RESULTS FROM SIMULATING AND MODELING COMPRESSIBLE TURBULENT FLOWS

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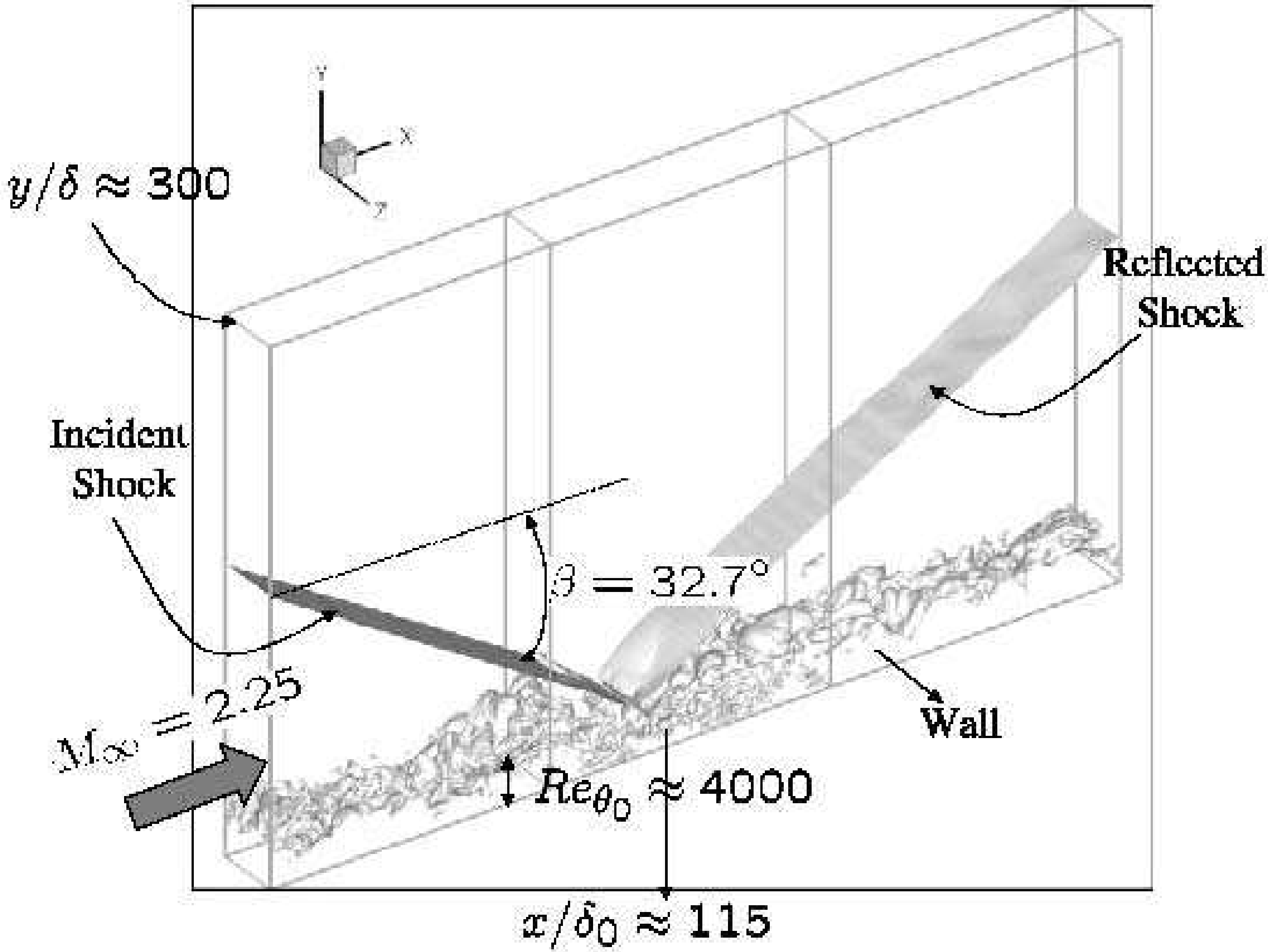
DIRECT NUMERICAL SIMULATIONS

- Impinging shock/boundary layer interaction simulation
  - Extends shock-free spatial simulation results
    - *Phys. Fluids, Vol. 16(3), pp. 530-545, 2004*, with S. Pirozzoli and F. Grasso
  - Numerical Algorithm
  - Physical/Computational Domains
  - Mean field simulation results

DOMAIN PARAMETERS FOR IMPINGING SHOCK CASE



NUMERICAL ALGORITHM/COMPUTATIONAL DOMAIN

- Full 3D, compressible, unsteady NS equations solved
  - 7th-order WENO discretization of Euler fluxes
  - 4th-order compact dicretization of viscous fluxes
  - 4th order RK scheme for time advancement
- Grid resolution: $2650 \times 111 \times 255$
- Domain size: $L_x^+ = 100000$, $L_y^+ = 30000$, $L_z^+ = 1.380$
- Grid spacings (interaction zone): $\Delta_x^+ = 15$, $\Delta_{yw}^+ = 1$, $\Delta_z^+ = 6$

BOUNDARY CONDITIONS

- Boundary Conditions
  - Inflow: Compressible, Blasius boundary layer solution
  - Outflow:
    - buffer domain used with coarsening in x-direction
      - first order extrapolation in the supersonic region;
      - uniform pressure in the subsonic sublayer
  - Far-field: nonreflecting boundary conditions

BOUNDARY CONDITIONS (cont.)

- Wall

$$u = w = 0 \; , \; v = v_{bs} \; , \; T = T_\infty (1 + \mathrm{Pr}^{1/3} \frac{\gamma - 1}{2} M_\infty^2)$$

$$v_{bs}(x, z, t) = \begin{cases} 0 & x < x_a, \quad x > x_b \\ \mathcal{A} u_\infty f(x) g(z) h(t) & x_a \le x \le x_b \end{cases} ,$$

$$\begin{aligned} f(x) &= 4 \sin\theta \, (1 - \cos\theta) / \sqrt{27}, \quad \theta = 2\pi \, (x - x_a) / (x_b - x_a) \\ g(z) &= \sum_{l=1}^{l_{max}} Z_l \sin(2\pi l \, (z/z_{max} + \phi_l)) \, , \sum_{l=1}^{l_{max}} Z_l = 1, \quad Z_l = 1.25 \, Z_{l+1} \\ h(t) &= \sum_{m=1}^{m_{max}} V_m \sin(2\pi \, (\beta t + \phi_m)) \, , \sum_{m=1}^{m_{max}} V_m = 1, \quad V_m = 1.25 \, V_{m+1} \end{aligned}$$

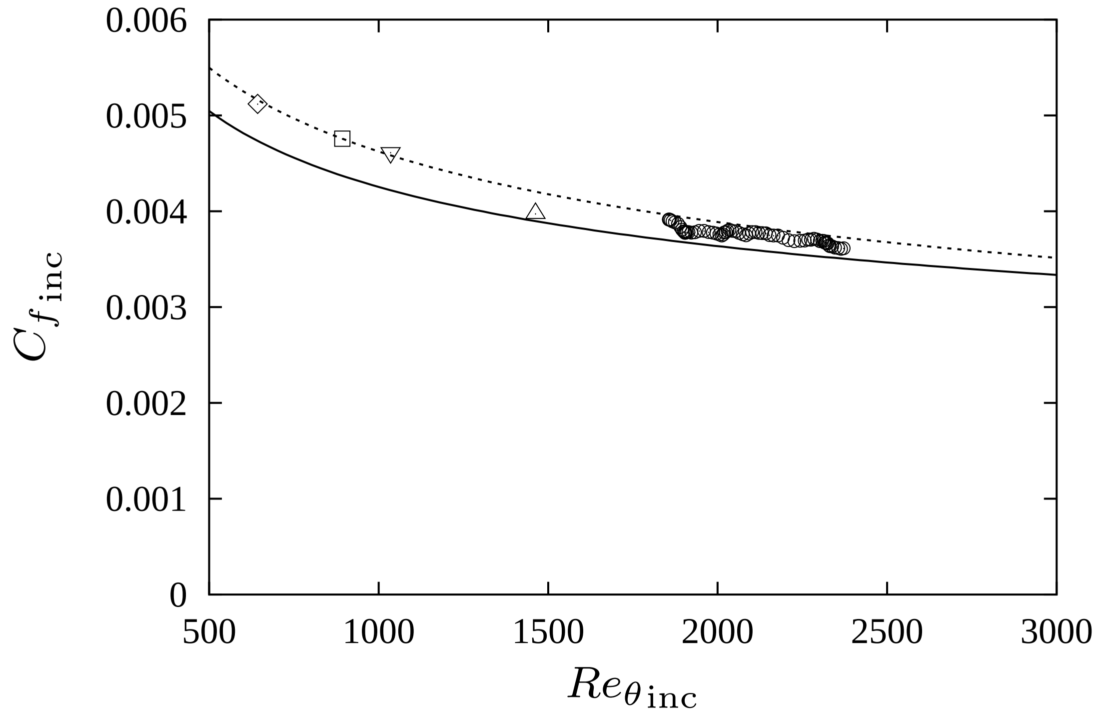
IMPINGING SHOCK SIMULATION RESULTS

- Streamwise mean pressure variation
- Streamwise skin-friction variation
- Streamwise variation of Van Driest velocity
- Contours in $x - y$ plane
  - Instantaneous and mean density
  - Instantaneous and mean pressure
  - Instantaneous and mean velocity
  - Turbulent shear stress
  - Turbulent kinetic energy

