Data-based modelling of low-order modes for AO control: what do on-sky experiments tell us?

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Adaptive optics for astronomy

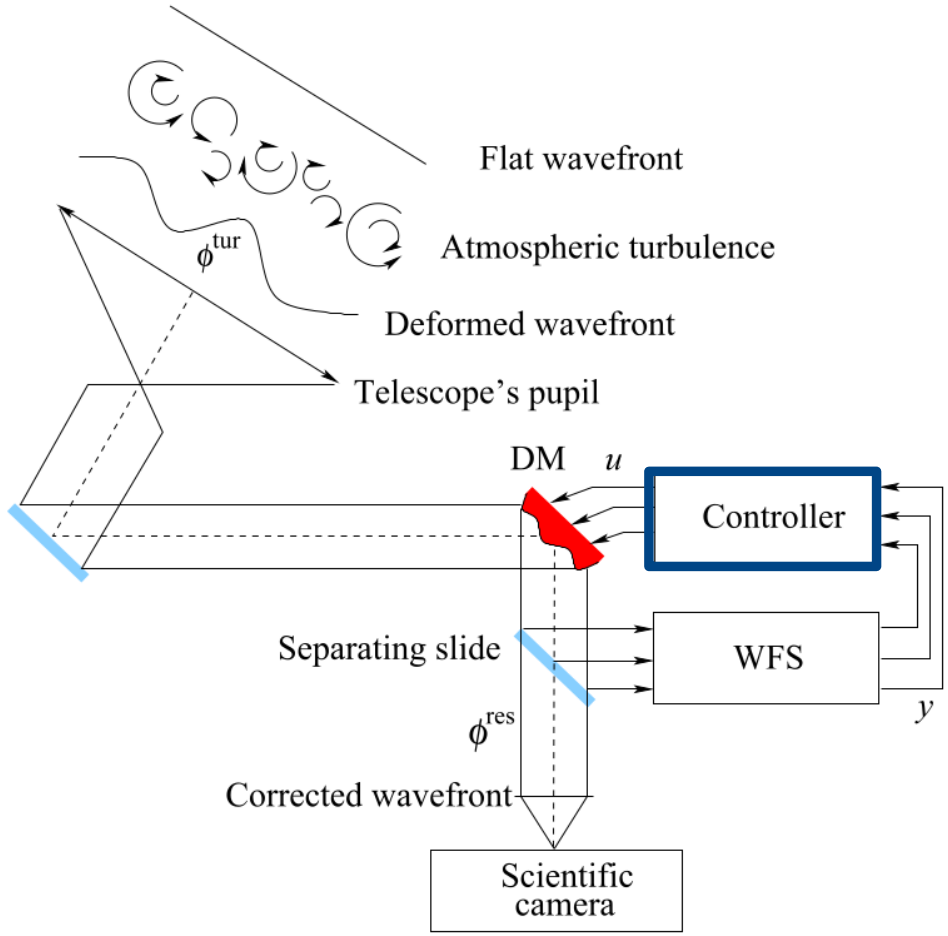


Figure 1: AO system schematics

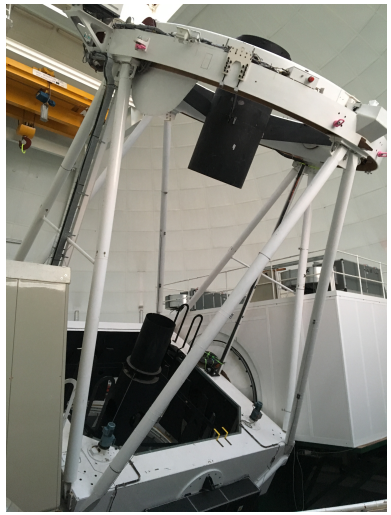
I. SCAO on CANARY

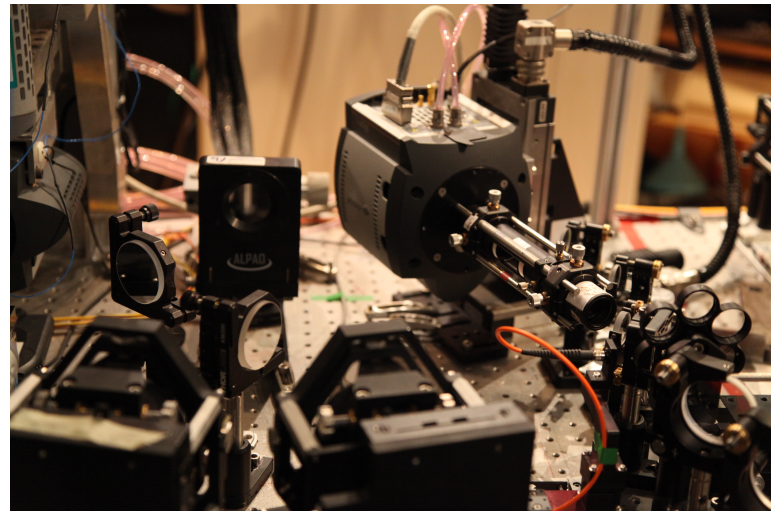
II. Disturbance model and the Linear-Quadratic-Gaussian (LQG) regulator

III. On-sky experiments
- Case 1: large wind speed
- Case 2: low wind speed

IV. Performance analysis

SCAO on CANARY



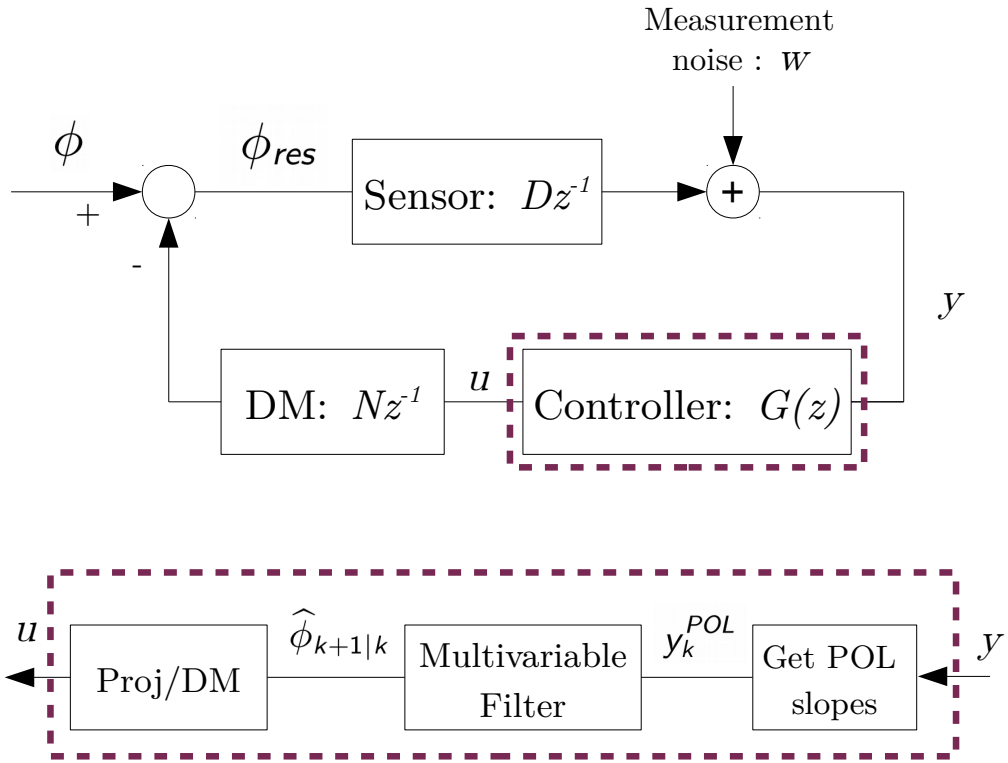


Run from July 18th to 22nd, 2019	
Telescope	William Herschel
Diameter of prime mirror [m]	4.2
Central obscuration [m]	1.2
Shack-Hartmann	14×14
DM	15×15
Coupling	0.45
Control frequency [Hz]	200
Delay in the loop	2.25 frames

Seeing and wind profiles available in real-time [1].

