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Imperfect Knowledge Management, Information Fusion Applied to Risk Assessment & Decision-Making

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Works in collaboration with

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ONERA

THE FRENCH AEROSPACE LAB

Workshop “Towards a dynamic and holistic approach to disaster risk reduction”

Context: Natural risks management

Rapid mass movements in mountains (avalanches, floods, rockfalls,…)



… **threaten** people and infra structures



We try to **get protected** against them by taking good decisions and actions.

Context: Decision-making and natural risks management

Many **decisions** have to be taken to **assess** and **manage risks**

PREVENTION

Decisions for **nonstructural mitigation measures**

St Etienne de Tinée – 2009 (L. Bernard/National Park of Mercantour)

Torrent St Antoine – Modane - Savoie – 1987(M. Meunier – *Cemagref)*

What are the **hazard, risk levels**?

Land-use planning: where should **urbanisation** be **prohibited**, **regulated** or **fully allowed**?

PROTECTION

Decisions for choice of **protection works design** and **maintenance strategies**

Photo: *Cemagref*

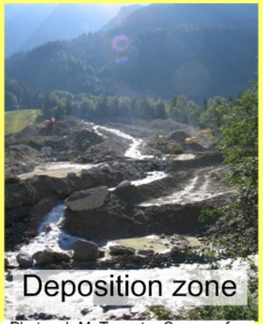


Photo: J.-M. Tacnet – *Cemagref*

Photo: J.-M. Tacnet – *Cemagref*

Which **protection** is **needed**?
Is it **effective**?

EVENT MANAGEMENT

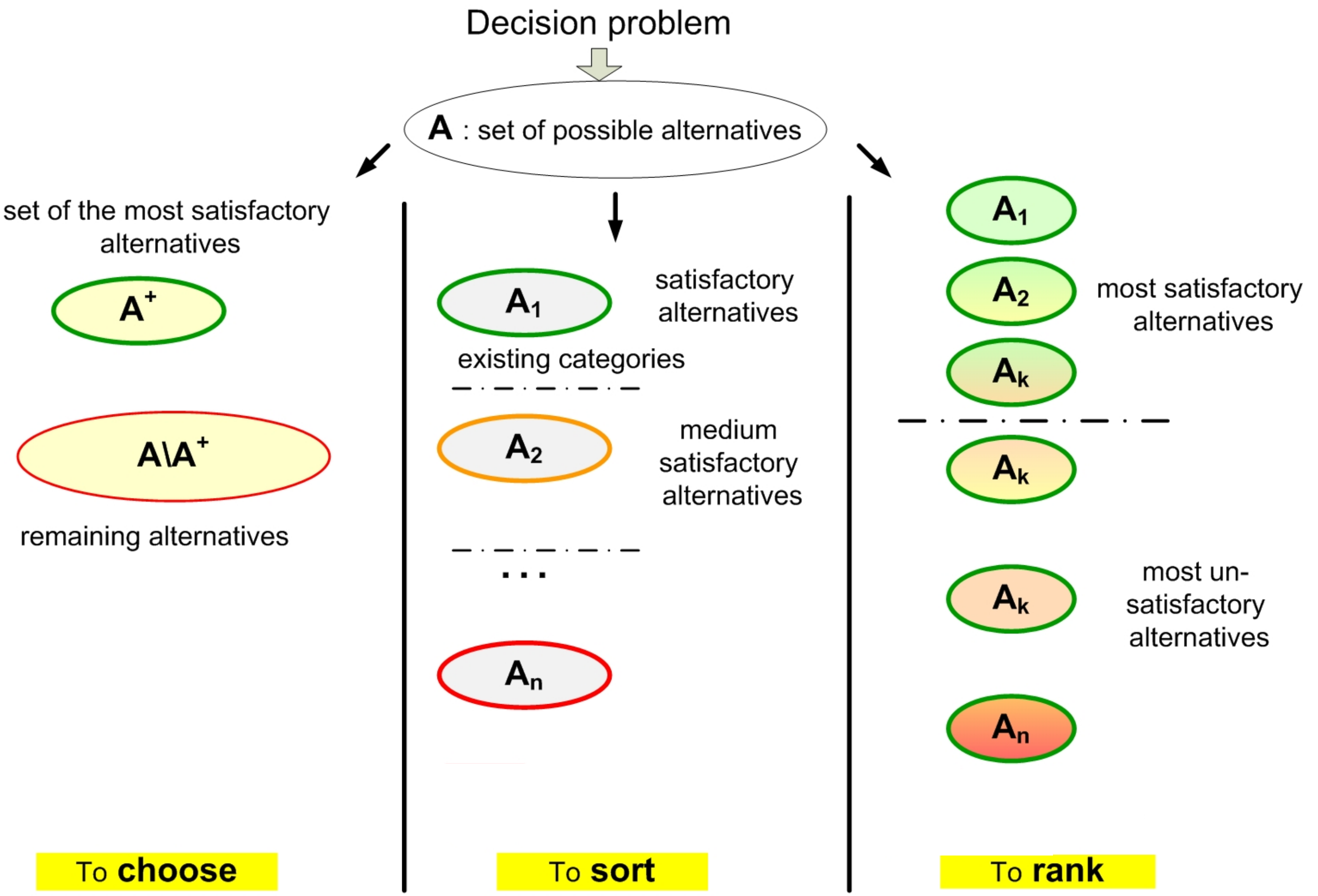
Decisions for **(railroad) infrastructure** management

Photo: J.F. Casanova –Dauphiné Libéré

Photo: J.-M. Tacnet – *Cemagref*

Should we **close**, **re-open**, **monitor** this road?

Context: Decision-making and natural risks management



Soft ELECTRE TRI BF-TOPSIS

Objectives of the approach

Risk management is based on **complex, multi-actors decision processes**

The goal is to design **decision-aiding methods** in a context of **heterogeneous** and **imperfect information** provided by **more** or **less reliable sources**…

We use **belief function theory** to improve **multicriteria decision-making methods**

and apply them to real life problems…. [Carladous PhD. Thesis 2017]

Part 1 - Belief Functions

… or how to go beyond probabilities

Belief = State of mind in which one thinks something to be true

Paradigm shift

Beliefs often are related with singular event or evidence, and are **not necessarily related with statistical data** and generic knowledge.

Part 1 - Belief functions [Dempster 1967,Shafer 1976]

Frame of discernment (FoD) $\Theta = \{\theta_i, i = 1, \ldots, n\}$ **Power-set** $2^\Theta \triangleq \{X | X \subseteq \Theta\}$

Impossibility — partial ignorances — full ignorance

Example

$$2^\Theta = \{\emptyset, \theta_1, \theta_2, \theta_3, \theta_1 \cup \theta_2, \theta_1 \cup \theta_3, \theta_2 \cup \theta_3, \theta_1 \cup \theta_2 \cup \theta_3\}$$

Basic belief assignment (BBA) $m(\cdot) : 2^\Theta \mapsto [0,1]$ s.t. $m(\emptyset) = 0$ and $\sum_{A \in 2^\Theta} m(A) = 1$

Vacuous BBA : $m_v(\Theta) = 1$ and $m_v(A) = 0, \ \forall A \neq \Theta$

Bayesian BBA : if focal elements of m(.) are singletons

Belief in A: $$Bel(A) \triangleq \sum_{B \in 2^\Theta | B \subseteq A} m(B) = Pl(\Theta) - Pl(\bar{A}) = 1 - Pl(\bar{A})$$ Degree of support of A

Plausibility of A: $$Pl(A) \triangleq \sum_{B \in 2^\Theta | B \cap A \neq \emptyset} m(B) = Bel(\Theta) - Bel(\bar{A}) = 1 - Bel(\bar{A})$$ Degree of non contradiction of A

Interpretation $$0 \leqslant Bel(A) \leqslant P(A) \leqslant Pl(A) \leqslant 1$$ Lower and upper bounds of (subj.) unknown proba P(A)

Uncertainty of A = Pl(A)-Bel(A)

Part 1 - Discounting a Source of Evidence (SoE)

Reliability discounting [Shafer 1976]

$$\boxed{\begin{cases} m^{\alpha}(A) \triangleq \alpha \cdot m(A) & \forall A \neq \Theta \\ m^{\alpha}(\Theta) \triangleq \alpha \cdot m(\Theta) + (1-\alpha) \end{cases}}$$

