New **B**elief **F**unction **T**heory (BFT) based methods for multicriteria decision making

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Outlines

- Introduction
  - Multicriteria decision making in context of risk, uncertainty: e.g. natural hazards in mountains
- Three recent developments associating multicriteria decision methods and belief function theory (BFT) are described
  - ER-MCDA
  - DSmT-AHP
  - Cautious OWA
- Conclusion

AHP and BFT…

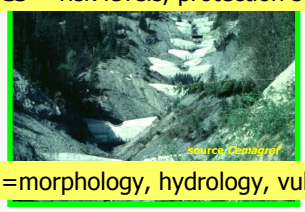
Decision under uncertainty

Example of application context: risk management related to **rapid mass movements** in **mountains** (avalanches, mountain river floods, rockfalls)

DM =expert, community mayor…

…They threaten people and **infrastructures**

Alternatives = risk levels, protection strategies …



Criteria =morphology, hydrology, vulnerability…

…and we try to **get protected** against them

Aid-decision methods considering information imperfection are **expected**

The ER-MCDA* methodology

**Evidential Reasoning and Multi Criteria Decision Analysis*

…to help **decisions** based on **imperfect information** provided by **more or less reliable** (and important) **sources**

Uncertainty in not the only kind of information imperfection

(unconsistency, imprecision, uncompleteness, uncertainty)

p.m. Probabilities

Fuzzy sets theory (to link numerical and linguistic values)

Possibility theory (to represent both imprecision and uncertainty)

Belief function theory (to represent imprecision, uncertainty and consider heterogeneous sources)

Backgrounds of belief function theory

Information fusion requires a **common frame of discernment**

$\Theta = \{\theta_1, \theta_2\}$ ← **Frame of discernment**

The set of exhaustive and exclusive hypotheses

θ_1 θ_2 $\theta_1 \cap \theta_2 \equiv \emptyset$

Powerset → $2^\Theta = \{\emptyset, \theta_1, \theta_2, \theta_1 \cup \theta_2\}$

Basic belief assignment (BBA)

$$\begin{cases} m(.) : 2^\Theta \to [0,1] \\ m(\emptyset) = 0 \\ \sum_{\theta_i \in 2^\Theta} m(\theta_i) = 1 \end{cases}$$

$$m(\theta_1) + m(\theta_2) + \underbrace{m(\theta_1 \cup \theta_2)}_{uncertainty} = 1$$

Fusion of 2 sources

$\mathcal{B}_1$ $\mathcal{B}_2$

BBA → m1 BBA → m2

Dempster **fusion rule** → m12

$$m(X) \triangleq [m_1 \oplus m_2] = \frac{\sum_{A \cap B = X} m_1(A) m_2(B)}{1 - \sum_{A \cap B = \emptyset} m_1(A) m_2(B)}$$

+ many other fusion rules (PCR...)

DSmT

Existing links

possibility distribution

e.g. Application to a decision context

Choice of the **sensitivity** (exposure) **level** of a site **exposed** to a **natural hazard**

$\Theta = \{\theta_1, \theta_2, \theta_3, \theta_4\}$

Set of 4 exhaustive and exclusive hypotheses (sensitivity levels)

{NoS} θ_1 {No sensitivity}

{LS} θ_2 {Low sensitivity}

{MS} θ_3 {Medium sensitivity}

{HS} θ_4 {High sensitivity}

The **frame** of **discernment**: set of possible decisions, alternatives...