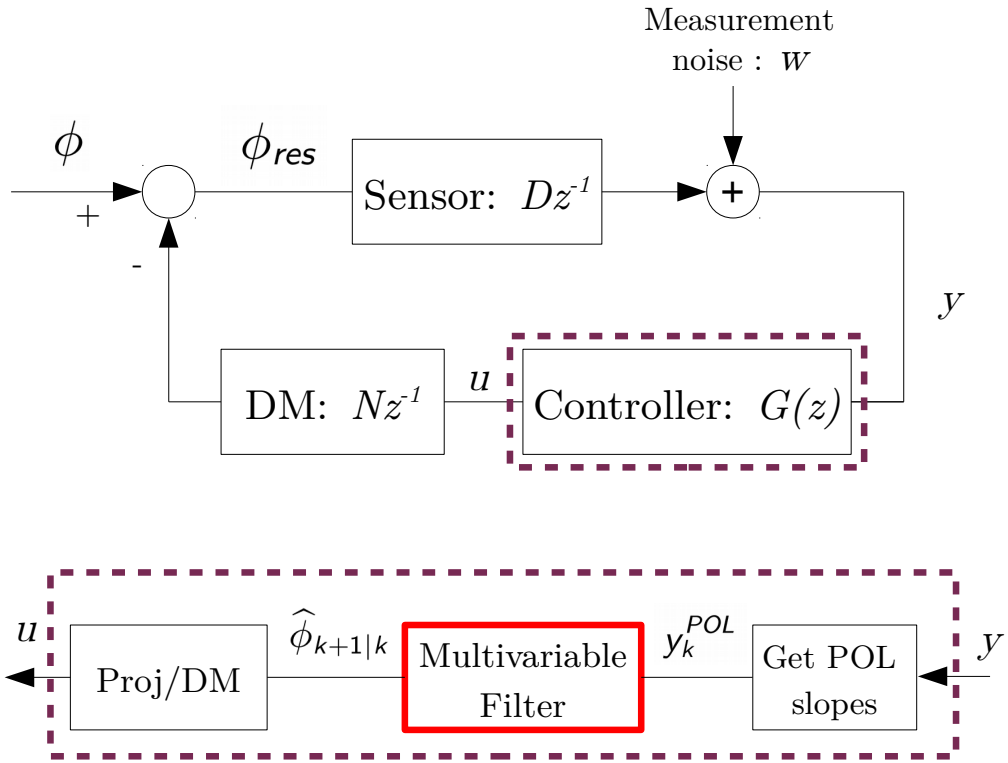
[1] J. Osborn, "Characterising atmospheric turbulence using SCIDAR techniques," Imaging and Applied Optics 2018.

The LQG regulator



1. Minimum variance prediction of the wavefront, $\widehat{\phi}_{k+1|k}$
2. Projection on the mirror's actuators, $u_k = M_{com} D \widehat{\phi}_{k+1|k}$

The LQG regulator



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Stochastic disturbance model

Wavefront expressed in a Zernike basis with 495 modes.

Total disturbance: $\phi_k + \phi_k^{LO}$

Prior-based model[2] (all modes):

$$\phi_{k+1} = A_1\phi_k + A_2\phi_{k-1} + v_k \tag{1}$$

Data-driven model (for the first n_{LO} modes):

$$\begin{cases} x_{k+1}^{LO} = A^{LO}x_k^{LO} + v_k \\ \phi_k^{LO} = C^{LO}x_k^{LO} + \eta_k \end{cases} \tag{2}$$

[2] G. Sivo, et al., "First on-sky SCAO validation of full LQG control with vibration mitigation on the CANARY pathfinder," Opt. Express 22, 23565-23591 (2014)

Stochastic disturbance model

y_k^{OL}: wavefront sensor measurement in open-loop
w_k: measurement noise
v_k: process noise

Concatenating prior-based and data-based models in state-space form:

$$\begin{cases} \begin{bmatrix} \phi_{k+1} \\ \phi_k \\ x_{k+1}^{LO} \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & A_2 & 0 \\ I & 0 & 0 \\ 0 & 0 & A_{LO} \end{bmatrix}}_{A} \begin{bmatrix} \phi_k \\ \phi_{k-1} \\ x_k^{LO} \end{bmatrix} + v_k \\ y_k^{OL} = \underbrace{\begin{bmatrix} 0 & D & D_{LO} \end{bmatrix}}_{C} \begin{bmatrix} \phi_k \\ \phi_{k-1} \\ x_k^{LO} \end{bmatrix} + w_k \end{cases} \tag{3}$$

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A linear multivariable filter:

$$\begin{cases} x_{k+1} = Ax_k + v_k \\ y_k^{OL} = Cx_k + w_k \end{cases} \tag{4}$$

The LQG controller - minimum variance control

1. Minimum variance prediction of the wavefront, $\widehat{\phi}_{k+1|k}$, with a Kalman filter:

$$\begin{cases} \widehat{x}_{k+1|k} = (A - L_\infty C)\widehat{x}_{k|k-1} + L_\infty y_k^{POL} \\ \widehat{\phi}_{k+1|k} = C_\phi \widehat{x}_{k+1|k} \end{cases} \tag{5}$$

2. Projection on the mirror inputs:

$$u_k = M_{com} D \widehat{\phi}_{k+1|k} \tag{6}$$

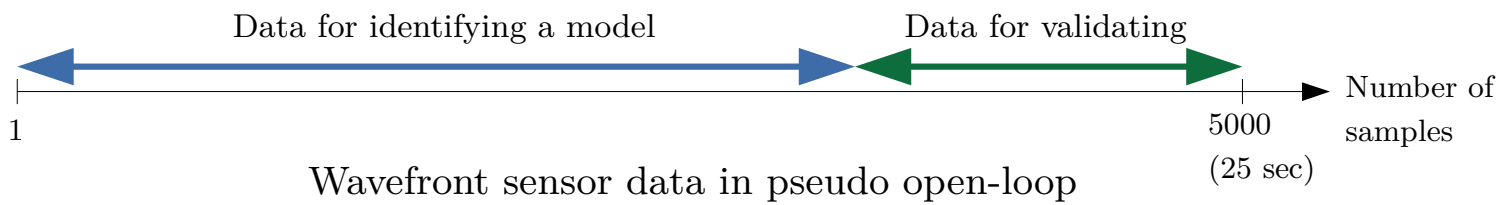
Model identification for the low-orders

Low-order disturbance model in state-space form,

$$\begin{cases} x_{k+1}^{LO} = A^{LO} x_k^{LO} + v_k \\ \phi_k^{LO} = C^{LO} x_k^{LO} + \eta_k \end{cases} \tag{7}$$

Now, estimate its matrices.