$\alpha = 0$ means "the SoE is 100% unreliable"

$\alpha = 1$ means "the SoE is 100% reliable"

Importance discounting [Smarandache-Dezert-Tacnet 2010]

$$\boxed{\begin{cases} m^{\beta}(A) \triangleq \beta \cdot m(A) & \forall A \neq \emptyset \\ m^{\beta}(\emptyset) \triangleq \beta \cdot m(\emptyset) + (1-\beta) \end{cases}}$$

$\beta = 0$ means "the SoE is not important at all"

$\beta = 1$ means "the SoE is 100% important"

Part 1 - Belief functions - Dempster-Shafer rule

Dempster-Shafer (DS) rule of combination [Dempster 1967,Shafer 1976]

If we consider two independent SOE with respect to same FoD, then

Conjunctive rule $= m_{12}^{\text{Conj}}(X)$

$$m_{12}^{DS}(X) = [m_1 \oplus m_2](X) \triangleq \frac{\sum_{X_1,X_2 \in 2^\Theta | X_1 \cap X_2 = X} m_1(X_1)m_2(X_2)}{1 - \sum_{X_1,X_2 \in 2^\Theta | X_1 \cap X_2 = \emptyset} m_1(X_1)m_2(X_2)}$$

Degree of conflict = m(ø)

DS rule extends to the fusion of n>2 sources

DS rule is commutative and associative, and vacuous BBA has no impact

Shafer Conditioning [Shafer 1976] $m_Z(Z) = 1$ (one knows Z for sure)

$$m(X|Z) = [m \oplus m_Z](X) \Rightarrow \begin{cases} Bel(X|Z) = \frac{Bel(X \cup \bar{Z}) - Bel(\bar{Z})}{1 - Bel(\bar{Z})} \\ Pl(X|Z) = \frac{Pl(X \cap Z)}{Pl(Z)} \end{cases}$$

⚠ **Only** apparent compatibility with Bayes rule!

Part 1 - Belief functions - Dempster-Shafer rule

Advantage: Associativity

Drawbacks of DS rule

Not defined when conflict is total, and numerically not robust to input changes

⚠ **Counter intuitive results** when conflict is high [Zadeh 1979]

⚠ **Counter intuitive results** when conflict is low [Dezert-Wang-Tchamova 2012]

$\Theta = \{A, B, C\}$, with $m_1 \neq m_2 \neq m_v$

Focal elem. \ bba's	$m_1(.)$	$m_2(.)$
A	a	0
$A \cup B$	$1-a$	b_1
C	0	$1-b_1-b_2$
$A \cup B \cup C$	0	b_2

$$\begin{aligned} m_{12}(\emptyset) &= m_1(A)m_2(C) + m_1(A \cup B)m_2(C) \\ &= a(1-b_1-b_2) + (1-a)(1-b_1-b_2) = 1 - b_1 - b_2 \end{aligned}$$

$$m_{12}^{DS}(\cdot) = [m_1 \oplus m_2](\cdot) = m_1(\cdot)$$

Informative source m_2 does not impact DS result !

⚠ The bounds of conditional belief interval $[Bel(A|B), Pl(A|B)]$ can be incompatible with the lower and upper bounds of $P(A|B)$!!!

see Ellsberg's example in [Dezert-Tchamova-Han 2018]

Part 1 - Belief functions - PCR fusion rules

Principle of Proportional Conflict Redistribution (PCR) rules [DSmT Book Vol2]

Redistribute each partial conflict to elements involved in it proportionally to their mass

Principle of Proportional Conflict Redistribution (PCR) rules [DSmT Book Vol2]

PCR5 rule presented by Smarandache and Dezert

PCR6 rule presented by Martin and Osswald

Toolboxes and code http://www.bfasociety.org

[Smarandache-Dezert-Tacnet 2010]

PCR5 and PCR6 formulas for 2 sources

$$\boxed{m_{12}^{PCR5/6}(X) = m_{12}^{Conj}(X) + \sum_{\substack{Y \in 2^{\Theta} \\ X \cap Y = \emptyset}} \left[\frac{m_1(X)^2 m_2(Y)}{m_1(X) + m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_2(X) + m_1(Y)}\right]}$$

PCR5=PCR6 for the fusion of 2 Sources. General formulas exist for n>2.

Part 1 - Example of PCR fusion

Example

$\Theta = \{A, B\}$

	A	B	$A \cup B$
$m_1(.)$	0.6	0.3	0.1
$m_2(.)$	0.2	0.3	0.5
$m_{12}(.)$	0.44	0.27	0.05

$$m_{12}(A \cap B = \emptyset) = m_1(A)m_2(B) + m_1(B)m_2(A) = 0.18 + 0.06 = 0.24$$

$$x_1/0.6 = y_1/0.3 = (x_1 + y_1)/(0.6 + 0.3) = 0.18/0.9 = 0.2 \rightarrow \begin{cases} x_1 = 0.6 \cdot 0.2 = 0.12 \\ y_1 = 0.3 \cdot 0.2 = 0.06 \end{cases}$$

$$x_2/0.2 = y_2/0.3 = (x_2 + y_2)/(0.2 + 0.3) = 0.06/0.5 = 0.12 \rightarrow \begin{cases} x_2 = 0.2 \cdot 0.12 = 0.024 \\ y_2 = 0.3 \cdot 0.12 = 0.036 \end{cases}$$

PCR5/6 result

$$\begin{cases} m_{12}^{PCR5/6}(A) = \boxed{0.44} + \boxed{0.12} + \boxed{0.024} = 0.584 \\ m_{12}^{PCR5/6}(B) = \boxed{0.27} + \boxed{0.06} + \boxed{0.036} = 0.366 \\ m_{12}^{PCR5/6}(A \cup B) = \boxed{0.05} + 0 = 0.05 \end{cases}$$

DS result

$$\begin{cases} m_{12}^{DS}(A) \approx 0.579 \\ m_{12}^{DS}(B) \approx 0.355 \\ m_{12}^{DS}(A \cup B) \approx 0.066 \end{cases}$$

The mass of ignorance is reduced with PCR rules

Advantages of PCR rules

It does not increase uncertainty more than justified

It works with any level of conflict

It is numerically robust to input changes

Drawbacks

Complexity

Non associativity

Part 1 - Approximation of a BBA in a proba measure

Simplest method

⚠ Take the mass of each element of Θ and normalize, but it does not take into account partial ignorances

$$P_m(A) = \frac{m(A)}{\sum_{B\in\Theta} m(B)}$$

Cobb-Shenoy method [Cobb Shenoy 2006]

⚠ Take the plausibility of each element of Θ and normalize, but it is inconsistent with belief interval

$$P_{Pl}(A) = \frac{Pl(A)}{\sum_{B\in\Theta} Pl(B)}$$

Pignistic transform [Smets 1990]

Redistribute the mass of partial ignorances **equally** to singletons included in them

$$BetP(A) = \sum_{X\in 2^\Theta} \frac{|X\cap A|}{|A|} m(X)$$

higher entropy obtained with $BetP(\cdot)$

DSmP transform [Dezert Smarandache 2008]

Redistribute mass of partial ignorances **proportionally to masses of singletons** included in them. $\epsilon > 0$ is a small parameter to prevent division by zero in some cases.

$$DSmP_\epsilon(A) = \sum_{Y\in 2^\Theta} \frac{\sum\limits_{\substack{Z\subseteq A\cap Y\\ |Z|=1}} m(Z) + \epsilon|A\cap Y|}{\sum\limits_{\substack{Z\subseteq Y\\ |Z|=1}} m(Z) + \epsilon|Y|} m(Y)$$

smaller entropy obtained with $DSmP(\cdot)$

Part 1 - Distances between BBAs

Tessem distance [Tessem 1993] ⚠ this is not a strict metric!

$$d_T(m_1, m_2) = \max_{A \subseteq \Theta} \{|\mathrm{BetP}_1(A) - \mathrm{BetP}_2(A)|\}$$

