

Compressible turbulent boundary layers with non-adiabatic walls

- Mean gradients effects : $T(y)-\rho(y)-\mu(y)-\lambda(y)$
- Morkovin hypothesis : $p_{rms} \ll \langle P \rangle - T_{trms} \leq 20\% \langle T_t \rangle$

'The influence of compressibility that Morkovin's hypothesis does not treat are the effect of viscosity, in regions where mean viscous stresses are important.'

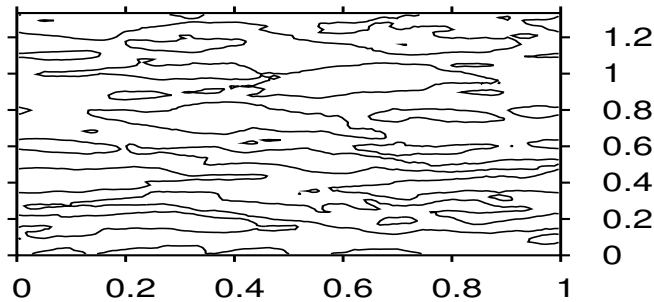
(Bradshaw Ann. Rev. Fluid Mech. 1977)

- Strong Reynolds Analogy
- Inner layer scaling

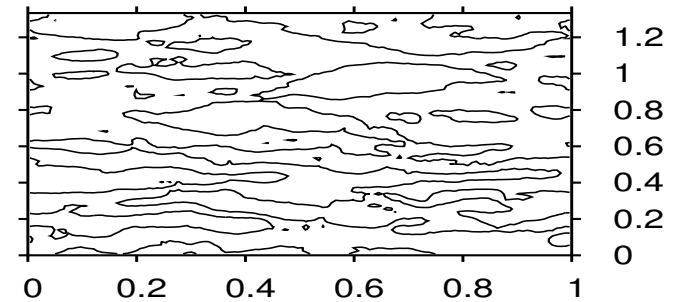
'The large scale motion should be statistically coupled to the thermal field almost exclusively through mean values of ρ , μ , λ , and the generalized law of the wall so that with a variable lateral stretching factor, it may resemble the incompressible motion.'

(M.V. Morkovin Colloques Internationaux CNRS 1962)

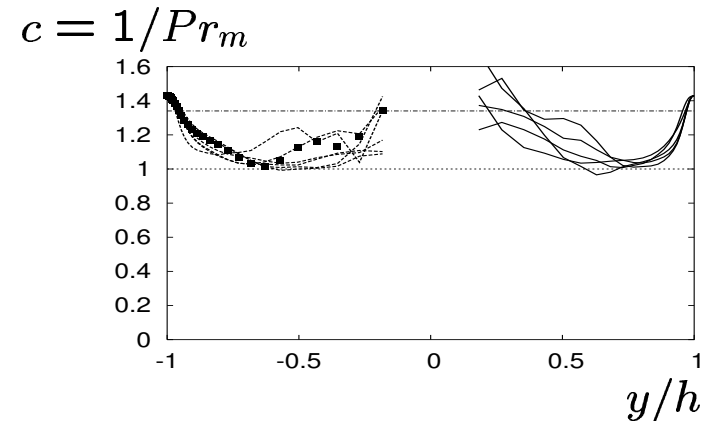
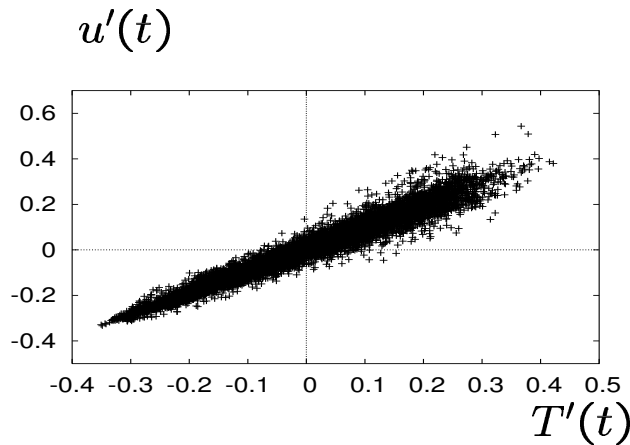
Strong Reynolds Analogy



wall friction : $\tau_w = \tilde{\mu} \frac{\partial \tilde{u}}{\partial y} \Big|_w = \rho_w u_\tau^2$



