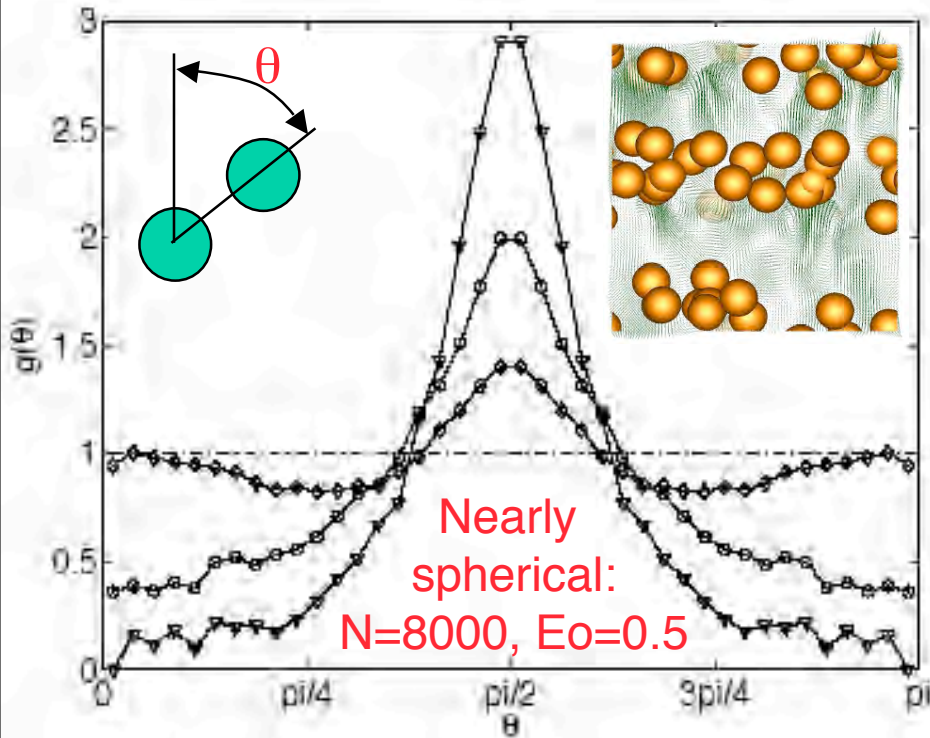
DNS of Multiphase Flows

Applications of DNS to bubbly flows

- A. Esmaeeli and G. Tryggvason, "An Inverse Energy Cascade in Two-Dimensional, Low Reynolds Number Bubbly Flows." *J. Fluid Mech.* 314 (1996), 315-330.
- E.A. Ervin and G. Tryggvason, "The Rise of Bubbles in a Vertical Shear Flow." *ASME J. Fluid Engineering* 119 (1997), 443-449.
- A. Esmaeeli and G. Tryggvason, "Direct Numerical Simulations of Bubbly Flows. Part I—Low Reynolds Number Arrays." *J. Fluid Mech.* 377 (1998), 313-345.
- A. Esmaeeli and G. Tryggvason, "Direct Numerical Simulations of Bubbly Flows. Part II—Moderate Reynolds Number Arrays" *J. Fluid Mech.* 385 (1999), 325-358.
- B. Bunner and G. Tryggvason. Direct Numerical Simulations of Three-Dimensional Bubbly Flows. *Phys. Fluids*, 11 (1999), 1967-1969.
- B. Bunner and G. Tryggvason. Dynamics of Homogeneous Bubbly Flows: Part 1. Rise Velocity and Microstructure of the Bubbles. *J. Fluid Mech.* 466 (2002), 17-52.
- B. Bunner and G. Tryggvason. Dynamics of Homogeneous Bubbly Flows. Part 2, Fluctuations of the Bubbles and the Liquid. *J. Fluid Mech* 466 (2002), 53-84.
- B. Bunner and G. Tryggvason. G. Bunner and G. Tryggvason. "Effect of Bubble Deformation on the Stability and Properties of Bubbly Flows." *J. Fluid Mech.* 495 (2003), 77-118.

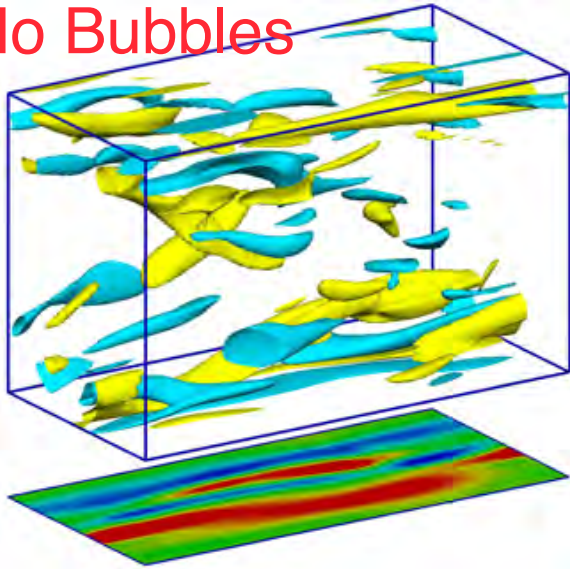


Angular pair probability distribution

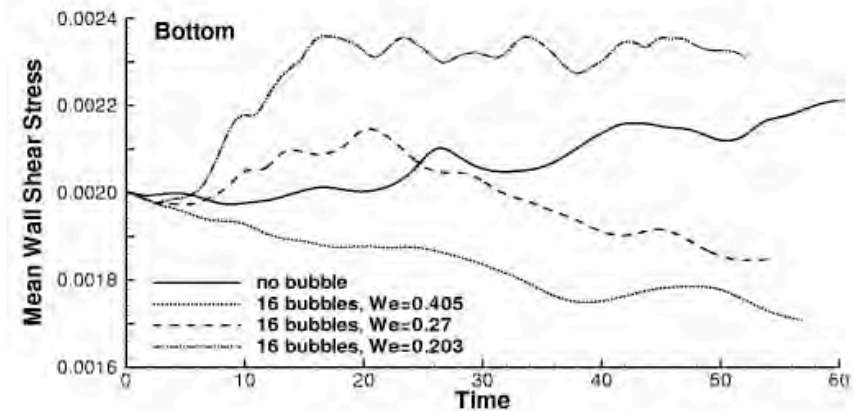
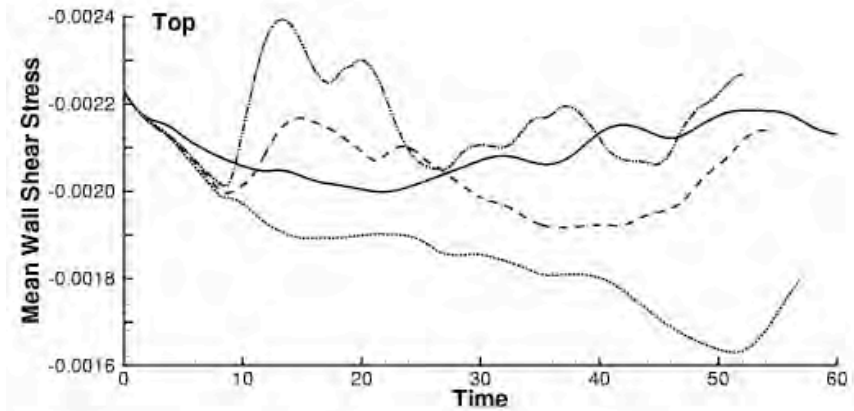
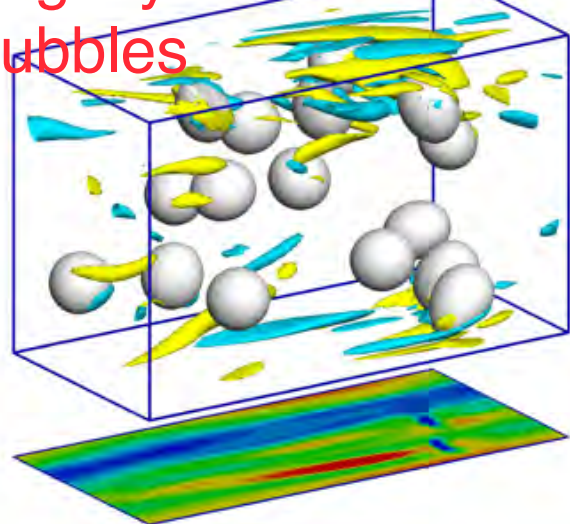


$$G(\bar{r}) = \frac{V}{N(N-1)} \left\langle \sum_i \sum_{j \neq i} \delta(\bar{r} - \bar{r}_{ij}) \right\rangle$$

No Bubbles



Slightly deformable bubbles



DNS of bubbles injected near the wall in a turbulent channel flow show that the deformability of the bubbles plays a major role. Bubbles with a deformability comparable to what is seen experimentally can lead to drag reduction



