Modal Wavefront Sensorless Adaptive Optics with Karhunen–Loève Functions

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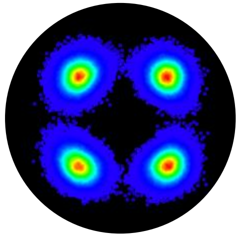
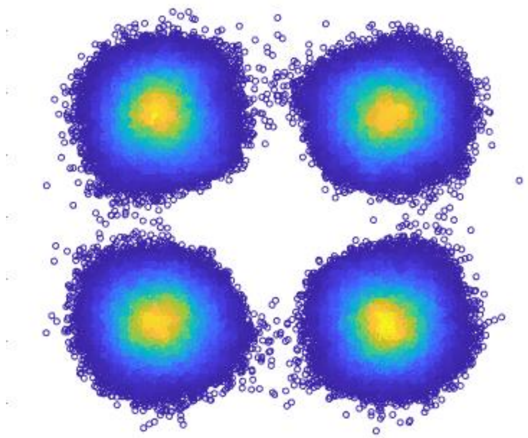
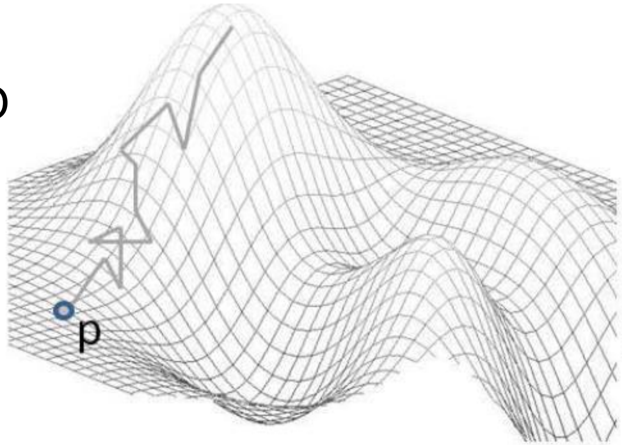


QPSK



Outline

- Why sensorless adaptive optics (AO)?
- Mean squared error minimized wavefront (WF) decomposition in Kolmogorov turbulence
  - Modal decomposition and residuals
  - Karhunen-Loève modes (KL)
- Simulations
  - Turbulence and modal stochastic parallel gradient descent framework (SPGD)
  - Convergence analysis
  - Optima analysis
- 6.3 GBit/s free-space optical (FSO) communication link with sensorless AO
  - Experimental framework
  - Bit error rate (BER) evaluation
- Conclusion & outlook



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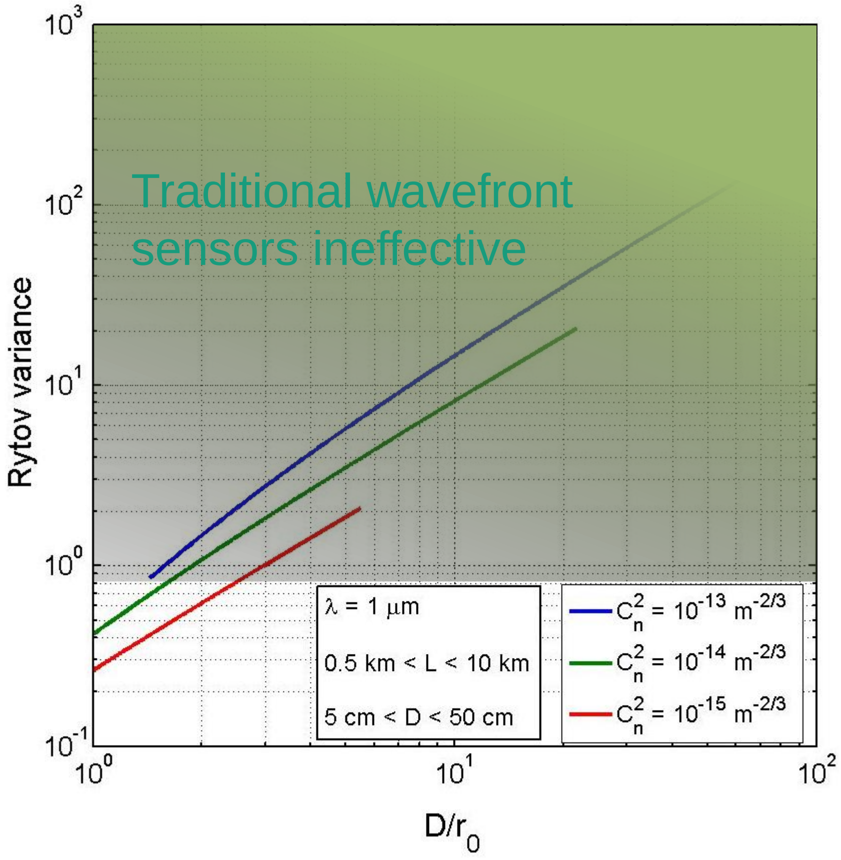
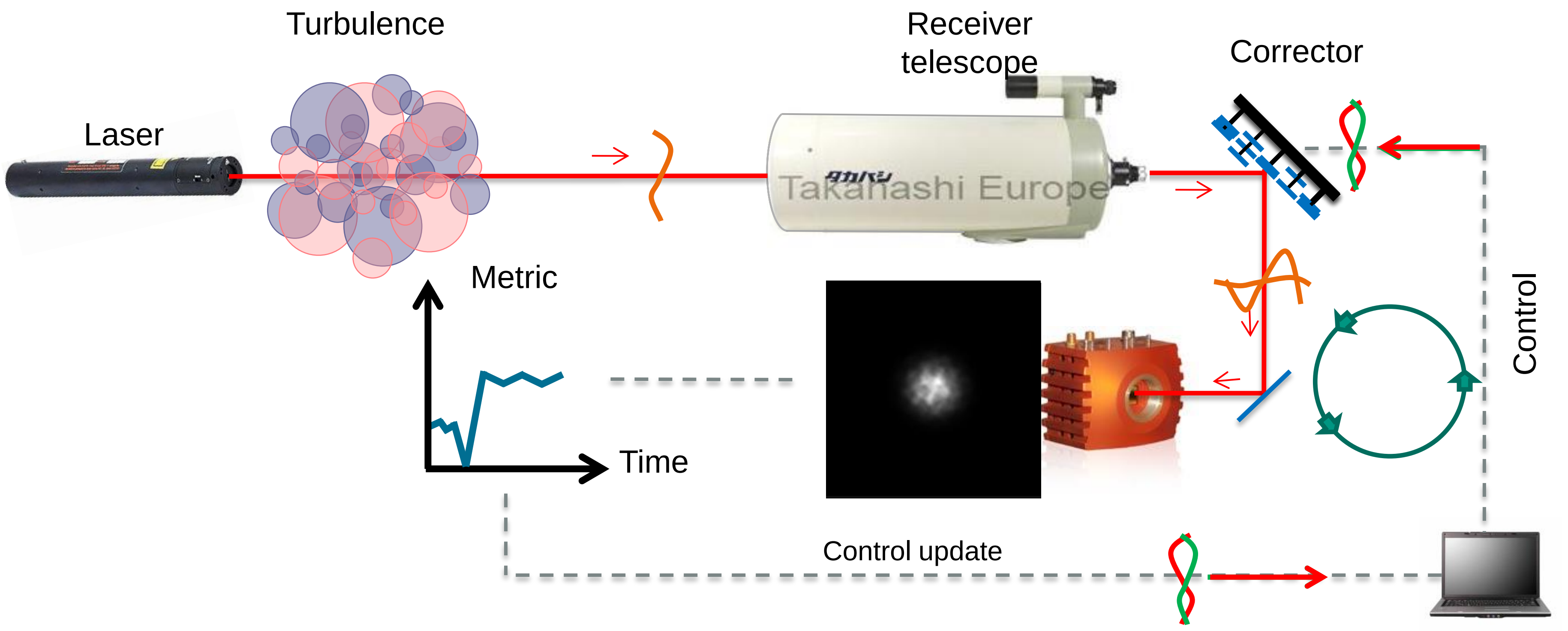
Why sensorless adaptive optics ?

