Anisotropy of transport properties in Magnetohydrodynamics turbulent flows

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(Turbulence and Interactions 2006 - Porquerolles)

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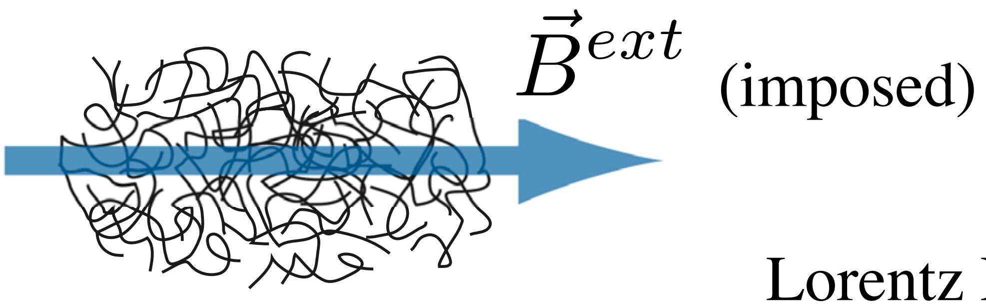
Conductive flows in the limit Rm<<1

Problem:



Lorentz Force

$$\left(\frac{\partial}{\partial t} + \vec{u}\cdot\vec{\nabla}\right)\vec{u} = -\vec{\nabla}\left(p/\rho\right) + \nu\Delta\vec{u} + \frac{1}{\rho}\vec{J}\times\vec{B}$$

$$R_{\mathrm{m}} \ll 1:\ \frac{1}{\rho}\overrightarrow{J}\times\overrightarrow{B} \approx -\frac{\sigma B_z^2}{\rho}\Delta^{-1}\partial_z\partial_z\overrightarrow{u}$$

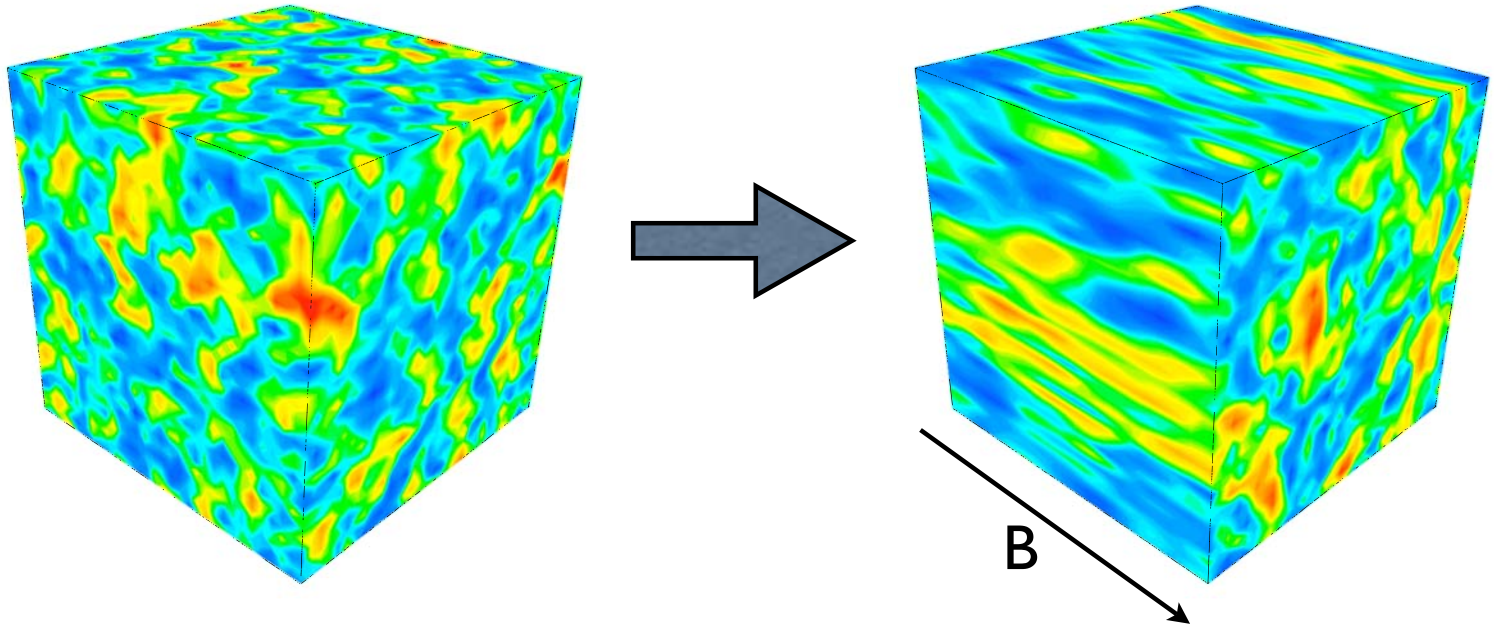
$$N = \frac{\sigma B_z^2 L}{\rho u}$$

Interaction parameter

Quasi-static approximation

$$\left(\frac{\partial}{\partial t} + \overrightarrow{u} \cdot \overrightarrow{\nabla}\right) \overrightarrow{u} = -\overrightarrow{\nabla}(p/\rho) + \nu \Delta \overrightarrow{u} - \frac{\sigma B_z^2}{\rho} \Delta^{-1} \partial_z \partial_z \overrightarrow{u}$$