ER-MCDA* methodology

**Evidential Reasoning and Multi Criteria Decision Analysis*

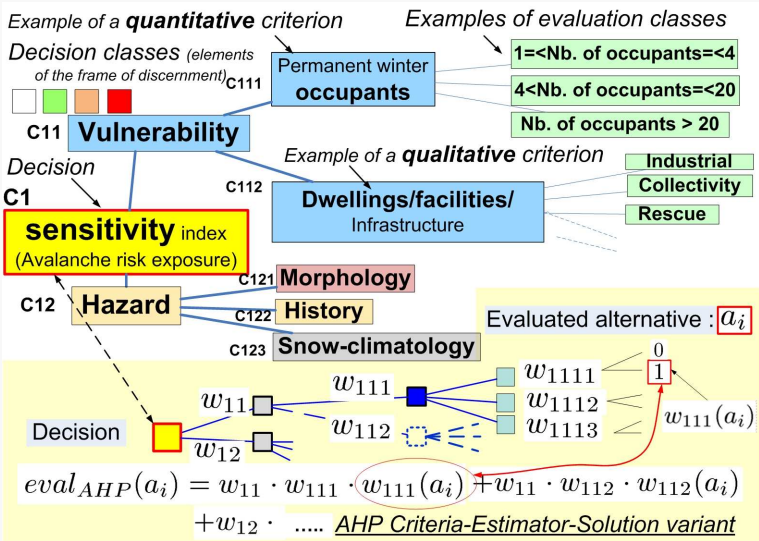
- The multi-criteria decision analysis (**AHP**) allows an analytic **breaking down** (into criteria) of the decision problem (+ **traceability**)
- Uncertainty theories are used :
  - to represent **different kinds** of **information imperfection** (unconsistency, imprecision, uncompletness and uncertainty)
  - to **evaluate** the **criteria** (**possibility theory** and **belief function theory**)
  - to **"map" the evaluations** into the frame of discernment (**fuzzy sets theory**)
  - To consider the multiple information **sources** and **produce a decision** (belief function theory)

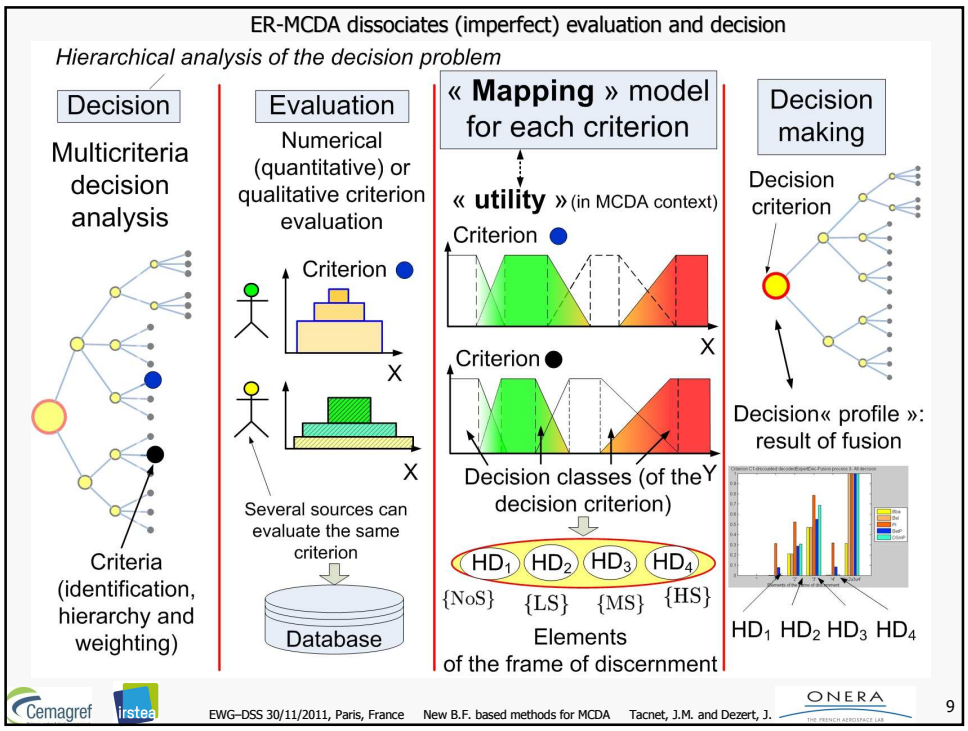
(Tacnet et al., 2009) - *Information Fusion for natural hazards in mountains* in Advances and Applications of DSmT for Information Fusion. Vol. 3

(Tacnet et al., 2010) – *A two-step fusion process for multi-criteria decision applied to natural hazards in mountains* , Int. Workshop on Belief Functions theory proceedings , 1-2 april 2010, Brest, France

Decision context : assessment of a sensitivity index (sorting decision problem)

The Analytic Hierarchy Process **(AHP)** is used as a **conceptual framework** (hierarchy, weights...)