Barchers et al., Appl. Opt., 2002

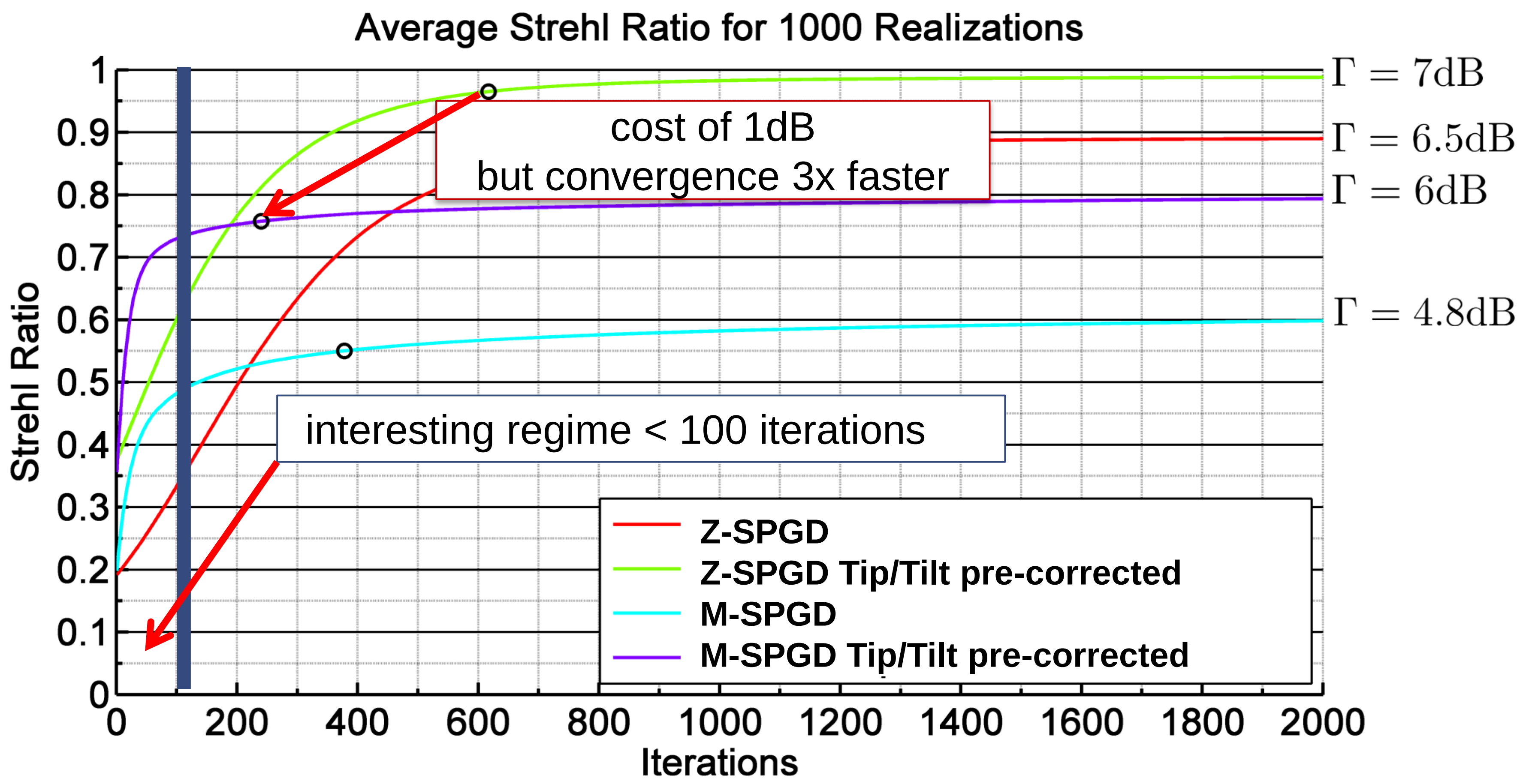


- Free-space optical communication is all about power
  - Link budget, antenna gain, propagation losses
  - Receiver sensitivity, error resilience
- Traditional WF sensing methods require energy redirection -> already limited resource diminished even more
- Gradient-based sensorless AO
  - Communication detector metric is evaluated
  - Higher scintillation resilience
  - Low-level hardware implementation enables woofer-tweeter system without power loss
- But: high correction bandwidth demand -> Focus on improving convergence speed

Sensorless AO concept for free-space optical communications



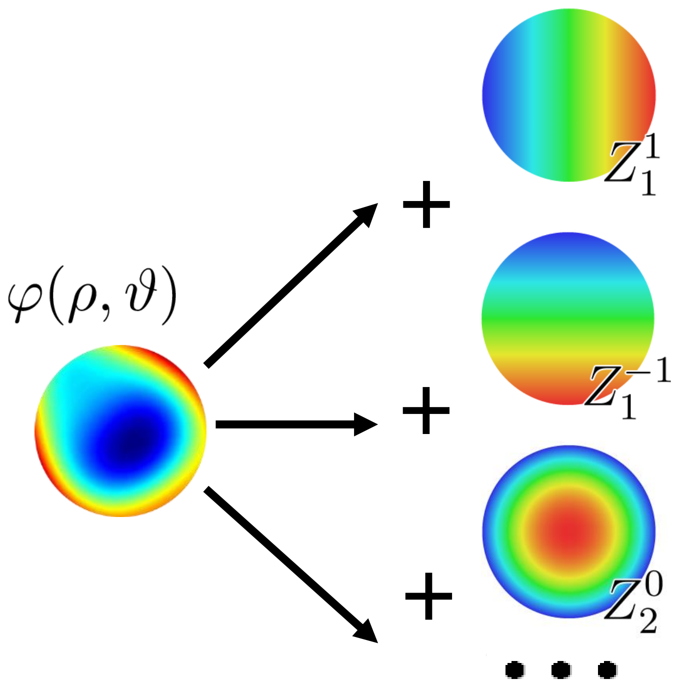
Zonal-SPGD (Z-SPGD) vs. Modal (M-SPGD) comparison



- Z-SPGD (M. Vorontsov) slower but more reliable
- M-SPGD sacrifices performance for speed

Modal decomposition and residual error analysis

- Decomposition:



Zernike polynomials and atmospheric turbulence*

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The Perkin-Elmer Corporation, Norwalk, Connecticut 06856

(Received 3 October 1975)

This paper discusses some general properties of Zernike polynomials, such as their Fourier transforms, integral representations, and derivatives. A Zernike representation of the Kolmogoroff spectrum of turbulence is given that provides a complete analytical description of the number of independent corrections required in a wave-front compensation system.

$$\Delta_J = \langle \varphi^2 \rangle - \sum_{j=1}^{J} \langle |a_j|^2 \rangle$$

$$Z_j(r,\vartheta) = \sqrt{n+1}\, R_n^m(r) \cos m\vartheta, \qquad m \neq 0, \quad j \text{ even}$$

$$Z_j(r,\vartheta) = \sqrt{n+1}\, R_n^m(r) \sin m\vartheta, \qquad m \neq 0, \quad j \text{ odd}$$

$$Z_j(r,\vartheta) = \sqrt{n+1}\, R_n^0(r), \qquad m = 0$$

$$R_n^m(r) = \sum_{k=0}^{(n-m)/2} \frac{(-1)^k (n-m)!}{k!\left(\frac{n+m}{2}-k\right)!\left(\frac{n-m}{2}-k\right)!} r^{n-2k}$$

$\Delta_1 = 1.0299\ (D/r_0)^{5/3}$	$\Delta_{12} = 0.0352\ (D/r_0)^{5/3}$
$\Delta_2 = 0.582\ (D/r_0)^{5/3}$	$\Delta_{13} = 0.0328\ (D/r_0)^{5/3}$
$\Delta_3 = 0.134\ (D/r_0)^{5/3}$	$\Delta_{14} = 0.0304\ (D/r_0)^{5/3}$
$\Delta_4 = 0.111\ (D/r_0)^{5/3}$	$\Delta_{15} = 0.0279\ (D/r_0)^{5/3}$
$\Delta_5 = 0.0880\ (D/r_0)^{5/3}$	$\Delta_{16} = 0.0267\ (D/r_0)^{5/3}$
$\Delta_6 = 0.0648\ (D/r_0)^{5/3}$	$\Delta_{17} = 0.0255\ (D/r_0)^{5/3}$
$\Delta_7 = 0.0587\ (D/r_0)^{5/3}$	$\Delta_{18} = 0.0243\ (D/r_0)^{5/3}$
$\Delta_8 = 0.0525\ (D/r_0)^{5/3}$	$\Delta_{19} = 0.0232\ (D/r_0)^{5/3}$
$\Delta_9 = 0.0463\ (D/r_0)^{5/3}$	$\Delta_{20} = 0.0220\ (D/r_0)^{5/3}$
$\Delta_{10} = 0.0401\ (D/r_0)^{5/3}$	$\Delta_{21} = 0.0208\ (D/r_0)^{5/3}$
$\Delta_{11} = 0.0377\ (D/r_0)^{5/3}$	

Karhunen-Loève modes motivation

- Zernike modes are correlated
  - WF mode amplitude modifications coupled
  - Changing tilt -> coma changes as well
- Correlations are given in Noll's matrix
  (covariance between Zernike polynomials)
- KL mode -> independent coefficients (diagonal cov. matrix)
- Approach: diagonalize Noll's matrix
  -> obtain numerical expression for constructing
  KL from radial Zernike polynomials (N. Roddier 1990)

Noll's index j

$$\begin{array}{c}
\begin{array}{cccccccc} 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\
\begin{pmatrix}
0,4536 & 0 & 0 & 0 & 0 & 0 & -0,0143 & 0 \\
0 & 0,4536 & 0 & 0 & 0 & -0,0143 & 0 & 0 \\
0 & 0 & 0,0235 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0,0235 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0,0235 & 0 & 0 & 0 \\
0 & -0,0143 & 0 & 0 & 0 & 0,0063 & 0 & 0 \\
-0,0143 & 0 & 0 & 0 & 0 & 0 & 0,0063 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0063
\end{pmatrix}
\end{array}$$