Joule dissipation: $-\frac{\sigma B_z^2}{\rho}[\cos^2\theta(\overrightarrow{B}_0, \overrightarrow{k})]\, \hat{u}_i$



Passive scalar transport

$$\left(\frac{\partial}{\partial t} + \overrightarrow{u} \cdot \overrightarrow{\nabla}\right)\Theta = \lambda \Delta \Theta$$

- scalar: temperature, concentration...
- not explicitly anisotropic, only through u
- no source term, no buoyancy

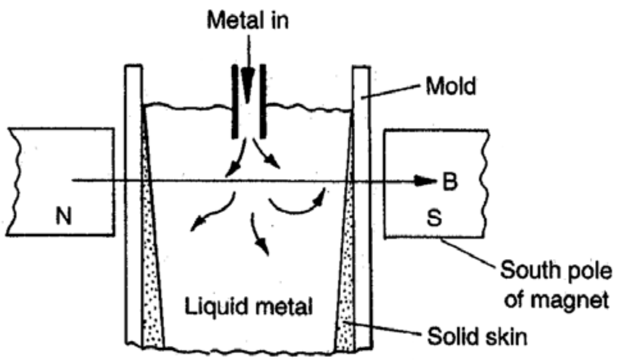
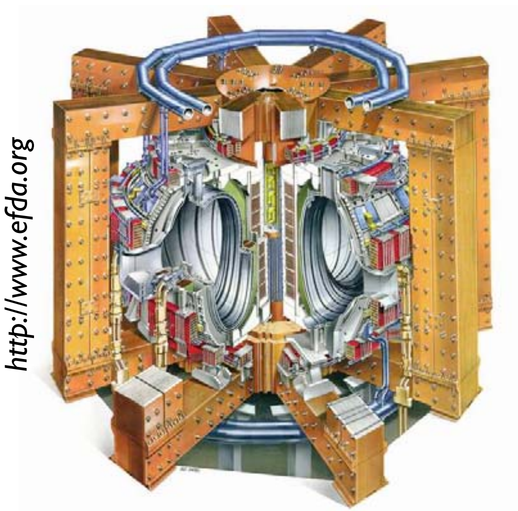


Figure I.5 Magnetic damping.
(Davidson, 2001)



http://www.efda.org

Semi-conductor crystal growth

unknown origin

Numerical simulations

Direct Numerical Simulations:

Homogeneous domain with periodic boundary conditions (spectral code, dealiased)

Resolution : 256^3

Forced velocity field : energy injected isotropically at large scales

The scalar field is initialized isotropically in the stationary flow,
then it decays freely in the turbulent system.