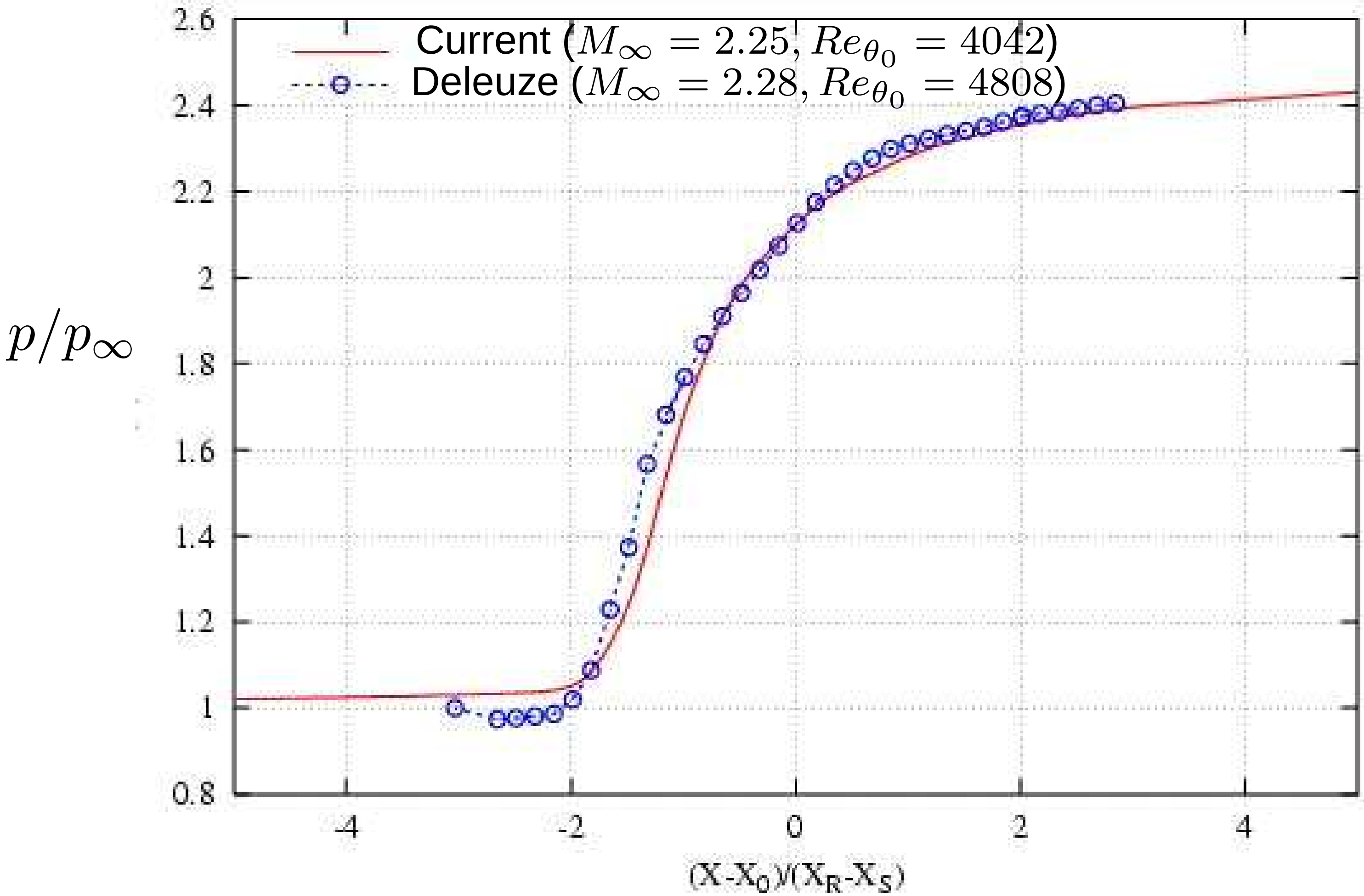
RELATED DNS STUDIES (WITHOUT SHOCK)

- Guarini, Moser, Shariff and Wray (2000)
  Approximate spatial DNS (under the approximation of slow boundary layer growth)
  Mixed Fourier and B-spline Galerkin method
  $M_\infty = 2.5$, $\mathrm{Re}_\theta = 1577$ Grid $256 \times 209 \times 192$
- Maeder, Adams and Kleiser (2001)
  Extended temporal DNS simulation
  Mixed compact-pseudospectral finite-difference method
  $3 \leq M_\infty \leq 6$, $\mathrm{Re}_\theta \approx 3000$ Grid $192 \times 144 \times 180$ (at $M_\infty = 3$)

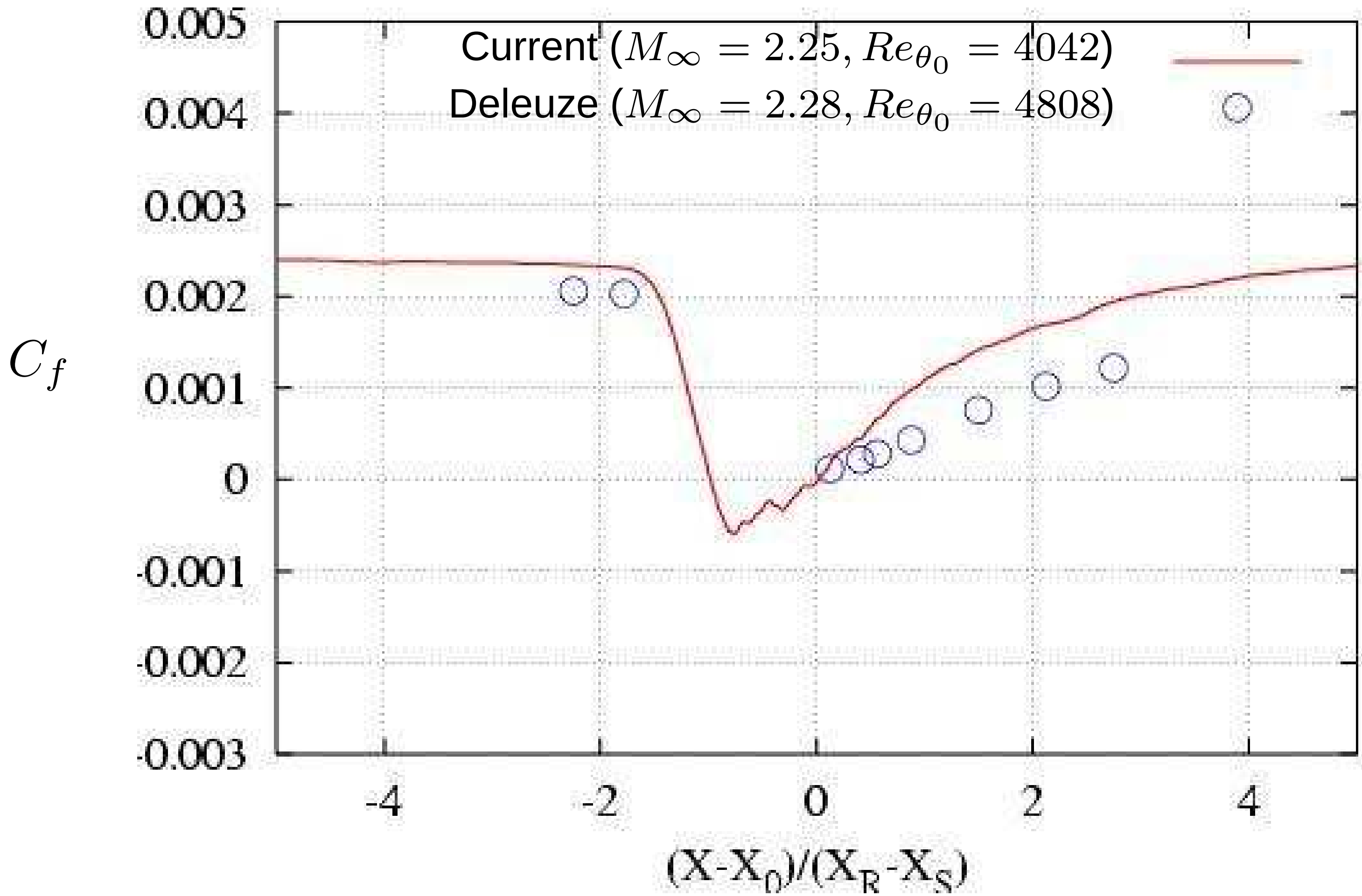
RELATED DNS STUDIES (WITHOUT SHOCK)