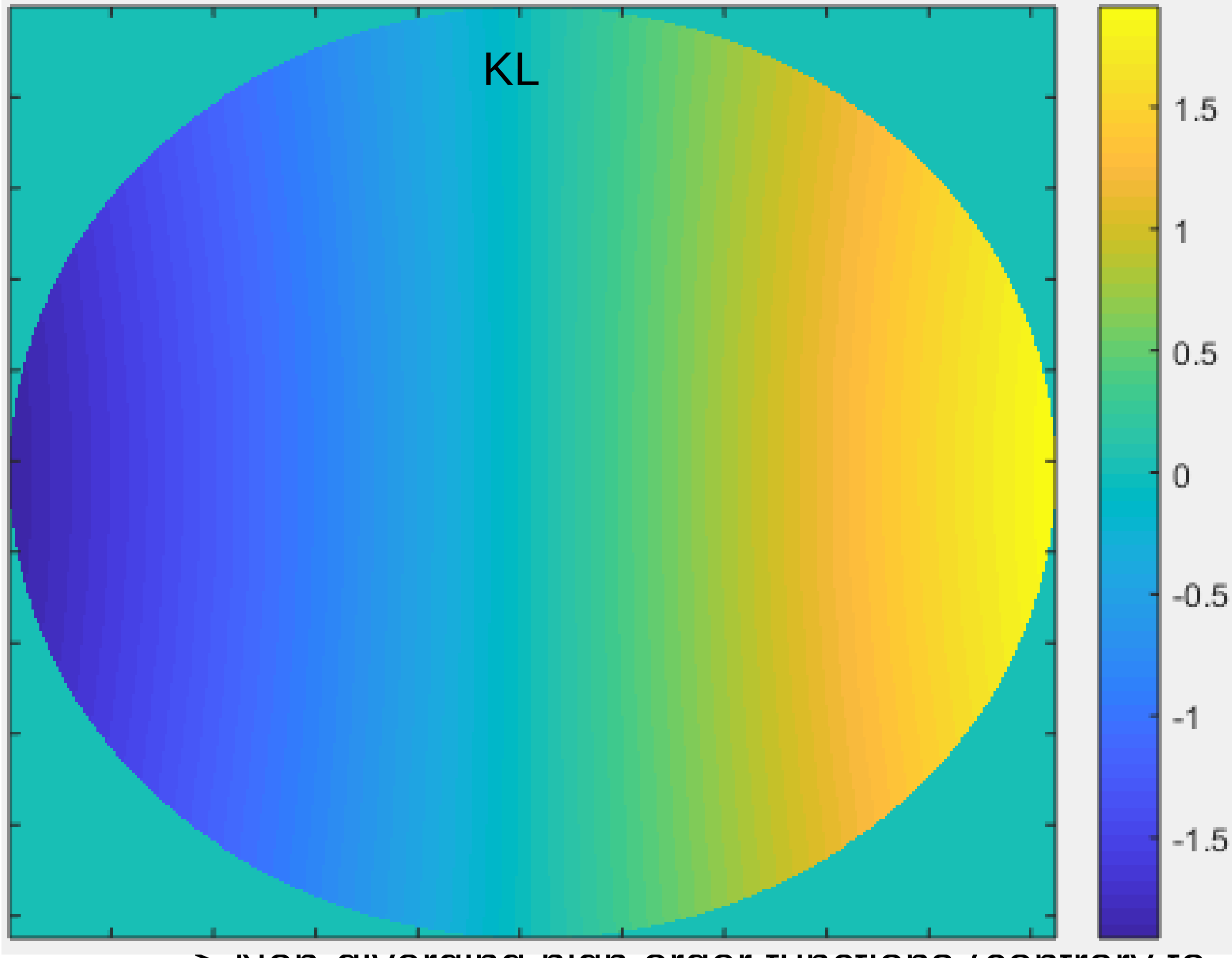
(a) Noll's matrix

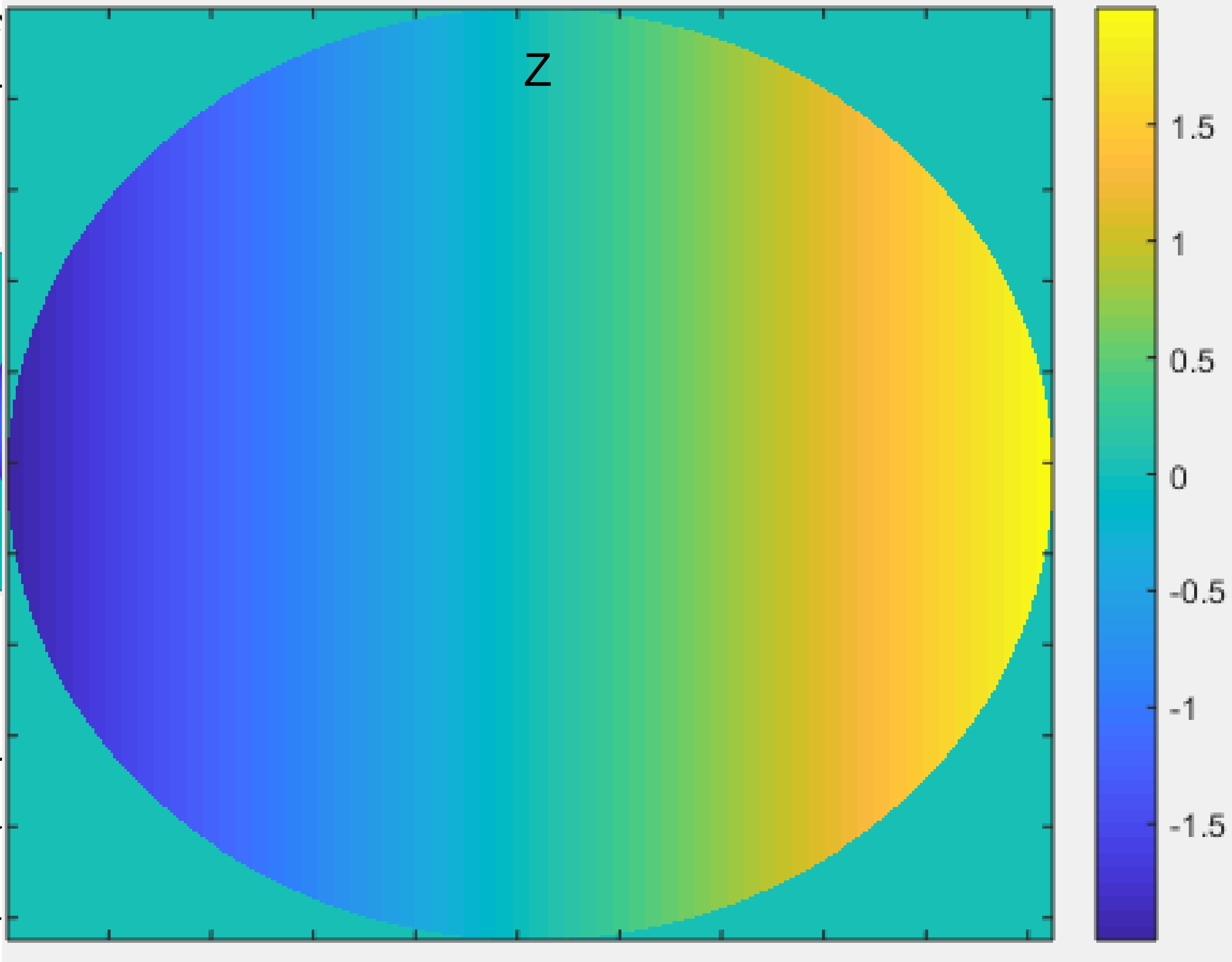
$$\begin{pmatrix}
0,4541 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0,4541 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0242 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0,0242 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0,0242 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0,0067 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0,0067 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0067
\end{pmatrix}$$

(b) KL covariance matrix

Karhunen-Loève modes – conversion from Zernike

$$K_l(r,\vartheta) = R_p^q(r)\Theta_p^q(\vartheta)$$





-> Non-diverging high order functions (contrary to Zernike)

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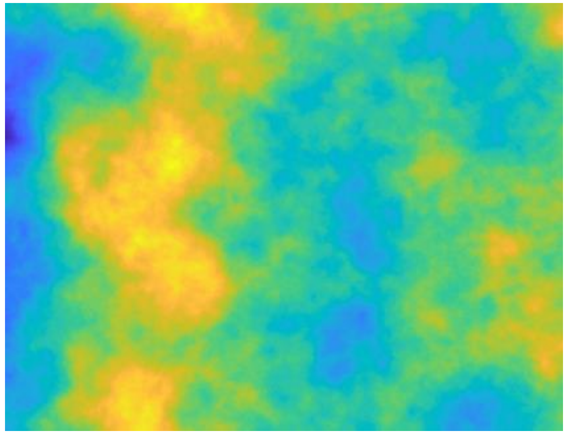
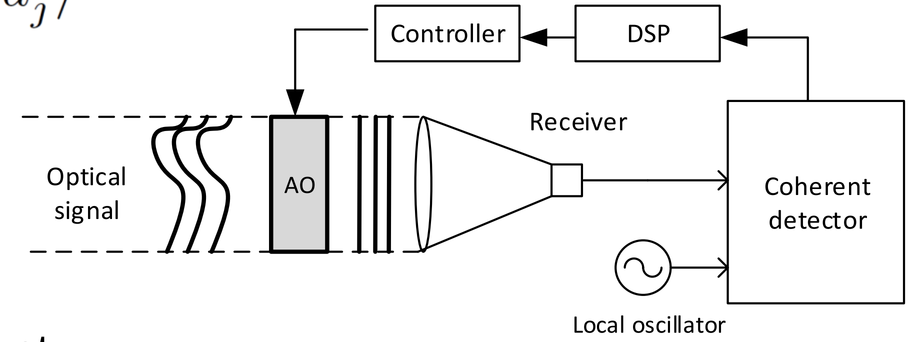
Simulation framework and modal SPGD configuration

- Turbulence emulation:
  - Fourier method with subharmonics (Johansson 1994)
  - Independent set of single layer screens (de-tilted)

$$\frac{D}{r_0} = 4$$



- Modal SPGD configuration: $$u_n^{m+1} = u_n^m + G \cdot \delta J^m \delta u \cdot \gamma_n^m$$
  - Optimize mode amplitudes instead of local phase deviation
  - Proposal: weighting mode perturbations according to spatial energy contribution $$\delta \tilde{u}_n = \delta u \cdot \sqrt{\langle a_j^2 \rangle}$$
  - Accurate spatial frequency representation
- Analysis:
  - Strehl ratio (SR) – Maréchal's approximation
  - Residual errors – squares of ensemble-mean mode coefficient