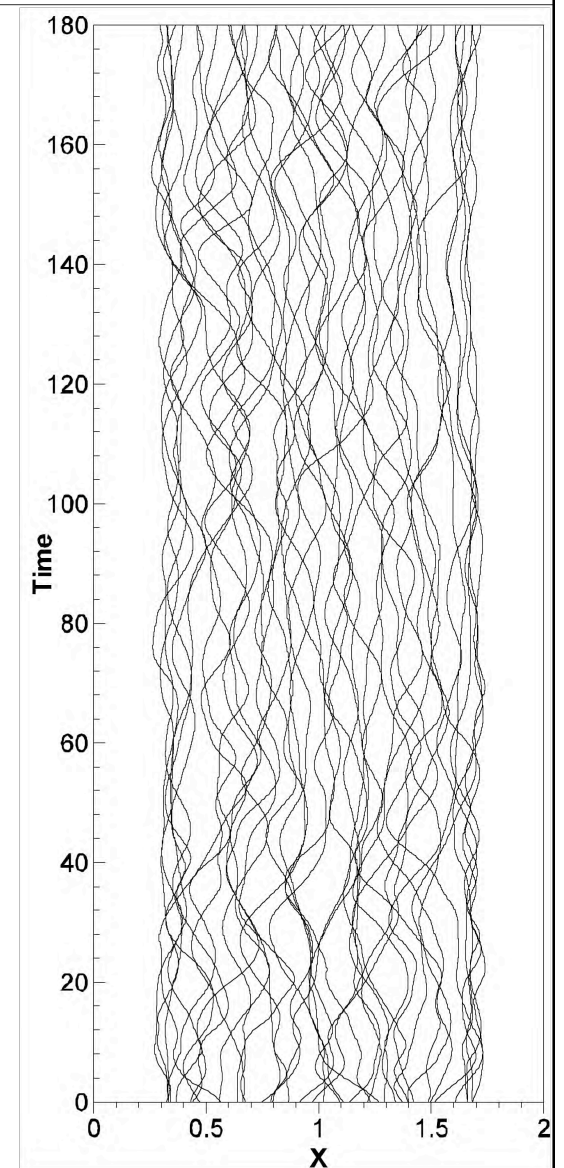
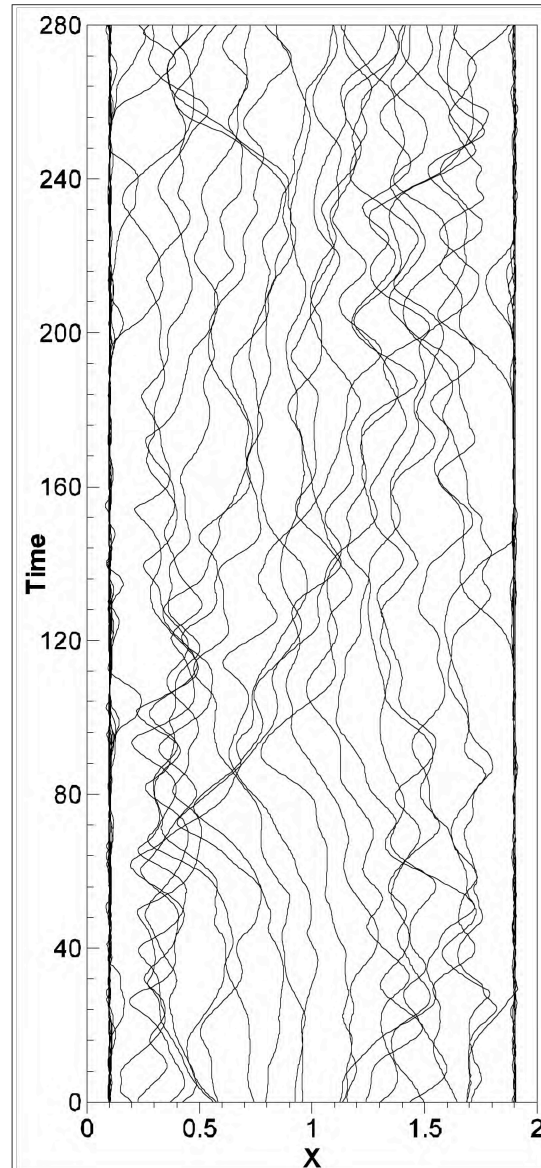
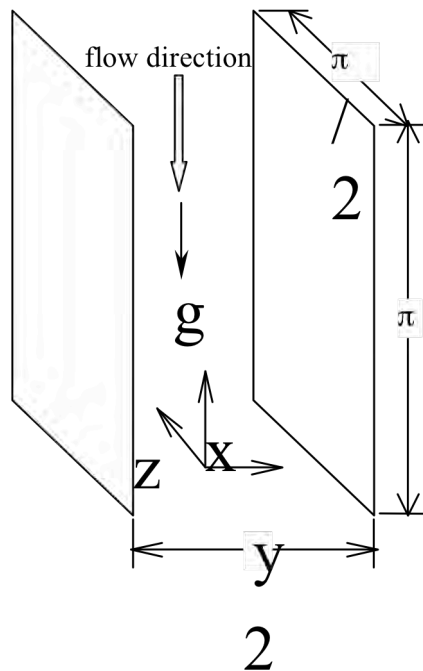
Bubbly Flows in Vertical Channels Laminar Flows



DNS of Multiphase Flows

Bubbles in Laminar Channel Flows

The path of bubbles. The cross-channel coordinate versus time for the upflow (left) and the downflow (right) at approximately steady state.

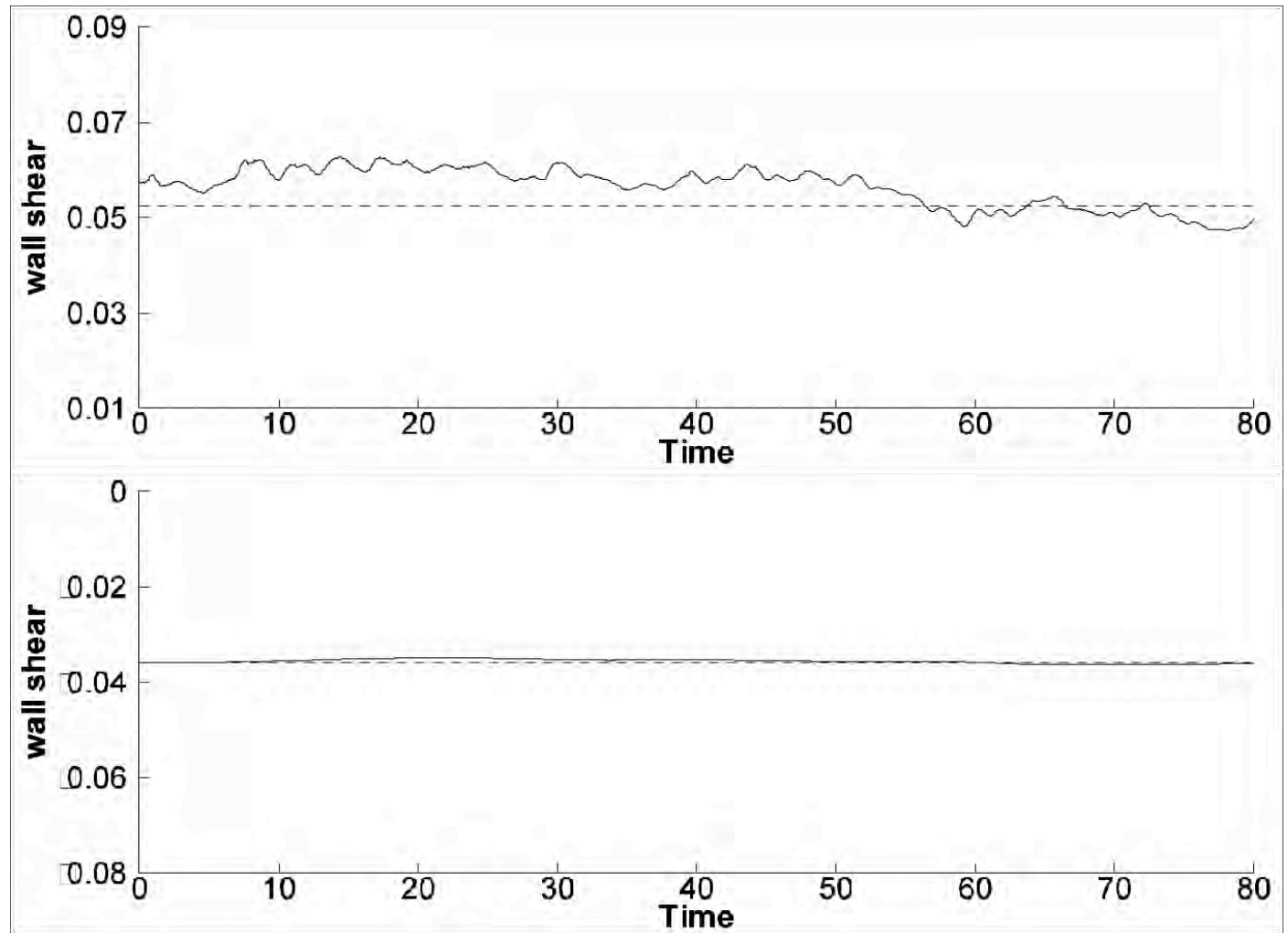




DNS of Multiphase Flows

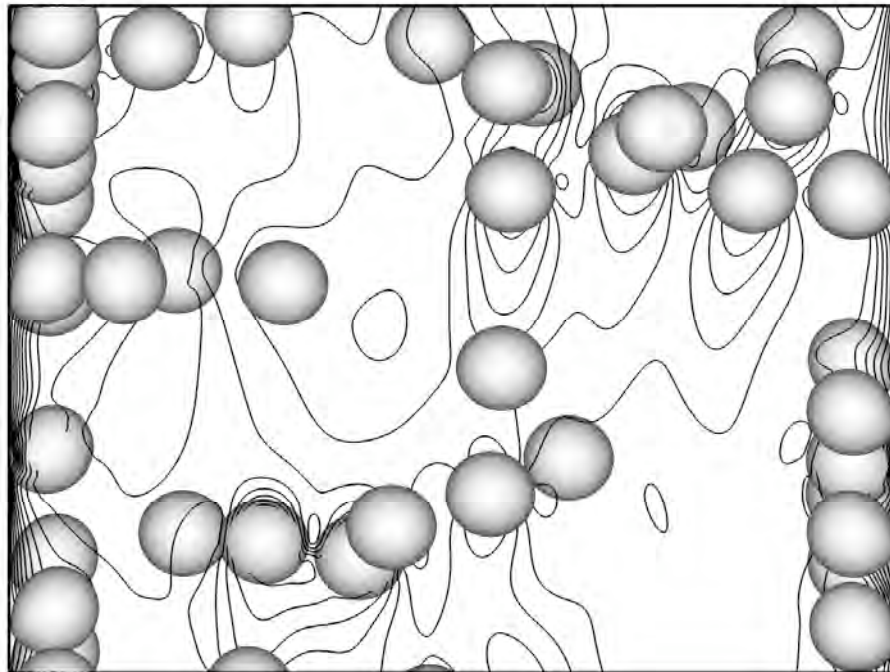
Bubbles in Laminar Channel Flows

The averaged wall shear stress versus time for the upflow (top) and the downflow (bottom). The dashed lines represent the corresponding the ideal wall shear stress needed to balance the pressure gradient.