ER-MCDA dissociates (imperfect) evaluation and decision
Hierarchical analysis of the decision problem
Decision
Multicriteria decision analysis
Criteria (identification, hierarchy and weighting)
Evaluation
Numerical (quantitative) or qualitative criterion evaluation
Criterion
X
Several sources can evaluate the same criterion
Database
« Mapping » model for each criterion
« utility » (in MCDA context)
Criterion
X
Criterion
Y
Decision classes (of the decision criterion)
HD1 HD2 HD3 HD4
{NoS} {LS} {MS} {HS}
Elements of the frame of discernment
Decision making
Decision criterion
Decision« profile »: result of fusion
HD1 HD2 HD3 HD4
Cemagref
irstea
EWG–DSS 30/11/2011, Paris, France
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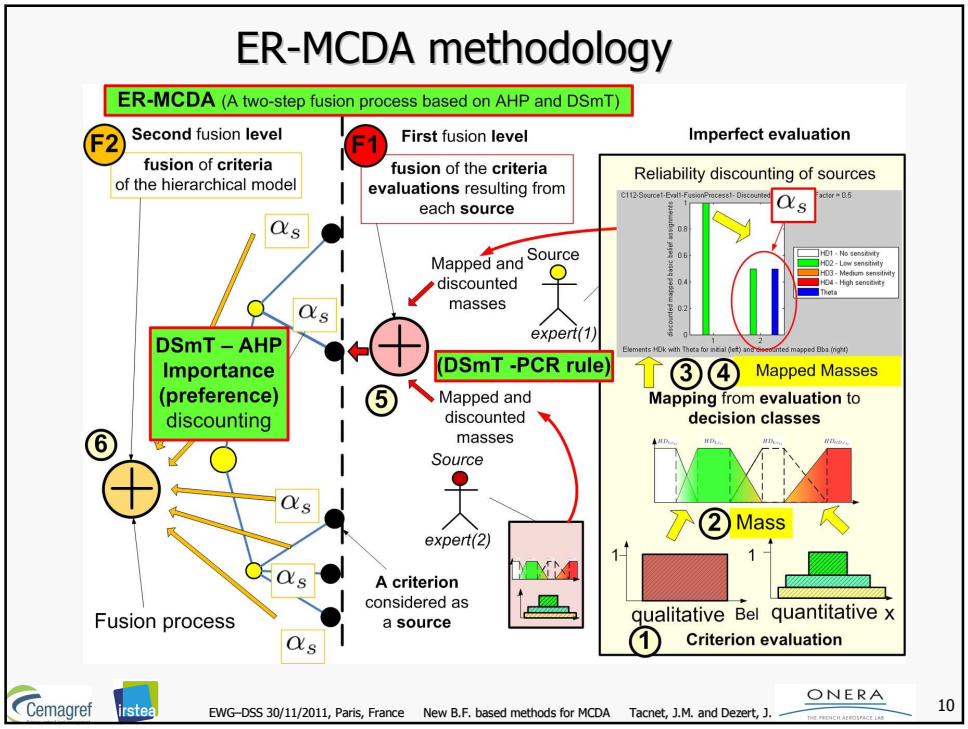
ER-MCDA methodology
ER-MCDA (A two-step fusion process based on AHP and DSmT)
F2 Second fusion level
fusion of criteria of the hierarchical model
F1 First fusion level
fusion of the criteria evaluations resulting from each source
Imperfect evaluation
Reliability discounting of sources
Mapped and discounted masses
Source
expert(1)
DSmT – AHP Importance (preference) discounting
(DSmT -PCR rule)
Mapped and discounted masses
Source
expert(2)
A criterion considered as a source
Fusion process
Mapped Masses
Mapping from evaluation to decision classes
Mass
qualitative Bel
quantitative x
Criterion evaluation
Cemagref
irstea
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The DSmT-AHP methodology