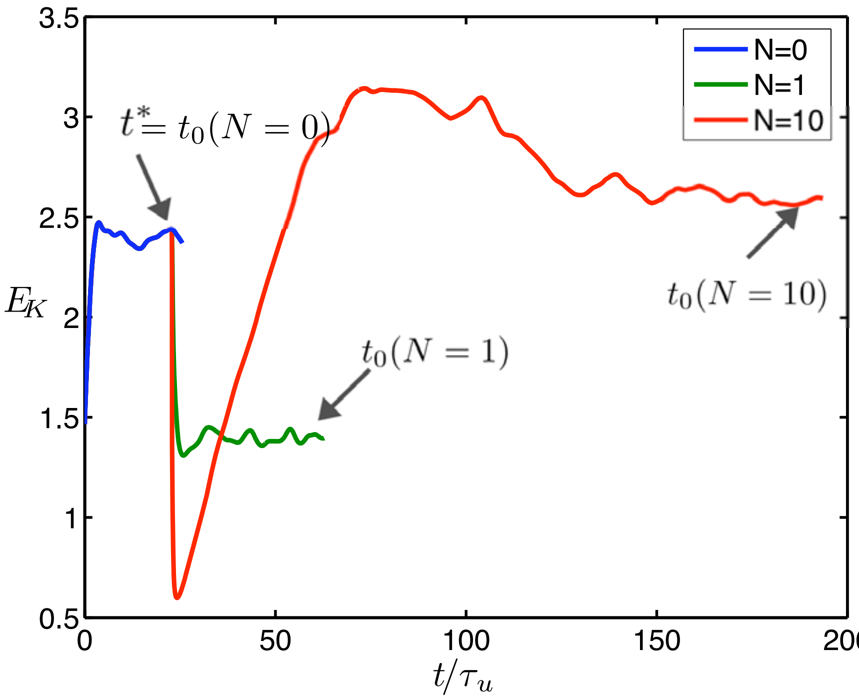
Interaction parameter : N=0, N=1, N=10

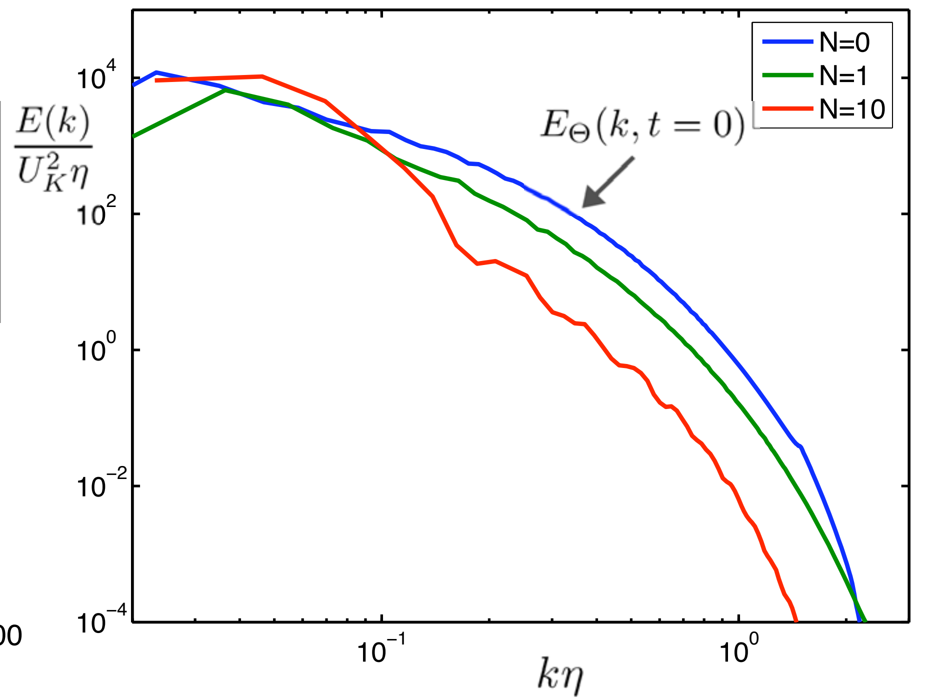
Initial condition: $E_\Theta(k, t=0) \propto E_u(k)$

$R_\lambda \sim 90$ (N=0)