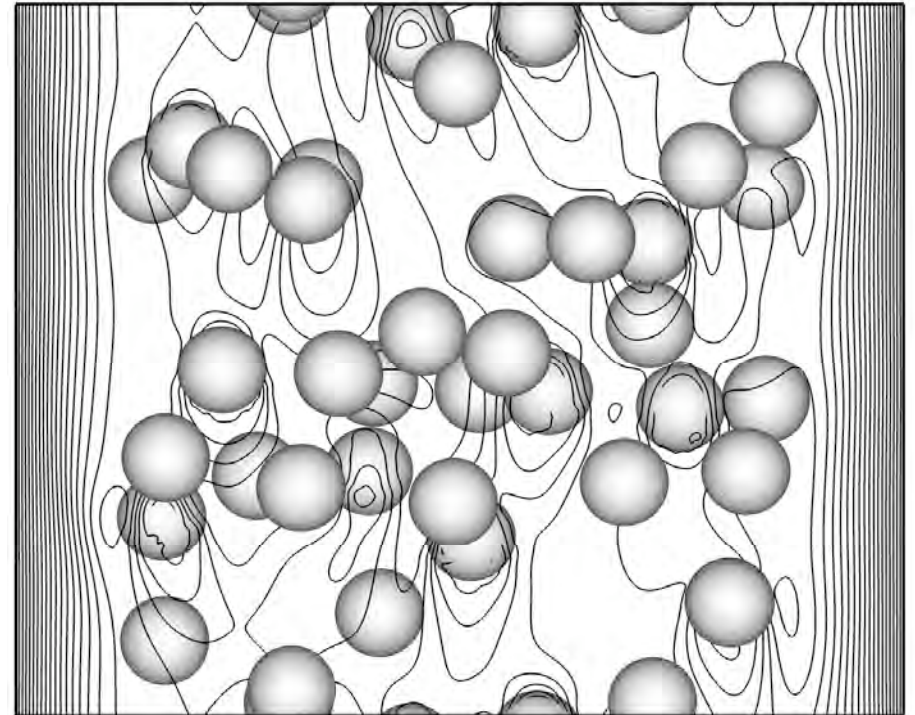


DNS of Multiphase Flows

Bubbles in Laminar Channel Flows



Upflow

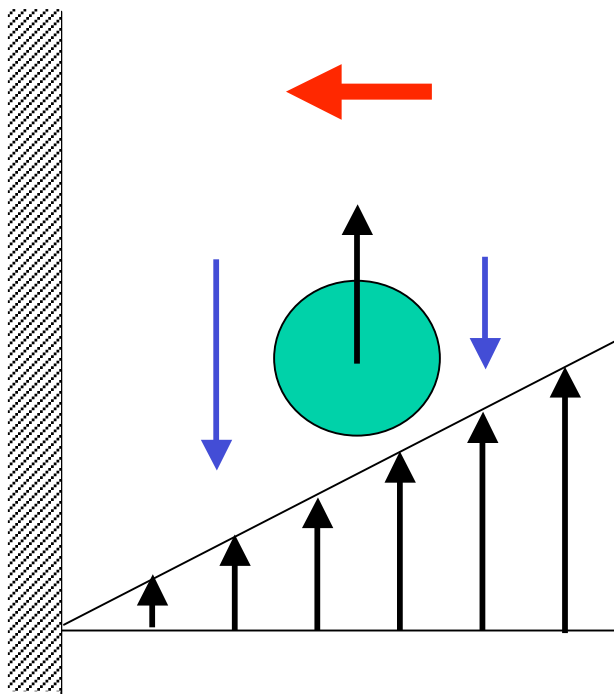


Downflow

The bubble distribution and isocontours of the vertical velocity in the middle plane for upflow on the left and dowflow on the right. The velocity contours are plotted with 0.05 intervals.

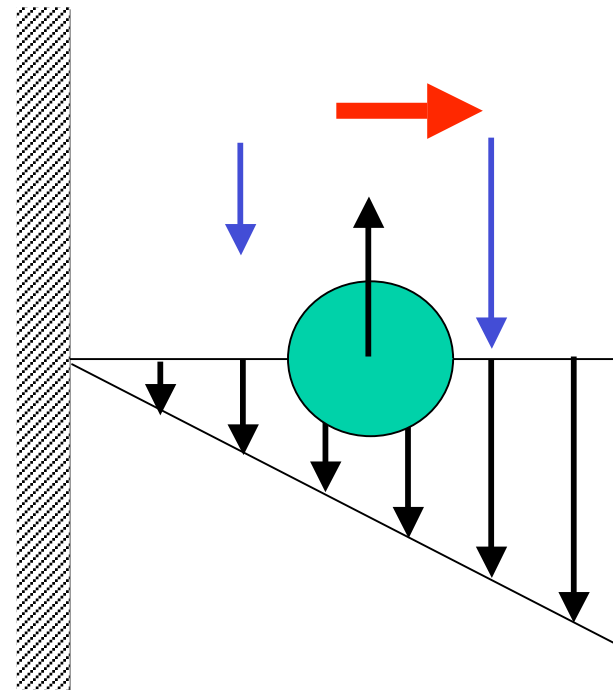
For a nearly spherical bubble

Lift toward wall



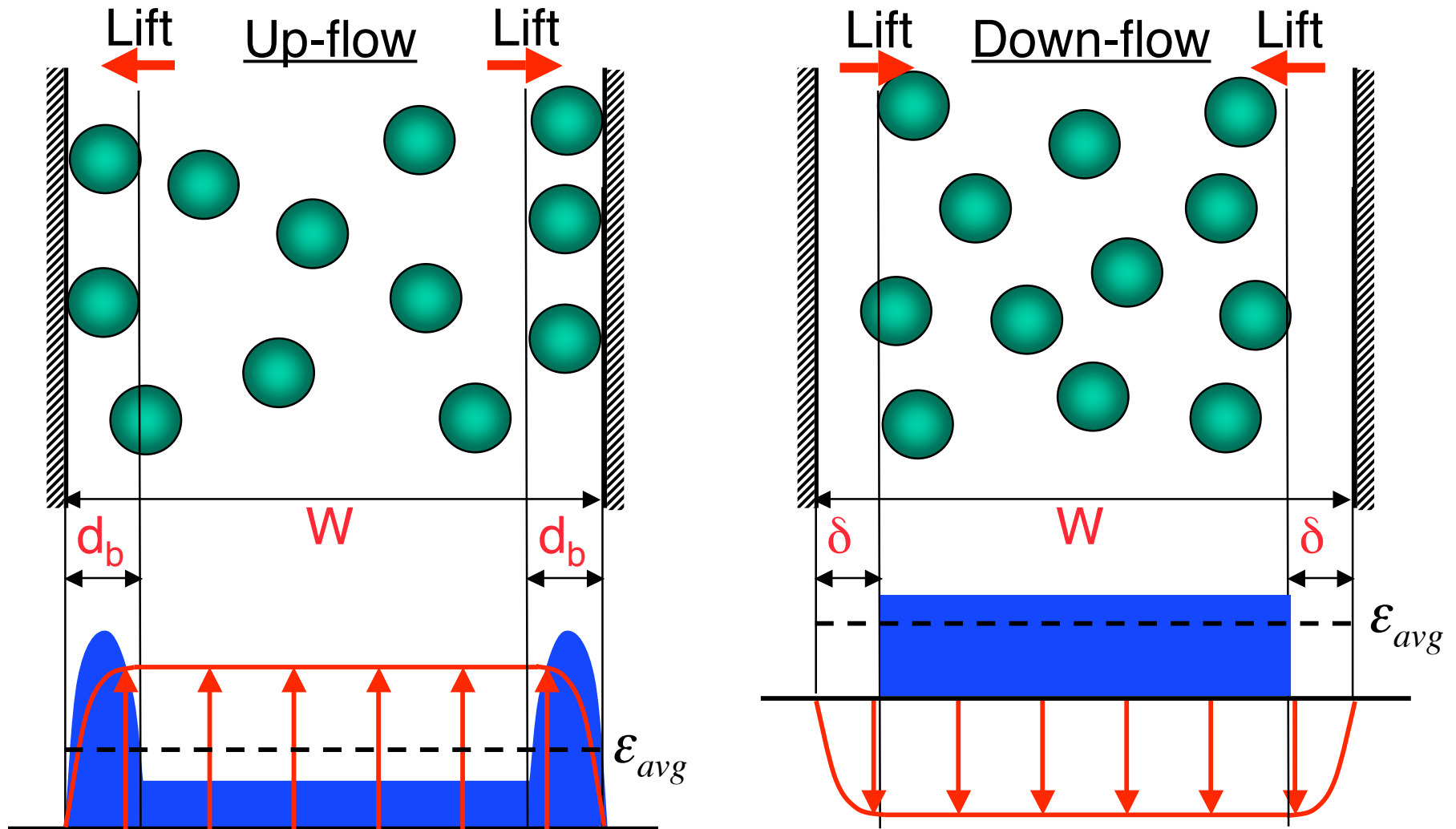
Upflow

Lift away from wall



Downflow

Simple two-fluid model for laminar multiphase flow





DNS of Multiphase Flows

Bubbles in Laminar Channel Flows

Force balance at steady state:

$$\frac{d\tau}{dx} - \beta - \Delta\rho g(\varepsilon_{avg} - \varepsilon(x)) = 0 \quad \beta = \frac{dp}{dy} + \rho_{avg}g$$

In the center of the channel there is no shear, therefore

$$\beta = -\Delta\rho g(\varepsilon_{avg} - \varepsilon_c) \Rightarrow \varepsilon_c = \varepsilon_{avg} + \frac{\beta}{\Delta\rho g}$$

$$\text{upflow: } \beta < 0 \Rightarrow \varepsilon_c < \varepsilon_{avg}$$

$$\text{downflow: } \beta > 0 \Rightarrow \varepsilon_c > \varepsilon_{avg}$$

Downflow: The wall layer is void of bubbles. To find its thickness, use the fact that the total bubble volume is conserved:

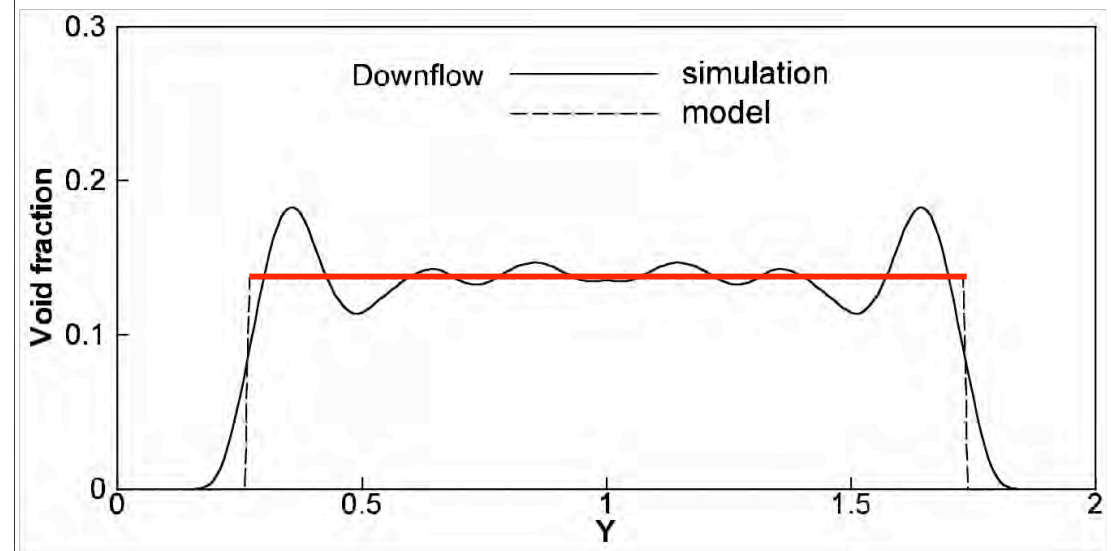
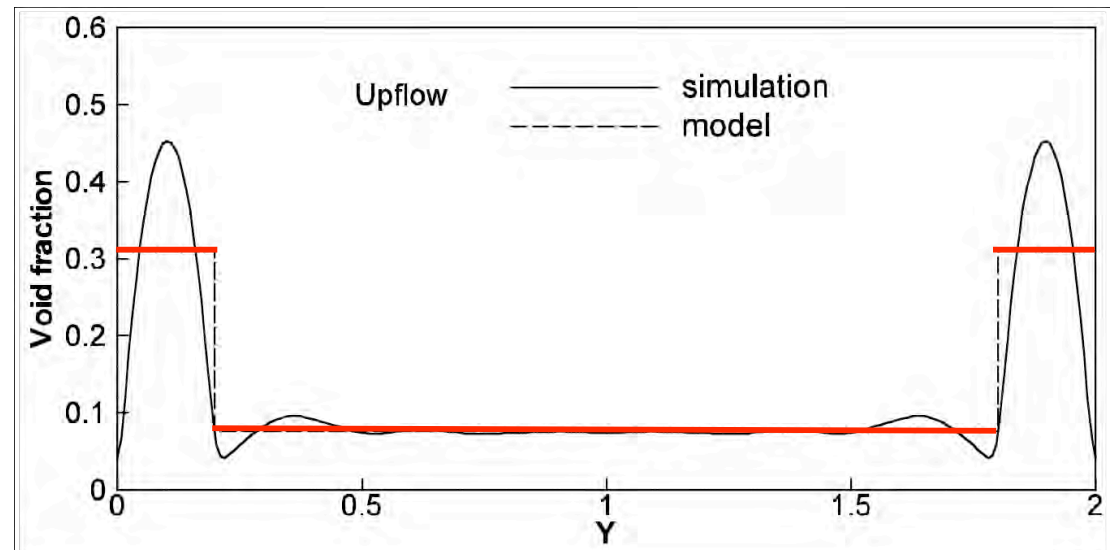
$$\varepsilon_{avg}H = \varepsilon_c(H - 2w) \Rightarrow w = \frac{H}{2}(\varepsilon_c - \varepsilon_{avg}) \Rightarrow w = \frac{-\beta H}{2\Delta\rho g}$$