...an adaptation of AHP to consider **sets of alternatives** evaluated by provided by **more or less reliable** (and important) **sources**

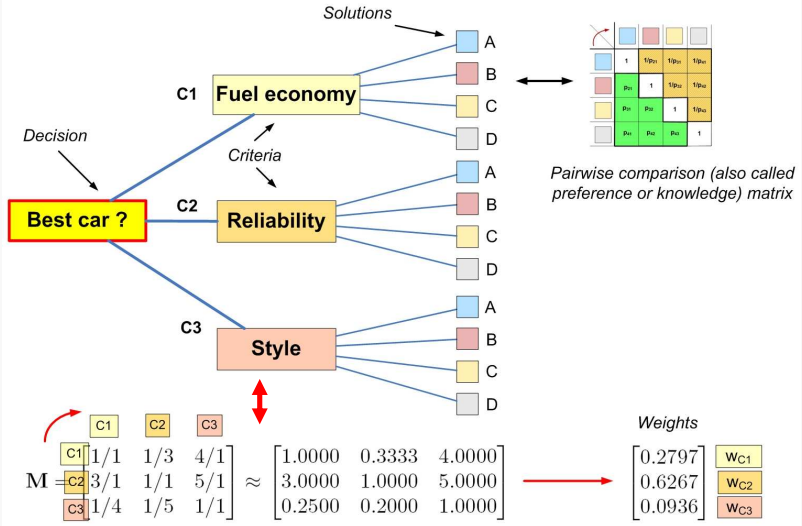
The DSmT-AHP methodology

- Original AHP is based on **pairwise comparisons** between solutions or alternatives
- Those comparisons can be **uncertain** or **uncomplete**
- A improvement of methods already proposed to use **belief functions** in the AHP framework (e.g. Beynon 2000, DS-AHP)

DSmT-AHP is an evolution that uses new fusion rules and discounting techniques

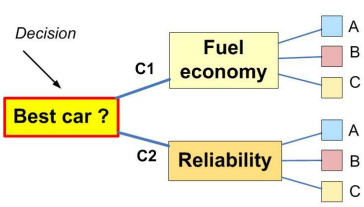
DSmT-AHP: a basic example

A simple decision problem: **choosing the best car** between four models A, B, C, D



DSmT-AHP : **preferences** are **assessed** on **sets of solutions** (including total ignorance)

DSmT-AHP introduces uncertainty in solutions evaluations
choosing the best car between three models A, B, C according to C1 and C2



$$\mathbf{M}(C1) = \begin{array}{c|ccc} & A & B\cup C & \Theta \\ \hline A & 1 & 0 & 1/3 \\ B\cup C & 0 & 1 & 2 \\ \Theta & 3 & 1/2 & 1 \end{array}$$

$$\mathbf{M}(C2) = \begin{array}{c|cccc} & A & B & A\cup C & B\cup C \\ \hline A & 1 & 2 & 4 & 3 \\ B & 1/2 & 1 & 1/2 & 1/5 \\ A\cup C & 1/4 & 2 & 1 & 0 \\ B\cup C & 1/3 & 5 & 0 & 1 \end{array}$$