Jousselme distance [Jousselme et al. 2001]

$$d_J(m_1, m_2) \triangleq \sqrt{0.5 \cdot (m_1 - m_2)^T \mathbf{Jac}\,(m_1 - m_2)}$$

$$\mathbf{Jac}(A, B) = |A \cap B| / |A \cup B|$$

Euclidean belief interval distance [Han Dezert Yang 2014]

$$d_{BI}(m_1, m_2) \triangleq \sqrt{\frac{1}{2^{|\Theta|-1}} \cdot \sum_{A \in 2^\Theta} d^I(BI_1(A), BI_2(A))^2}$$

$$BI_1(A) = [Bel_1(A), Pl_1(A)] \qquad BI_2(A) = [Bel_2(A), Pl_2(A)]$$

$$d^I([a_1, b_1], [a_2, b_2]) = \sqrt{\left[\frac{a_1 + b_1}{2} - \frac{a_2 + b_2}{2}\right]^2 + \frac{1}{3}\left[\frac{b_1 - a_1}{2} - \frac{b_2 - a_2}{2}\right]^2}$$

Part 1 - Decision-making based on belief functions

Maximum of belief strategy (pessimistic/cautious)

$$m(\cdot) \to Bel(\cdot) \quad \text{and} \quad \delta = \hat{\theta} = \arg\max_{\theta_i \in \Theta} Bel(\theta_i)$$

Maximum of plausibility strategy (optimistic)

$$m(\cdot) \to Pl(\cdot) \quad \text{and} \quad \delta = \hat{\theta} = \arg\max_{\theta_i \in \Theta} Pl(\theta_i)$$

Compromise strategy with proba transforms

$$m(\cdot) \to P(\cdot) \quad \text{and} \quad \delta = \hat{\theta} = \arg\max_{\theta_i \in \Theta} P(\theta_i)$$

Decision using min distance strategy [Han Dezert Yang 2014]

$\mathcal{X} = \{\text{admissible} X, X \in 2^{\Theta}\}$ is the set of possible admissible decisions

$$\delta = \hat{X} = \arg\min_{X \in \mathcal{X}} d_{BI}(m, m_X)$$

$$q(\hat{X}) = 1 - \frac{d_{BI}(m, m_{\hat{X}})}{\sum_{X \in \mathcal{X}} d_{BI}(m, m_X)} \in [0, 1]$$

→ Higher is the quality index, more confident we are in the decision

Part 1 - Total Belief Theorem and Fagin Halpern conditioning

Total Probability Theorem For any event B and any partition $\{A1, \ldots, A_k\}$ of Θ

(TPT)

$$\boxed{P(B) = P(B \cap A_1) + P(B \cap A_2) + \ldots + P(B \cap A_k)}$$

Total Belief and Total Plausibility Theorems [Dezert-Tchamova-Han 2018]

(TBT)

$$\boxed{Bel(B) = \sum_{X \in \mathcal{F}_\Theta(m) | X \subseteq B} m(X) = \sum_{i=1,\ldots,k} Bel(A_i \cap B) + U(A^* \cap B)}$$

where $\mathcal{F}_{A*}(m) \triangleq \mathcal{F}_\Theta(m) - \mathcal{F}_{A_1}(m) - \ldots - \mathcal{F}_{A_k}(m)$ ← set of focal elements of m(.) included in Ak

$$U(A^* \cap B) \triangleq \sum_{X \in \mathcal{F}_{A*}(m) | X \in \mathcal{F}_B(m)} m(X)$$

Fagin-Halpern conditioning from TBT [Dezert-Tchamova-Han 2018]

(FH)

$$Bel(A|B) = \frac{Bel(A \cap B)}{Bel(A \cap B) + Pl(\bar{A} \cap B)}$$

$$Pl(A|B) = \frac{Pl(A \cap B)}{Pl(A \cap B) + Bel(\bar{A} \cap B)}$$

⚠ Shafer's conditioning formulas are inconsistent with TBT and conditional proba bounds.

(see Ellsberg's urn example)

Part 1 - Generalized Bayes Theorem (GBT)

Generalized Bayes' Theorem (GBT): For any partition $\{A_1, \ldots, A_k\}$ of a FoD Θ, any belief function $Bel(\cdot) : 2^\Theta \mapsto [0, 1]$, and any subset B of Θ with $Bel(B) > 0$, then one has for $i \in \{1, \ldots, k\}$

$$\boxed{Bel(A_i|B) = \frac{Bel(B|A_i)q(A_i, B)}{\sum_{i=1}^{k} Bel(B|A_i)q(A_i, B) + U((\bar{A}_i \cap B)^*)}}$$

where

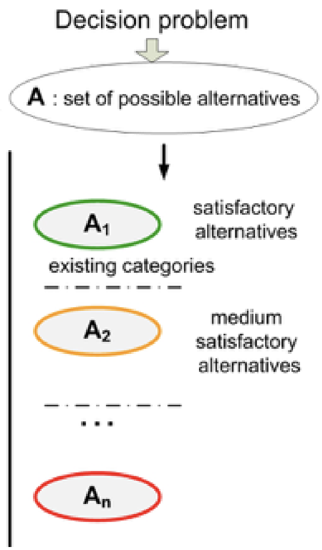
$$\begin{cases} U((\bar{A}_i \cap B)^*) \triangleq Pl(\bar{A}_i \cap B) - Bel(\bar{A}_i \cap B) = \sum_{X \in \mathcal{F}_{(\bar{A}_i \cap B)^*}(m)} m(X) \\ q(A_i, B) \triangleq Bel(A_i) + U((\bar{B} \cap A_i)^*) - U(B^* \cap A_i) \end{cases}$$

Lemma: GBT reduces to Bayes Theorem if $Bel(\cdot)$ is a Bayesian belief function

Proofs : [Dezert-Tchamova-Han 2018]

Part 2 - Soft ELECTRE TRI

for sorting alternatives into categories based on multi-criteria

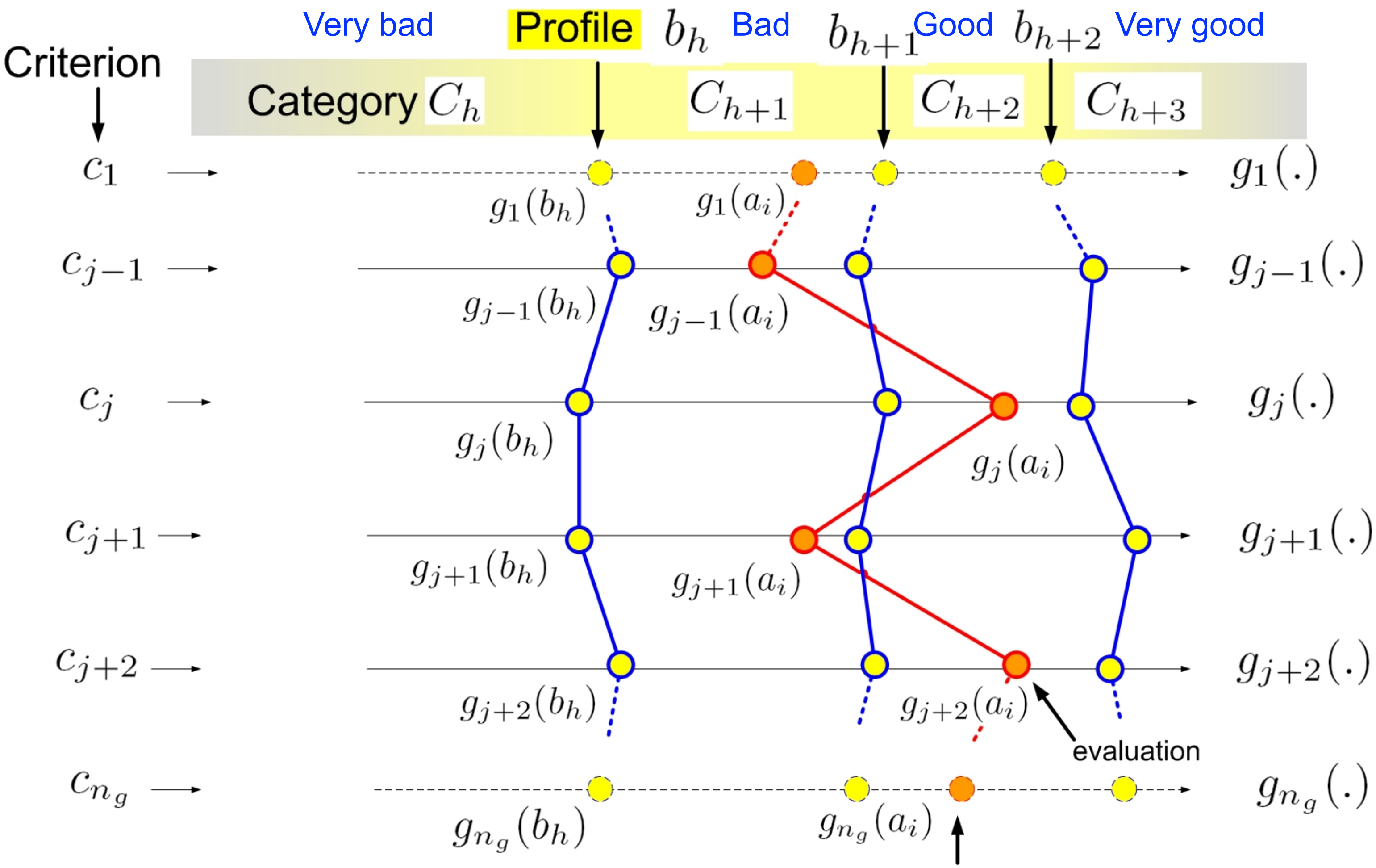


ELECTRE (1968) ➡ ELECTRE TRI (1992) ➡ Soft ELECTRE TRI [Dezert-Tacnet 2012]