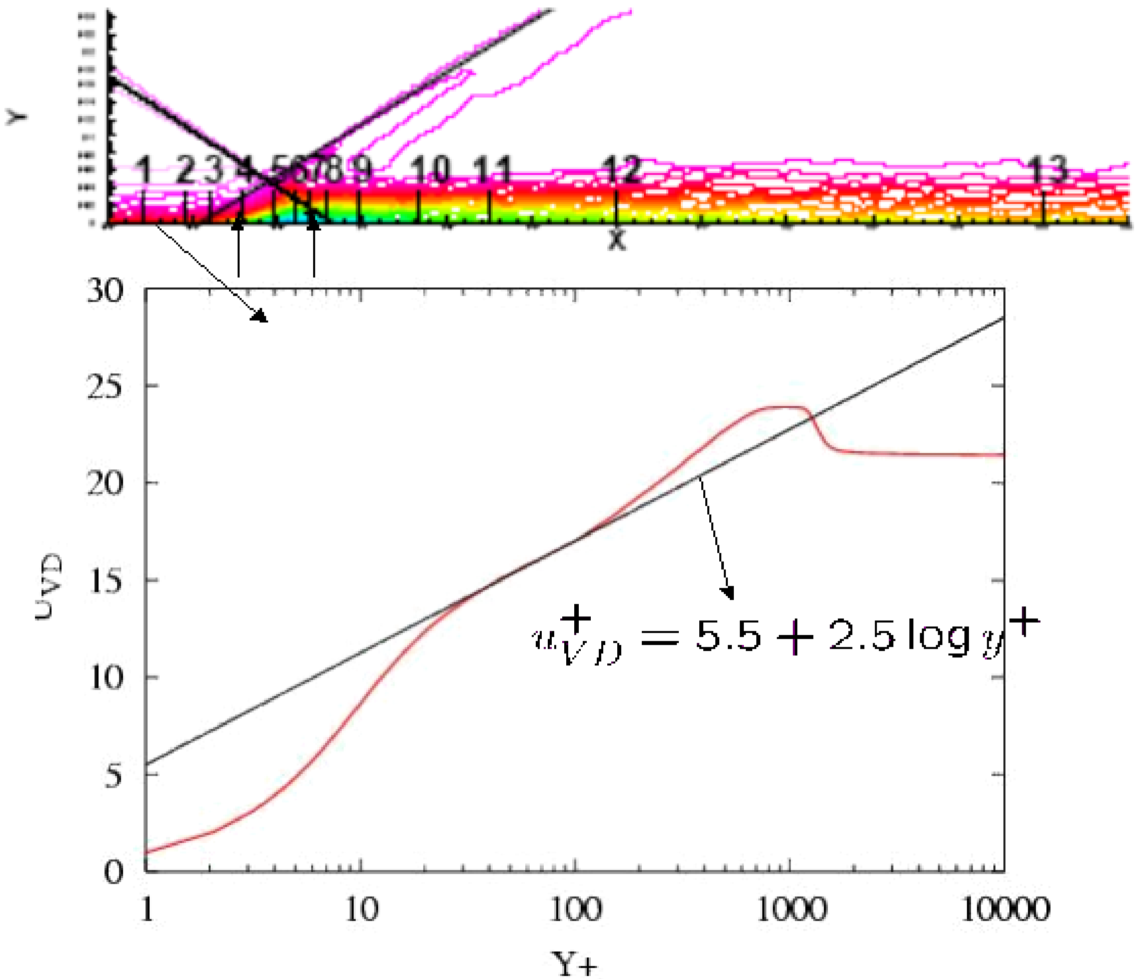
STREAMWISE MEAN PRESSURE VARIATION



STREAMWISE SKIN FRICTION VARIATION



STREAMWISE VARIATION OF VAN DRIEST VELOCITY



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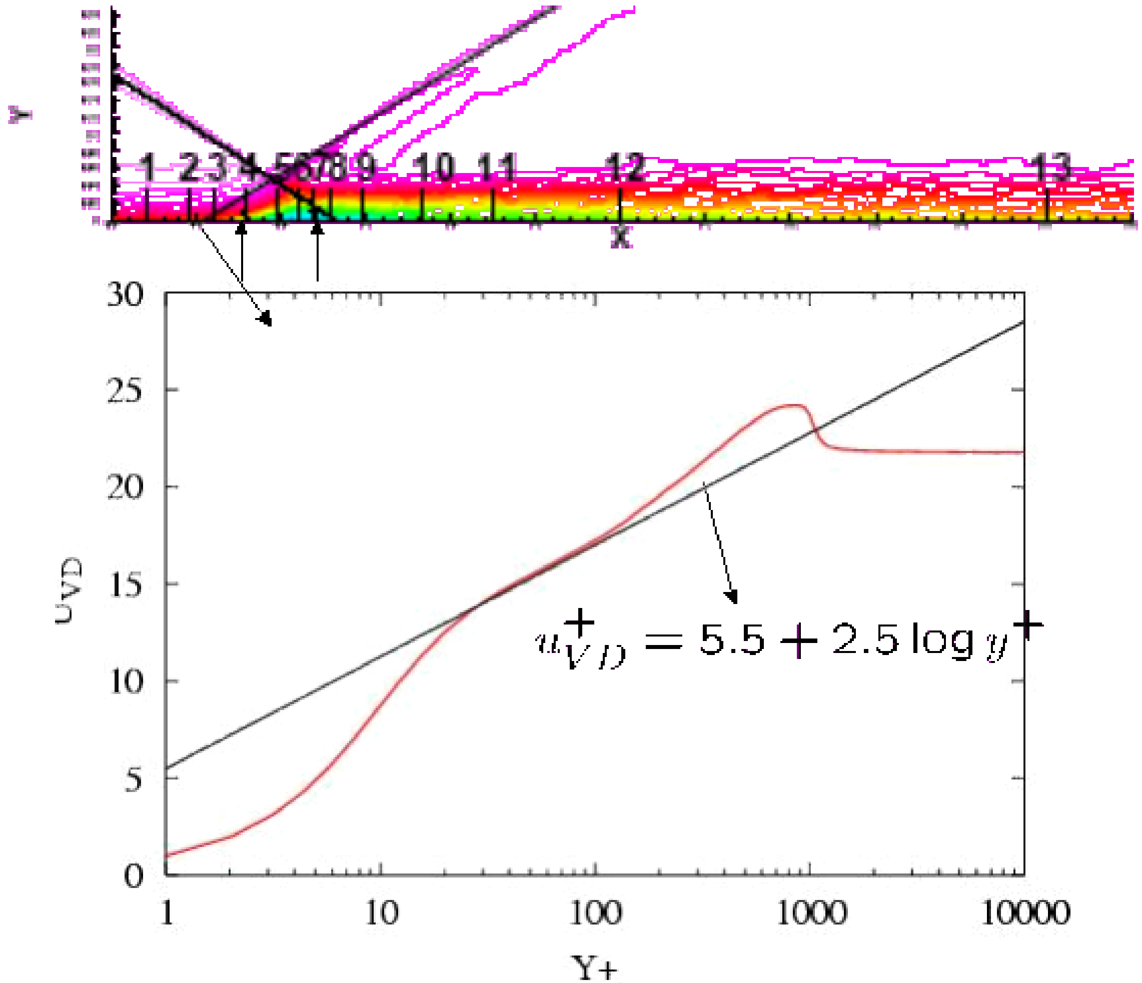

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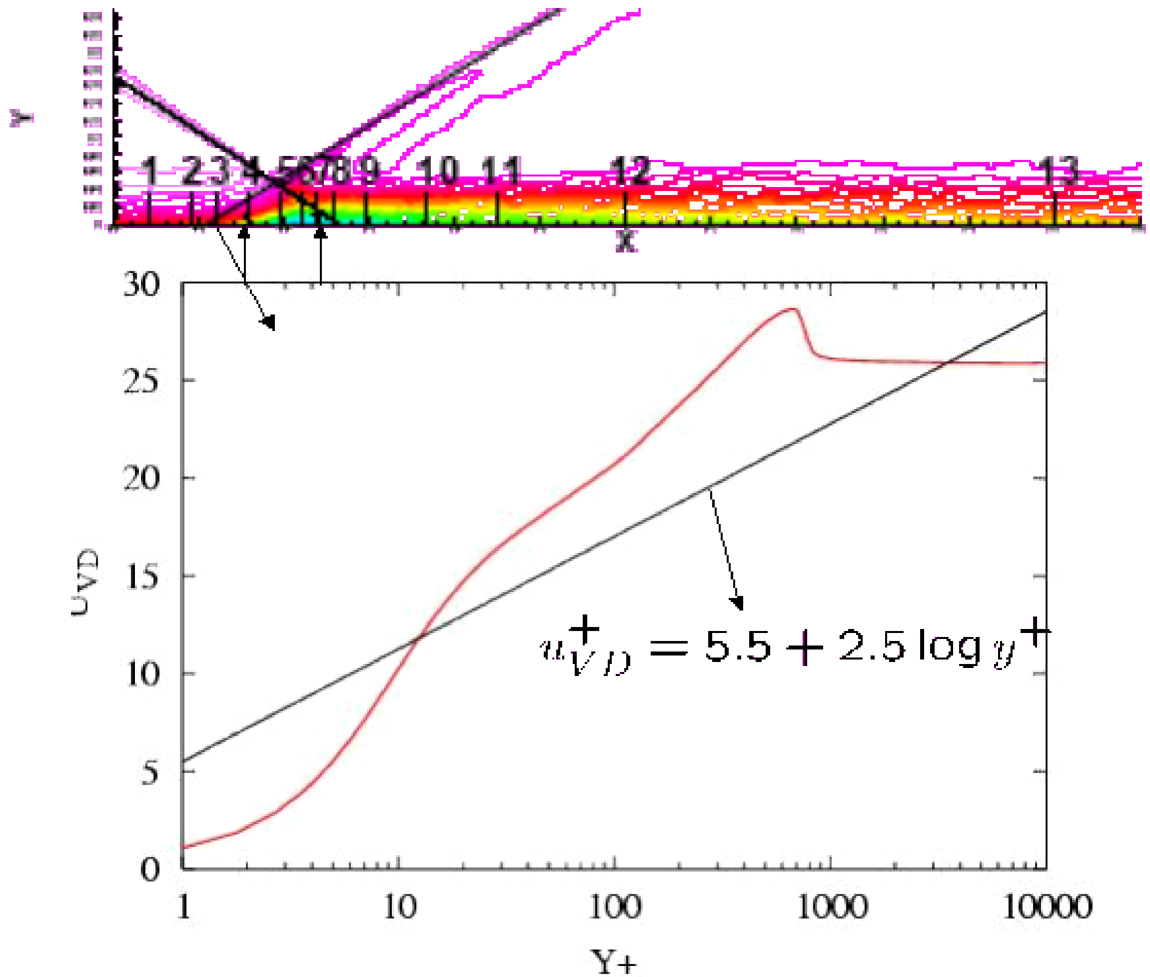