$$Sc = \frac{\nu}{\lambda} = 1$$

Velocity field





$\hookrightarrow \tau_u = L(N=0)/u(N=0)$

<u>Kolmogorov scaling :</u>

$$\eta = \left(\frac{\nu^3}{<\epsilon>}\right)^{1/4}$$

$$U_K = (\nu <\epsilon>)^{1/2}$$

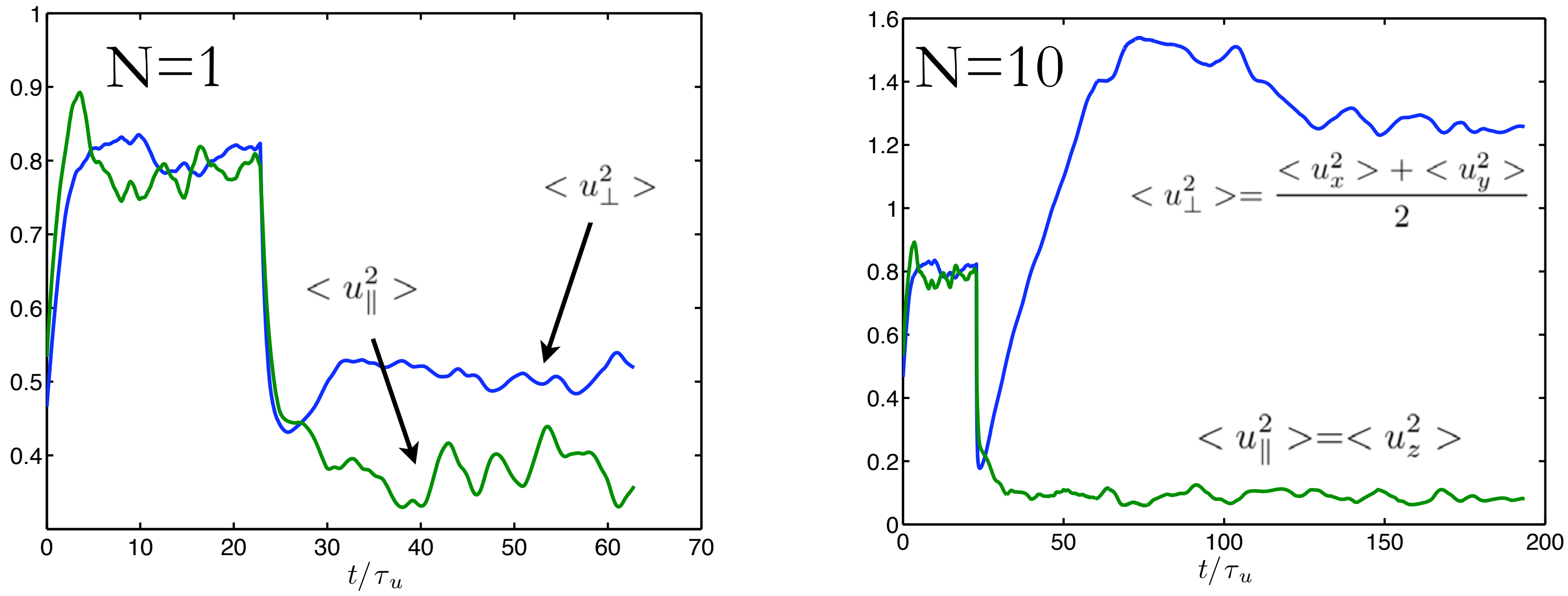
$\eta_{(N=0)} = 0.0117$

$\eta_{(N=1)} = 0.0182$

$\eta_{(N=10)} = 0.0231$

Velocity anisotropy

1) Velocity components anisotropy



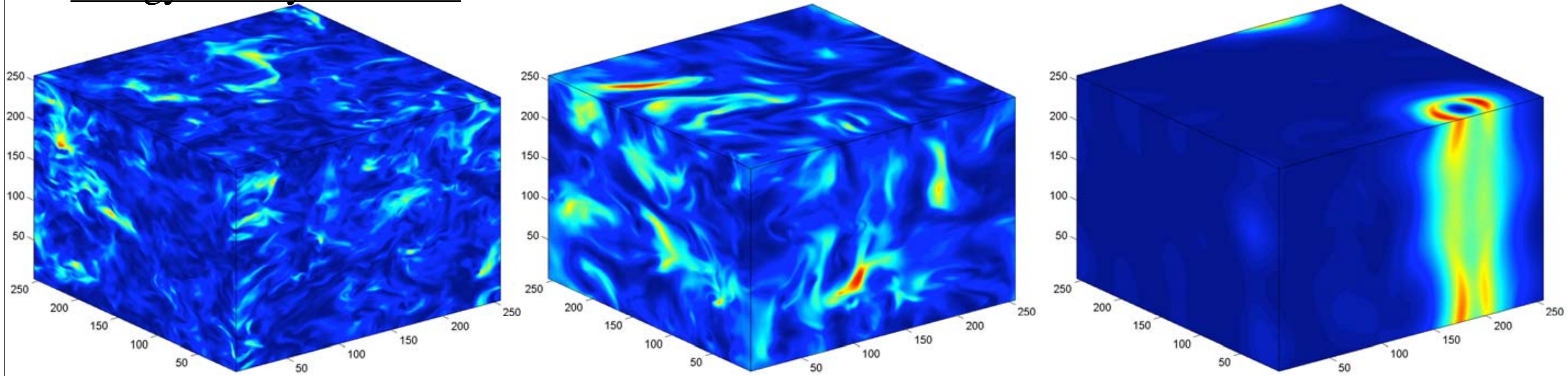
2) Velocity gradients anisotropy:

$$G = \frac{< (\partial_z u_x)^2 >}{< (\partial_y u_x)^2 >} \quad (\simeq 1 \text{ for isotropic turbulence})$$