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Bubbles in Laminar Channel Flows

The average void fraction profile across the channel for the upflow (top) and the downflow (bottom). The solid lines denote the results from the simulations and the dashed lines are the analytical results.

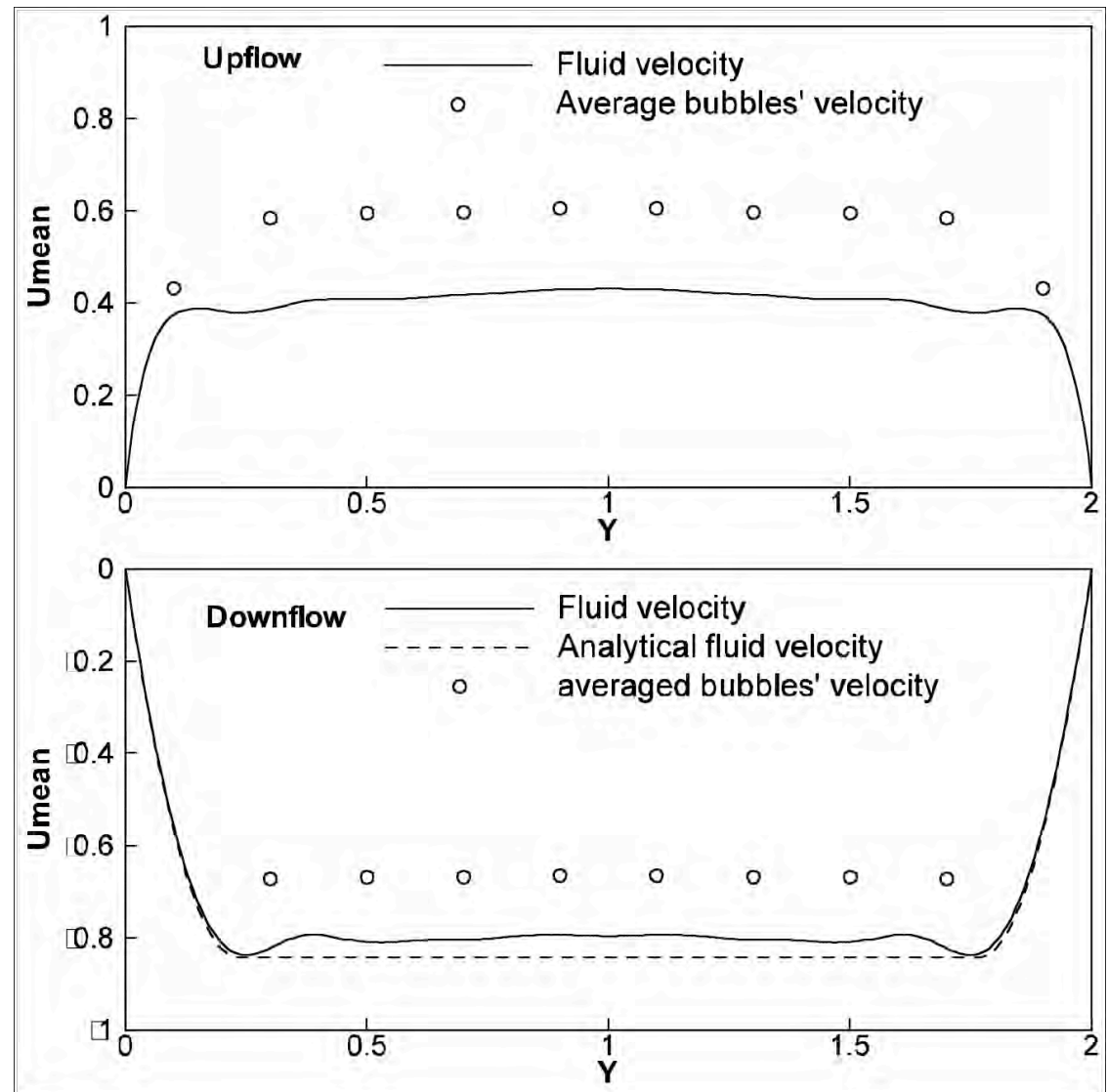




DNS of Multiphase Flows

Bubbles in Laminar Channel Flows

The average vertical liquid velocity profile across the channel for the upflow (top) and the downflow (bottom). The solid lines are the simulated results, averaged over 80 time units. The dashed line is the analytical results for the downflow. The circles represent the average bubble velocities in 10 equal sized bins across the channel.