ELECTRE = **EL**imination **E**t **C**hoix **T**raduisant la **RE**alité [Roy 1968]

Part 2 - Sorting alternatives in categories

For each criteria, we preset categories by some profile bounds



Which category does a_i belong to?

Profile of alternative a_i

Part 2 - Soft ELECTRE TRI (SET)

[Dezert Tacnet 2012]

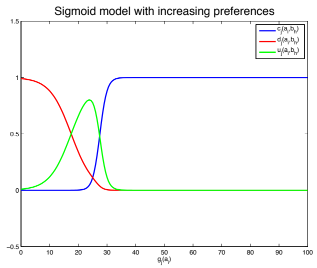
Purpose: We evaluate the assertion a_i is at least as good as b_h

$$\Theta \triangleq \{c, \bar{c}\}$$

$$c = a_i \text{ is concordant with assertion}$$
$$\bar{c} = a_i \text{ is discordant with assertion}$$

SET Step 1: Partial concordance and discordance indices are replaced by local BBAs

We use sigmoidal models
+ BBA PCR6 fusion



$$\begin{cases} c_j(a_i, b_h) \triangleq m_{ih}^j(c) \in [0,1] \\ d_j(a_i, b_h) \triangleq m_{ih}^j(\bar{c}) \in [0,1] \\ u_j(a_i, b_h) \triangleq m_{ih}^j(c \cup \bar{c}) \in [0,1] \end{cases}$$

SET Step 2: Global belief of assertion and global indices

PCR6 fusion + imp. Discounting

$$m_{ih}(\cdot) = m_{ih}^1 \oplus \ldots \oplus m_{ih}^{n_g}$$

$$\longrightarrow \begin{cases} c(a_i, b_h) \triangleq m_{ih}(c)\alpha(a_i, b_h) \\ d(a_i, b_h) \triangleq m_{ih}(\bar{c})\beta(a_i, b_h) \\ u(a_i, b_h) \triangleq 1 - c(a_i, b_h) - d(a_i, b_h) \end{cases}$$

SET Step 3: Probabilized outranking based on imprecise probability areas

SET Step 4: Soft (probabilistic) assignment of each alternative in a category

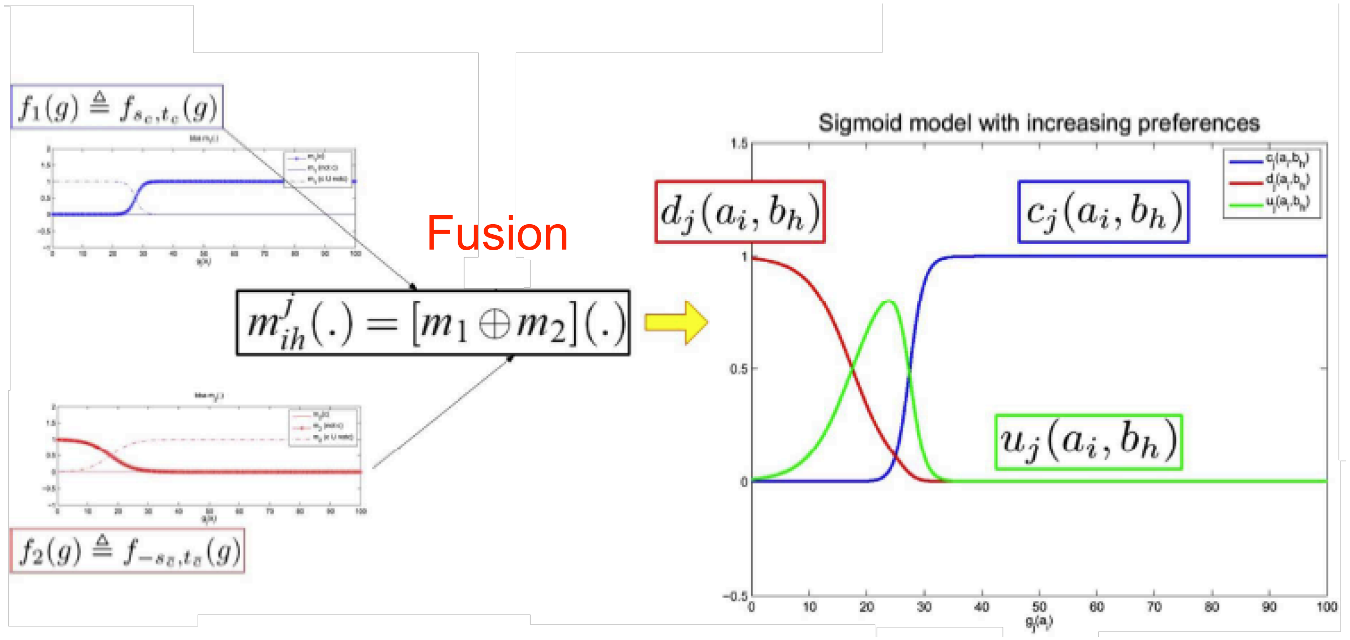
Soft ELECTRE TRI - Step 1 : Local BBAs

SET Step 1: Computation of local concordances, discordances and uncertainties

focal element	$m_1(.)$	$m_2(.)$
c	$f_{s_c,t_c}(g)$	0
$\bar{c}$	0	$f_{-s_{\bar{c}},t_{\bar{c}}}(g)$
$c \cup \bar{c}$	$1 - f_{s_c,t_c}(g)$	$1 - f_{-s_{\bar{c}},t_{\bar{c}}}(g)$

$$\begin{cases} c_j(a_i,b_h) \triangleq m_{ih}^j(c) \in [0,1] \\ d_j(a_i,b_h) \triangleq m_{ih}^j(\bar{c}) \in [0,1] \\ u_j(a_i,b_h) \triangleq m_{ih}^j(c \cup \bar{c}) \in [0,1] \end{cases}$$

$$f_{s,t}(g) \triangleq 1/(1+e^{-s(g-t)})$$

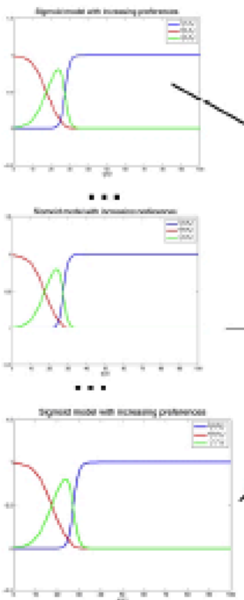


Soft ELECTRE TRI - Step 2 : Global BBA

SET Step2: Computation of **global** concordances, discordances and uncertainties

For all alternatives a_i, for each profile b_h

c_1 … c_j … c_{n_g}



ET concordance and partial indexes are replaced by bba's $\Theta \triangleq \{c, \bar{c}\}$

Local concordance

Local discordance

Uncertainty

Fusion process

$$\begin{cases} m_{ih}(.) = [m^1_{ih} \oplus m^2_{ih} \oplus \ldots \oplus m^{n_g}_{ih}](.) \\ m_{hi}(.) = [m^1_{hi} \oplus m^2_{hi} \oplus \ldots \oplus m^{n_g}_{hi}](.) \end{cases}$$