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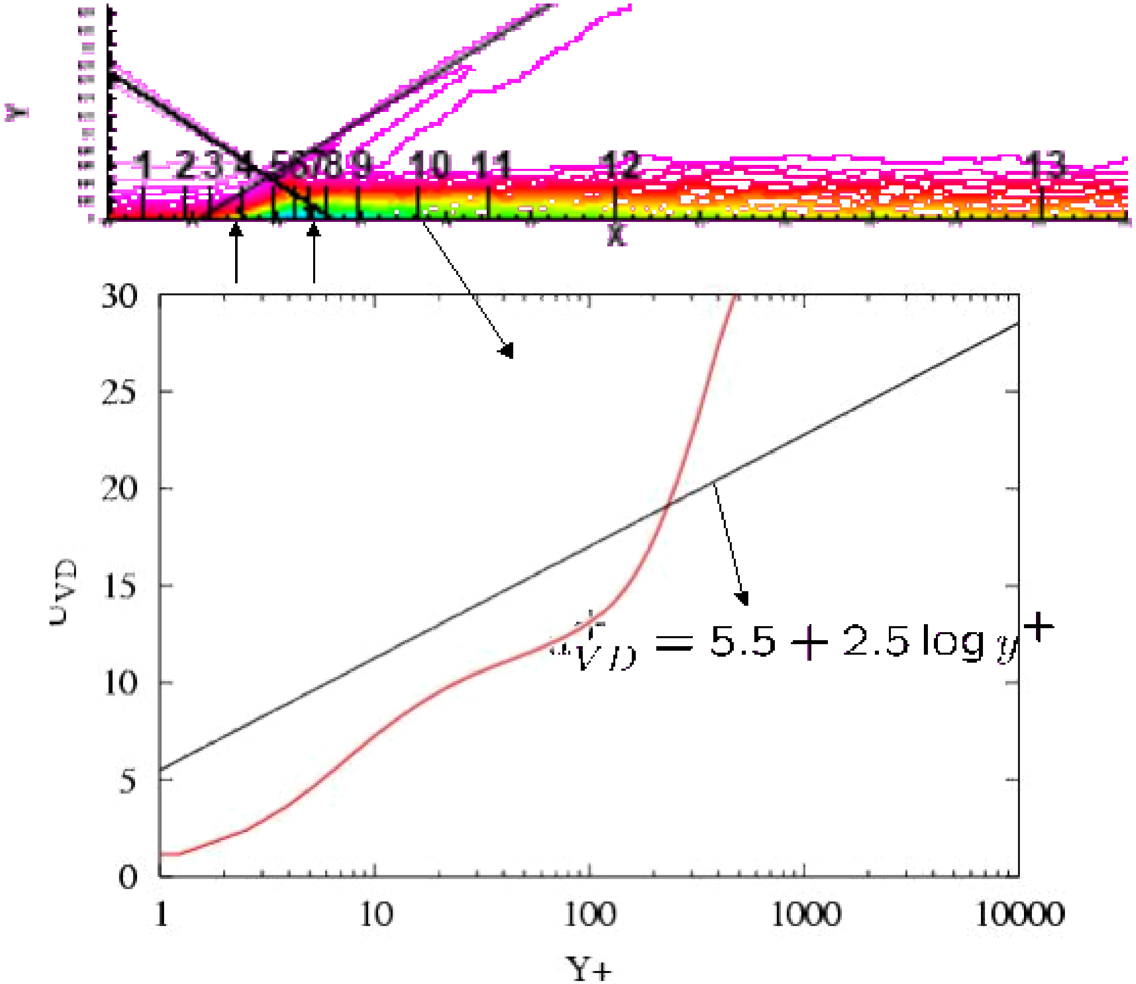

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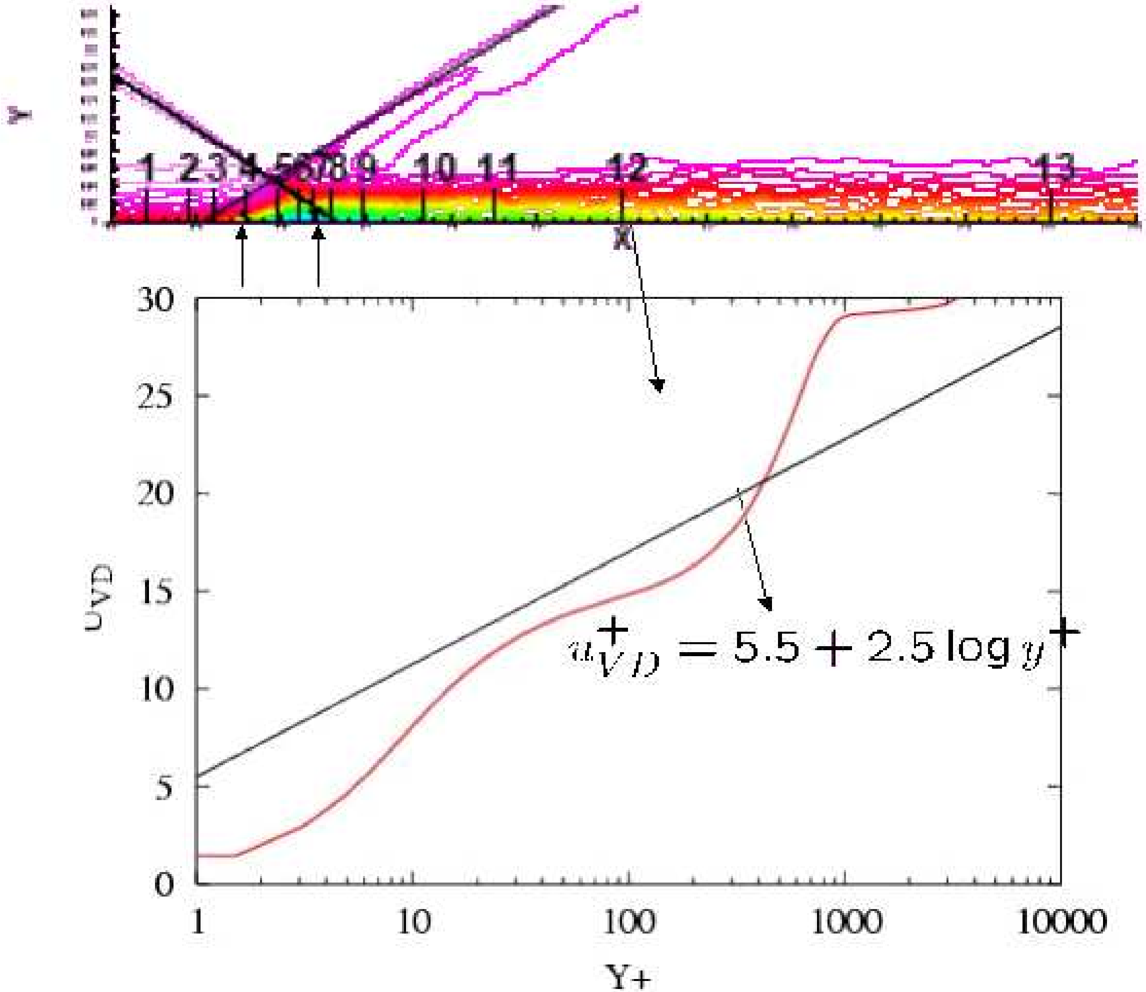