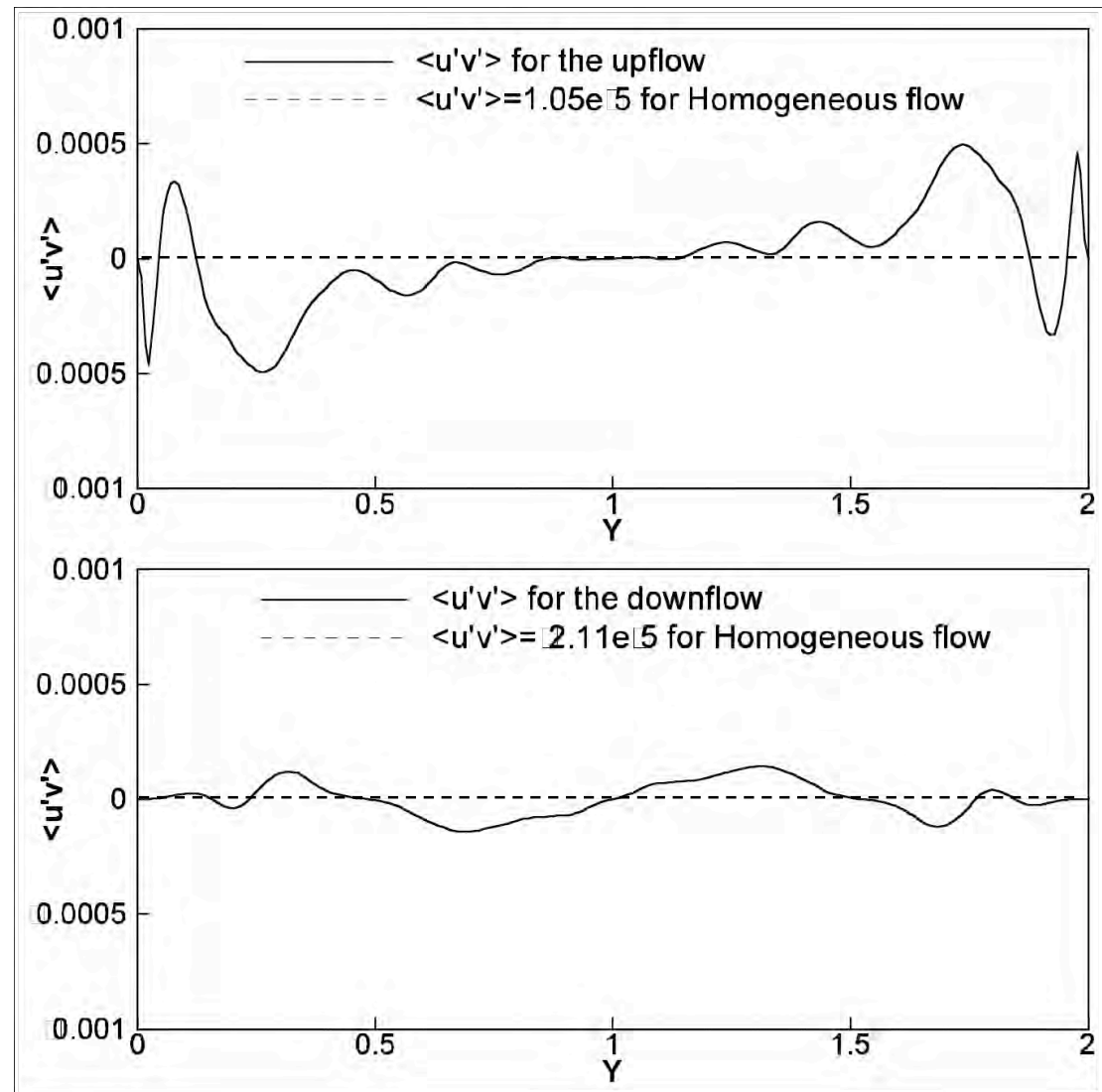




DNS of Multiphase Flows

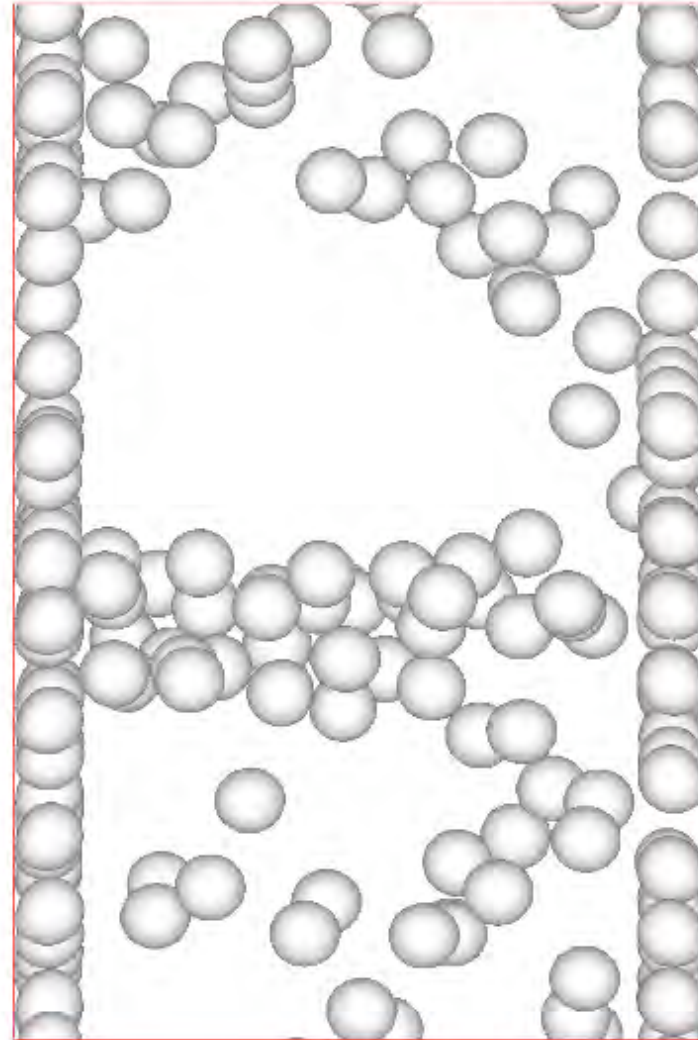
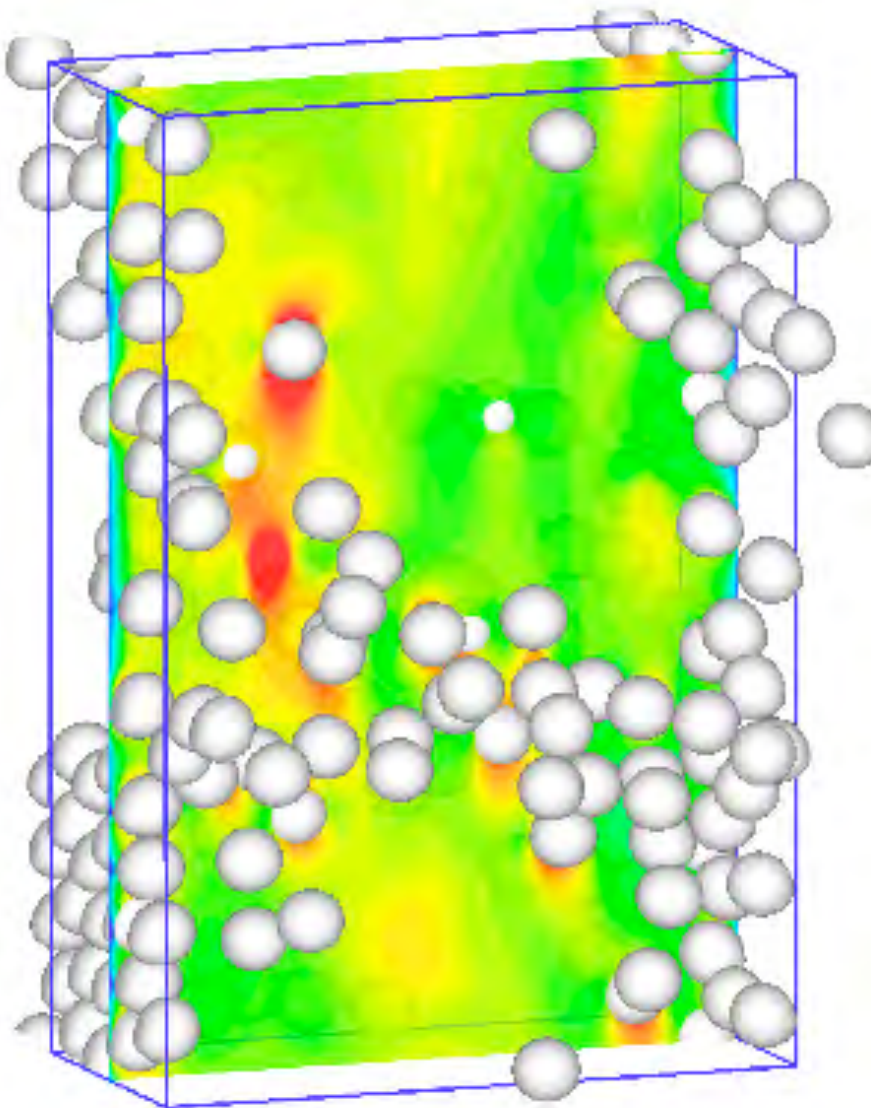
Bubbles in Laminar Channel Flows

The average Reynolds stress profile versus the cross-channel coordinate for the upflow (top) and the downflow (bottom). The solid lines are the results of averaging over 80 time units. The dashed lines denote the results from the corresponding homogeneous flow.