Strehl ratio evolution

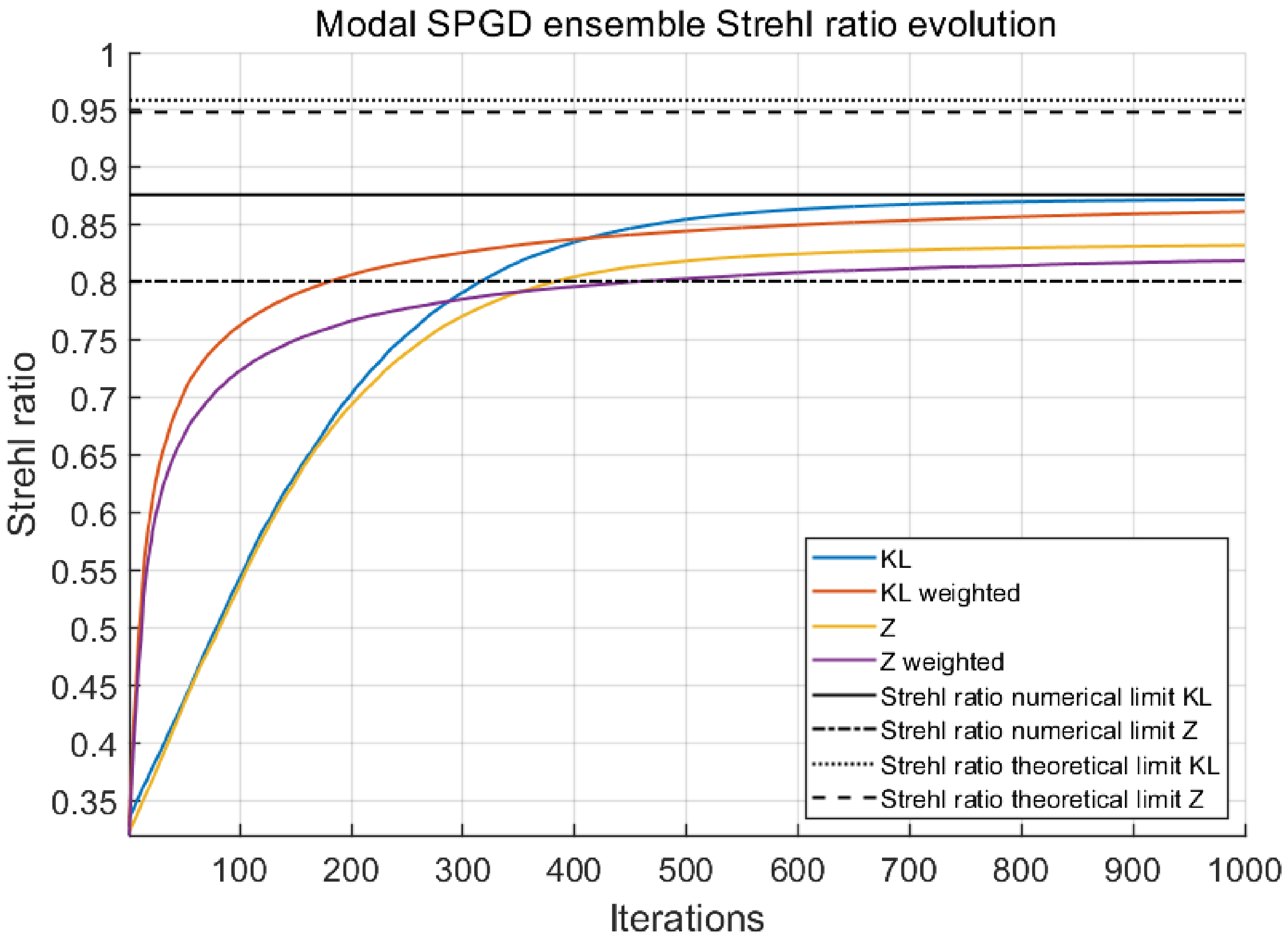
- Numerical limits:

$$SR_{\mathrm{opt,\ 105\ modes}} = \exp\left(-\Delta_{105} * (D/r_0)^{(5/3)}\right)$$

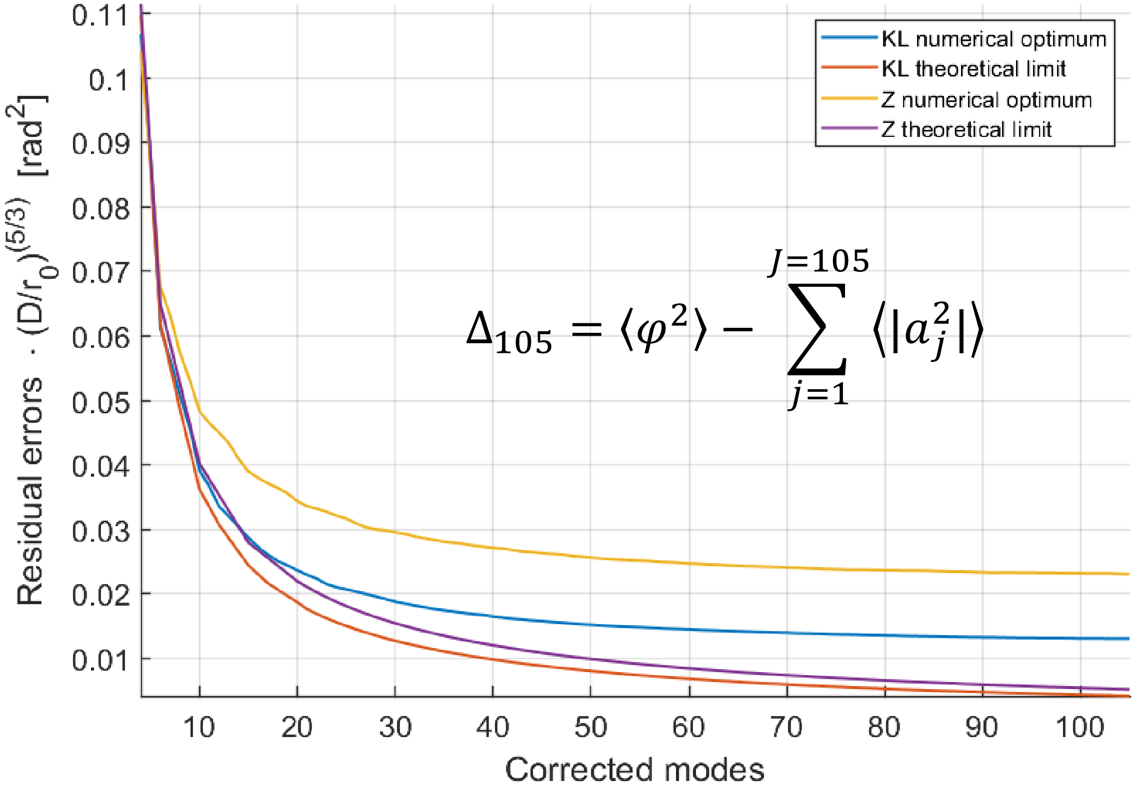
- 5 % SR gain with KL modes (“for free”)

- Weighting speeds up convergence by up to a factor of 4 (KL) and 3 (Z)

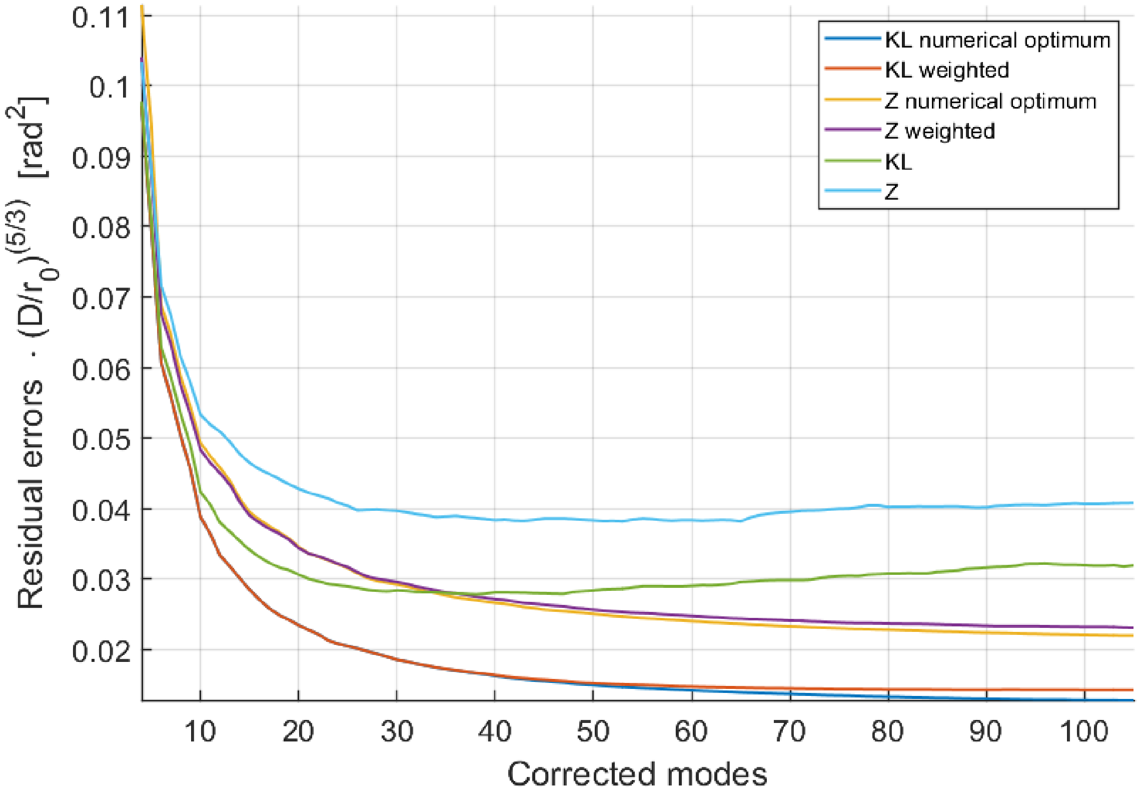
- Why Z-SPGD surpasses its numerical limit?