N=0 : 1.02 ; N=1 : 0.44 ; N=10 : 0.066

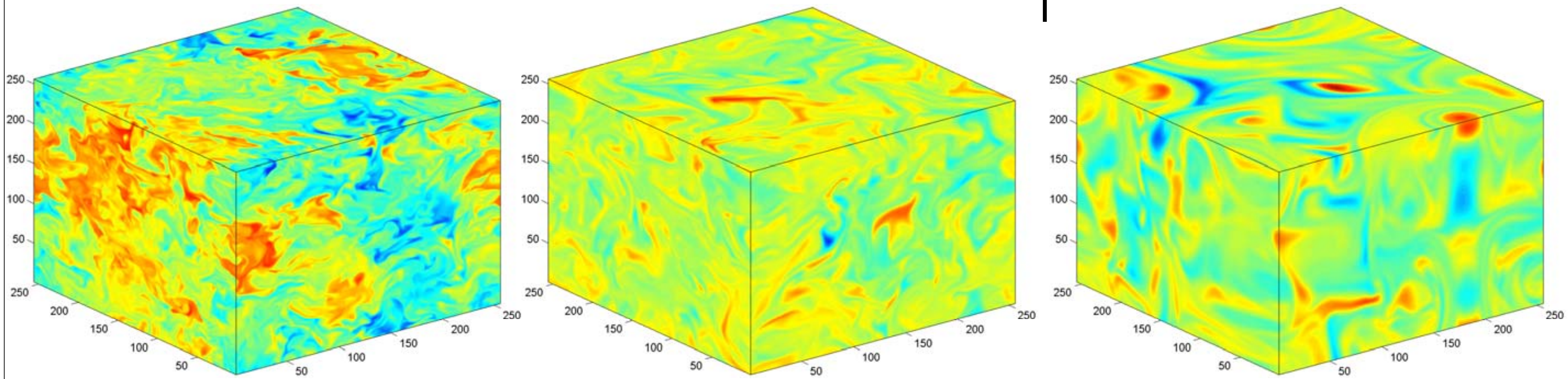
Fields structure

Energy density contours:



N=0 N=1 $\vec{B}$ N=10

Scalar variance contours:



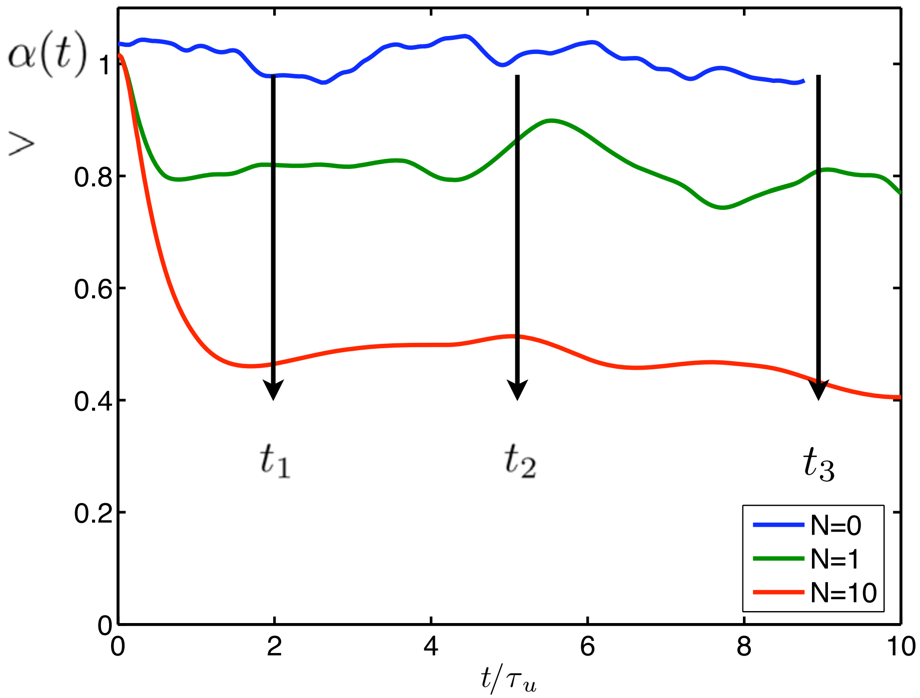
Scalar anisotropy

Anisotropy coefficient :

$$\alpha(t) = \frac{<(\partial_{||}\Theta)^2>}{<(\partial_{\perp}\Theta)^2>}$$

$$<(\partial_{||}\Theta)^2> = <(\partial_z\Theta)^2>$$

$$<(\partial_{\perp}\Theta)^2> = \frac{<(\partial_x\Theta)^2> + <(\partial_y\Theta)^2>}{2}$$



$\Longrightarrow$ Gradients in the direction parallel to $\vec{B}$ are damped faster than in the direction perpendicular.