Model identification:



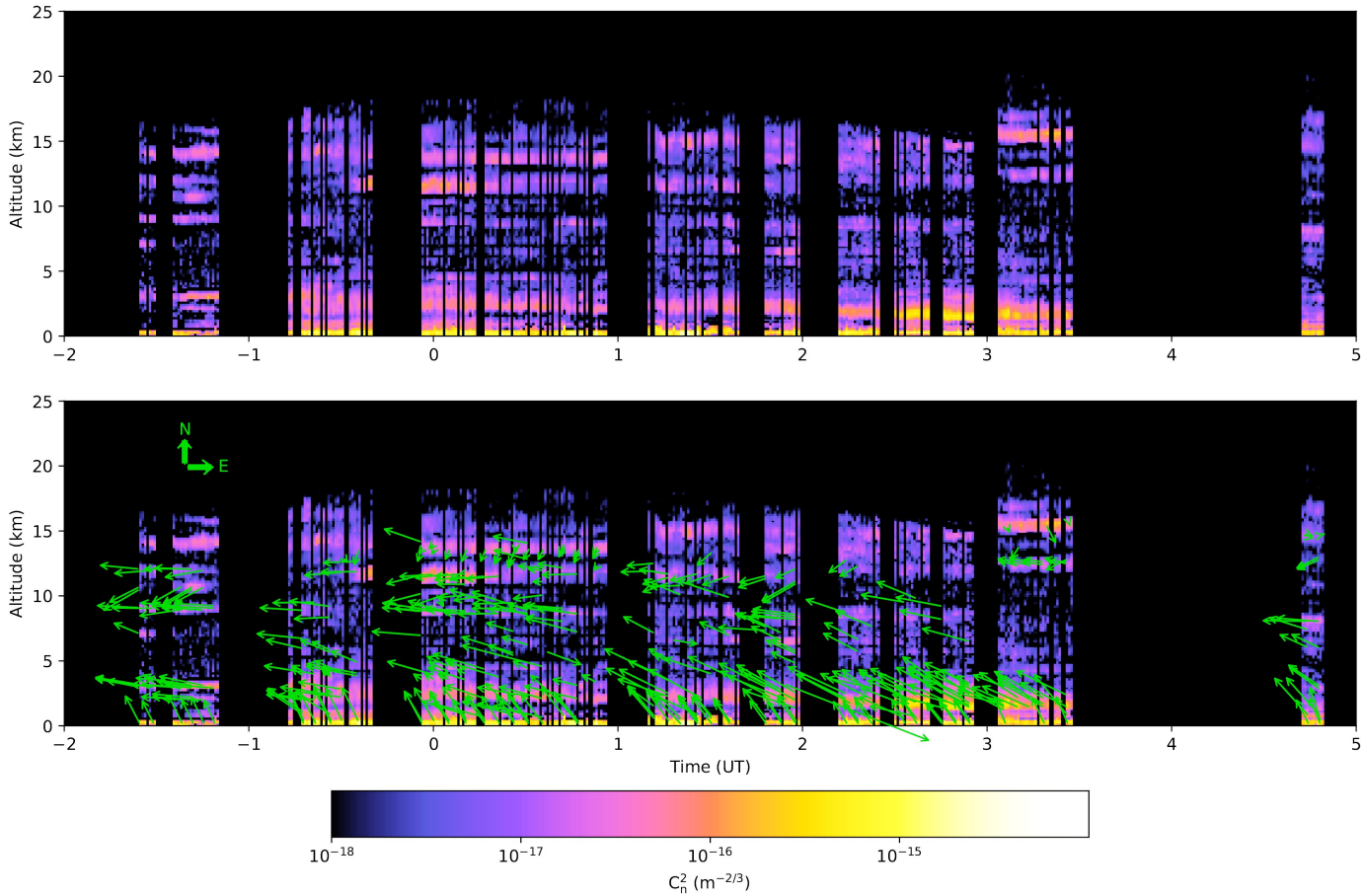
Wavefront sensor data in pseudo open-loop

1. Static wavefront reconstruction (Maximum A Posteriori): $\phi^{LO} = R_{MAP} y^{OL}$
2. Apply the subspace algorithm N4SID[3]

[3] P. Van Overschee, B. De Moor, "N4SID: Subspace algorithms for the identification of combined deterministic-stochastic systems", Automatica, Vol. 30, no. 1, pp. 75–93, 1994.

Case 1: large wind speed (1/3) - the atmosphere

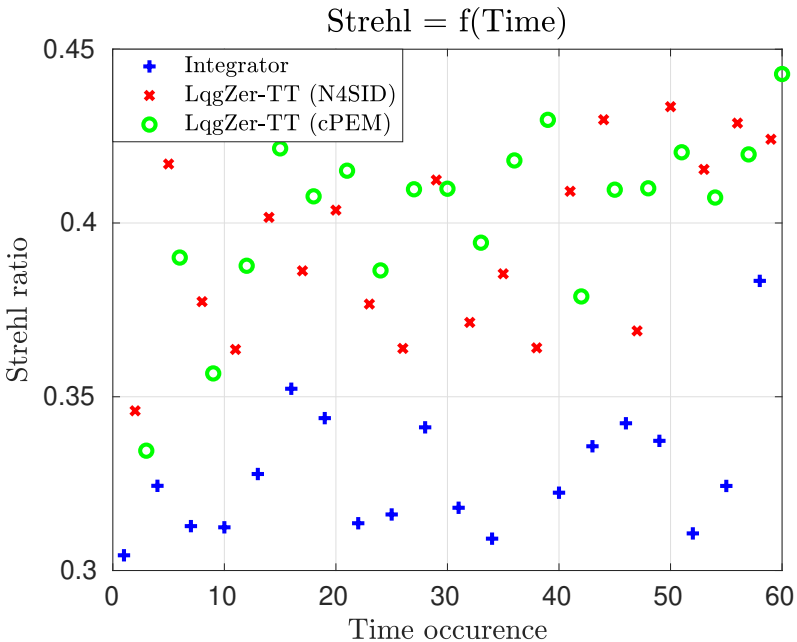
Evolution of the Cn^2 and windspeed from SLODAR

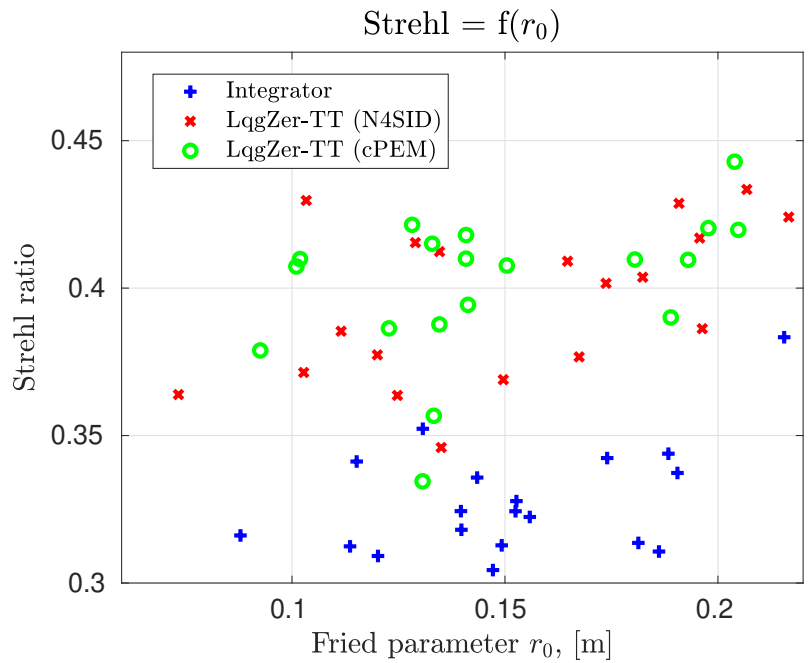


Length of the arrow in upper left position: 10m/s

Case 1: large wind speed (2/3) - on-sky results for $n_{LO} = 2$

Identification of tip and tilt dynamics either with N4SID, or cPEM (cascaded Prediction Error Method [4]).