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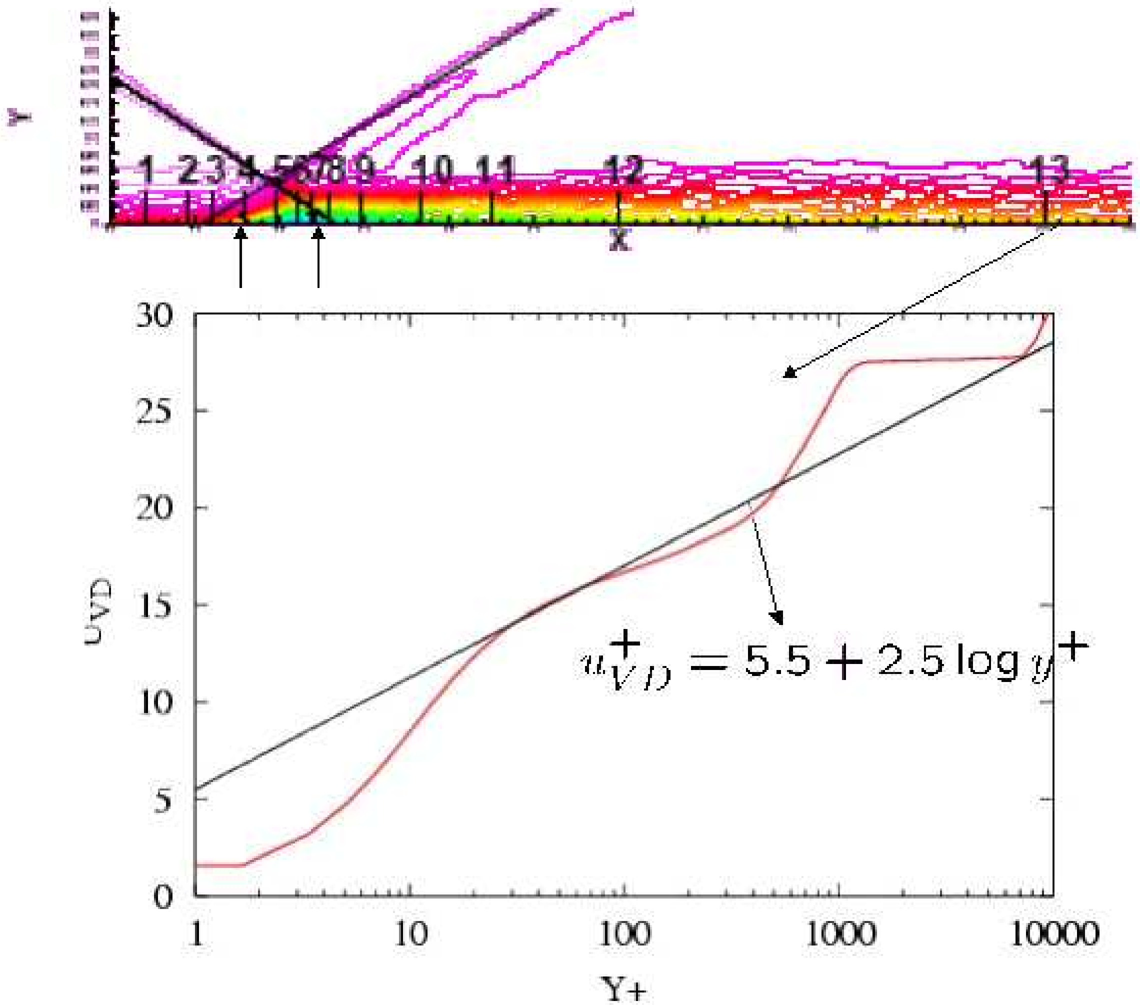
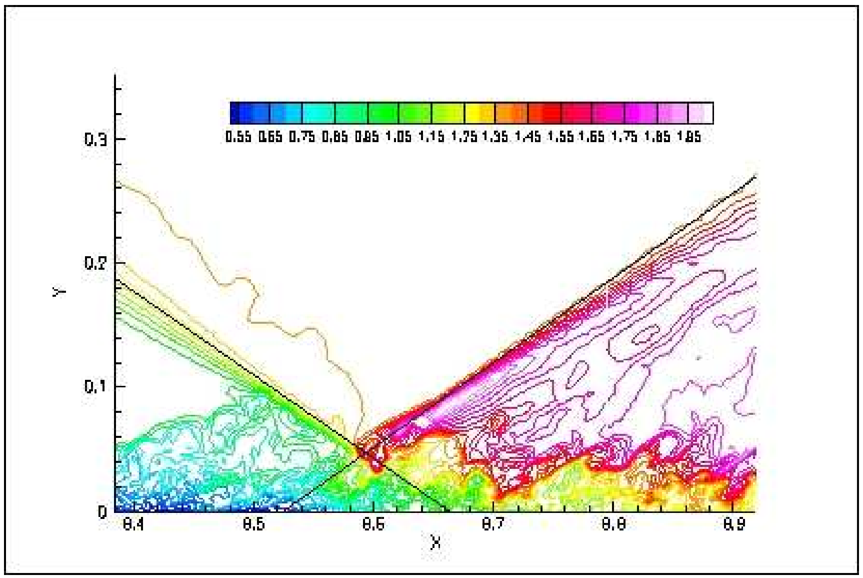
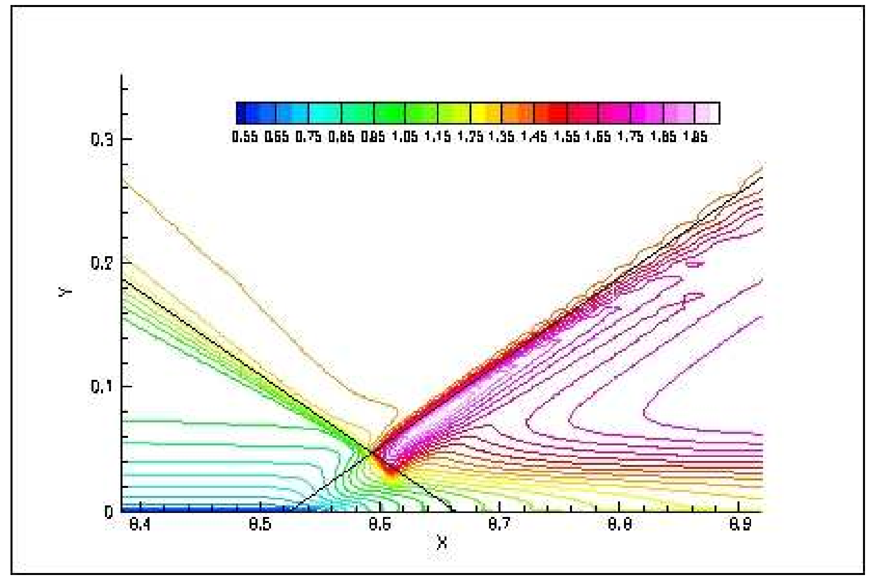