Pairwise comparison (also called preference or knowledge) matrices

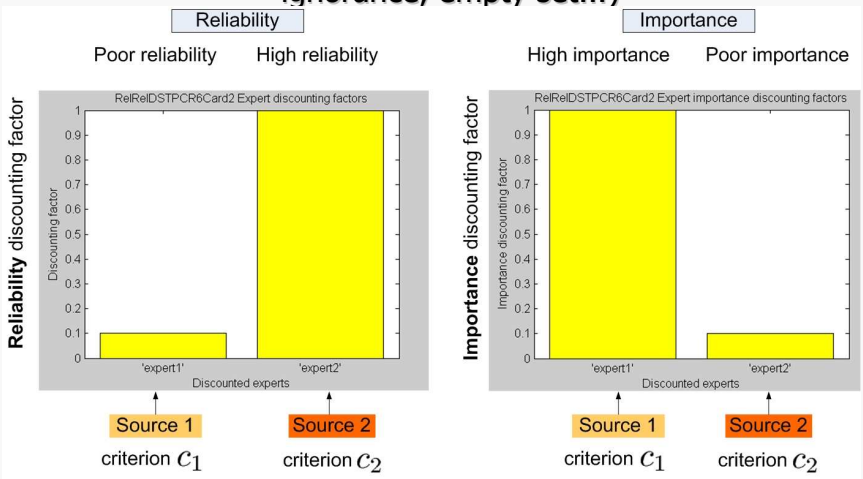
Priority vectors → **Basic belief assignments (bba's)**

Weights** are **derived** into **bbas

$$\begin{array}{cccc} A & B\cup C & \Theta & \\ \tilde{\mathbf{w}}(C1) \approx [0.0889 & 0.5337 & 0.3774]' & \longrightarrow m_{C1}(.) \end{array}$$

$$\begin{array}{cccc} A & B & A\cup C & B\cup C \\ \tilde{\mathbf{w}}(C2) \approx [0.5002 & 0.1208 & 0.1222 & 0.2568]' \longrightarrow m_{C2}(.) \end{array}$$

Importance and reliability are two different concepts: **specific "discounting" techniques** are proposed (transfert of bba's on ignorance, empty set...)



From (Tacnet et al., 2010) in *Advances and Applications of DSmT for information Fusion – Collected Works, volume 3.*

see (Smarandache, F., Dezert, J., Tacnet, J.M., 2010 – Fusion 2010 proceedings , July, Edimburgh, Scotland)

Conclusion DSmT-AHP

- DSmT-AHP is an evolution of AHP to consider **uncertainty** in comparison of solutions (criteria solution context) in case of **multiple sources**
- It **improves existing methods** (e.g. DS-AHP) by using
  - **high performance fusion rule** (PCR rule , DSmT framework) for
  - **New discounting methods** have been proposed to consider either **reliability** and **importance** discounting

The COWA-ER (Cautious Ordered Weighted Averaging – Evidential reasoning) methodology

… a multicriteria method for decision under uncertainty

Decision **M**aker (DM) evaluates a set $A = \{A_1, A_2, \ldots, A_q\}$ (alternatives) according to states of the world $S = \{S_1, S_2, \ldots, S_n\}$

Probabilities of the **states** of **nature** (e.g. phenomena scenarios) → p_1 … p_j … p_n

Evaluation of alternative A_i in the state of nature S_j

*"Classic" **Decision** under **risk***

$$\begin{array}{c|ccccc} & S_1 & \cdots & S_j & \cdots & S_n \\ \hline A_1 & C_{11} & \cdots & C_{11} & \cdots & C_{1n} \\ \vdots & & & \vdots & & \\ A_i & C_{i1} & \cdots & C_{ij} & \cdots & C_{in} \\ \vdots & & & \vdots & & \\ A_q & C_{q1} & \cdots & C_{qj} & \cdots & C_{qn} \end{array} \quad \begin{array}{l} E[C_1] = \sum_j p_j \cdot C_{1j} \\ \\ E[C_i] = \sum_j p_j \cdot C_{ij} \\ \\ E[C_q] = \sum_j p_j \cdot C_{qj} \end{array}$$