Residual error – modal optima and numerical results



$$\Delta_{105} = \langle \varphi^2 \rangle - \sum_{j=1}^{J=105} \left\langle |a_j^2| \right\rangle$$



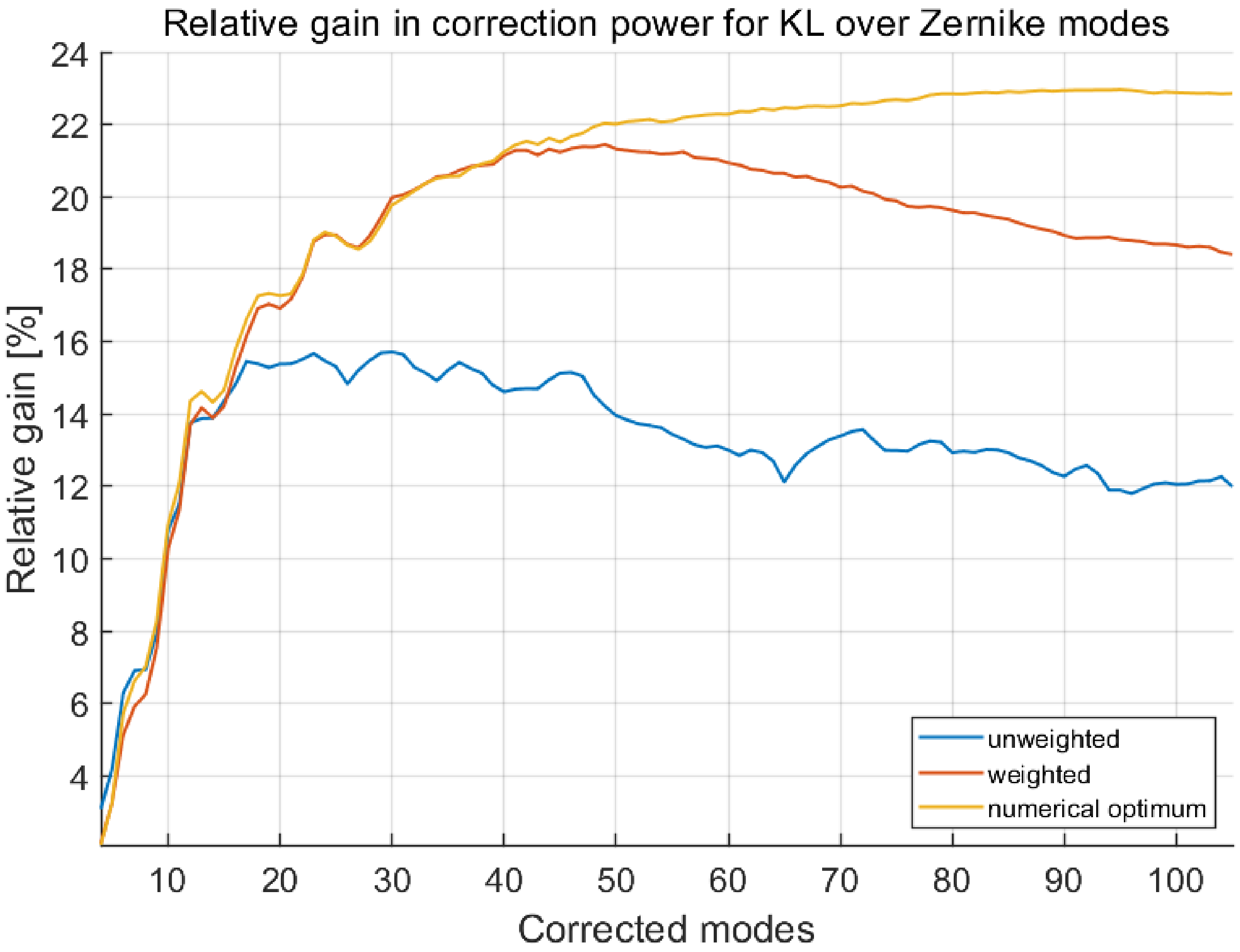
- Accurate spatial frequency representation pushes modal performance close to the limit
- Equally weighted modes correct „unseen“ modes -> SR contradiction to residual errors

Relative gain comparison

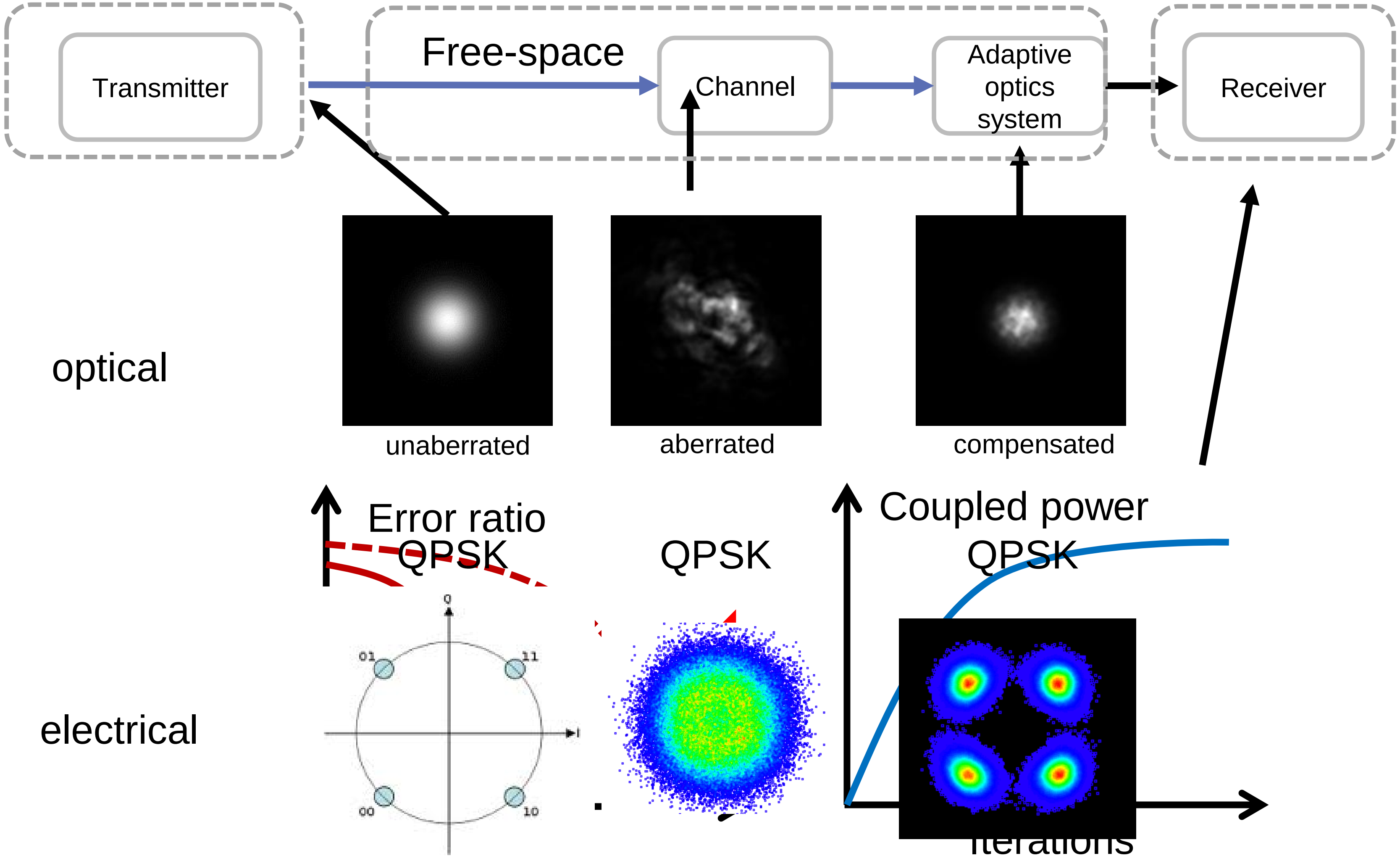
- Relative gain:

$$G_{\mathrm{rel}} = \frac{\sqrt{\Delta_{\mathrm{Z}}} - \sqrt{\Delta_{\mathrm{KL}}}}{\sqrt{\Delta_{\mathrm{Z}}}} * 100\%$$

- Incorporation of a minimal number of modes required
- Saturation peak gives number of modes to be included
- Accurate spatial frequency representation diminishes correction errors