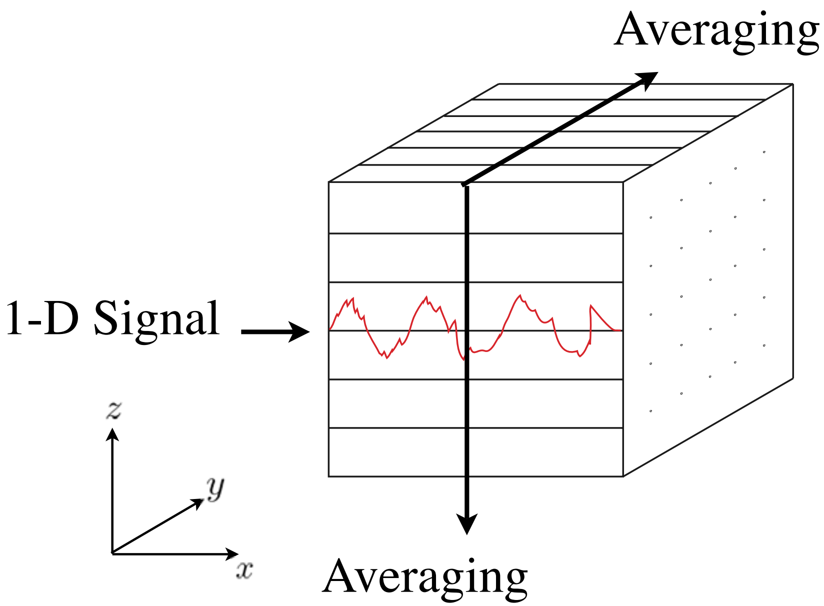
$\Longrightarrow$ The anisotropy coefficient decays quickly at the beginning, then stabilizes to a stationary value.

Anisotropy in spectral space

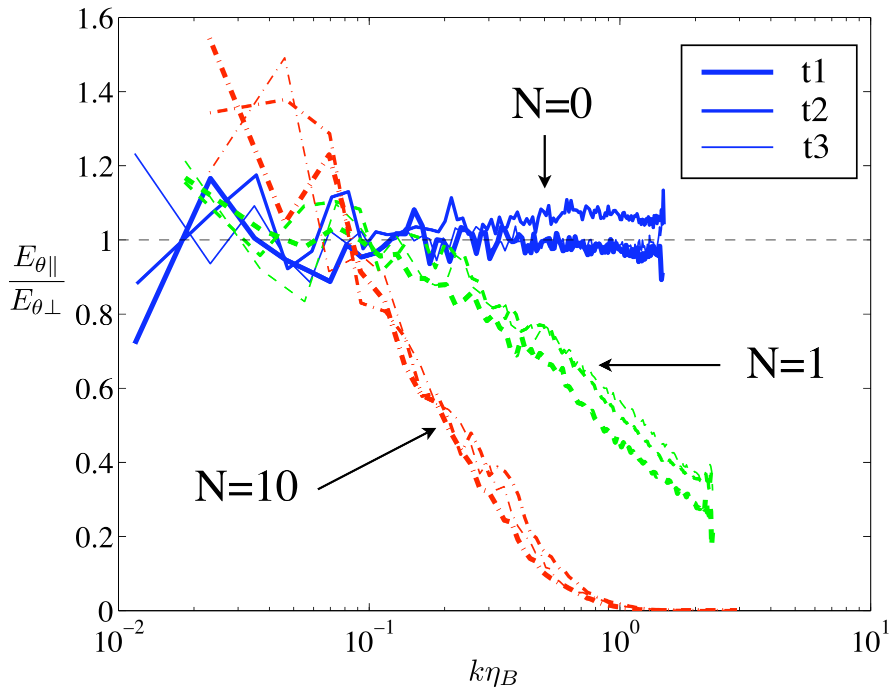
1-Dimensional spectra :

$$R_x(r) = < \Theta(x,y,z)\Theta(x+r,y,z) >$$

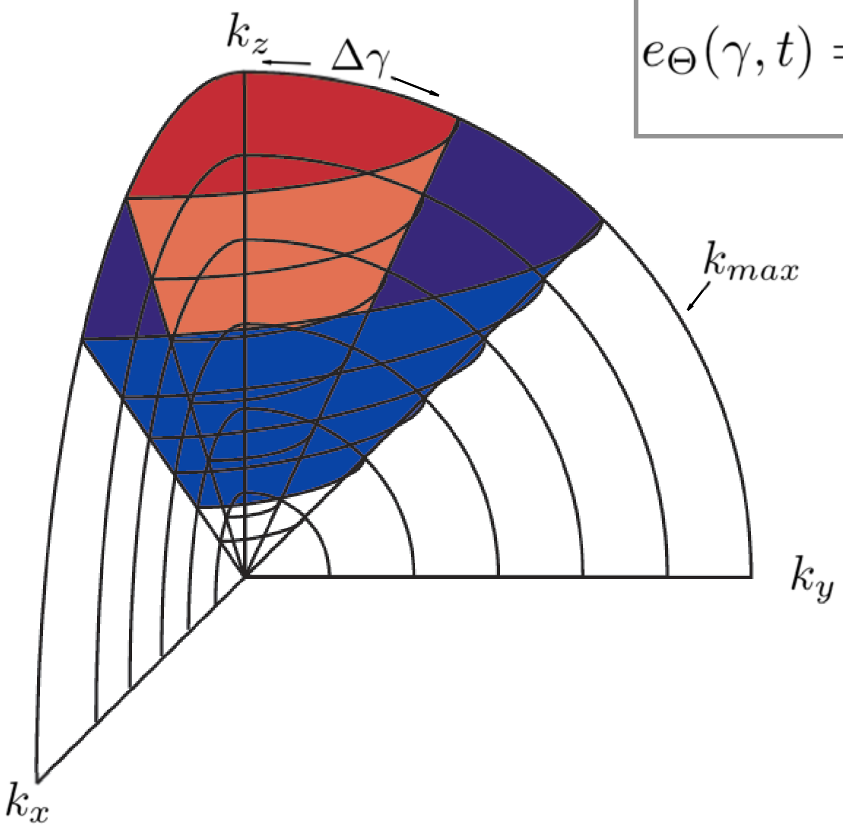
$$\Longrightarrow E_{\Theta\ x}(k_x) = \mathcal{F}\{R_x(r)\}$$