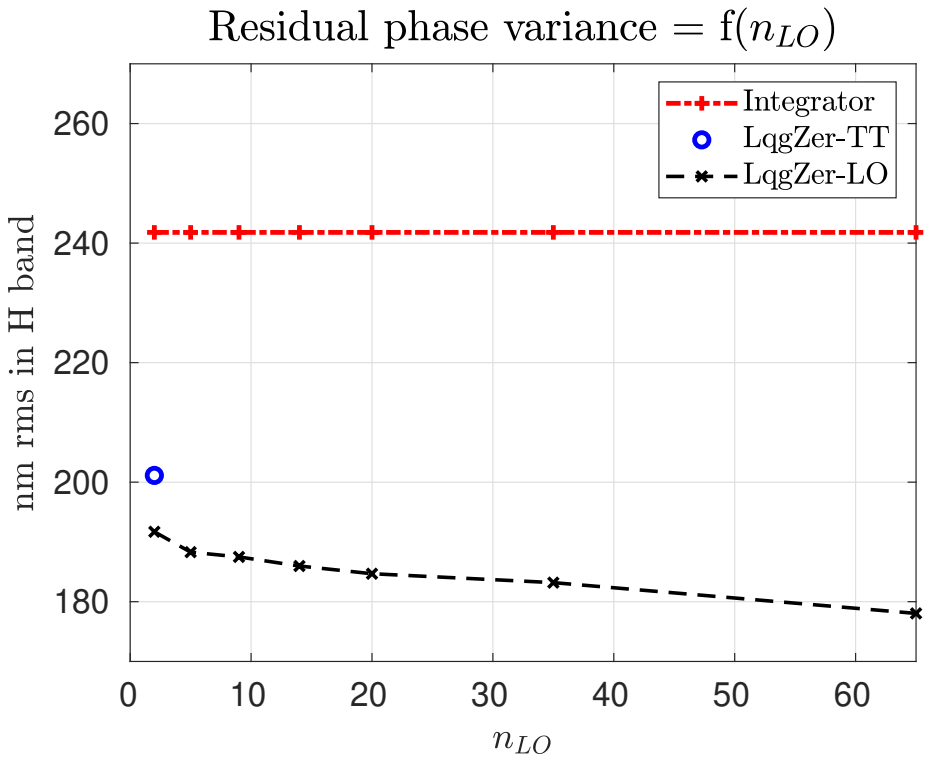




$\rightarrow$ performance improves when identifying the tip-tilt dynamics from data.
$\rightarrow$ cPEM slightly better than N4SID

[4] C. Kulcsár, P. Massioni, G. Sivo and H-F Raynaud, "Vibration mitigation in adaptive optics control," vol. 8447, Adaptive Optics Systems III SPIE, pp. 381 – 396, 2012

Case 1: large wind speed (2/2) - replay mode increasing n_{LO}

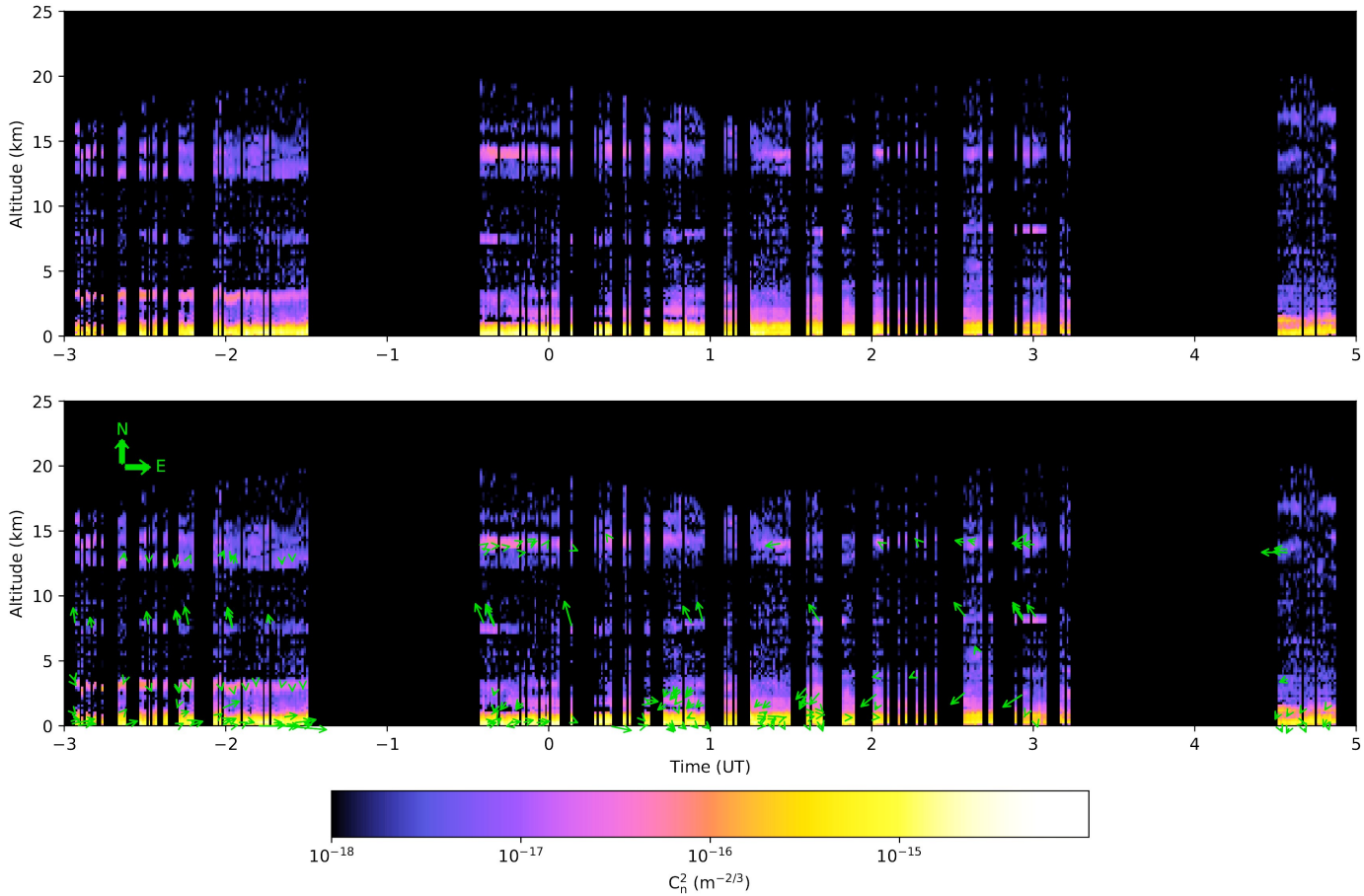


$\rightarrow$ lower residual variance with a minimum variance reconstruction of the tip and tilt

$\rightarrow$ the larger n_{LO}, the lower the residual phase variance

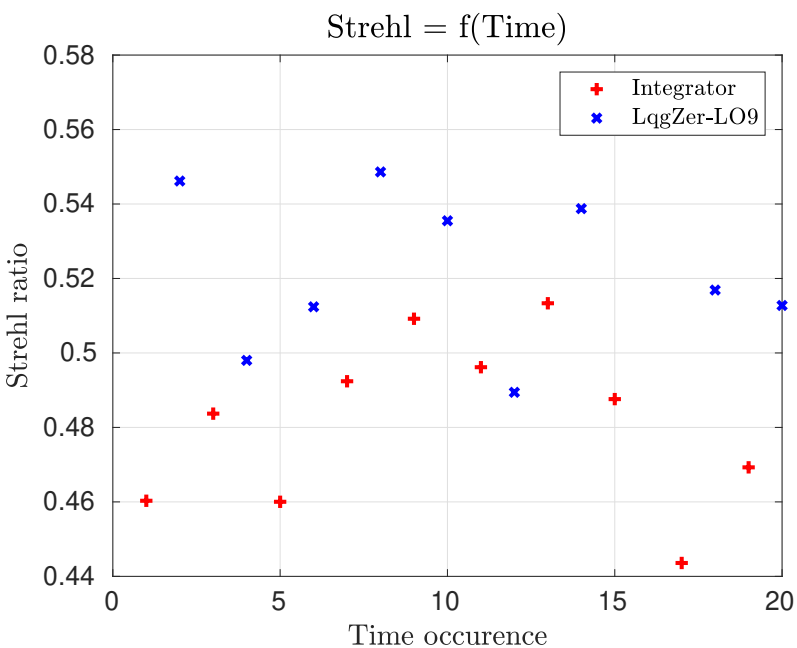
Case 2: low wind speed (1/4) - the atmosphere