INSTANTANEOUS AND MEAN DENSITY CONTOURS

ρ

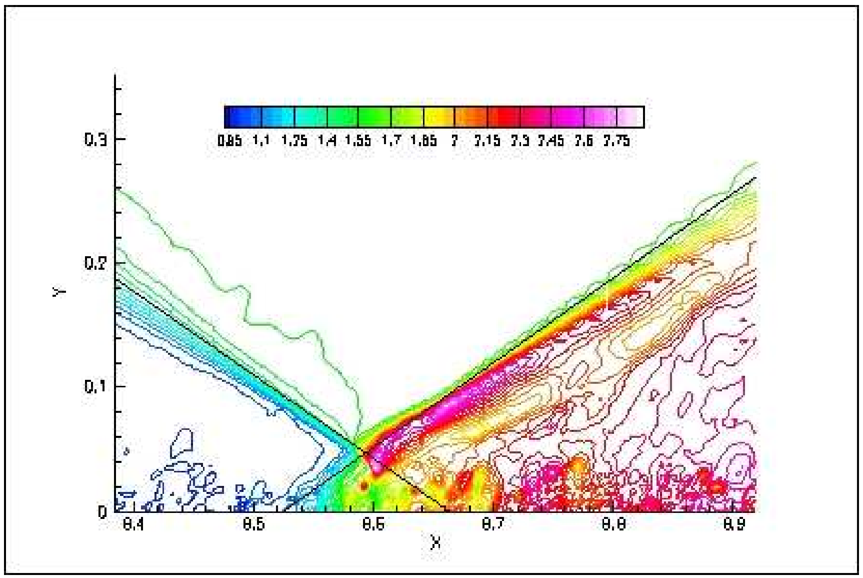


$\overline{\rho}$

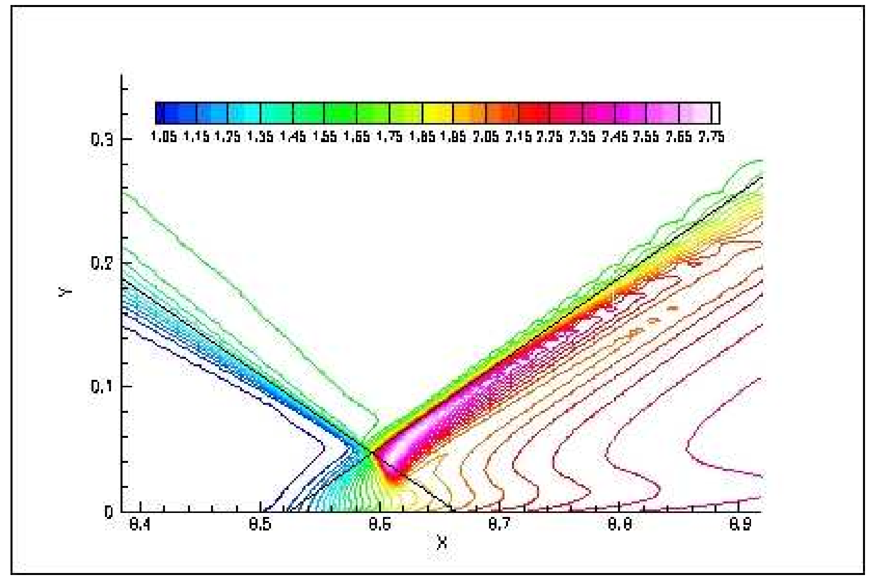


INSTANTANEOUS AND MEAN PRESSURE CONTOURS

p

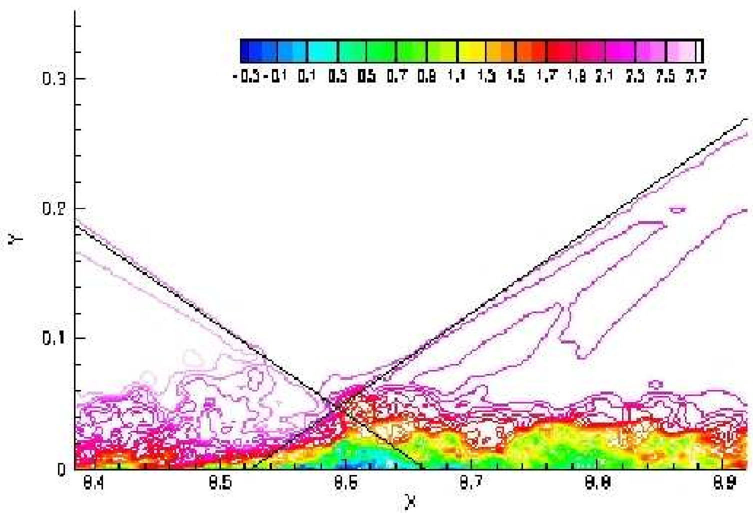


$\overline{p}$

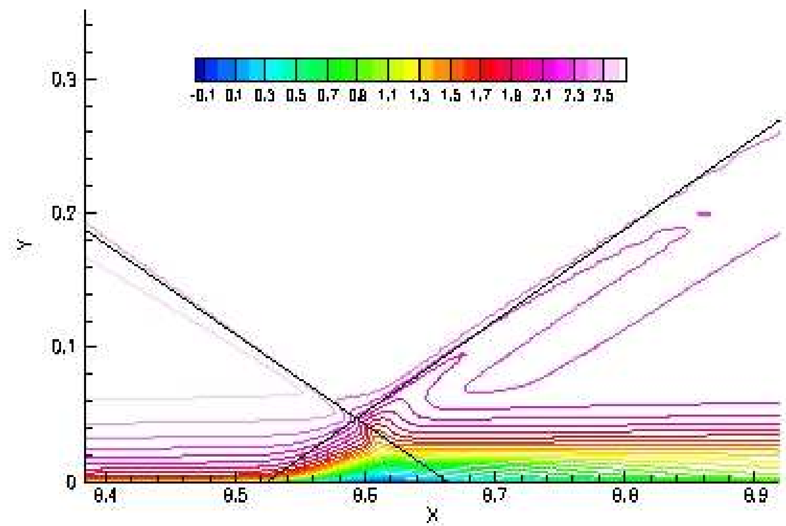


INSTANTANEOUS AND MEAN VELOCITY CONTOURS

u

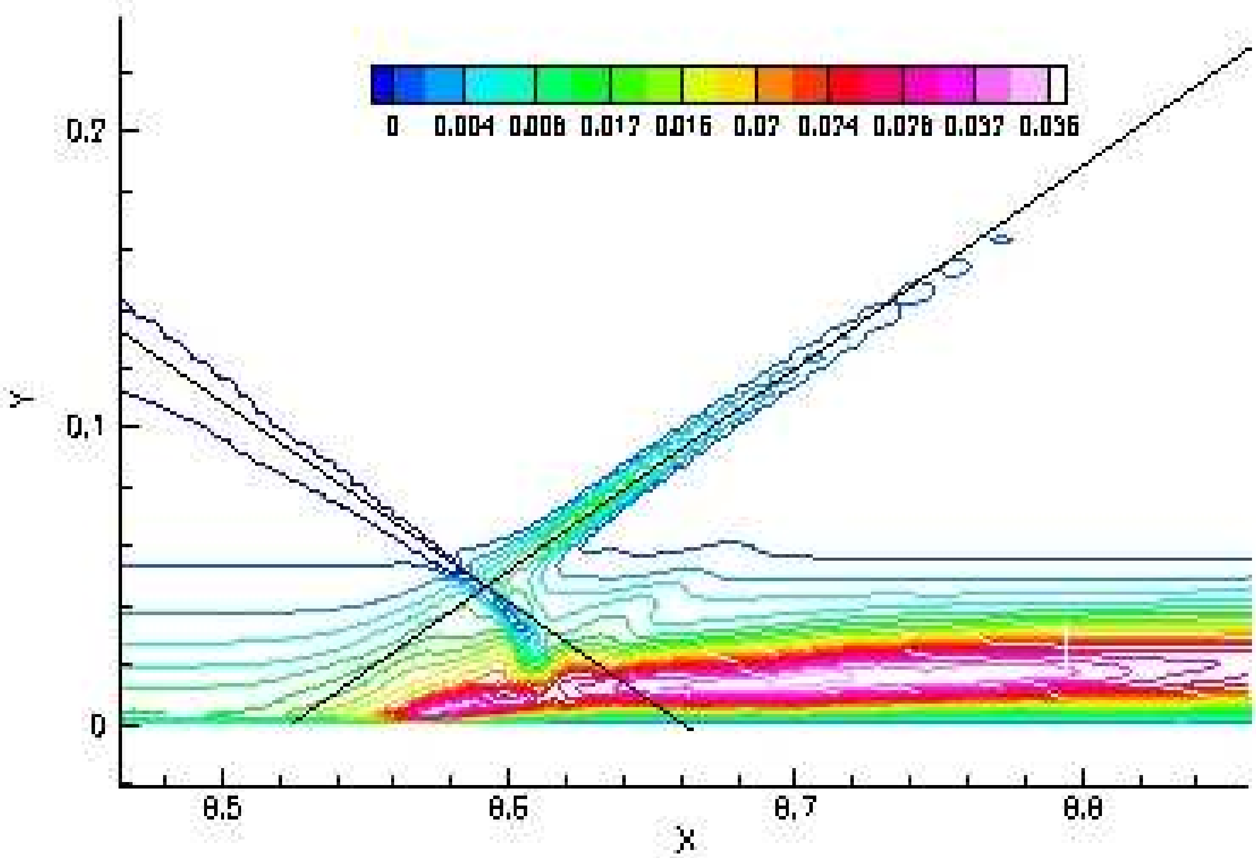


$\tilde{u}$



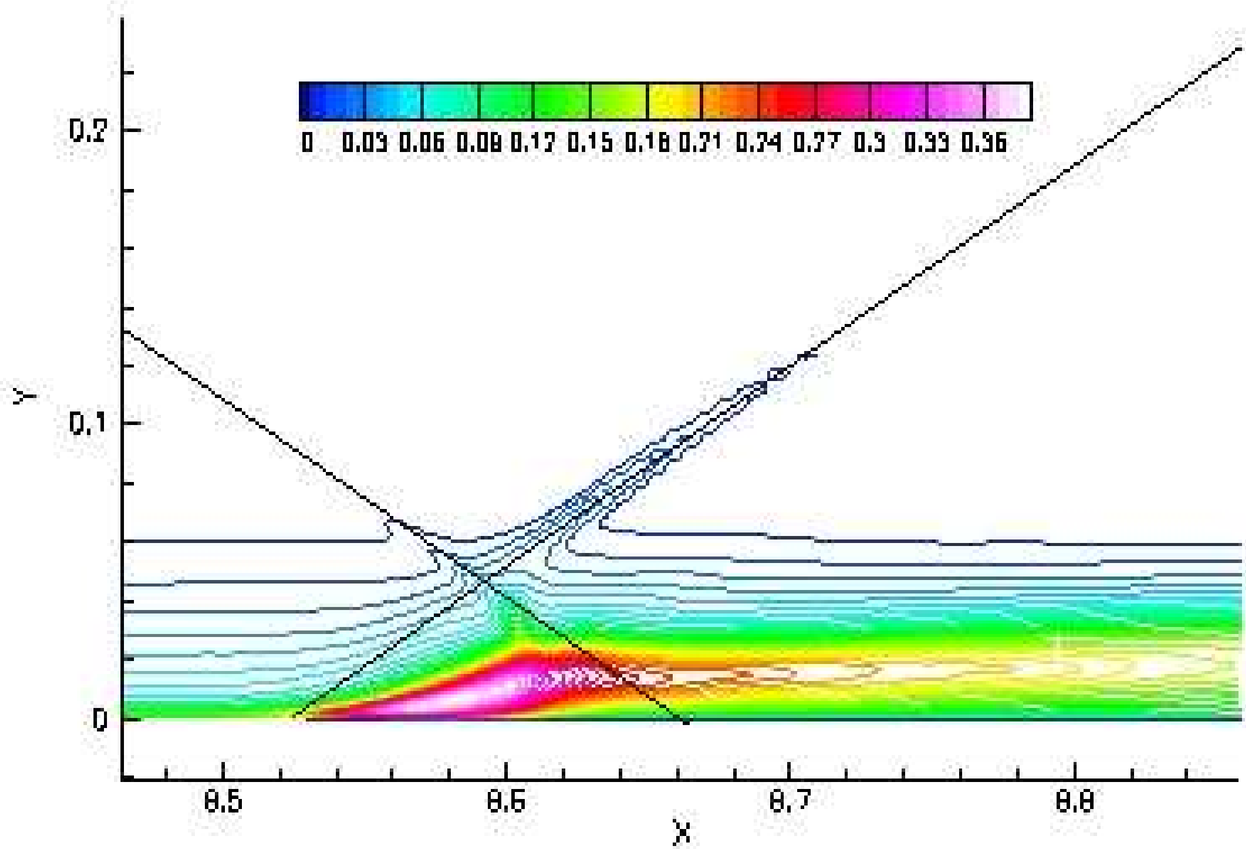
TURBULENT SHEAR STRESS CONTOURS

$$-\widetilde{u_1''u_2''}$$



TURBULENT KINETIC ENERGY CONTOURS

$$\widetilde{u_i'' u_i''}$$



PRESSURE-STRAIN RATE MODELING

- With increasing convective Mach number M_c, new compressible models should account for
  - Suppression of growth rate of turbulent shear layer
  - Reduction of turbulent kinetic energy of turbulent shear layer
- DNS studies of turbulent kinetic energy budget indicate primary factor is reduction in production term
  - Reduced turbulence production traced to decrease in magnitude of pressure-strain components

GOVERNING FLUCTUATING EQUATIONS

- Dimensional fluctuating mass and momentum equations

$$\frac{\partial \rho^{*\prime}}{\partial t^*} + \frac{\partial}{\partial x_j^*}\left(\rho^{*\prime}\tilde{u}_j^* + \rho^* u_j^{*\prime\prime}\right) = 0$$

$$\frac{\partial}{\partial t^*}\left(\rho^{*\prime}\tilde{u}_i^* + \rho^* u_i^{*\prime\prime}\right) \quad + \quad \frac{\partial}{\partial x_j^*}\Big[\rho^*\left(\tilde{u}_i^* u_j^{*\prime\prime} + u_i^{*\prime\prime}\tilde{u}_j^* + u_i^{*\prime\prime}u_j^{*\prime\prime}\right)$$
$$- \langle\rho^*\rangle \widetilde{u_i^{*\prime\prime}u_j^{*\prime\prime}} + \rho^{*\prime}\tilde{u}_i^*\tilde{u}_j^*\Big] = -\frac{\partial p^{*\prime}}{\partial x_i^*} \; .$$

- Nondimensionalize with
  - Fluctuating field: length scale l_e^* and velocity scale u_e^*
  - Mean field: velocity scale $\mathcal{S}^* l_e^*$
  - Thermodynamic variables: density and pressure scales ρ_e^* and $\rho_e^* u_e^{*2}$, respectively