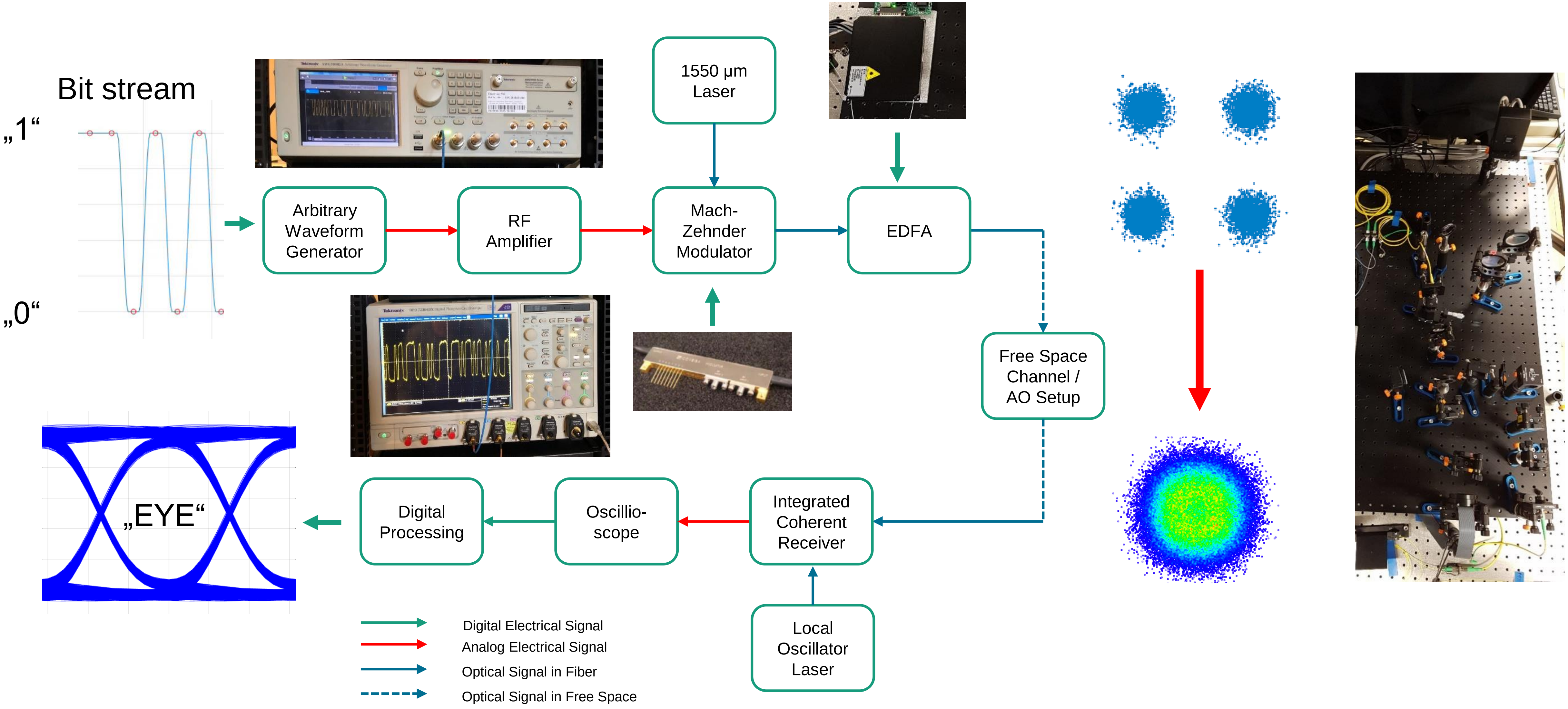


Coherent free space optical communication scheme



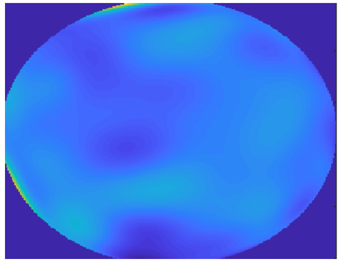
QPSK – quadrature phase shift keying

6.25 Gbit/s coherent FSO communication link with sensorless AO



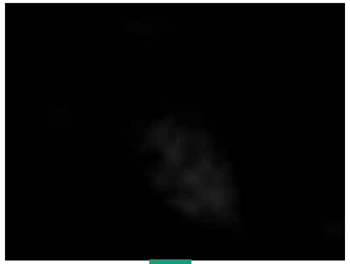
Closed loop sensorless AO setup

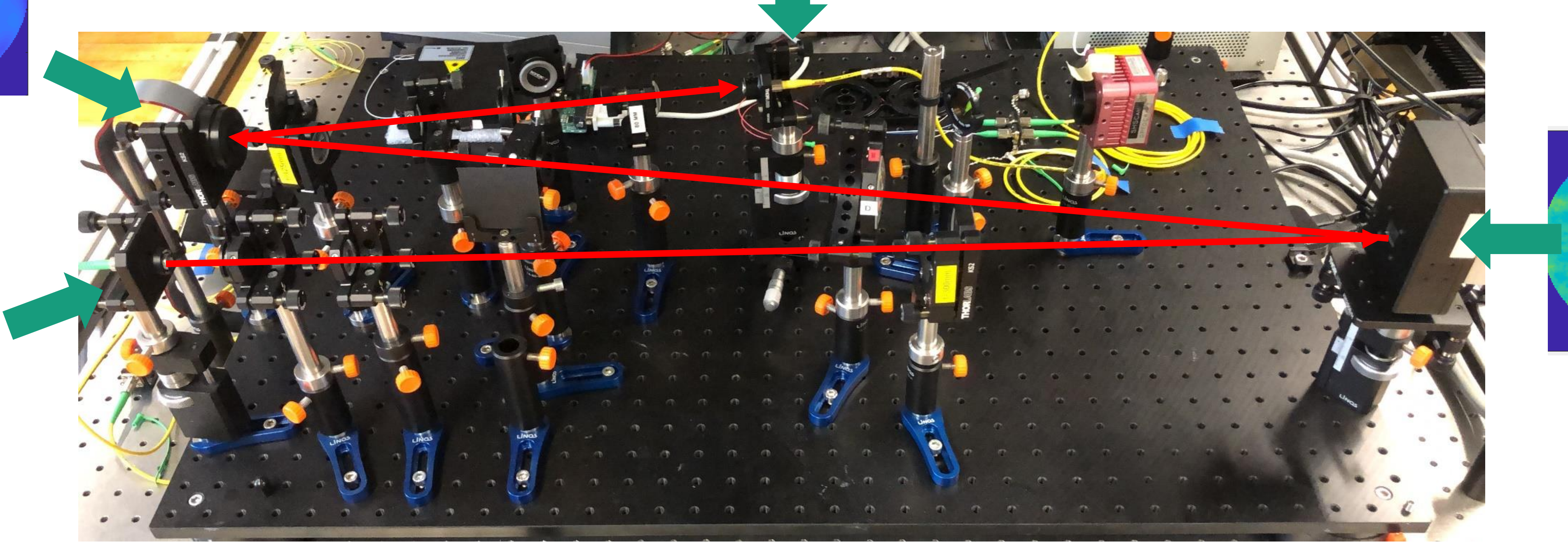
DM



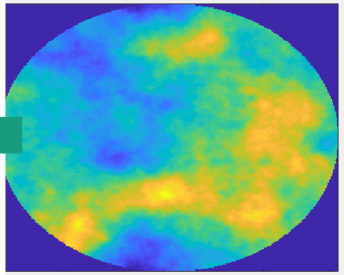