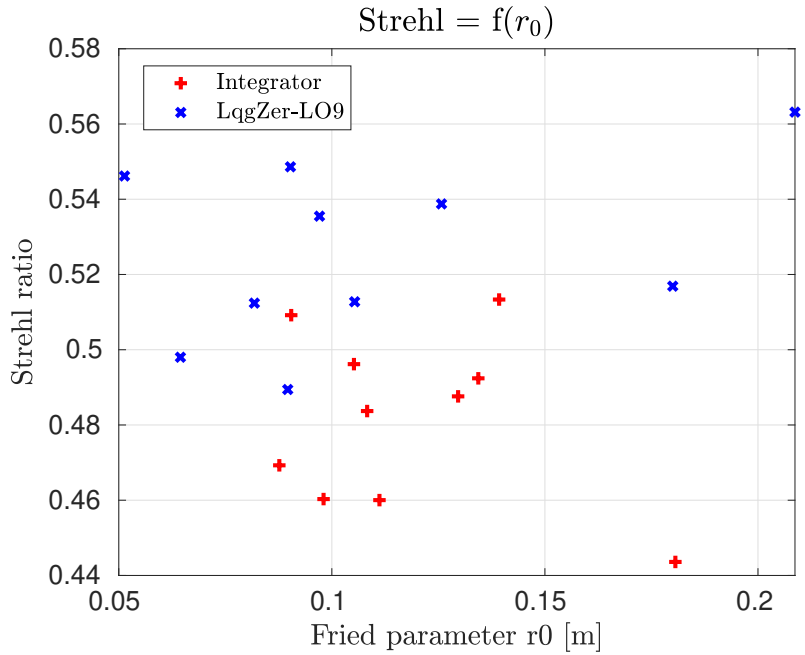
Evolution of the Cn^2 and windspeed from SLODAR



Length of the arrow in upper left position: 10m/s

Case 2: low wind speed (2/4) - on-sky results with $n_{LO} = 9$

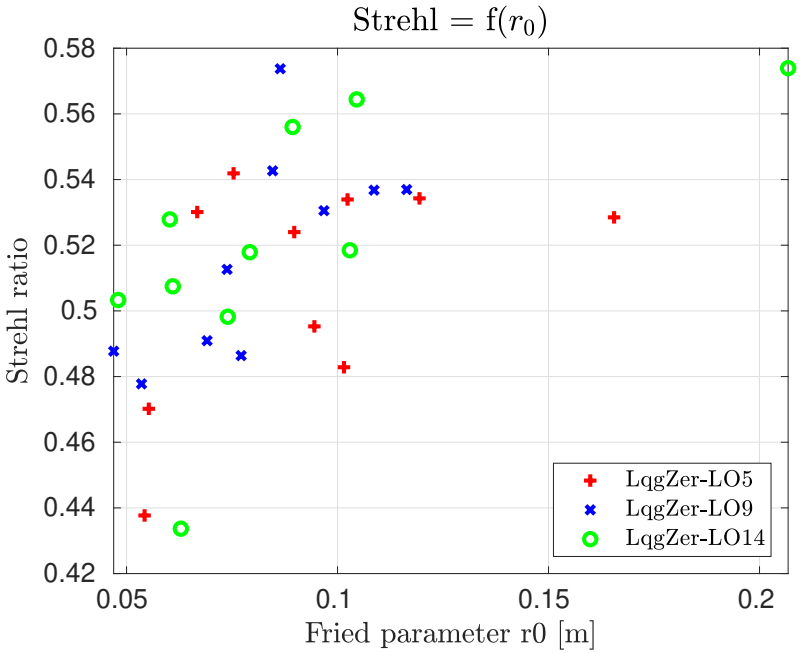


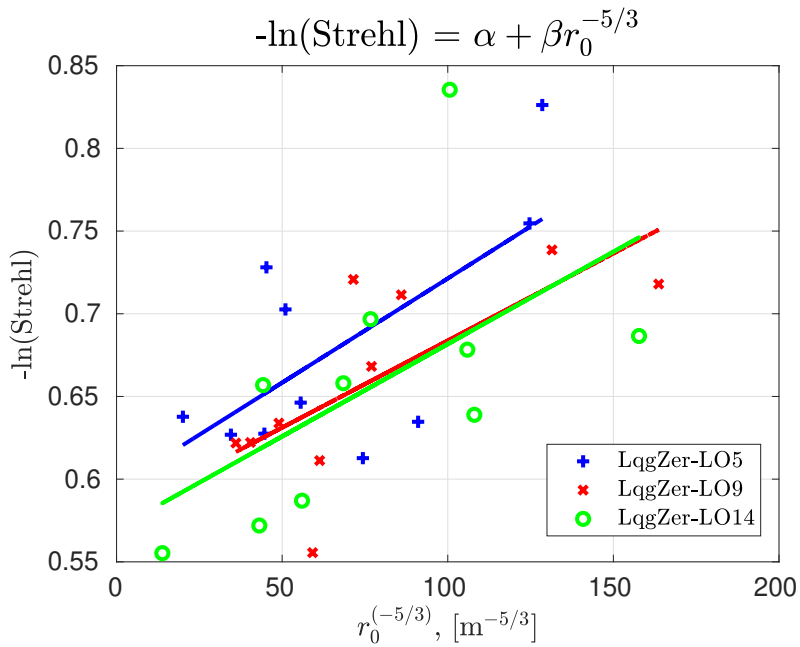


$\rightarrow$ strong performance improvement when $n_{LO} = 9$

Case 2: low wind speed (3/4) - on-sky results for increasing n_{LO}

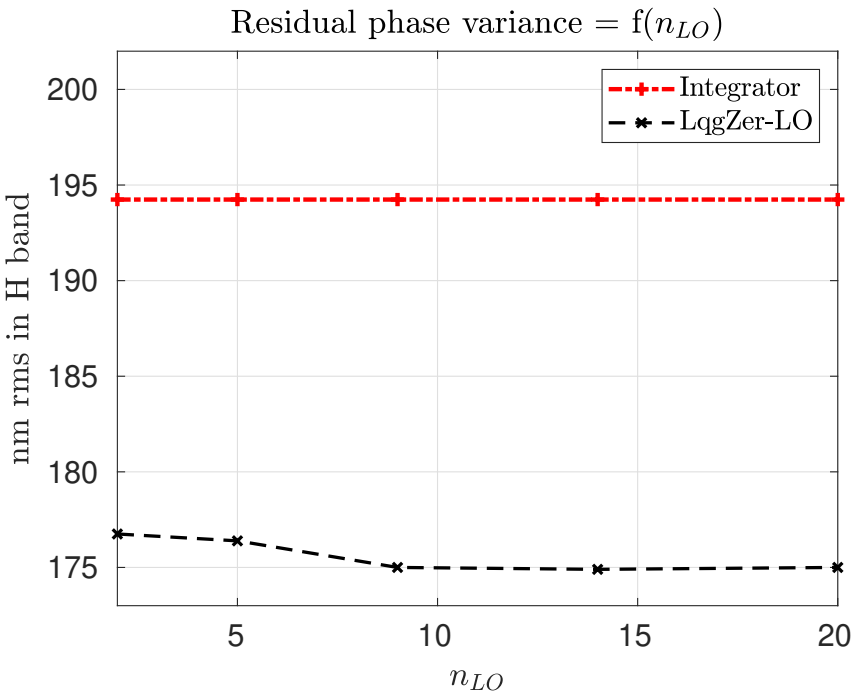
Maréchal approximation, $SR = e^{-\sigma^2_{res}}$
Showing trends via: $\sigma^2_{res} \approx \alpha + \beta r_0^{-5/3}$