WAVE EQUATION FOR FLUCTUATING PRESSURE

$$\left[\left(M_t\frac{\partial}{\partial t}+M_g\tilde{u}_j\frac{\partial}{\partial x_j}\right)^2 - \frac{\partial^2}{\partial x_j \partial x_j}\right]p' =$$

$$+2\left(\frac{M_g}{M_t}\right)\frac{\partial \tilde{u}_i}{\partial x_j}\frac{\partial(\rho u_j'')}{\partial x_i}+\left(\frac{M_g}{M_t}\right)^2\frac{\partial \tilde{u}_i}{\partial x_j}\frac{\partial \tilde{u}_j}{\partial x_i}\rho'$$

$$M_t = u_e^*/c^*, \qquad M_g = \mathcal{S}^* l_e^*/c^*$$

- Formal solution given in physical and wave-vector space by

$$p'(\mathbf{x},t)=\int_0^t dt' \int d^3\mathbf{x}' G(\mathbf{x}-\mathbf{x}',t-t')f(\mathbf{x}',t')$$

$$\hat{p}(\mathbf{k},t)=\int_0^t dt' \widehat{G}(\mathbf{k},t-t')\hat{f}(\mathbf{k},t')$$

GREEN'S FUNCTION FOR HOMOGENEOUS FLOW

- Green's function $G(\mathbf{x},t)$ given by

$$G(\mathbf{x},t) = \frac{1}{(2\pi)^3}\int d^3\mathbf{k}e^{\iota\boldsymbol{\mathcal{K}}(\mathbf{k},t)\cdot\mathbf{x}}\widetilde{G}(\mathbf{k},t)$$

where moving wave-vector $\boldsymbol{\mathcal{K}}$ is given by

$$\boldsymbol{\mathcal{K}}(\mathbf{k},t) = \mathbf{k}\left[\mathbf{I} - \left(\frac{M_g}{M_t}t\right)\boldsymbol{\lambda}\right]$$

- Isothermal homogeneous shear flow $\tilde{u}_i(\mathbf{x}) = \lambda_{ij}x_j$

$$\boldsymbol{\lambda} = [\lambda_{ij}] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

GREEN'S FUNCTION FOR HOMOGENEOUS FLOW

- $\widetilde{G}(\mathbf{k},t)$ satisfies the equation

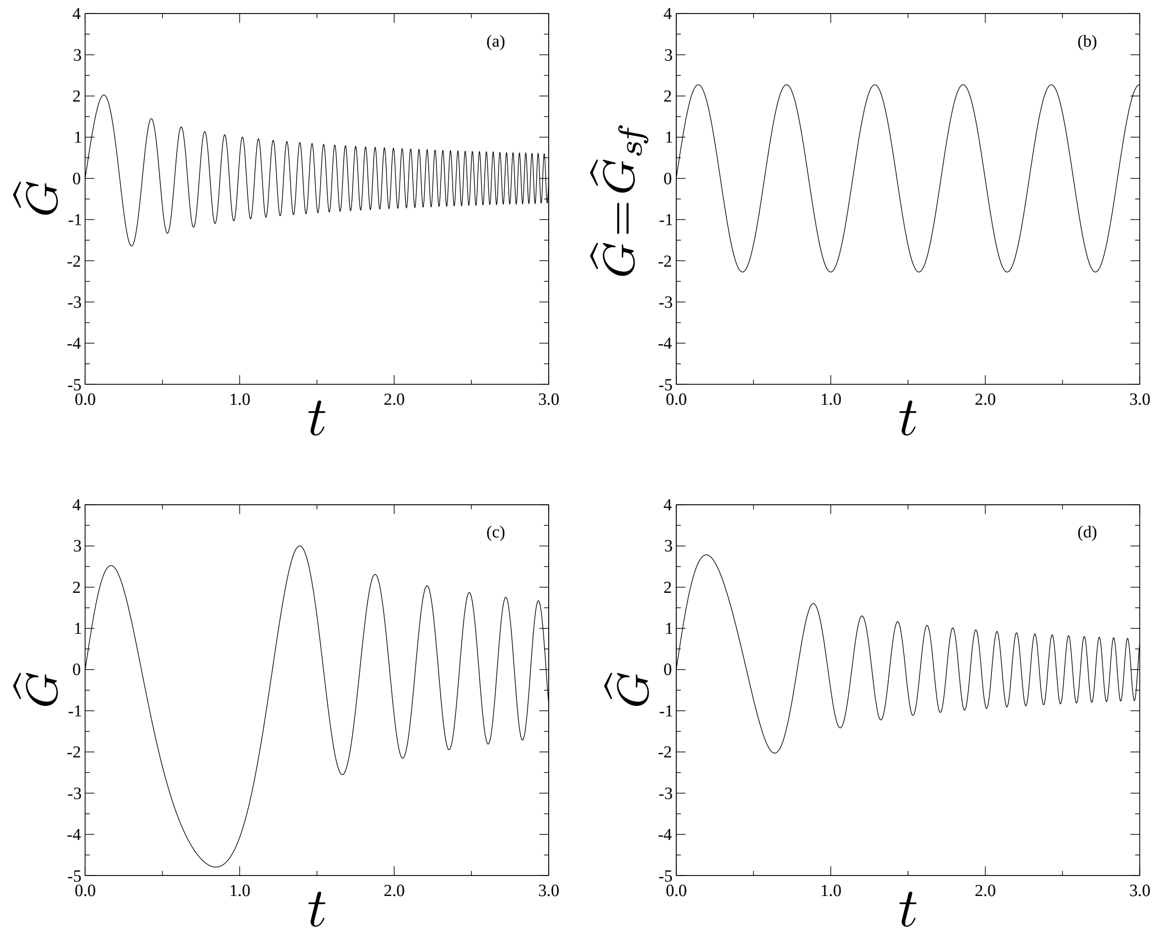
$$\left[M_t^2\frac{\partial^2}{\partial t^2}+\left(\frac{M_g k_1}{M_t}\right)^2\left(t-\frac{k_2}{k_1}\frac{M_t}{M_g}\right)^2+\left(k_1^2+k_3^2\right)\right]\widetilde{G}(\mathbf{k},t)=\delta(t)$$

  - Limiting case of shear-free flow ($M_g=0$) studied previously (Pantano and Sarkar, 2002)

$$\widetilde{G}_{sf}(k,t)=\frac{\mathcal{H}(t)}{M_t k}\sin\left(kt/M_t\right)$$

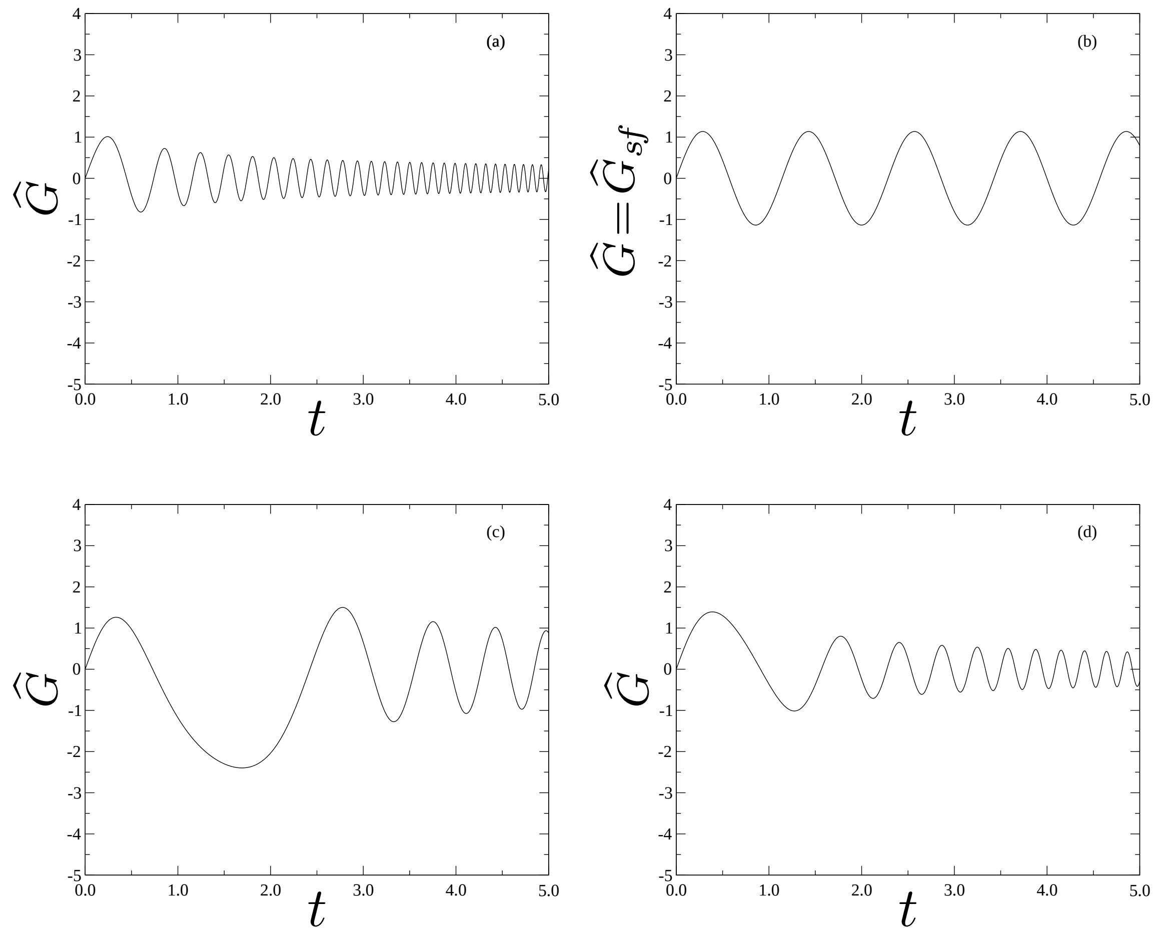
where $k^2=k_ik_i$, $\mathcal{H}(t)$ the Heaviside function