A^* is the **chosen alternative**. It corresponds to $C^* = Max_i(E[C_i])$

New COWA-ER method introduces **uncertain evaluations** of **scenarios** and new **cautious decision rules** based on credibility and plausibility *(see Tacnet and Dezert, 2011 – Fusion 2011 conference)*

States of the world $S = \{S_1, S_2, S_3, S_4, S_5\}$ Alternatives $A = \{A_1, A_2, A_3, A_4\}$

$m(S_1 \cup S_3 \cup S_4) = 0.6$ $\quad m(S_2 \cup S_5) = 0.3$ $\quad m(S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5) = 0.1$

$X_1 \leftarrow$ partial ignorances $\rightarrow X_2$ $\quad$ full ignorance X_3

Original OWA (**O**rdered **W**eighted **A**veraging) method (Yager, 1988)

$F : R^n \longrightarrow R$

$w_i \in [0,1] \qquad \sum_{i=1}^{n} w_i = 1$

$$W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

Weight of the highest value in $(a_1, \ldots, a_n)$

Weight of the lowest value in $(a_1, \ldots, a_n)$

$$F(a_1, \ldots, a_n) = \sum_{i=1}^{n} (w_i \cdot b_i)$$

b_i is the ith largest element in the collection $a_1, a_2, \ldots, a_n$

$$C = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{pmatrix} 10 & 0 & 20 & 30 \\ 1 & 10 & 20 & 30 \\ 30 & 10 & 2 & 5 \end{pmatrix} \end{array} \qquad W = \begin{bmatrix} 0.3 \\ 0.4 \\ 0.2 \\ 0.1 \end{bmatrix}$$

$$F(C_{11}, C_{12}, C_{13}, C_{14}) = F(10, 0, 20, 30) = 0.3 \cdot 30 + 0.4 \cdot 20 + 0.2 \cdot 10 + 0.1 \cdot 0 = 19$$

Adaptation of OWA (Ordered Weighted Averaging)

- The **original OWA** approach considers several alternatives evaluated in the context of different **uncertain scenarii**: it includes several ways (**pessimistic, optimistic, hurwicz, normative**) to interpret and aggregate the evaluations with respect to a given scenario.

Two evolutions are proposed :

- An **OWA approach** using the classical concept of **expected utility** based on **DSmP** or **BetP** has been proposed (p.m.).
- **COWA-ER** allows to take easily a decision in a context of **uncertain scenarios**. COWA-ER uses **simultaneously** the **two extreme pessimistic** and **optimistic decision attitudes** combined with an efficient fusion rule (no complex programming to get weights as in classic OWA approach) : **decision** is more **cautious**.

Main steps of COWA-ER approach

Pessimistic and **optimistic** valuations of the expected payoffs

1 Pessimistic choice 2 Optimistic choice

$$C = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{ccccc} S_1 & S_2 & S_3 & S_4 & S_5 \end{array} \\ \begin{pmatrix} 7 & 5 & 12 & 13 & 6 \\ 12 & 10 & 5 & 11 & 2 \\ 9 & 13 & 3 & 10 & 9 \\ 6 & 9 & 11 & 15 & 4 \end{pmatrix} \end{array} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 6.2 \\ 3.8 \\ 4.8 \\ 5.2 \end{bmatrix} \quad \begin{bmatrix} 10.9 \\ 11.4 \\ 11.2 \\ 13.2 \end{bmatrix} \quad E[C] = \begin{bmatrix} E[C_1] \\ E[C_2] \\ E[C_3] \\ E[C_4] \end{bmatrix} \subset \begin{bmatrix} [6.2; 10.9] \\ [3.8; 11.4] \\ [4.8; 11.2] \\ [5.2; 13.2] \end{bmatrix}$$