$\rightarrow$ $n_{LO} = 9$ is better than $n_{LO} = 5$
$\rightarrow$ $n_{LO} = 14$ does not improve significantly compared to $n_{LO} = 9$

Case 2: low wind speed (4/4) - replay mode



$\rightarrow$ trends observed on-sky are confirmed in replay: data-driven model for 9 modes is most suitable

Performance analysis - rejection transfer functions

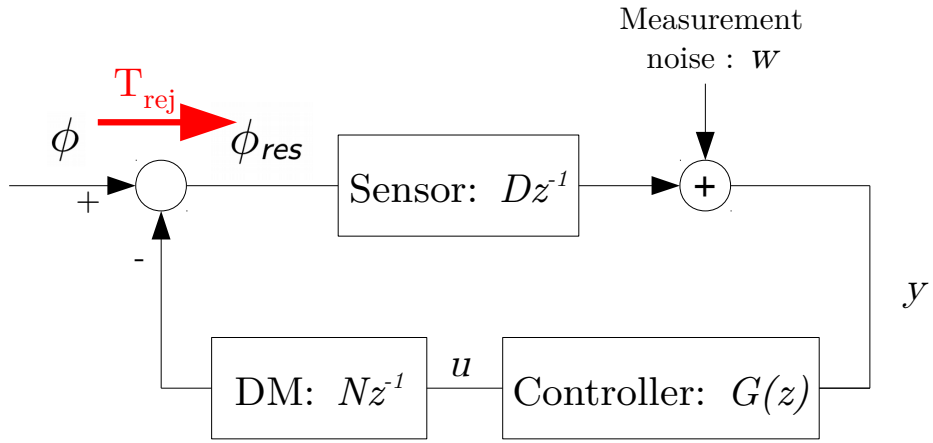
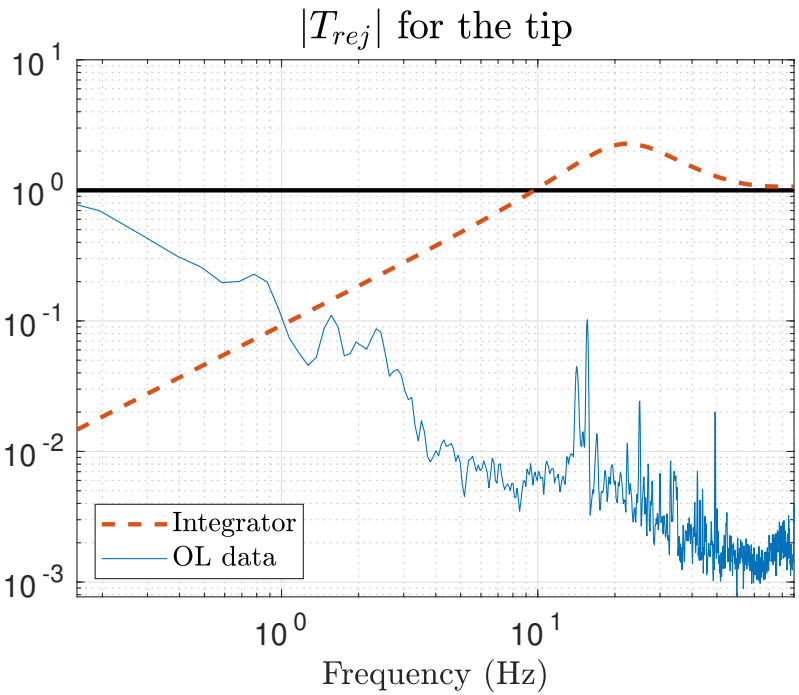
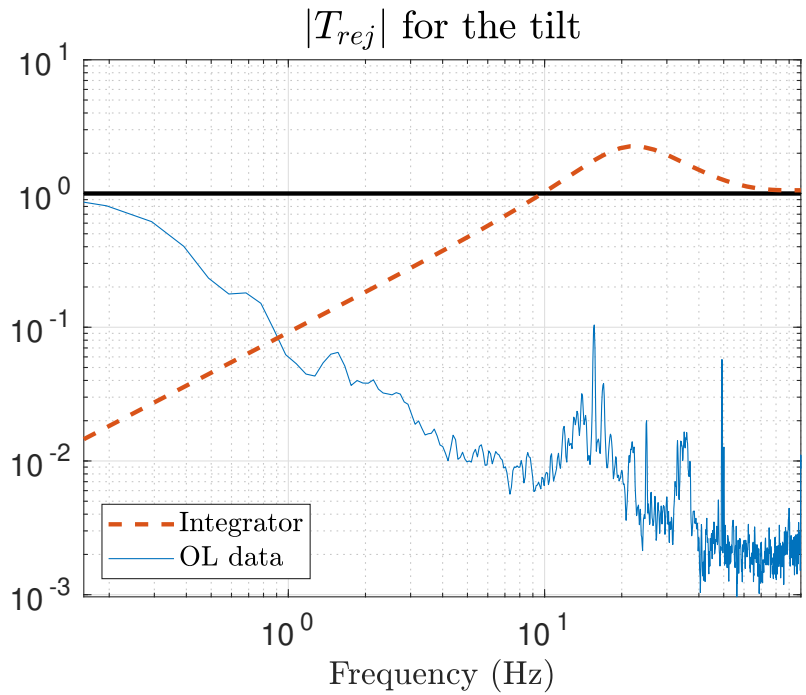


Figure 2: $\phi_{res} = T_{rej}\phi + T_{noise}w$





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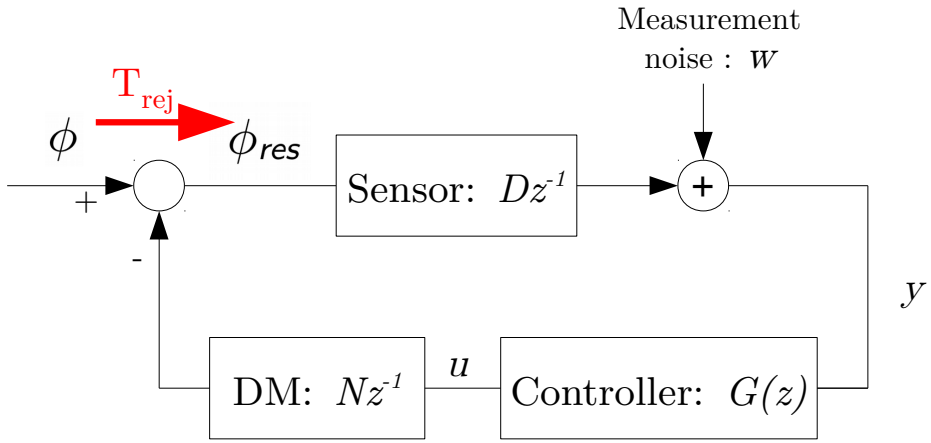