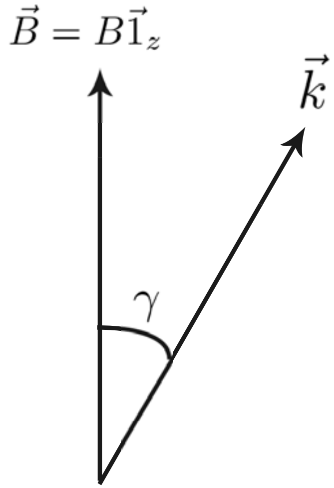
$$E_{\Theta\|}(k) = E_{\Theta z}(k)$$
$$E_{\Theta\perp}(k) = \frac{1}{2}(E_{\Theta x}(k) + E_{\Theta y}(k))$$

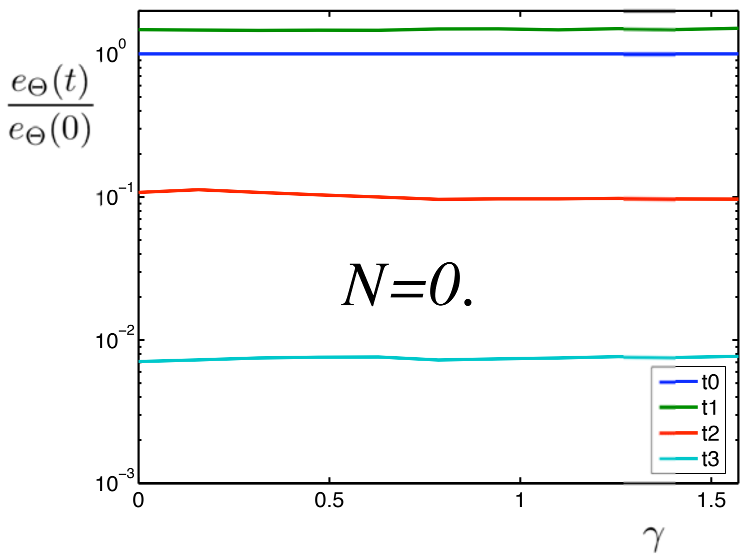


Angular energy partition (1)