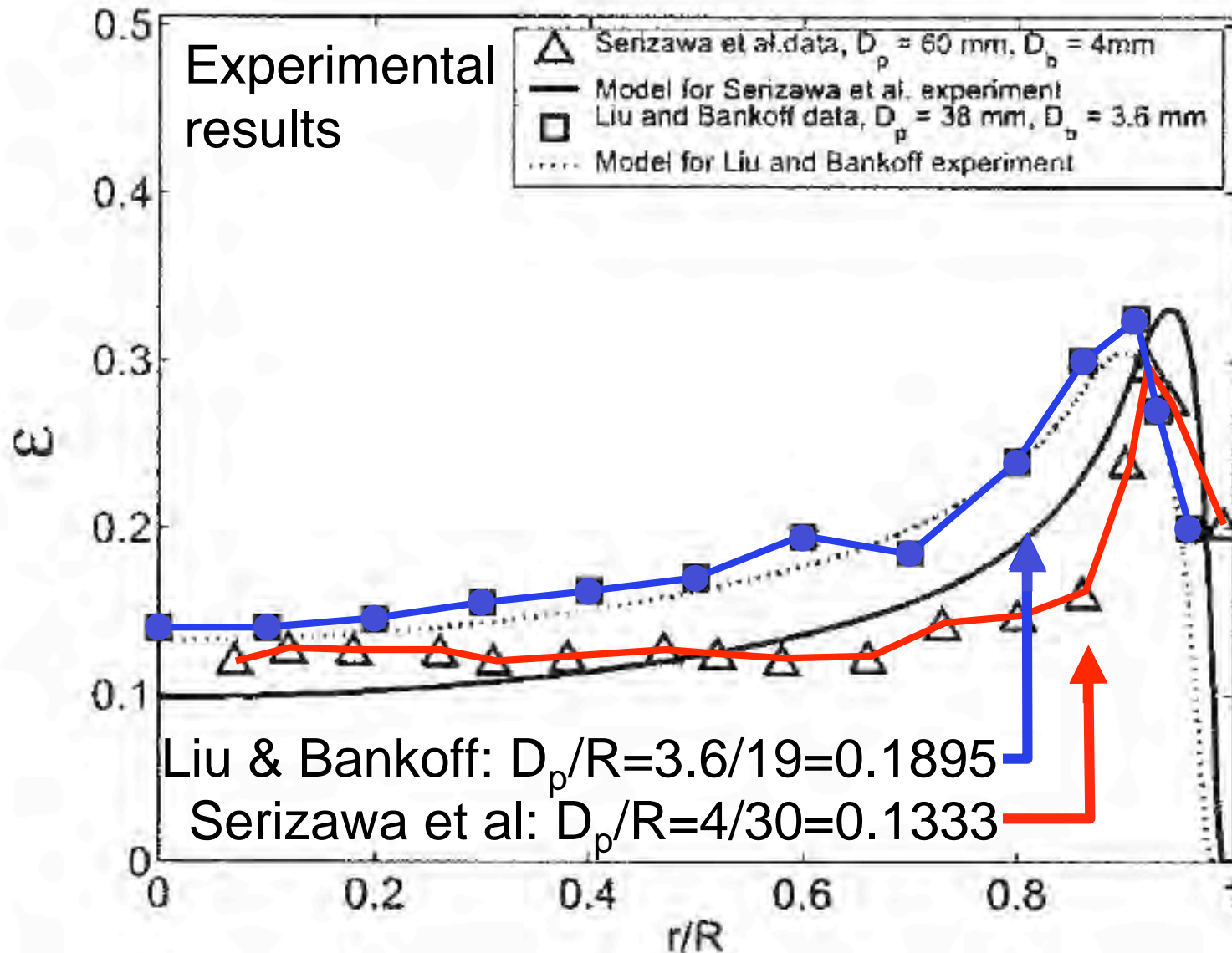
DNS of Multiphase Flows

Bubbles in Laminar Channel Flows



DNS of Multiphase Flows

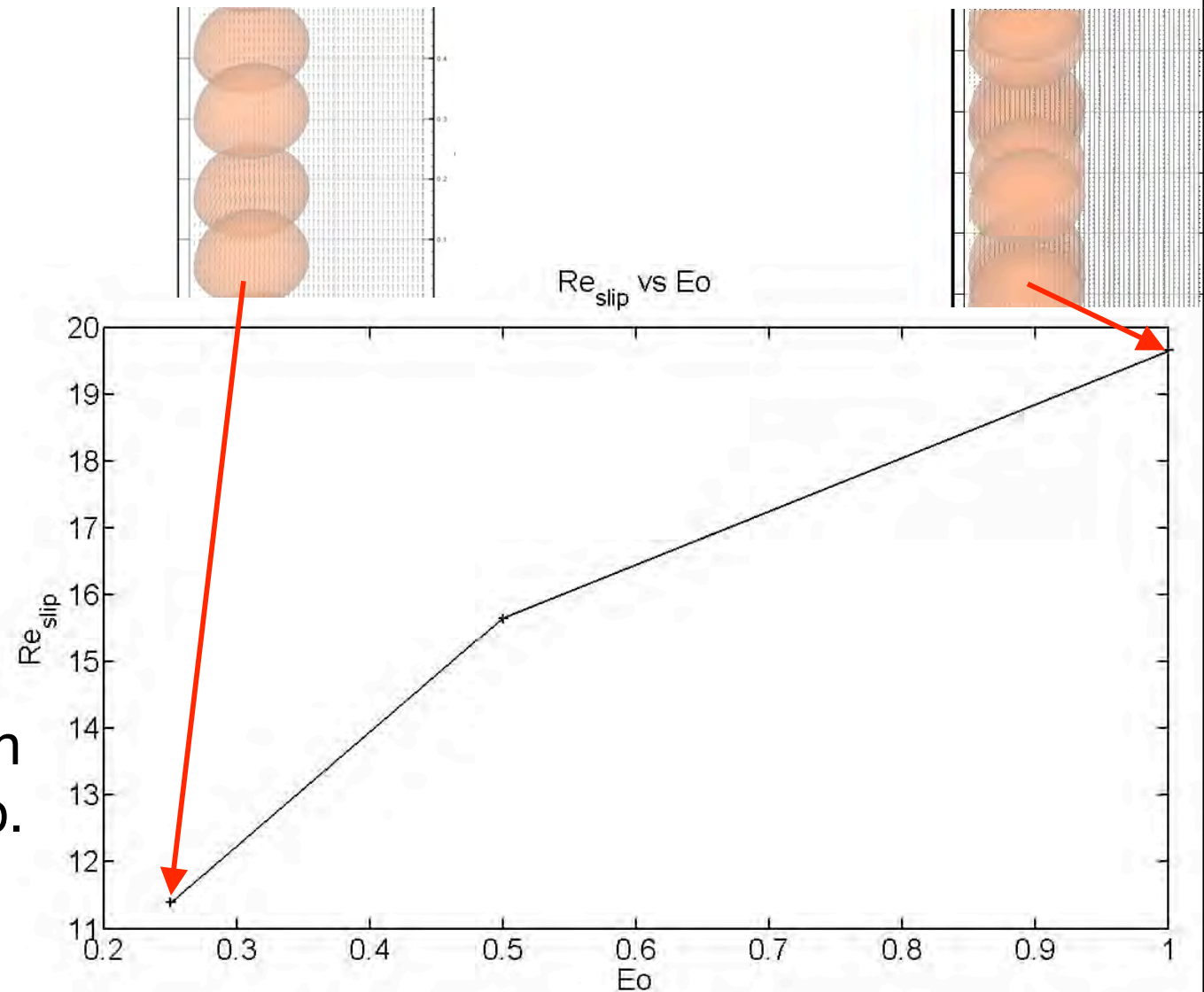
Bubbles in Laminar Channel Flows



DNS of Multiphase Flows

Bubbles in Laminar Channel Flows

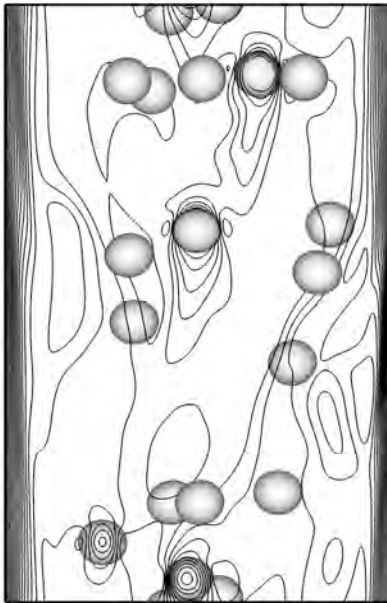
Effect of bubble deformability:
Velocity jump
across the wall-
layer versus
surface tension.
Relatively minor
changes in the
shape of the
bubbles leads to
a large change in
the velocity jump.



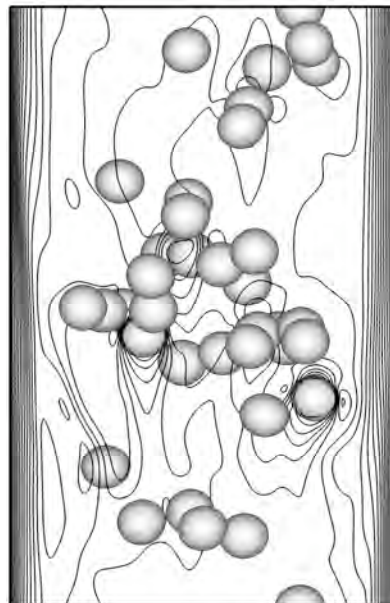


Bubbly Flows in Vertical Channels Turbulent Flows

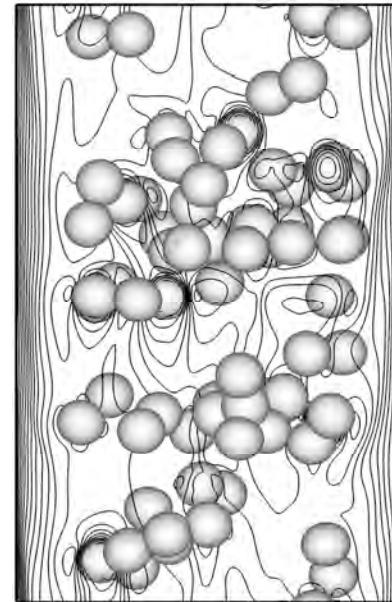
$\varepsilon=1.5\%$



$\varepsilon=3\%$



$\varepsilon=6\%$



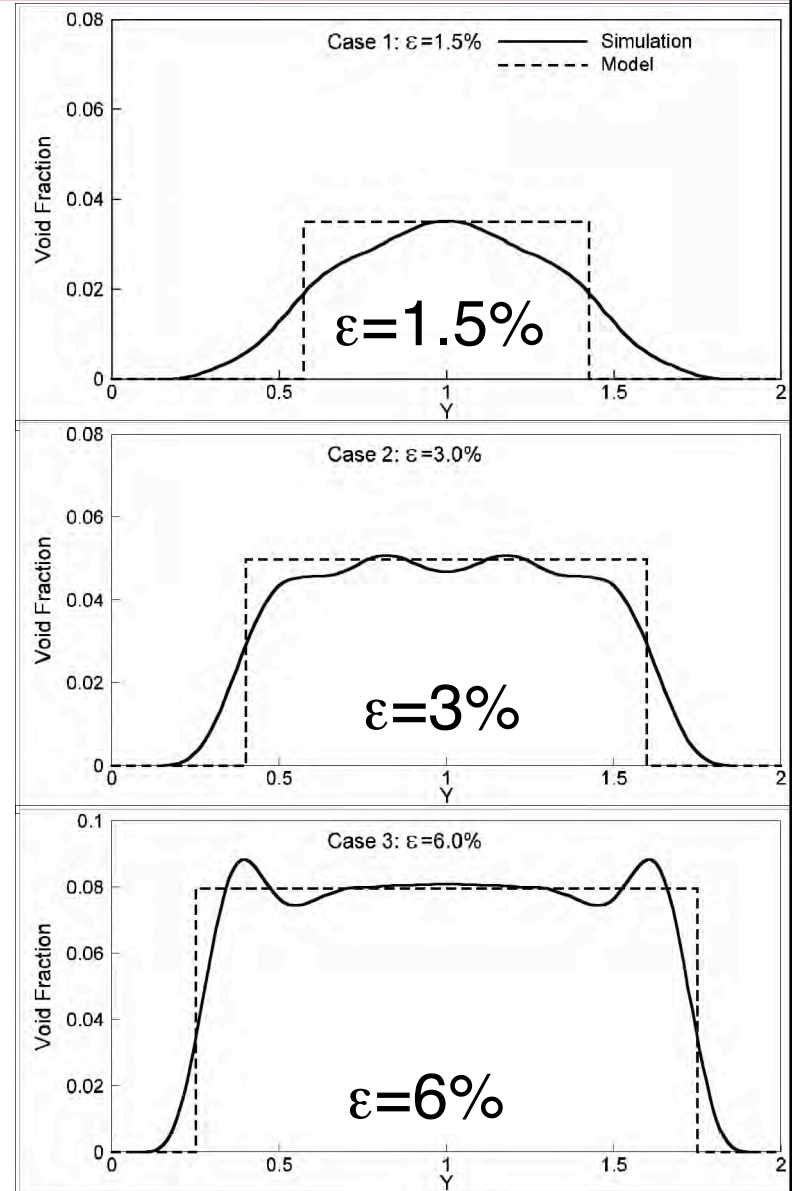
The bubble distribution and iso-contours of the vertical velocity in the middle plane of the channel, after the flow has reached an approximate steady state.



DNS of Multiphase Flows

Turbulent Downflow—Spherical Bubbles

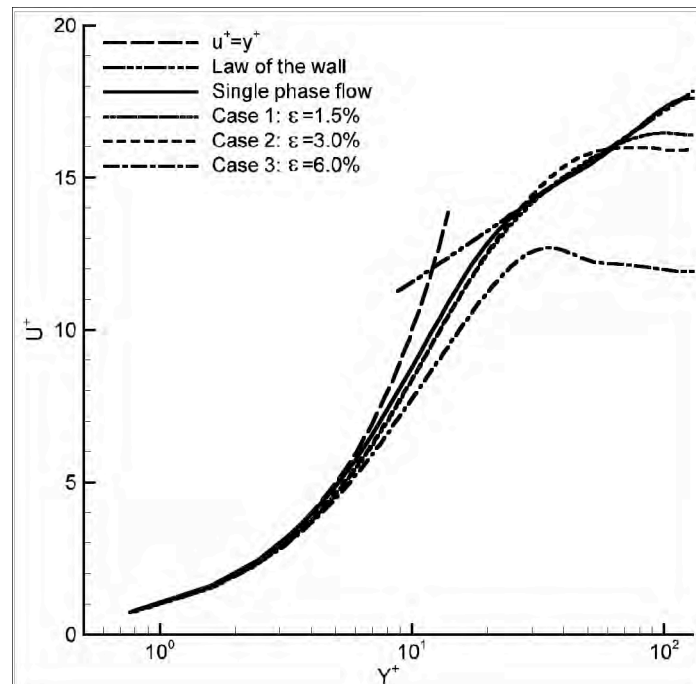
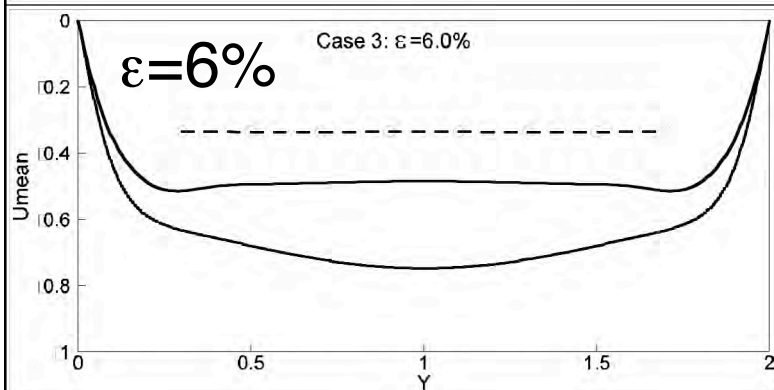
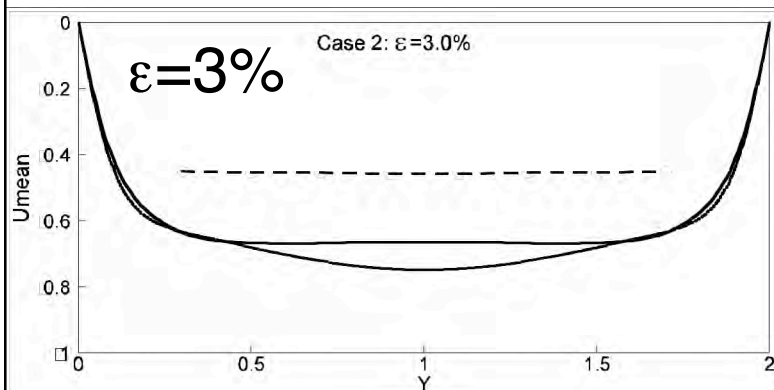
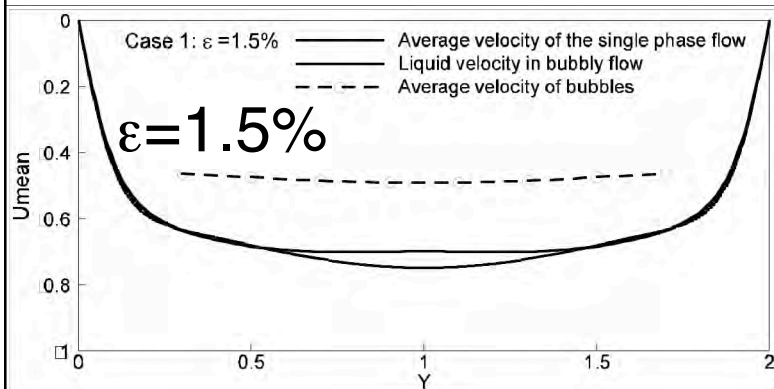
The average void fraction profile across the channel for all three cases. The solid lines denote the simulation results and the dashed lines are the analytical predictions.