Fiber

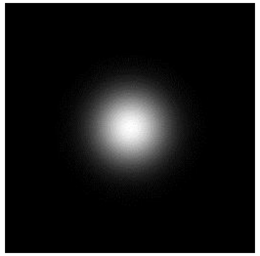
Input





SLM





Fiber

Output

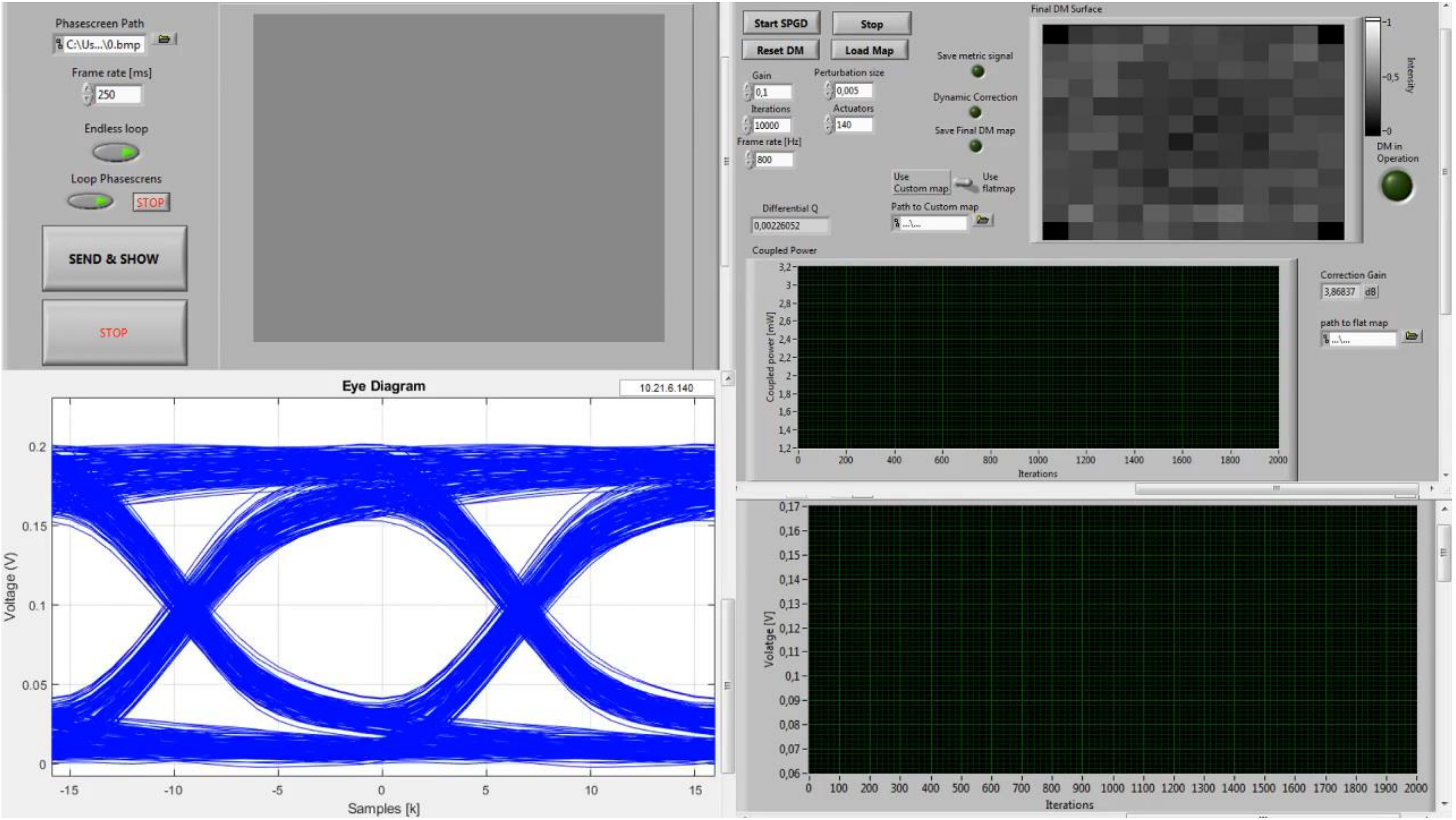
DM – deformable mirror

SLM – spatial light modulator

Setup demonstration

SLM control

SPGD control



Live EYE

Live metric

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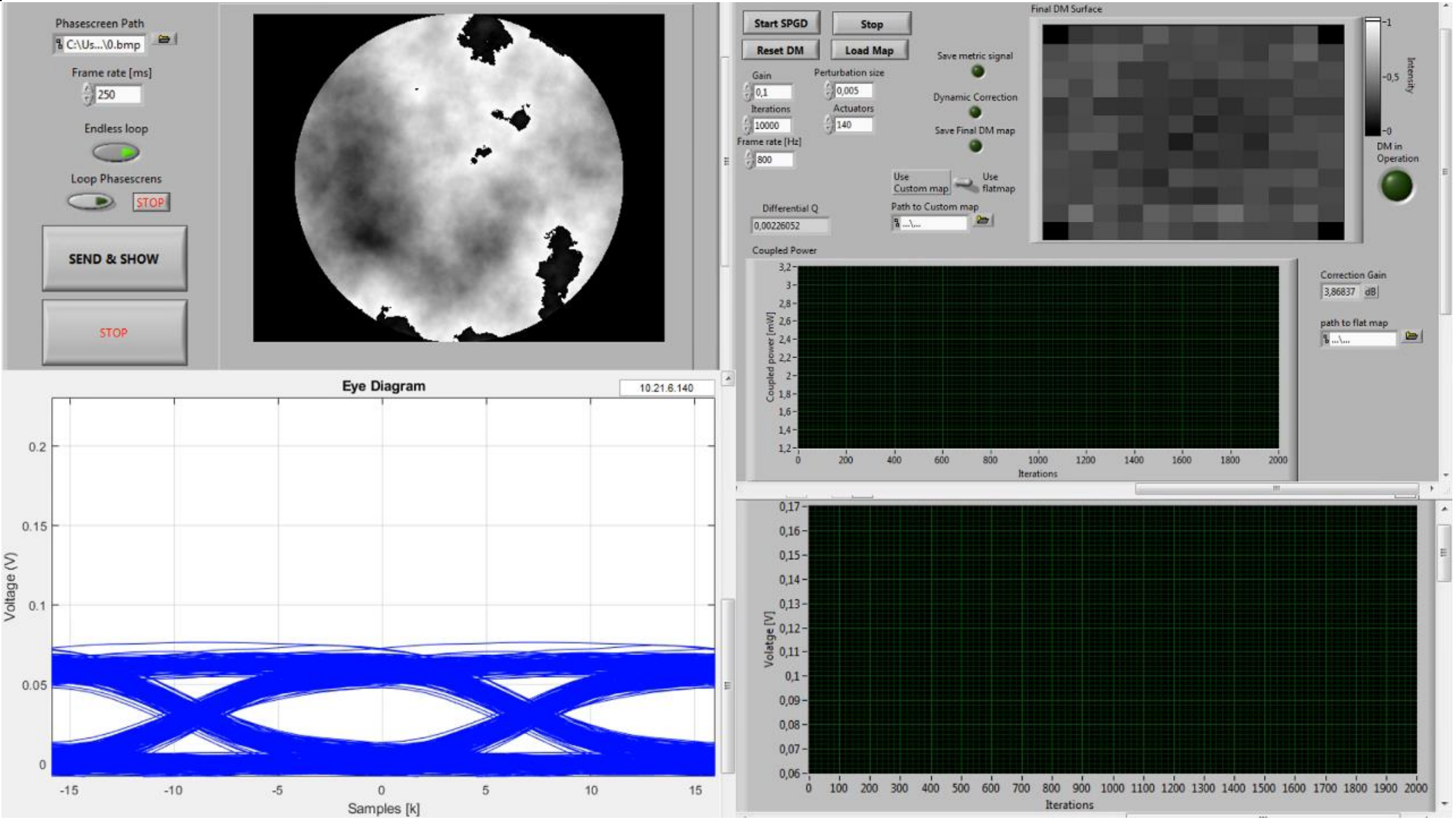
Setup demonstration – tilt removed