$[C_i^{\min}, C_i^{\max}]$

Four steps of the COWA-ER* approach

***C**autious **O**rdered **W**eighted **A**veraging-**E**vidential **R**easoning

- Step 1 : normalization of imprecise values in $[0, 1]$
- Step 2 : conversion of each normalized imprecise value into elementary bba $m_i(.)$
- Step 3 : fusion of bba $m_i(.)$ with an efficient combination rule (typically PCR5)
- Step 4 : choice of the final decision based on the resulting combined bba

COWA-ER - Step 2

COWA-ER Step 2 Conversion of each imprecise value into its bba (Dezert,1991)

Method

Frame of discernment $\Theta = \{A_1, A_2, A_3, A_4\}$

We use an existing method to **define bba** from **pessimistic** and **optimistic strategies** results

Modeling for computing a bba associated to the hypothesis A_i from any imprecise value $[a; b]$

$$\begin{cases} m_i(A_i) = a \\ m_i(\bar{A}_i) = 1 - b \\ m_i(A_i \cup \bar{A}_i) = m_i(\Theta) = b - a \end{cases}$$

$Bel(A_i) = a$

$\bar{A}_i$ is the complement of A_i in $\Theta = \{A_1, A_2, A_3, A_4\}$

$Pl(A_i) = b$

The uncertainty is represented by the length of the interval $[a; b]$

Imprecision of the variable (expected payoff C_i) on which is defined the belief function for A_i

Application

$$E^{Imp}[C] \approx \begin{bmatrix} [0.47; 0.82] \\ [0.29; 0.86] \\ [0.36; 0.85] \\ [0.39; 1.00] \end{bmatrix}$$

Basic belief assignments of the alternatives

Alternatives A_i	$m_i(A_i)$	$m_i(\bar{A}_i)$	$m_i(A_i \cup \bar{A}_i)$
A_1	0.47	0.18	0.35
A_2	0.29	0.14	0.57
A_3	0.36	0.15	0.49
A_4	0.39	0	0.61

COWA-ER – Step 4

COWA-ER **Step 4**

Decision-making from the resulting bba's of the fusion

Decision attitudes

- Maximum of credibility — Pessimistic choice
- Maximum of pignistic probability, DSmP — Compromise choice
- Maximum of plausibility — Optimistic choice

A_i	$Bel(A_i)$	$BetP(A_i)$	$DSmP(A_i)$	$Pl(A_i)$
A_1	0.2488	0.3126	0.3364	0.4850
A_2	0.1142	0.1863	0.1623	0.3729
A_3	0.1600	0.2299	0.2242	0.4130
A_4	0.1865	0.2712	0.2771	0.4521

$\boxed{A_1}$ *decision*

A_1 is always chosen: **a more cautious choice…**

Comparison with pessimistic and optimistic OWA approaches: ambiguity is reduced

1 Pessimistic choice

2 Optimistic choice

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 6.2 \\ 3.8 \\ 4.8 \\ 5.2 \end{bmatrix} \leftarrow \boxed{A_1} \qquad \begin{bmatrix} 10.9 \\ 11.4 \\ 11.2 \\ 13.2 \end{bmatrix} \leftarrow \boxed{A_4}$$

Conclusion : towards a **global framework** associating belief function theory and MCDA methods…

*Decision under **certainty*** — *Decision under **risk*** — *Decision under **uncertainty** or **ignorance***

Imperfection in **preferences** (importance) evaluation

(different decision models, aggregation principles are used)

Imperfection of **alternatives evaluation**

Alternative$_i$

(e.g. groups of alternatives, uncompleteness of alternatives, imprecise evaluations…)

Known (or objective) **probabilities** on the states of the world (conditionning the consequences of alternatives)

Unknown or **(subjective) probabilities** on the states of the world

Classical methods (e.g.)	MAUT, AHP, Outranking methods…	Expected Utility Theory (EUT)		Subjective Expected Utility Theory (SEUT)
Methods mixing ER and MCDM (e.g.)	DS-AHP, DSmT-AHP, ER-MCDA…		OWA (2008)	**COWA-ER**

Developments are under progress to introduce BFT in outranking methods…

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