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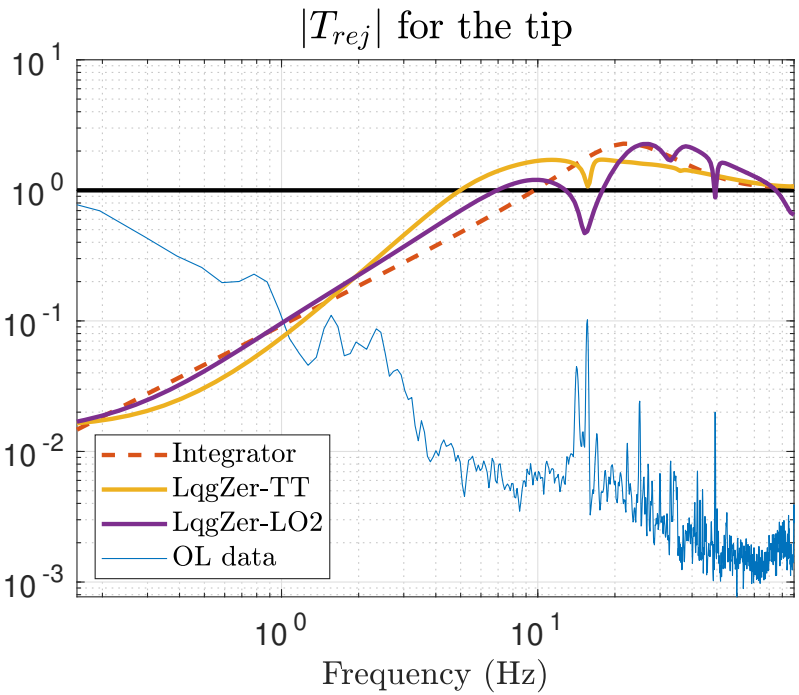

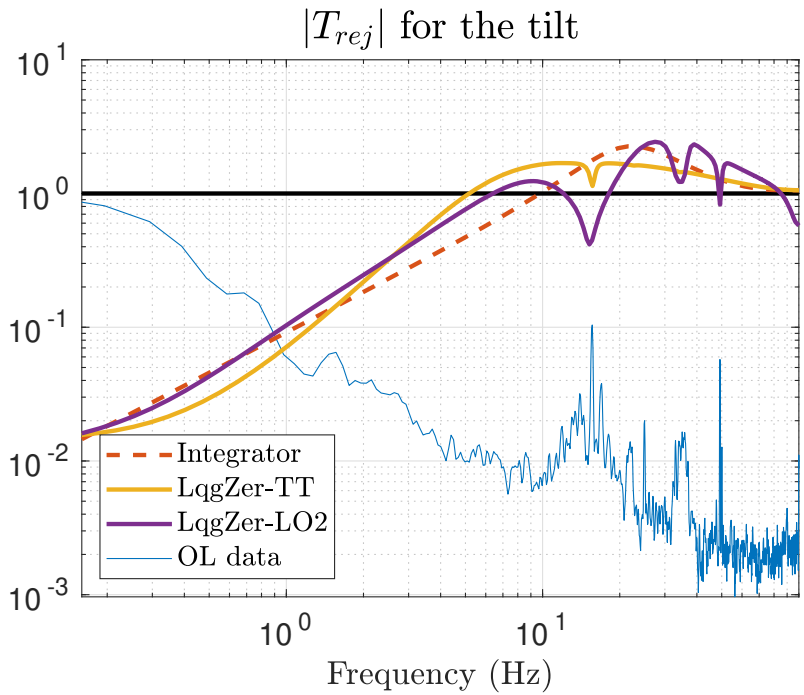

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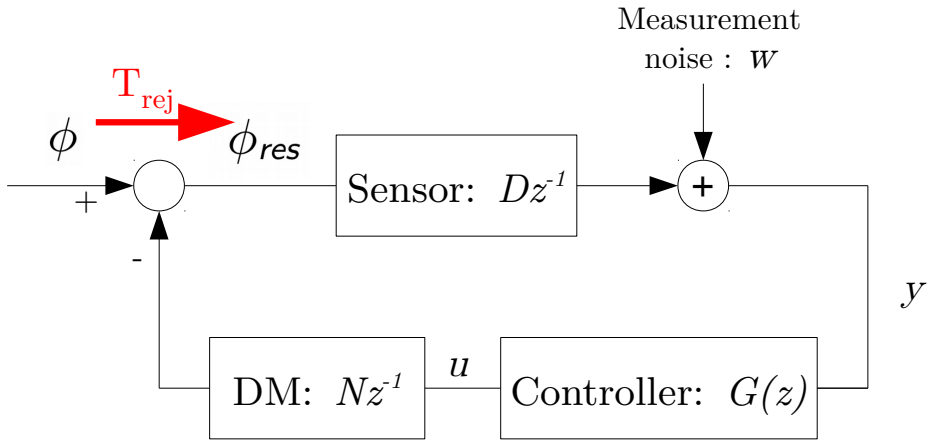

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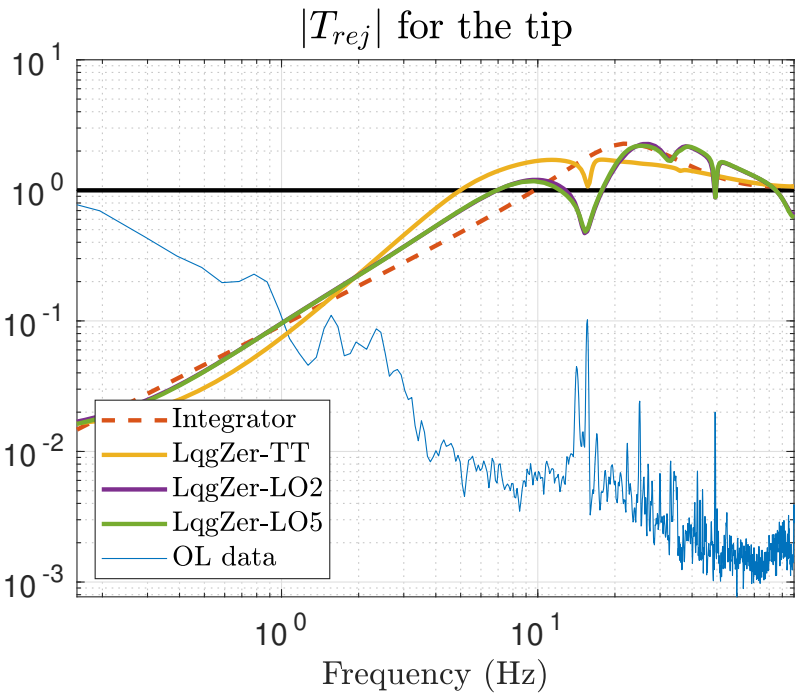