ET Global indexes are replaced by fusion results

$$\begin{cases} c(a_i, b_h) \triangleq m_{ih}(c)\alpha(a_i, b_h) \\ d(a_i, b_h) \triangleq m_{ih}(\bar{c})\beta(a_i, b_h) \\ u(a_i, b_h) \triangleq 1 - c(a_i, b_h) - d(a_i, b_h) \end{cases}$$

where

$$\alpha(a_i, b_h) \triangleq \begin{cases} 1 & \text{if} \quad \mathbf{V}_\alpha = \emptyset \\ \prod_{j \in \mathbf{V}_\alpha} \frac{1 - d_j(a_i, b_h)}{1 - m_{ih}(c)} & \text{if} \quad \mathbf{V}_\alpha \neq \emptyset \end{cases}$$

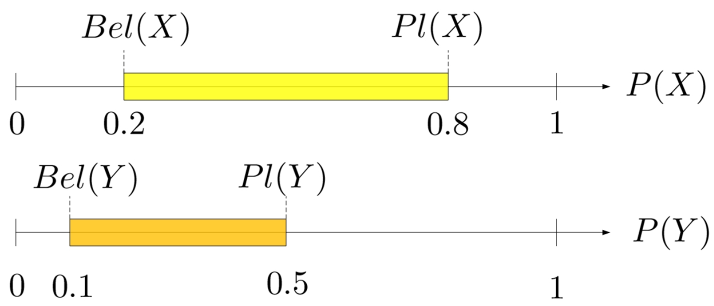
$$\beta(a_i, b_h) \triangleq \begin{cases} 1 & \text{if} \quad \mathbf{V}_\beta = \emptyset \\ \prod_{j \in \mathbf{V}_\beta} \frac{1 - c_j(a_i, b_h)}{1 - m_{ih}(\bar{c})} & \text{if} \quad \mathbf{V}_\beta \neq \emptyset \end{cases}$$

with $\begin{cases} \mathbf{V}_\alpha \triangleq \{j \in \mathbf{J} | d_j(a_i, b_h) > m_{ih}(c)\} \\ \mathbf{V}_\beta \triangleq \{j \in \mathbf{J} | c_j(b_h, a_i) > m_{ih}(\bar{c})\} \end{cases}$

Soft ELECTRE TRI - Step 3 : Probabilized outranking

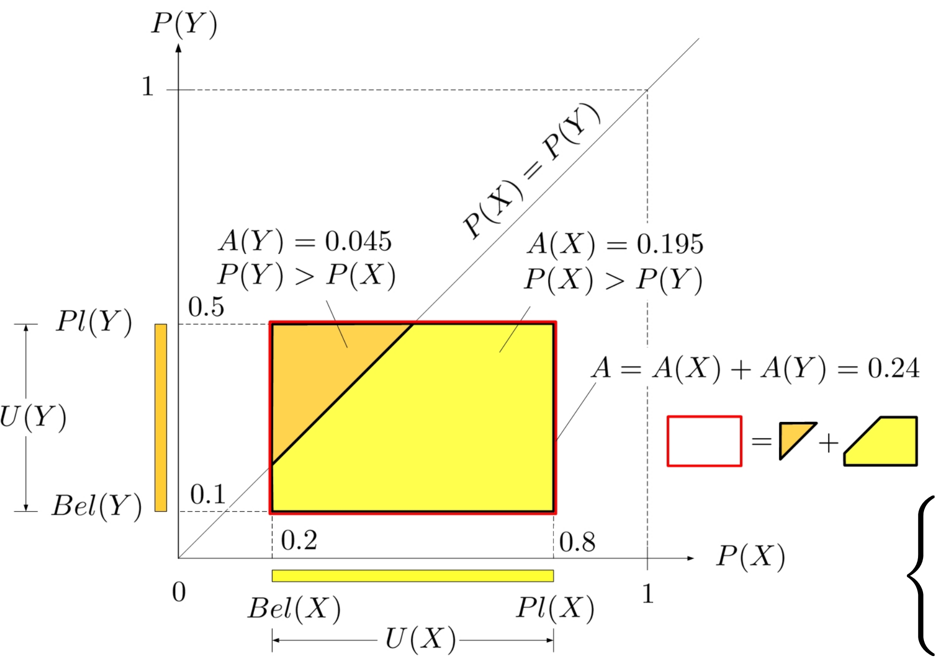
SET Step3: Probabilized outranking

$X = \text{"}a_i > b_h\text{"} \qquad Y = \text{"}b_h > a_i\text{"}$



$$\begin{cases} \text{Bel}(X) = c(a_i, b_h) \\ \text{Bel}(Y) = c(b_h, a_i) \end{cases}$$

$$\begin{cases} \text{Pl}(X) = 1 - d(a_i, b_h) = c(a_i, b_h) + u(a_i, b_h) \\ \text{Pl}(Y) = 1 - d(b_h, a_i) = c(b_h, a_i) + u(b_h, a_i) \end{cases}$$



$$\begin{cases} P_{X>Y} = \dfrac{A(X)}{A(X)+A(Y)} = \dfrac{0.195}{0.24} = 0.8125 \\ P_{Y>X} = \dfrac{A(Y)}{A(X)+A(Y)} = \dfrac{0.045}{0.24} = 0.1875 \end{cases}$$

$$\begin{cases} a_i > b_h \text{ with proba } P_{ih} = P_{X>Y} \approx 0.81 \\ b_h > a_i \text{ with proba } P_{hi} = P_{Y>X} \approx 0.19 \end{cases}$$

Soft ELECTRE TRI - Step 4 : Soft assignment

SET Step 4: Final assignment of alternative in a category

We consider all possible outranking sequences with their probabilities

Suppose at SET step 3 one gets for alternative ai

Profiles $b_h \rightarrow$ **Outranking probas** $\downarrow$	b_0	b_1	b_2	b_3
P_{ih}	1	0.7	0.2	0

$P_{i0} = P(X_{i0} = "a_i > b_0") = 1$

$P_{i3} = P(X_{i3} = "a_i > b_3") = 0$

All possible outranking sequences with their probas are

Profiles $b_h \rightarrow$ **Outrank sequences** $\downarrow$	b_0	C1	b_1	C2	b_2	C3	b_3	$P(S_k(a_i))$ $\downarrow$
$S_1(a_i)$	$>$		$>$		$>$		$<$	0.14
$S_2(a_i)$	$>$		$>$		$<$		$<$	0.56
$S_3(a_i)$	$>$		$<$		$<$		$<$	0.24
$S_4(a_i)$	$>$		$<$		$>$		$<$	0.06

$$P(S_1(a_i)) = 1 \times 0.7 \times 0.2 \times 1 = 0.14$$
$$P(S_2(a_i)) = 1 \times 0.7 \times (1 - 0.2) \times 1 = 0.56$$
$$P(S_3(a_i)) = 1 \times (1 - 0.7) \times (1 - 0.2) \times 1 = 0.24$$
$$P(S_4(a_i)) = 1 \times (1 - 0.7) \times 0.2 \times 1 = 0.06$$