ANISOTROPY OF GREEN'S FUNCTION



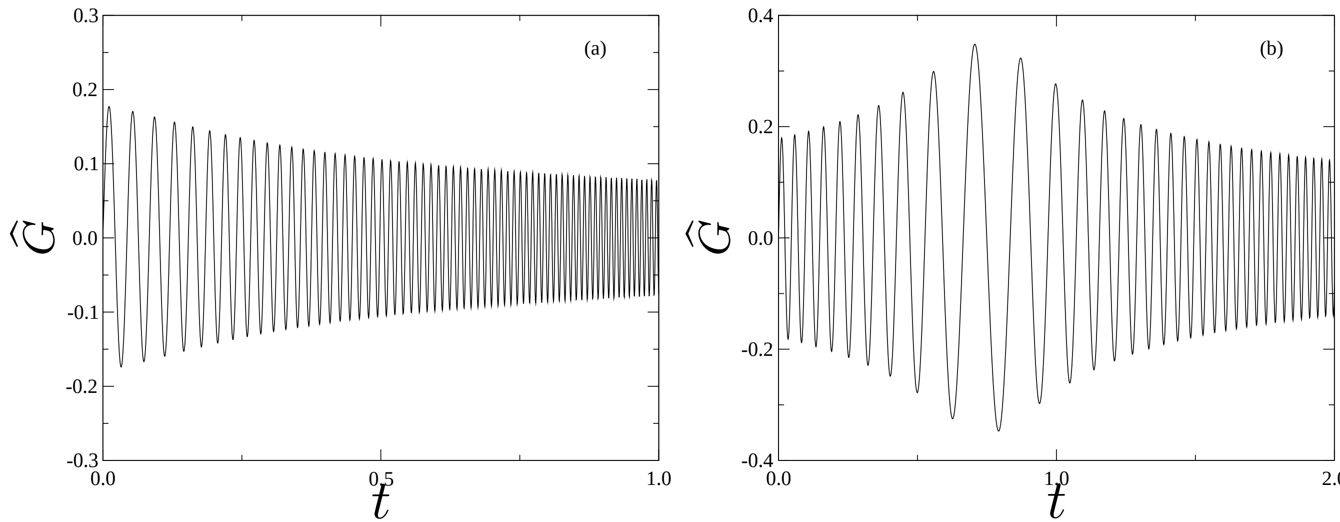
$M_t = 0.2$ and $M_g = 1.0$: Green's function at various values of the azimuthal angle ϕ on a circle of radius $k = 2.2$ in the shear plane $\theta = \pi/2$ ($k_3 = 0$): (a) $\phi = \pi/12$, (b) $\phi = \pi/2$, (c) $\phi = 7\pi/12$, (d) $\phi = 2\pi/3$

ANISOTROPY OF GREEN'S FUNCTION



$M_t = 0.4$ and $M_g = 1.0$: Green's function at various values of the azimuthal angle ϕ on a circle of radius $k = 2.2$ in the shear plane $\theta = \pi/2$ ($k_3 = 0$): (a) $\phi = \pi/12$, (b) $\phi = \pi/2$, (c) $\phi = 7\pi/12$, (d) $\phi = 2\pi/3$

ANISOTROPY OF GREEN'S FUNCTION



$M_t = 0.2$ and $M_g = 1.0$: Green's function on circle of radius $k = 28.0$ in the shear plane $\theta = \pi/2$ ($k_3 = 0$) with wave-vector azimuthal angle: (a) $\phi = \pi/12$, (b) $\phi = 7\pi/12$

PRESSURE-STRAIN RATE CORRELATION

- With determination of Green's function, now possible to obtain solution to convective wave equation for (rapid part) of fluctuating pressure
- Expression for two-point pressure-strain rate correlation Π_{ij},

$$\Pi_{ij}(\mathbf{r},t) = \left\langle \overline{p'(\mathbf{x}+\mathbf{r},t)\left(\frac{\partial}{\partial x_j}u_i''(\mathbf{x},t) + \frac{\partial}{\partial x_i}u_j''(\mathbf{x},t)\right)} \right\rangle$$

$$\begin{aligned} \Pi_{ij}(\mathbf{r},t) &= \frac{\iota}{\mathscr{V}} \int d^3\mathbf{x} \int d^3\mathbf{k}' d^3\mathbf{k}'' e^{\iota \mathbf{k}'\cdot(\mathbf{x}+\mathbf{r})} e^{\iota \mathbf{k}''\cdot\mathbf{x}} \\ &\quad \times \left\langle \hat{p}(\mathbf{k}',t)\left(k_j''\hat{u}_i(\mathbf{k}'',t) + k_i''\hat{u}_j(\mathbf{k}'',t)\right)\right\rangle \end{aligned}$$

OTHER FLUCTUATING PRESSURE CORRELATIONS

- Expression for two-point pressure-velocity correlation,

$$\left\langle \overline{p'(\mathbf{x}+\mathbf{r},t)u_i''(\mathbf{x},t)} \right\rangle = \frac{\iota}{\mathscr{V}} \int d^3\mathbf{x} \int d^3\mathbf{k}' d^3\mathbf{k}'' e^{\iota \mathbf{k}'\cdot(\mathbf{x}+\mathbf{r})} e^{\iota \mathbf{k}''\cdot\mathbf{x}} \times \left\langle \hat{p}(\mathbf{k}',t)\hat{u}_i(\mathbf{k}'',t) \right\rangle$$

- Expression for two-point pressure correlation

$$\left\langle \overline{p'(\mathbf{x},t)p'(\mathbf{x}+\mathbf{r},t)} \right\rangle = \frac{\iota}{\mathscr{V}} \int d^3\mathbf{x} \int d^3\mathbf{k}' d^3\mathbf{k} e^{\iota \mathbf{k}'\cdot(\mathbf{x}+\mathbf{r})} e^{\iota \mathbf{k}\cdot\mathbf{x}} \times \left\langle \hat{p}(\mathbf{k}',t)\hat{p}(\mathbf{k},t) \right\rangle$$

PRESSURE-STRAIN RATE CORRELATION

- In terms of the energy spectrum tensor

$$\Pi_{ij} = 2\overline{\rho}\left(\frac{M_g}{M_t}\right)\int_0^\infty dk\int_0^\pi \sin\theta d\theta\int_0^{2\pi} d\phi R(k,\theta,\phi;\tau_I) \times k_l\lambda_{lm}\left[E_{mi}(k,\theta,\phi)k_j + E_{mj}(k,\theta,\phi)k_i\right]$$

- $R(k,\theta,\phi;\tau_I)$ dependent on the anisotropic Green's function and temporal decorrelation time τ_I
  - For incompressible flow: $R(k,\theta,\phi;\tau_I)$ not present
  - For compressible, shear-free flow:

$$R(k,\theta,\phi;\tau_I) = \left(1 + \frac{M_t^2}{\tau_I^2 k^2}\right)^{-1}$$

- Representation for energy spectrum tensor E_{ij} needed

ENERGY SPECTRUM TENSOR

- Linear representation for $E_{ij}(\mathbf{k};\mathbf{b})$
  - Basis δ_{ij}, $\hat{k}_i\hat{k}_j$, b_{ij}, and $(b_{in}\hat{k}_n\hat{k}_j + \hat{k}_i\hat{k}_n b_{nj})$

$$\hat{k}_i = k_i/k$$

$$b_{ij} \equiv \frac{\left\langle u_i'' u_j'' \right\rangle}{2K} - \frac{\delta_{ij}}{3}$$

- Constraints: (i) trace of E_{ij} is proportional to isotropic energy spectral density $E(k)$ ($E_{ii} = E(k)/2\pi k^2$); (ii) continuity $k_i E_{ij} = 0$

ENERGY SPECTRUM TENSOR

- Four-term representation

$$E_{ij}(\mathbf{k};\mathbf{b}) = \frac{E(k)}{4\pi k^2}\left(\delta_{ij} - \hat{k}_i\hat{k}_j\right) + \frac{E_a(k)}{8\pi k^2}\left(\hat{k}_n b_{nm}\hat{k}_m\right)\left(\delta_{ij} + \hat{k}_i\hat{k}_j\right) + \frac{E_a(k)}{4\pi k^2}\left[b_{ij} - \left(b_{in}\hat{k}_n\hat{k}_j + \hat{k}_i\hat{k}_n b_{nj}\right)\right]$$

- Expansion coefficient $E_a(k)$ is anisotropic energy spectral density

ENERGY SPECTRUM TENSOR

- For incompressible flow ($R(k, \theta, \phi)$ not present)
  - Shih, Reynolds, Mansour (1990) used a similar expression to obtain a closure model for the LRR pressure-strain rate correlation
  - Exact forms for $E(k)$ and $E_a(k)$ not required
    - Obtained from conditions imposed by integral relation between the Reynolds stress energy spectrum and Reynolds stress anisotropy tensor

$$4\pi \int_0^\infty k^2 E_{ij} dk = 2K \left(b_{ij} + \frac{\delta_{ij}}{3} \right)$$

ENERGY SPECTRAL DENSITIES

- For compressible flow ($R(k, \theta, \phi)$ present)
- Modeled forms for both $E(k)$ and $E_a(k)$ required
  - Isotropic energy spectral density

$$E(k) = C\varepsilon^{2/3}k^{-5/3}\left[\frac{kL}{[(kL)^2 + C_L]^{1/2}}\right]^{17/3}$$

  - Anisotropic energy spectral density

$$E_a(k) = C_a\left(\frac{M_g}{M_t}\right)\varepsilon^{1/3}k^{-7/3}\left[\frac{kL}{[(kL)^2 + C_{aL}]^{1/2}}\right]^{19/3}$$

SUMMARY

- The next step is to determine the procedure to follow to extract a closure model from

$$\Pi_{ij} = 2\lambda_{lm}\left(M_{imlj} + M_{jmli}\right)$$

where

$$\lambda_{lm} = S_{lm} + W_{lm}$$

$$M_{ijlm} = \overline{\rho}\left(\frac{M_g}{M_t}\right)\int_0^\infty dk \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi R(k,\theta,\phi;\tau_I) k_l k_m E_{ij}(k,\theta,\phi)$$