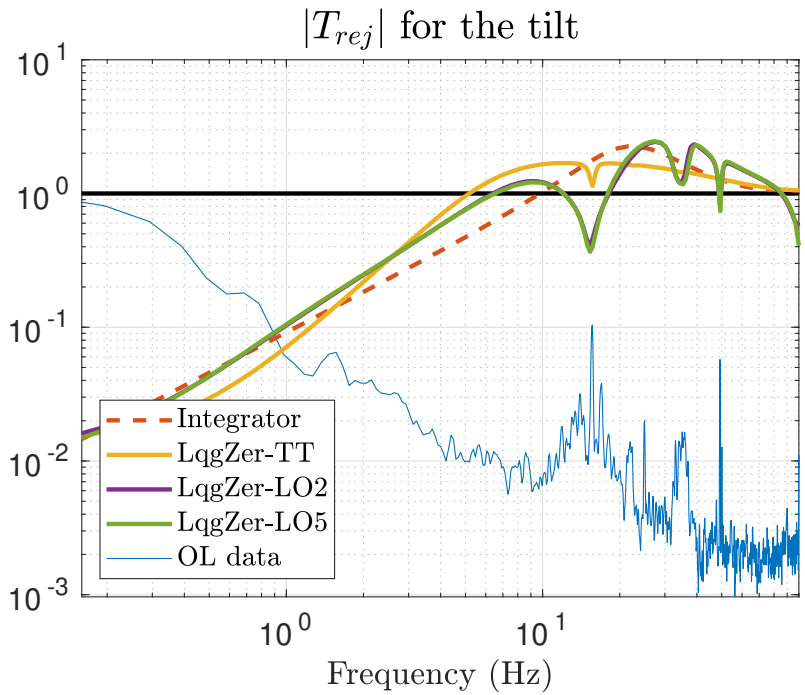

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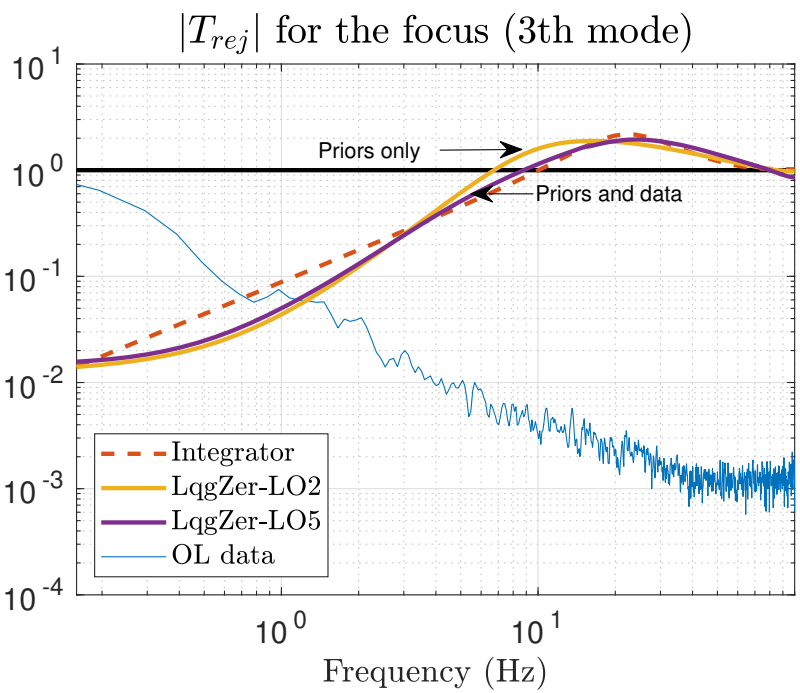
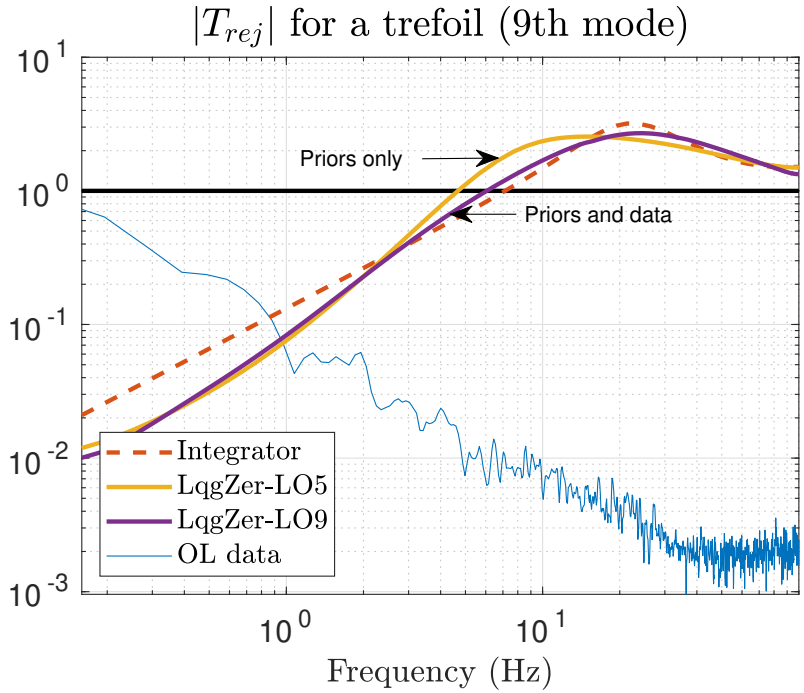


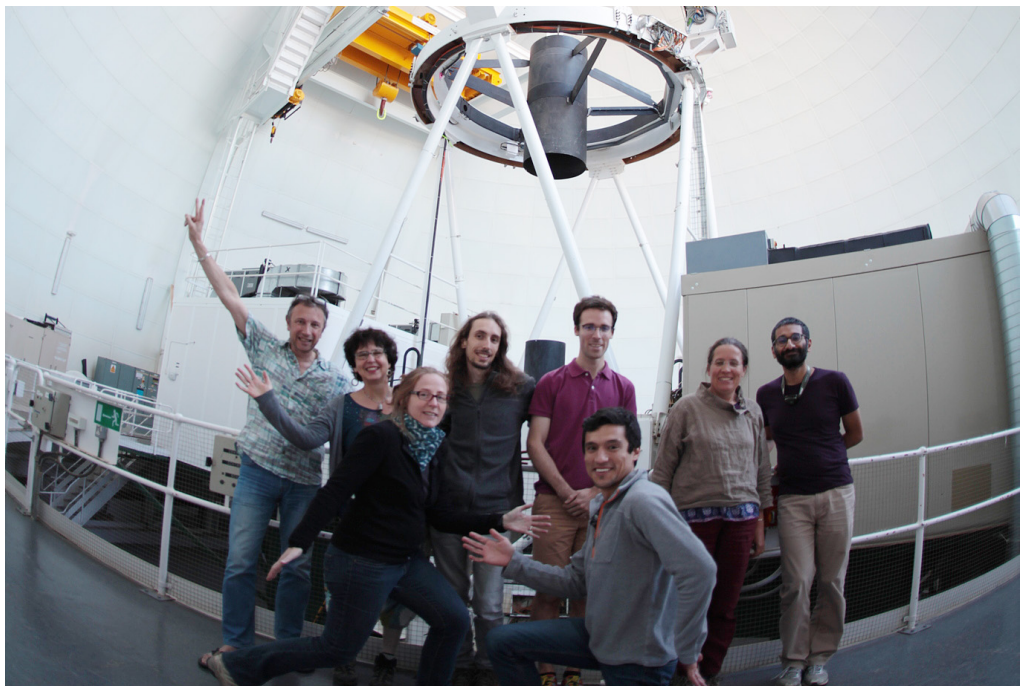


$\rightarrow$ better rejection for LQG at low frequencies, worse at high frequencies

$\rightarrow$ rejection from data better suited to the PSD of OL data

Conclusions from on-sky CANARY experiments at WHT

1. Identifying dynamics of more modes $\rightarrow$ gain in performance even when without vibrations
2. Controller with 9 identified modes successfully used by other teams[5][6]
3. Model-based control using wind profiles in tracking satellites (upcoming papers OSA AO + Optro2020)



[5] S. Haffert (Universiteit Leiden, NL) et al., "The multi-core integral-field unit (MCIFU): Overview and first light", under review for SPIE 2020.

[6] M. Langlois (CRAL-Lyon, France) al. "Mach-Zehnder Wavefront sensor for XAO: From laboratory tests to on sky measurements using the SCAO capability of CANARY at the William Hershel Telescope," under review for SPIE 2020.

Credit for the picture: Ali Bharmal