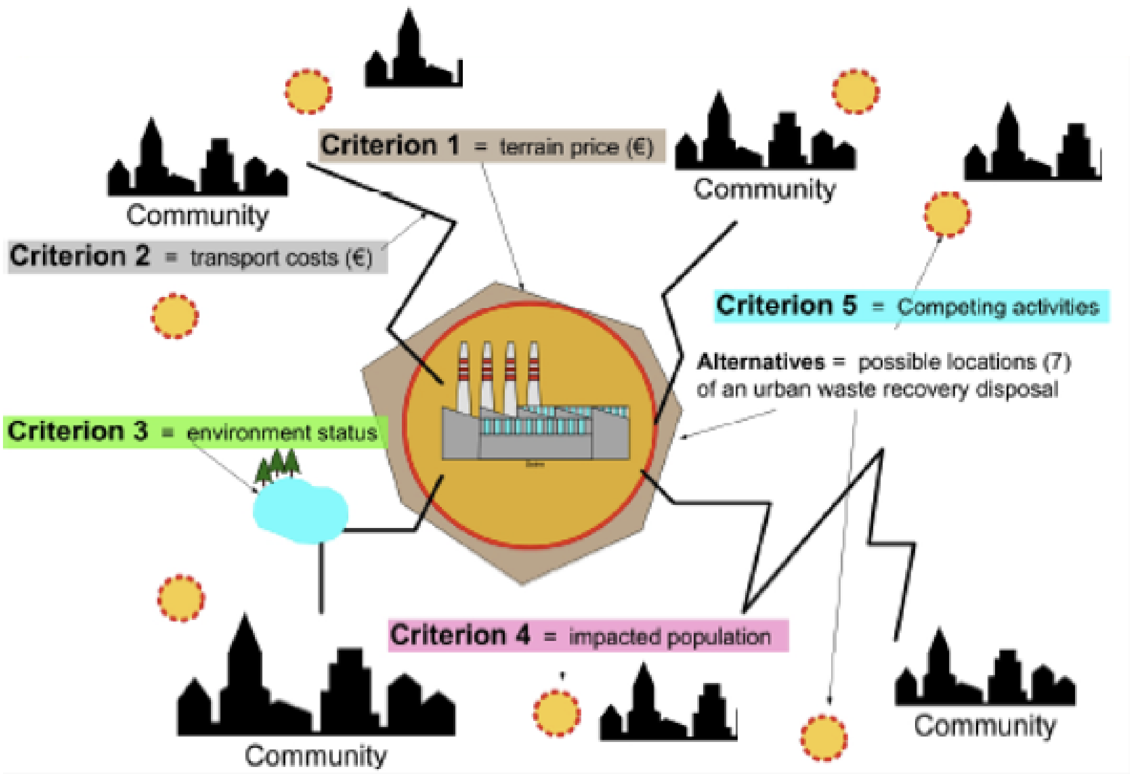
Final soft assignment (and hard assignment is possible from soft assignment)

Categories $C_h \rightarrow$ **Assignment probas** $a_i \downarrow$	C_1	C_2	C_3	$\emptyset$
$P(a_i \rightarrow C_h)$	0.24	0.56	0.14	$\delta_i = 0.06$

Inconsistency indicator

Soft ELECTRE TRI - Application example

We consider 7 possible locations a1,…, a7 for a future waste recovery disposal



We consider 5 criteria g1,…g5

Where should we settle the future urban waste recovery disposal?

Soft ELECTRE TRI - Application example (cont'd)

$g_1 =$ Terrain price ($\searrow$ preference);

the **lower** is g_1, the higher is the preference

$g_2 =$ Transport costs ($\searrow$ pref.);

the **lower** is g_2, the higher is the preference

$g_3 =$ Environment status expressed by population ($\nearrow$ pref.);

the **higher** is g_3, the lower are the negative effects

$g_4 =$ Impacted population ($\nearrow$ pref.);

the **higher** is g_4, the lower are the negative effects

$g_5 =$ Competition activities ($\nearrow$ pref.)

the **higher** is g_5, the lower is the competition with other activities (tourism, sport, etc)

Soft ELECTRE TRI - Application example (cont'd)

Input of the problem

Criteria $g_j \rightarrow$ / Choices $a_i \downarrow$	Terrain price $g_1 \searrow$ ($€/m^2$)	Transport cost $g_2 \searrow$ ($t \cdot km/\text{year}$)	Env. status $g_3 \nearrow$ $\{0, 1, \ldots, 10\}$	Impacted pop. $g_4 \nearrow$ $[0, 10]$	Competing activ. $g_5 \nearrow$ $\{0, 1, \ldots, 100\}$
a_1	-120	-284	5	3.5	18
a_2	-150	-269	2	4.5	24
a_3	-100	-413	4	5.5	17
a_4	-60	-596	6	8.0	20
a_5	-30	-1321	8	7.5	16
a_6	-80	-734	5	4.0	21
a_7	-45	-982	7	8.5	13

Profile definition for 3 categories (bad,medium,good)

Profiles $b_h \rightarrow$ / Criteria $g_j \downarrow$	b_1	b_2
g_1: $€/m^2$	-100	-50
g_2: $t \cdot km/\text{year}$	-1000	-500
g_3: $\{0, 1, \ldots, 10\}$	4	7
g_4: $[0, 10]$	4	7
g_5: $\{0, 1, \ldots, 100\}$	15	20

Weights and thresholds used

Thresholds $\rightarrow$ / Criteria $g_j \downarrow$	w_j (weight)	q_j (indifference)	p_j (preference)	v_j (veto)
g_1: $€/m^2$	0.25	15	40	100
g_2: $t \cdot km/\text{year}$	0.45	80	350	850
g_3: $\{0, 1, \ldots, 10\}$	0.10	1	3	5
g_4: $[0, 10]$	0.12	0.5	3.5	4.5
g_5: $\{0, 1, \ldots, 100\}$	0.08	1	5	8

Soft ELECTRE TRI - Application example (cont'd)

Final assignment of locations in categories based on classical ELECTRE TRI

	C_1	C_2	C_3
a_1	0	1	0
a_2	1	0	0
a_3	0	1	0
a_4	0	1	0
a_5	1	0	0
a_6	0	1	0
a_7	0	1	0

(a) Pessimistic attitude.

We use hard assignment with

$$\lambda = 0.75$$

= Inconsistency between ET and SET

= consistency between ET and SET

	C_1	C_2	C_3
a_1	0	1	0
a_2	0	0	1
a_3	0	1	0
a_4	0	0	1
a_5	0	1	0
a_6	0	1	0
a_7	0	1	0

(b) Optimistic attitude.

Final assignment of locations in categories based on SOFT ELECTRE TRI

	C_1	C_2	C_3	$\emptyset$
a_1	0.0054	0.3735	0.6123	$\delta_1 = 0.0088$
a_2	0.0894	0.7294	0.1614	$\delta_2 = 0.0198$
a_3	0.0001	0.9429	0.0570	$\delta_3 = 0$
a_4	0	0.9193	0.0807	$\delta_4 = 0$
a_5	0.7744	0.2111	0.0031	$\delta_5 = 0.0114$
a_6	0.0004	0.9990	0.0006	$\delta_6 = 0$
a_7	0.0025	0.9869	0.0106	$\delta_7 = 0$

Soft ELECTRE TRI - Conclusions

Advantages of SET versus ET

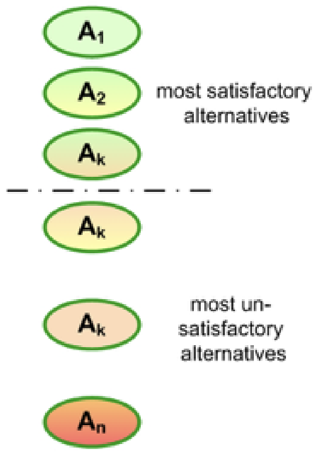
SET method uses the same inputs as ET (same criteria and thresholds definitions)

SET method is more effective

- it avoids lambda-cutting for hard assignment
- it avoids arbitrary choice of decision strategy (optimistic or pessimistic)
- it provides soft (probabilized) with inconsistency indicator

Part 3 - BF-TOPSIS

for ranking alternatives based on multi-criteria



BF-TOPSIS = **B**elief-**F**unctions based of **T**echnique for **O**rder **P**reference by **S**imilarity to **I**deal **S**olution

Part 3 - MDCM modeling

How to make a choice among several alternatives based on different criteria?

Car example: How to buy a car based on some criteria (i.e. cost, safety, etc.)?

Several methods exist depending on the problem modeling.