SLM control

$$\frac{D}{r_0} = 4$$

Live EYE



SPGD control

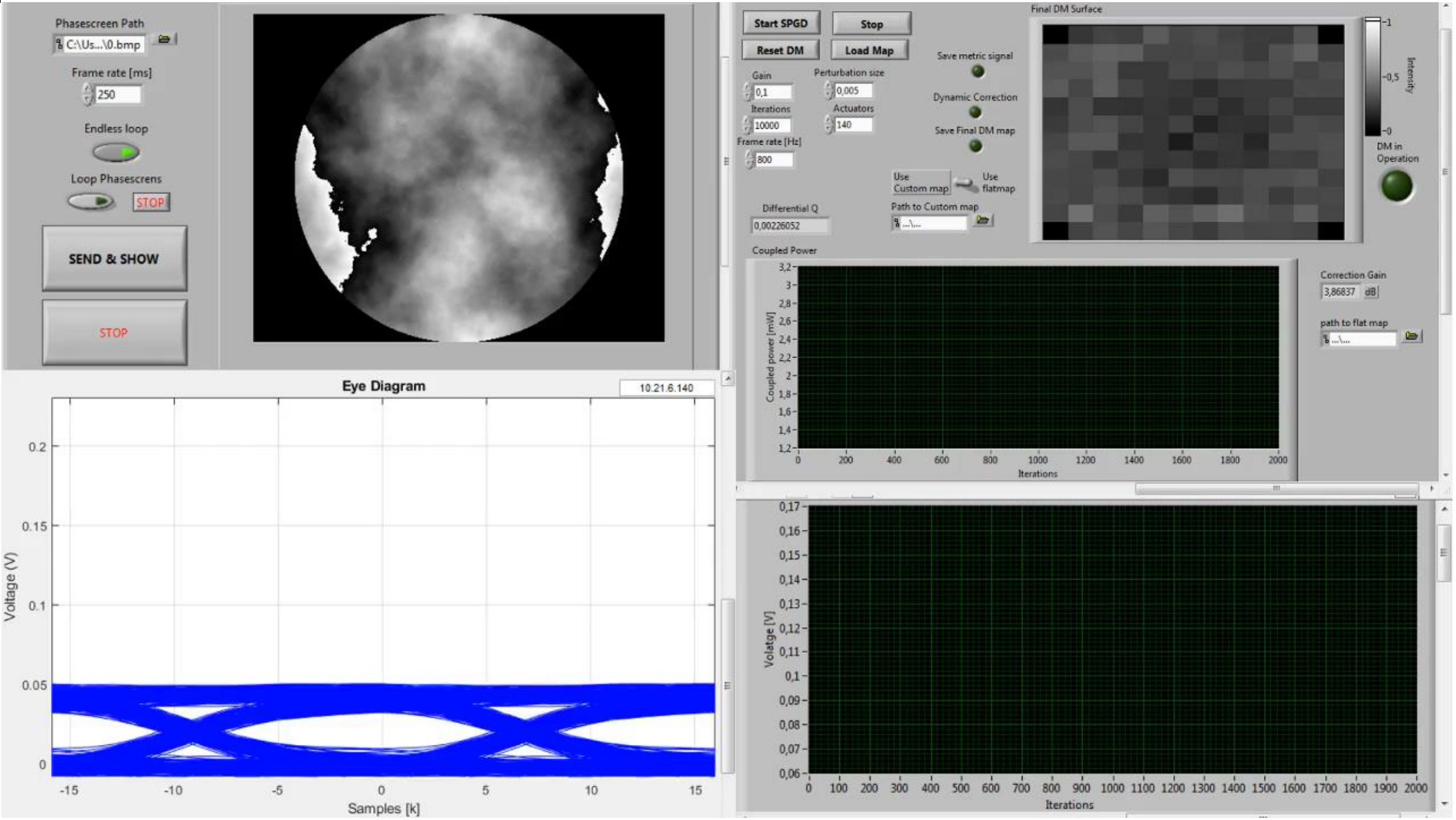
Live metric

Setup demonstration – tilt removed

SLM control

$$\frac{D}{r_0} = 4$$

Live EYE



SPGD control

Live metric

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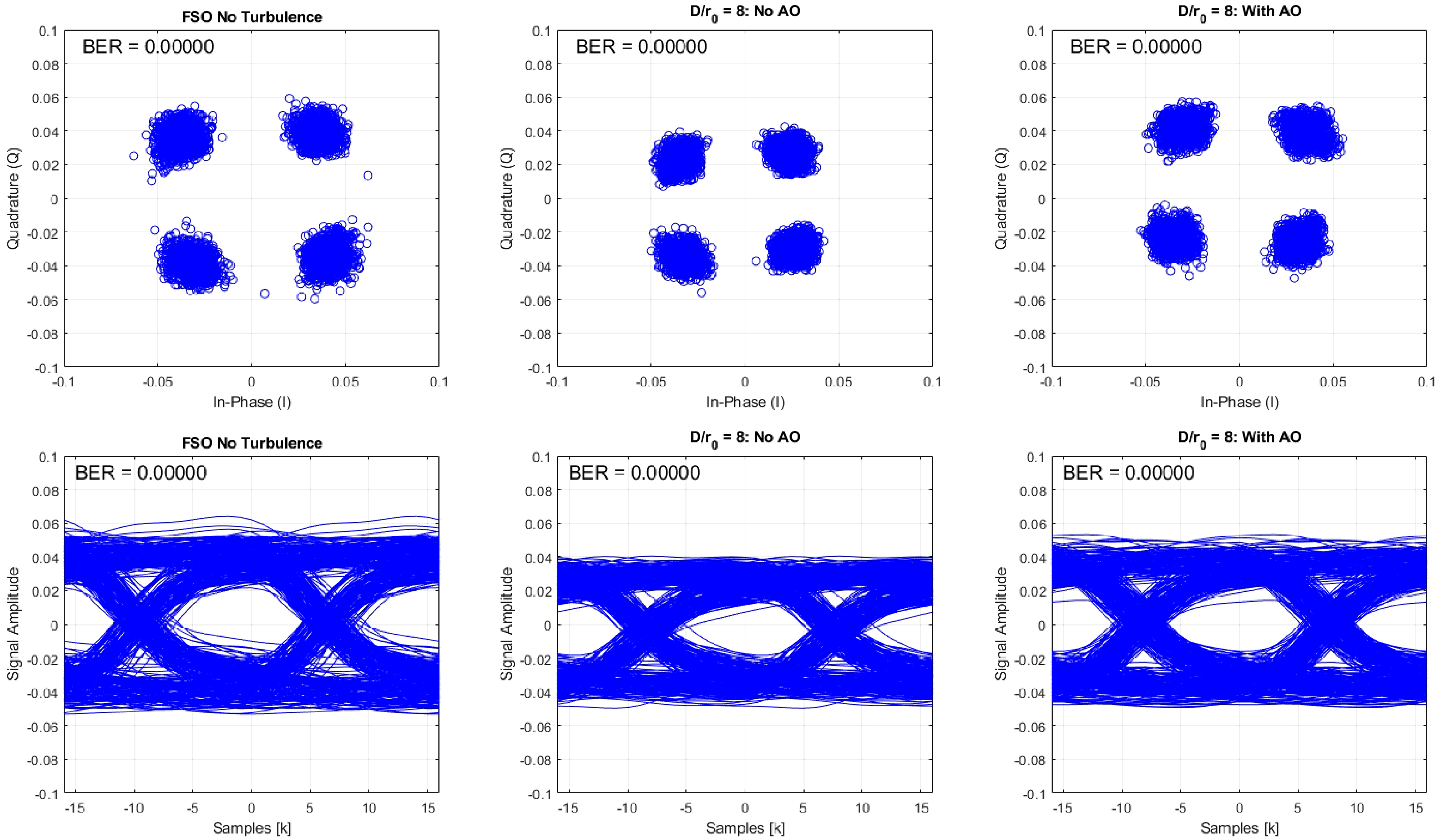
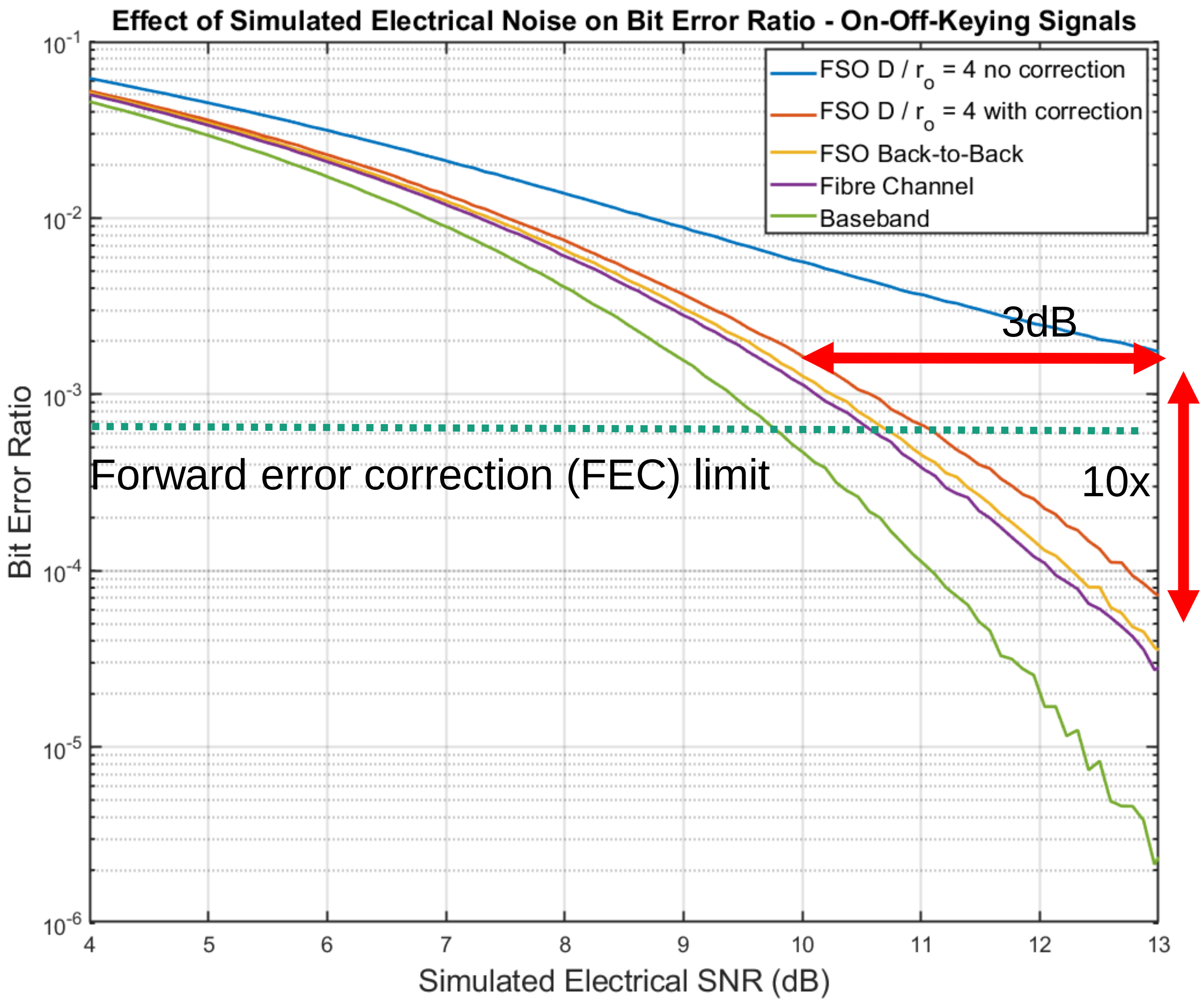
Coherent FSO communications system at 6.25 Gbit/s



Bit Error Ratio (BER) evaluation – On-Off Keying

- How to read the plot
  - Achievable BER for a certain SNR
  - Goal: reduce BER below FEC
- Turbulence increases power requirements for achieving a certain BER
- AO correction pushes BER below FEC limit



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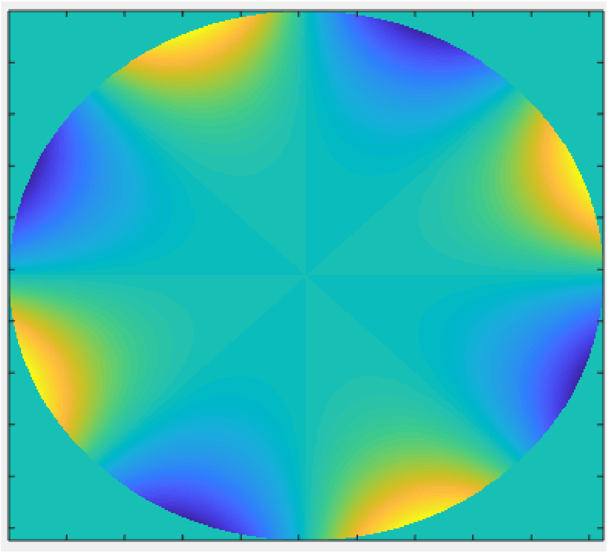
Conclusion & outlook



- Implementation of mean squared minimized modal wavefront expansion
- *a priori* incorporation of knowledge of the Kolmogorov turbulence
- 6.3 Gbit/s FSO com setup with sensorless AO
- BER evaluation of AO correction in weak turbulence