DNS of Multiphase Flows

Turbulent Downflow – Spherical Bubbles



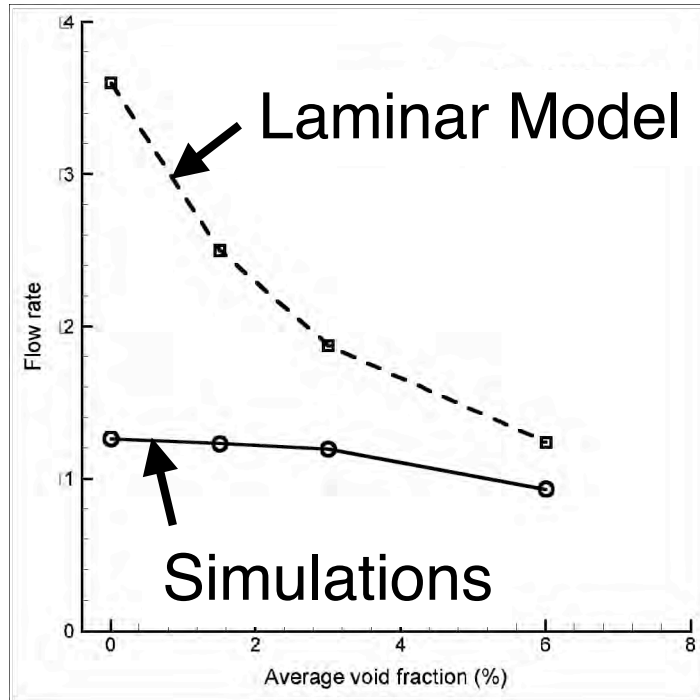
$\epsilon = 1.5\%$
 $\epsilon = 3\%$
 $\epsilon = 6\%$

The average vertical velocity (left) and a semi-log plot (top) of the average dimensionless vertical liquid velocities, normalized by the friction velocities, versus the distance from the wall, measured in wall units.

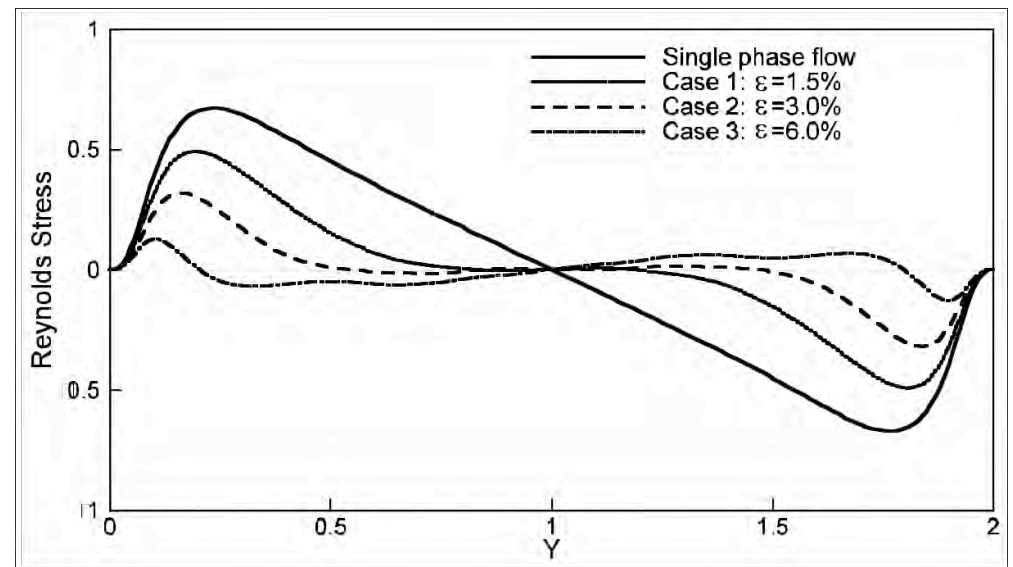


DNS of Multiphase Flows

Turbulent Downflow – Spherical Bubbles



A comparison of the total liquid flow rates from the simulations and those predicted by the laminar model

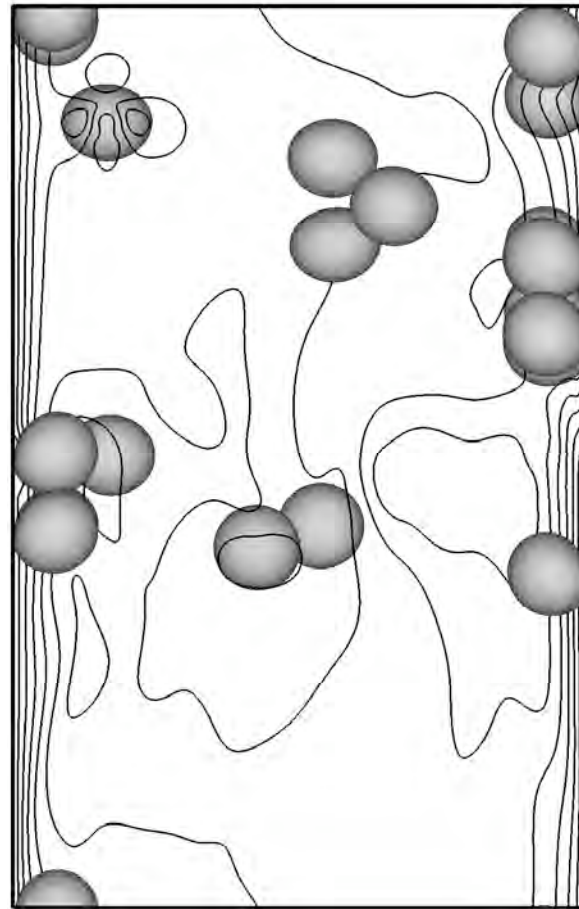


The average Reynolds stress profiles versus the normal-wall coordinate. The Reynolds stress is normalized by $(u^+)^2$, where u^+ is the friction velocity.



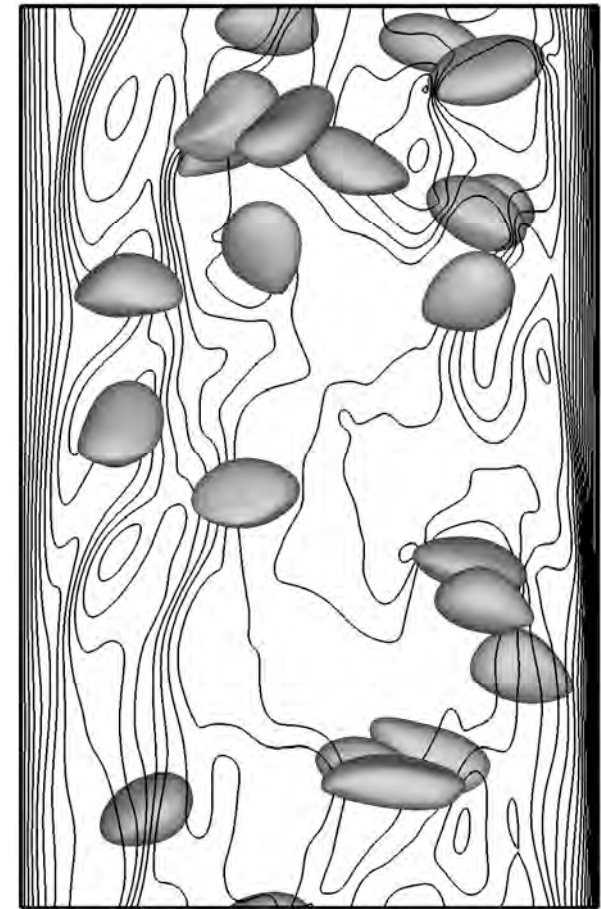
Effect of Bubble Deformability for Turbulent Upflow

The bubbles and iso-contours of the instantaneous vertical velocity in a plane through the middle of the channel for the upflow of nearly spherical (left) and much more deformable (right) bubbles at one time when the flow is approximately at steady state.