Here we classical modeling based on score matrix.

$$\mathbf{S} \triangleq \begin{array}{c} \\ A_1 \\ \vdots \\ A_i \\ \vdots \\ A_M \end{array} \begin{array}{c} \begin{array}{ccccc} C_1, w_1 & \dots & C_j, w_j & \dots & C_N, w_N \end{array} \\ \begin{bmatrix} S_{11} & \dots & S_{1j} & \dots & S_{1N} \\ & & \vdots & & \\ S_{i1} & \dots & S_{ij} & \dots & S_{iN} \\ & & \vdots & & \\ S_{M1} & \dots & S_{Mj} & \dots & S_{MN} \end{bmatrix} \end{array}$$

(Score matrix)

- A set of $M \geqslant 2$ alternatives $\mathcal{A} \triangleq \{A_1, \dots, A_M\}$
- A set of $N > 1$ Criteria $\mathbf{C} \triangleq \{C_1, \dots, C_N\}$
- A set of $N > 1$ criteria **importance weights** $W = \{w_1, \dots, w_N\}$, with $w_j \in [0, 1]$ and $\sum_j w_j = 1$

Part 3 - MDCM modeling

Some facts to recall

All MCDM methods developed so far suffer of Rank Reversal (RR)

Most methods require score normalization which is a source of RR

No MCDM method makes consensus for users, but some are very popular and simple

AHP (Analytic hierarchy process) [Saaty 1980] is not exempt of problems

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) [Hwang Yoon 1981] is very disputed because of choice of normalization

What we present here

A new **Belief-Function based** TOPSIS (**BF-TOPSIS**) to solve classical and non-classical MCDM problems [Dezert Han Yin 2016,Carladous et al. 2016]

Part 3 - Classical TOPSIS approach

Classical TOPSIS method [Hwang Yoon 1981]

1. Build the normalized score matrix $\mathbf{R} = [R_{ij}] = [S_{ij}/\sqrt{\sum_i S_{ij}^2}]$
2. Calculate the weighted normalized decision matrix $\mathbf{D} = [w_j \cdot R_{ij}]$
3. Determine the positive (best) ideal solution A^{best} by **taking the best/max value** in each column of $\mathbf{D}$
4. Determine the negative (worst) ideal solution A^{worst} by **taking the worst/min value** in each column of $\mathbf{D}$
5. Compute L2-distances $d(A_i, A^{best})$ of A_i, (i=1,. . . ,M) to A^{best}, and $d(A_i, A^{worst})$ of A_i to A^{worst}
6. Calculate the relative closeness of A_i to best ideal solution A^{best} by

$$C(A_i, A^{best}) \triangleq \frac{d(A_i, A^{worst})}{d(A_i, A^{worst}) + d(A_i, A^{best})}$$

When $C(A_i, A^{best}) = 1$, its means that $A_i = A^{best}$ because $d(A_i, A^{best}) = 0$

When $C(A_i, A^{best}) = 0$, its means that $A_i = A^{worst}$ because $d(A_i, A^{worst}) = 0$

7. Rank alternatives A_i according to $C(A_i, A^{best})$ in descending order, and select the highest preferred solution

Part 3 - BBA construction for BF-TOPSIS

$$\mathbf{S} = [S_{ij}] \Rightarrow \mathbf{M} = [(m_{ij}(A_i), m_{ij}(\bar{A}_i), m_{ij}(A_i \cup \bar{A}_i))]$$

How to get the BBA matrix M from the score matrix

$$\mathrm{Sup}_j(A_i) \triangleq \sum_{k\in\{1,\dots M\}|S_{kj}\leqslant S_{ij}} |S_{ij} - S_{kj}|$$

This measures how much Ai is higher (better) than other alternatives based on Cj

$$\mathrm{Inf}_j(A_i) \triangleq -\sum_{k\in\{1,\dots M\}|S_{kj}\geqslant S_{ij}} |S_{ij} - S_{kj}|$$

This measures how much Ai is lower (worse) than other alternatives based on Cj

Important inequality

$$\mathrm{Bel}_{ij}(A_i) \longleftarrow \boxed{\frac{\mathrm{Sup}_j(A_i)}{A^j_{max}}} \leqslant 1 - \boxed{\frac{\mathrm{Inf}_j(A_i)}{A^j_{min}}} \longrightarrow \mathrm{Bel}_{ij}(\bar{A}_i)$$

One always has $0 \leqslant \mathrm{Bel}_{ij}(A_i) \leqslant (\mathrm{Pl}_{ij}(A_i) = 1 - \mathrm{Bel}_{ij}(\bar{A}_i)) \leqslant 1$

BBA used for M matrix

$$m_{ij}(A_i) = \mathrm{Bel}_{ij}(A_i) \qquad m_{ij}(\bar{A}_i) = \mathrm{Bel}_{ij}(\bar{A}_i) = 1 - \mathrm{Pl}_{ij}(A_i)$$

$$m_{ij}(A_i \cup \bar{A}_i) = \mathrm{Pl}_{ij}(A_i) - \mathrm{Bel}_{ij}(A_i) = 1 - m_{ij}(A_i) - m_{ij}(\bar{A}_i)$$

Part 3 - BBA construction for BF-TOPSIS (cont'd)

Advantages of this BBA construction

1. if all S_{ij} are the same for a given column, we get $\forall A_i$, $Sup_j(A_i) = Inf_j(A_i) = 0$ and therefore $m_{ij}(A_i \cup \bar{A}_i) = 1$ which is the vacuous BBA, which makes sense.
2. it is invariant to the bias and scaling effects of score values. Indeed, if S_{ij} are replaced by $S'_{ij} = a \cdot S_{ij} + b$, with a scale factor $a > 0$ and a bias $b \in \mathbb{R}$, then $m_{ij}(\cdot)$ and $m'_{ij}(\cdot)$ remain equal.
3. if a numerical value S_{ij} is missing or indeterminate, then we use the vacuous belief assignment $m_{ij}(A_i \cup \bar{A}_i) = 1$.
4. We can also discount the BBA $m_{ij}(\cdot)$ by a reliability factor using the classical Shafer's discounting method if one wants to express some doubts on the reliability of $m_{ij}(\cdot)$.

In summary

From $[S_{ij}]$, we know how to build the matrix $\mathbf{M} = [(m_{ij}(A_i), m_{ij}(\bar{A}_i), m_{ij}(A_i \cup \bar{A}_i))]$

How to use these BBAs to rank A_i to make a decision? → BF-TOPSIS methods

Part 3 - BF-TOPSIS1 method (simplest method)

Steps of BF-TOPSIS1 [Dezert Han Yin 2016]

1. From $\mathbb{S}$, compute BBAs $m_{ij}(A_i)$ $m_{ij}(\bar{A}_i)$, and $m_{ij}(A_i \cup \bar{A}_i)$
2. Set $m_{ij}^{\text{best}}(A_i) \triangleq 1$, and $m_{ij}^{\text{worst}}(\bar{A}_i) \triangleq 1$ and compute distances $d_{BI}(m_{ij}, m_{ij}^{\text{best}})$ and $d_{BI}(m_{ij}, m_{ij}^{\text{worst}})$ to ideal solutions.
3. Compute the weighted average distances of A_i to ideal solutions

$$d^{\text{best}}(A_i) \triangleq \sum_{j=1}^{N} w_j \cdot d_{BI}(m_{ij}, m_{ij}^{\text{best}})$$

$$d^{\text{worst}}(A_i) \triangleq \sum_{j=1}^{N} w_j \cdot d_{BI}(m_{ij}, m_{ij}^{\text{worst}})$$

4. Compute the relative closeness of A_i with respect to ideal best solution A^{best}

$$C(A_i, A^{\text{best}}) \triangleq \frac{d^{\text{worst}}(A_i)}{d^{\text{worst}}(A_i) + d^{\text{best}}(A_i)}$$

5. Rank A_i by $C(A_i, A^{\text{best}})$ in descending order.

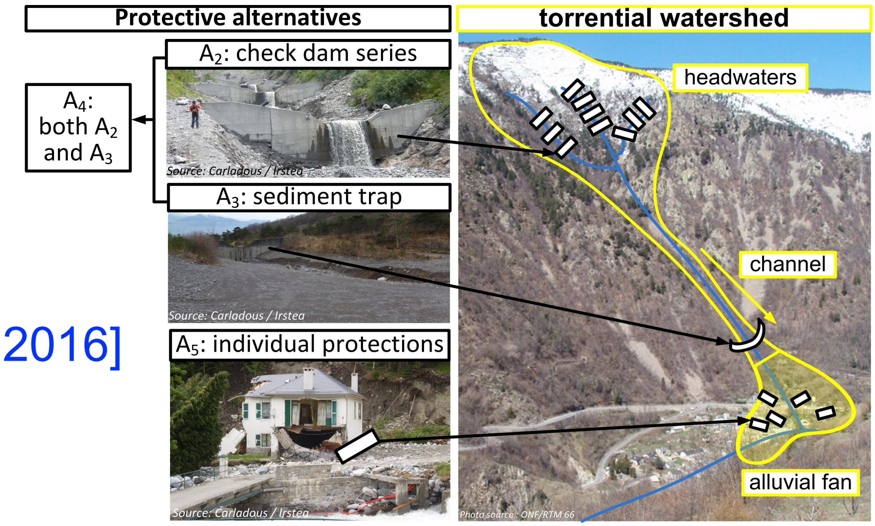
What protective action to take within a torrential watershed?