Wall heat flux : $q_w = -\tilde{\lambda} \frac{\partial \tilde{T}}{\partial y} \Big|_w = -\rho_w c_p u_\tau T_\tau$



boundary layer : MSRA (Gaviglio 1987, Rubesin 1990, Huang et al. 1995):

$$\frac{T'(t)}{\langle \tilde{T} \rangle} = \frac{1}{c(\partial \langle \tilde{T}_i \rangle / \partial \langle \tilde{T} \rangle - 1)} (\gamma - 1) M_l^2 \frac{u'(t)}{\langle \tilde{u} \rangle}$$

Inner Layer Scaling

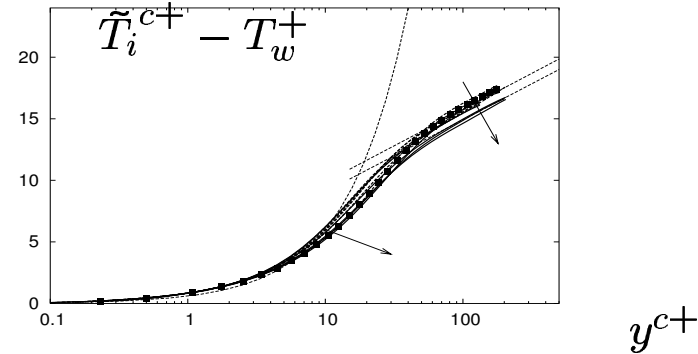
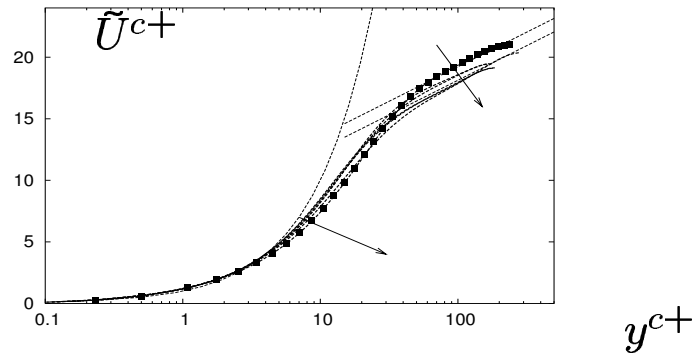
Integral wall scaling: $y^{c+} = \int_0^{y^+} \frac{\mu_w}{\bar{\mu}} dy^+$

$Re_\tau^c = h^{c+} = \int_0^{h^+} \frac{\mu_w}{\bar{\mu}} dy^+$

'Semi-local scaling' $Re_{sl} = \frac{\rho_e u_\tau^e h}{\mu_e}$

(Huang & Coleman *J. Fluid Mech.* 1995)

van Driest Transformation : density & viscosity correction



$$\langle \tilde{u} \rangle^{c+} = \int_0^{\langle \tilde{u} \rangle^+} \frac{y^+ \mu_w}{y^{c+} \langle \bar{\mu} \rangle} \sqrt{\frac{\langle \bar{\rho} \rangle}{\rho_w}} d\langle \tilde{u} \rangle^+ = \frac{1}{\kappa} \ln y^{c+} + C_u^c$$

$$\langle \tilde{T}_i \rangle^{c+} - T_w^+ = \frac{Pr_t}{\kappa} \ln y^{c+} + C_t^c$$