$$M=1.54 \times 10^{-10}$$

$$Eo=0.45$$



$$M=1.54 \times 10^{-7}$$

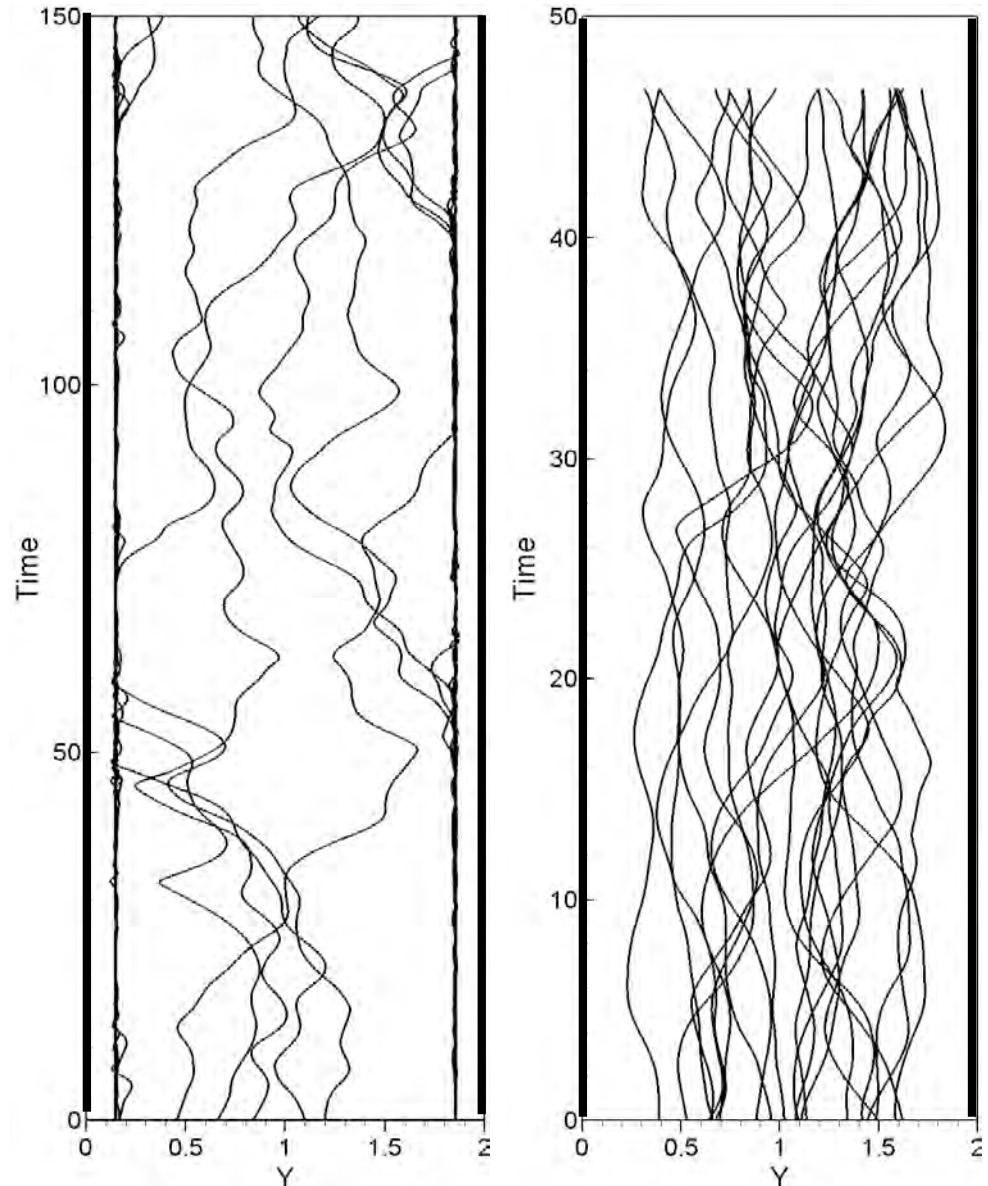
$$Eo=4.5$$



DNS of Multiphase Flows

Effect of Deformability

The path of the bubbles.
The cross-channel coordinate versus time for the upflow of nearly spherical (left) and much more deformable (right) bubbles at approximately steady state.

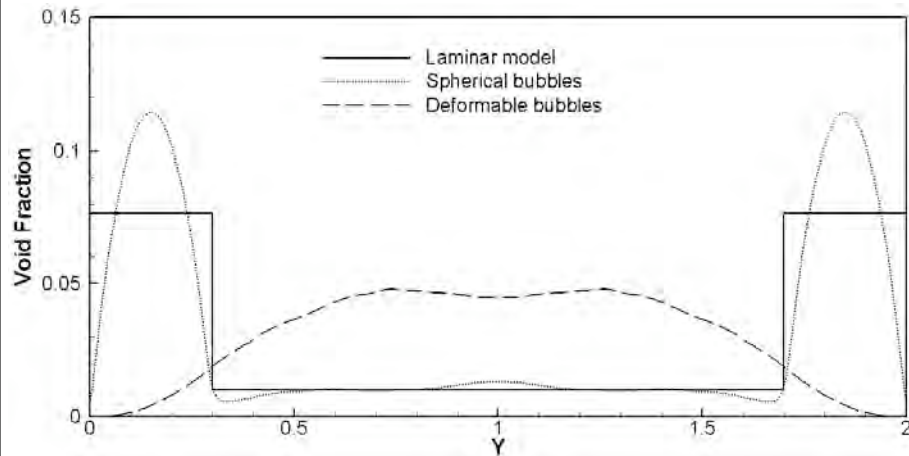




DNS of Multiphase Flows

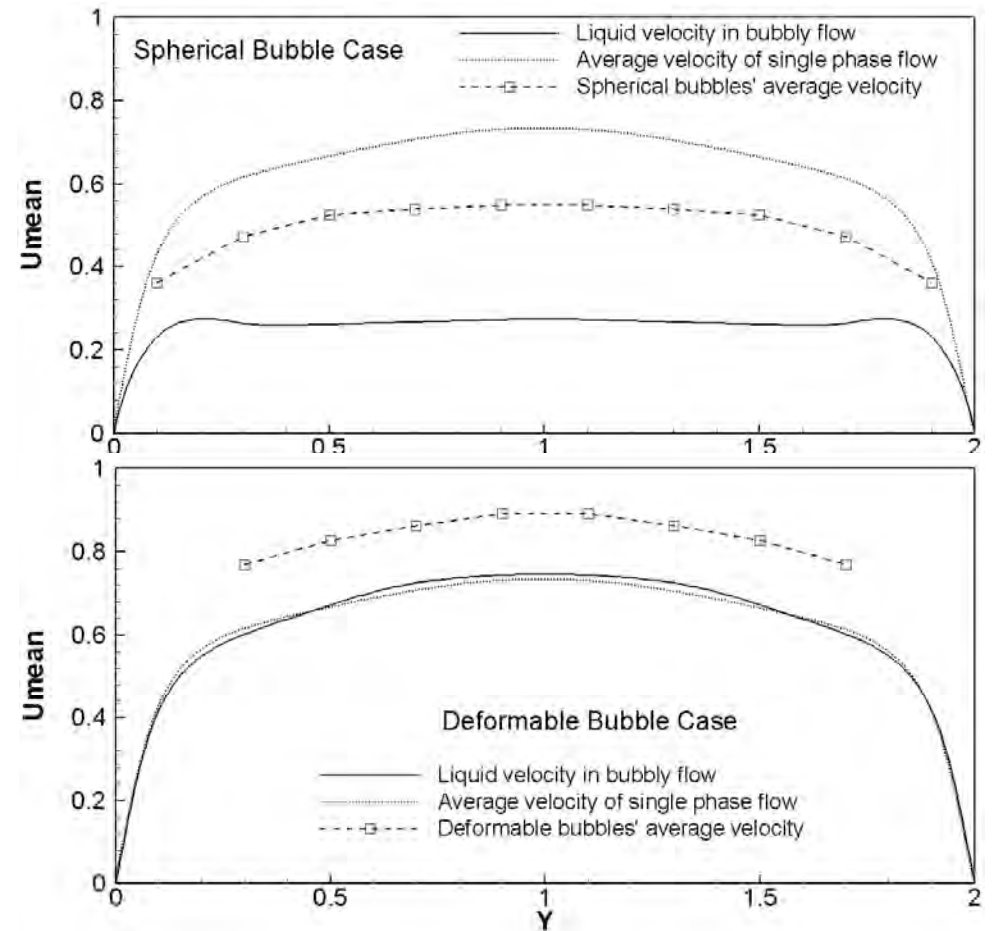
Effect of Deformability

Void fraction



The effect of bubble deformability on the statistics of turbulent bubbly upflow

Average velocity

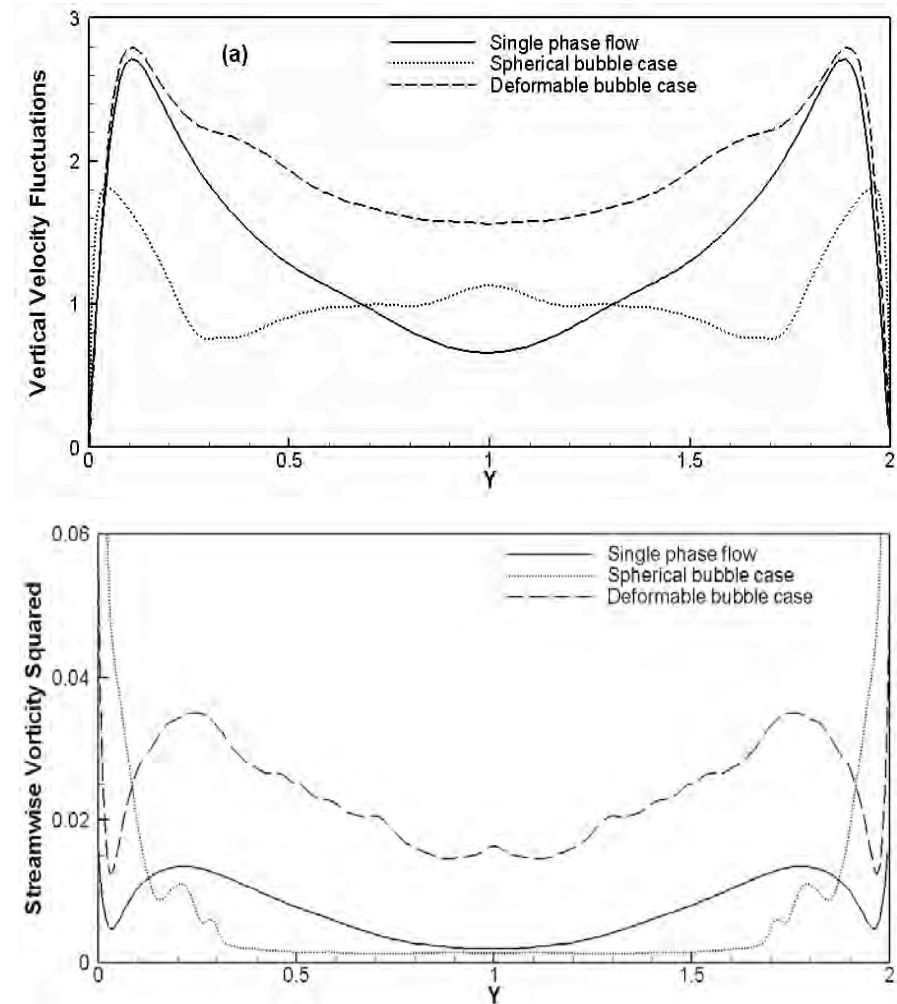




DNS of Multiphase Flows

Effect of Deformability

The average vertical velocity fluctuations (top) and the streamwise vorticity squared (bottom) for single phase flow, the nearly spherical bubbles and the more deformable bubbles versus the cross-channel coordinate at approximately steady state.





DNS of Multiphase Flows

Bubbly flows in channels—Summary

For nearly spherical bubbles the flow consists of a homogeneous core where the mixture is in hydrostatic equilibrium and a wall-layer.

For upflow the wall-layer is bubble rich and the total flow rate depends strongly on the deformability of the bubbles.

For downflow the wall-layer has no bubbles and the velocity profile is easily found for both laminar and turbulent flow. For downflow the exact size of the bubbles plays only a minor role, as long as they remain nearly spherical.

For upflow deformable bubbles stay away from walls, completely changing the flow structure



DNS of Multiphase Flows Conclusions

We now have the ability to conduct DNS of disperse multiphase flows for a fairly large range of situations. These simulations have been shown to yield insight of the same type obtained from DNS of single phase turbulent flows

Modeling of multiphase flows is far behind single phase flows and the development of more advanced models that can incorporate the new insight and data is perhaps the most urgent task in computational studies of multiphase flows

Numerical challenges still remain, but those are of critical importance primarily for flows that are more complex than those discussed here (churn-turbulent, phase change, etc.)