4 (or 5) possible actions

- A_1: doing nothing
- A_2: building check dam series
- A_3: building a sediment trap
- A_4: mixing A_2 and A_3
- A_5: adding individual protections

5 criteria

- C_1: investment cost
- C_2: annual maintenance cost
- C_3: Annual Risk Reduction (ARR) of houses damaged
- C_4: ARR of human casualties
- C_5: ARR of # of sites dangerous to environment



[Carladous et al.2016]

We want to reduce C1 and C2 and increase C3,C4 and C5

Part 3 - Application of BF-TOPSIS for risk management

Weighting factors of criteria are obtained by AHP (pairwise comparison matrix)

Initial score matrix for this problem Case 1 (4 actions) and Case 2 (5 actions)

		C_j	C_1	C_2	C_3	C_4	C_5
		w_j	0.08	0.04	0.10	0.46	0.32
$\mathbf{S}_{case2}$	$\mathbf{S}_{case1}$	A_1	0	0	0	0	0
		A_2	300 000	6 000	5	0.007	0.02
		A_3	300 000	1 500	5	0.008	0.04
		A_4	600 000	7 500	7	0.008	0.05
	A_5		1 000 000	0	7	0.008	0.1

All criteria are transformed into monetary value (in euros)

Transformation of score matrix (multiplication by -1 of C1 and C2)

		C_j	C_1	C_2	C_3	C_4	C_5
		w_j	0.08	0.04	0.10	0.46	0.32
$\mathbf{S}^{pref}_{case2}$	$\mathbf{S}^{pref}_{case1}$	A_1	0	0	0	0	0
		A_2	-300 000	-6 000	5	0.007	0.02
		A_3	-300 000	-1 500	5	0.008	0.04
		A_4	-600 000	-7 500	7	0.008	0.05
	A_5		-1,000,000	0	7	0.008	0.1

Hence the greater is better

Part 3 - Application of BF-TOPSIS for risk management

Solution in case of 4 possible actions

Methods		Ranking vectors	Preference orders
AHP	1	$[0.11, 0.18, 0.32, 0.40]$	$A_4 \succ A_3 \succ A_2 \succ A_1$
	2	$[0.12, 0.31, 0.40, 0.41]$	$A_4 \succ A_3 \succ A_2 \succ A_1$
BF-TOPSIS	1	$[0.12, 0.54, 0.79, 0.88]$	$A_4 \succ A_3 \succ A_2 \succ A_1$
	2	$[0.12, 0.54, 0.79, 0.88]$	$A_4 \succ A_3 \succ A_2 \succ A_1$
	3	$[0.03, 0.76, 0.96, 0.97]$	$A_4 \succ A_3 \succ A_2 \succ A_1$
	4	$[0.03, 0.76, 0.96, 0.97]$	$A_4 \succ A_3 \succ A_2 \succ A_1$

Solution in case of 5 possible actions

AHP	1	$[0.07, 0.12, 0.20, 0.24, 0.35]$	$A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$
	2	$[0.12, 0.25, 0.31, 0.30, 0.39]$	$A_5 \succ A_3 \succ A_4 \succ A_2 \succ A_1$
BF-TOPSIS	1	$[0.12, 0.49, 0.66, 0.69, 0.92]$	$A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$
	2	$[0.12, 0.49, 0.66, 0.69, 0.92]$	$A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$
	3	$[0.03, 0.68, 0.85, 0.88, 0.97]$	$A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$
	4	$[0.03, 0.68, 0.85, 0.88, 0.97]$	$A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$

rank reversal phenomena

more robust to rank reversal

Part 3 - BF-TOPSIS Conclusion

BF-TOPSIS improves TOPSIS thanks to Belief Functions [Dezert Han Yin 2016]

Advantages of BF-TOPSIS methods

No need for ad-hoc normalization of score matrix

Solid justification for BBA construction from score matrix

More robustness to rank reversal phenomena (although not exempt)

Complexity of BF-TOPSIS methods

BF-TOPSIS1: smallest complexity

BF-TOPSIS2: medium complexity

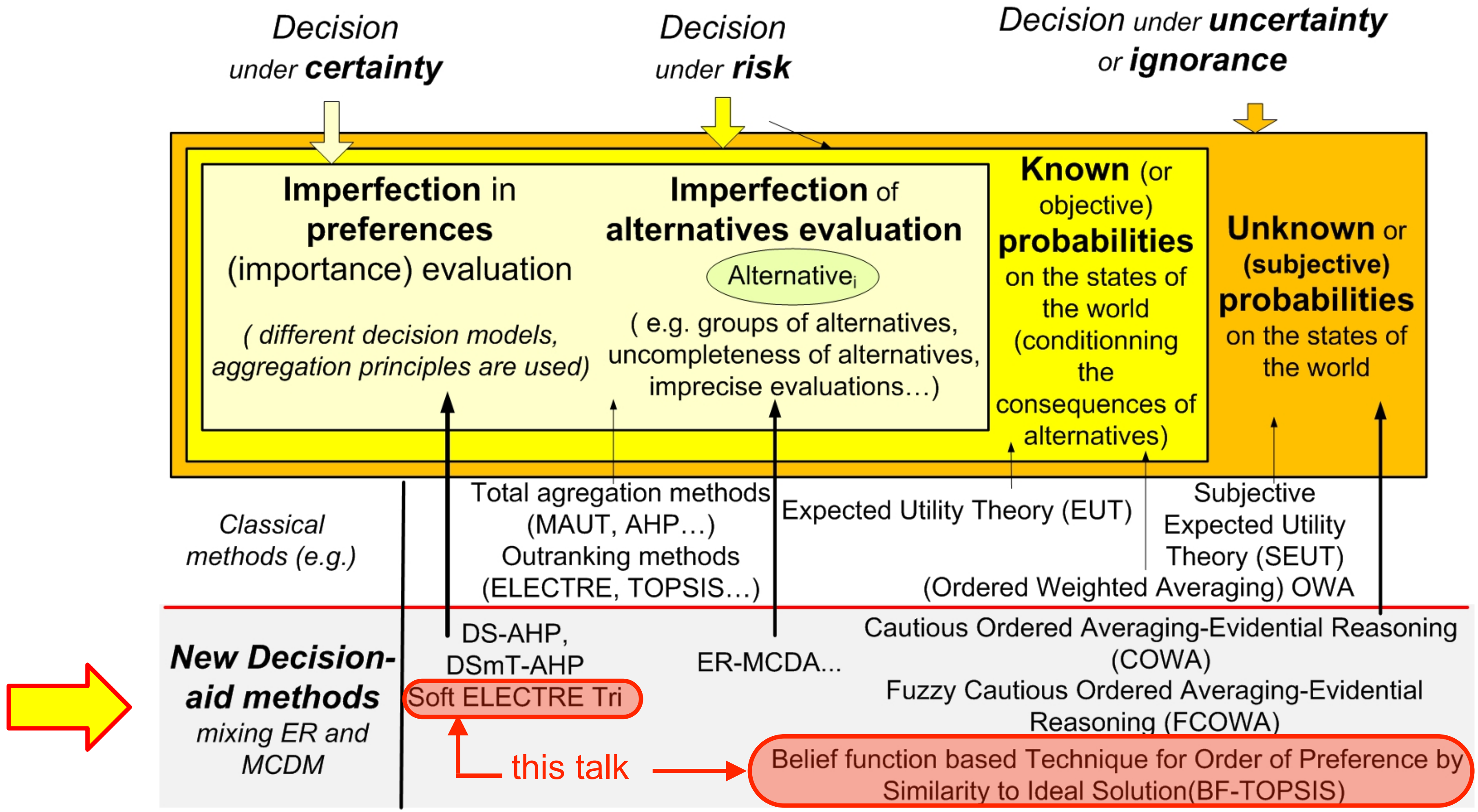
BF-TOPSIS3: high complexity (because of PCR6 rule)

BF-TOPSIS4: highest complexity (because of ZPCR6 rule)

BF-TOPSIS can work also with imprecise scores - see [Dezert Han Tacnet 2017]

Conclusion

A global framework to decide under imperfect information contexts mixing uncertainty theories and multicriteria decision-making methods



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Thank you for your attention.

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THE FRENCH AEROSPACE LAB

return on innovation

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