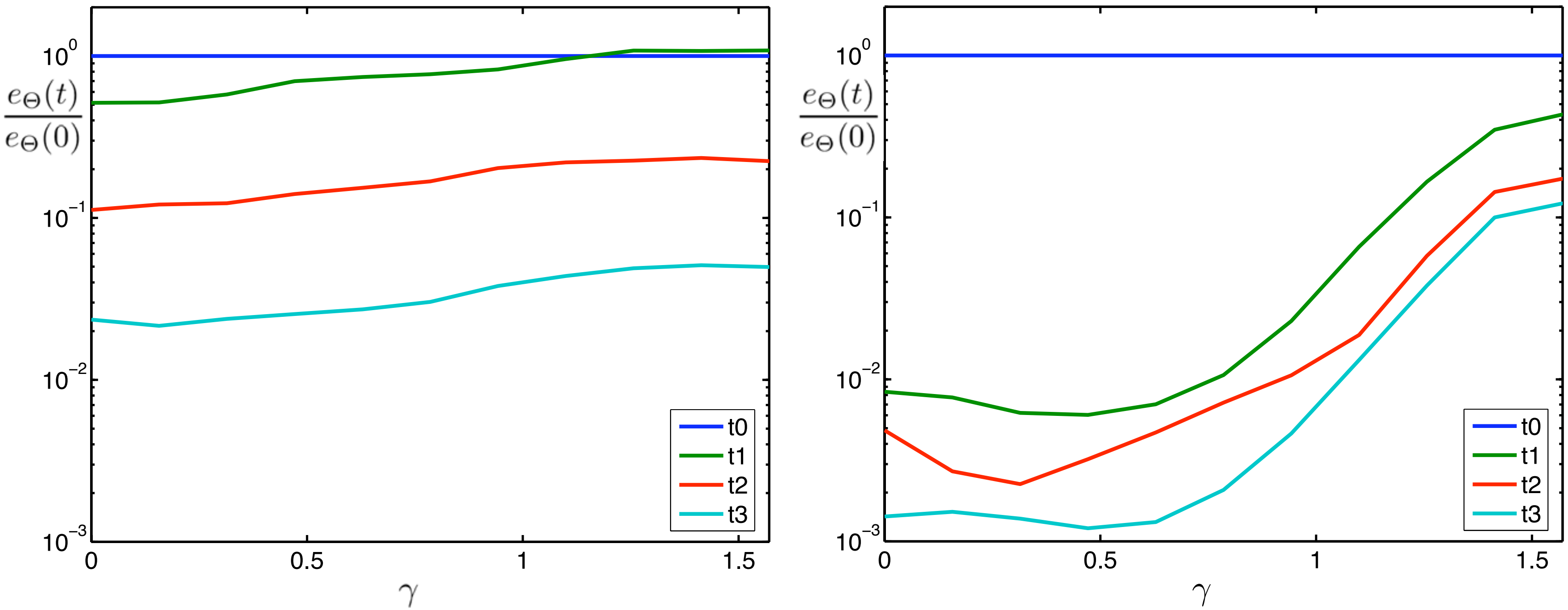


$$e_\Theta(\gamma,t) = \frac{\int_0^{k_{max}} E_\Theta(k,\gamma,t)\,dk}{N(\gamma)}$$



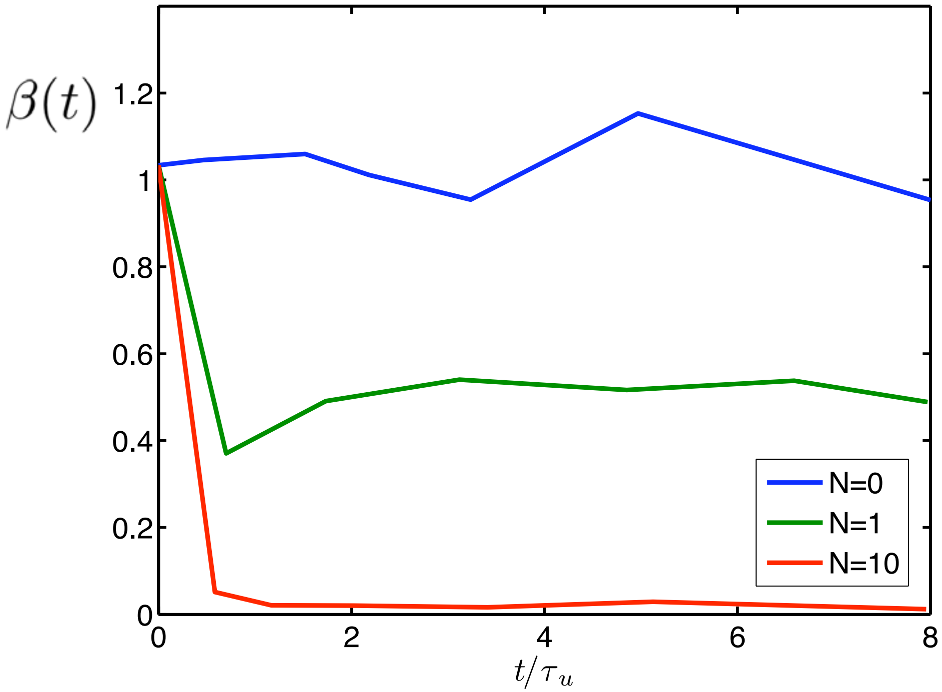


Angular energy partition (2)



N=1 *N=10*

Angular energy partition (3)



$$\beta(t) = \frac{e_\Theta(\gamma = 0, t)}{e_\Theta(\gamma = \pi/2, t)}$$

$\Longrightarrow$ Ratio of the scalar energy contained close to $\vec{B}$ axis, to scalar energy contained in the perpendicular plane.

Some conclusions

- Anisotropy of the scalar observed through anisotropy coefficient and angular spectra.

- Fourier modes having a component parallel to the magnetic field are dissipated quickly than others.

- Starting from an isotropic state, the scalar seems to evolve to an asymptotic level of anisotropy.

- Anisotropy strongest in the smallest scales of the scalar.

- Further study should include the Schmidt number effects.