A brief presentation of the Skew Mu Toolbox (SMT) v3

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Abstract

This short note gives a rough overview of the third version of the SMT Toolbox. More detailed discussions on μ analysis can be found in the documents associated to previous versions of the toolbox or in the papers cited at the end of this note (see the help of the routines). The main difference between this third release and previous ones is related to the computation of a guaranteed robustness margin via a new and more efficient routine called: **mu_margin.m**. As in previous versions, it is still based on a frequency segments elimination technique using enhanced sub-routines. Moreover, the robust performance problem is also treated in a very fast way.

Available on the web pages :
http://www.cert.fr/dcsd/idco/perso/Biannic/mypage.html
http://www.cert.fr/dcsd/idco/perso/Ferreres/index.html

After unzipping SMTv3.zip, three directories and an m-file $path2SMT.m$ will appear. First, $path2SMT.m$ is to be edited and modified. The three directories are:

- *Applications*/$*$ which contains the standard interconnection structures $M(s)$ - Δ for several applicative examples, and possibly the way to compute some of these interconnection structures.
- *Demos*/$*$ which contains several demos of classical and skew μ tools.
- *Subs*/$*$ which contains the Matlab main routines and sub-routines.

The objectives of the main routines in *Subs*/$*$ are listed in the following:

- **mixed_mu_lb_freq.m** computation of a mixed μ lower-bound at each point of a frequency gridding which is defined by the user.
- **mu_lb_with_freq.m**: computation of a μ lower bound over a frequency range and associated worst-case perturbations. This routine aims at finding the maximal value of the lower-bound for the given frequency range. Note that only real pertubations are accounted for in the current version.
- **mu_elim.m**: elminination of frequency intervals inside a given range for which the (skew) μ upper-bound is smaller than a test value.
- **mu_frequency_gridding.m**: computation of (skew) μ upper and lower bounds at each point of a frequency gridding. Several methods are available according to the problem to be solved.
- **mu_margin.m**: this new routine computes stability or performance robustness margins by estimating a reliable (skew) μ upper-bound over a frequency range. The routine is based on newly implemented tools for frequency segments elimination. Moreover, the procedure is fully automatized so that the user will only have to provide the standard plant and the structure of the uncertainty block. Advanced uses are also possible through optional inputs.
- **mu_max_1.m**: this routine implements a similar approach as in **mu_margin.m** with however a significant difference in the local μ upper-bound computation technique. Instead of considering a single frequency point for such a computation, two points are used here. This results *a priori* in scaling matrices (D and G) which should be valid on larger segments and may consequently reduce the number of iterations. When compared to the previous approach, there is here a risk to over-evaluate the μ upper-bound.

- **mu_max_2.m**: the aim of this routine is also to compute a guaranteed (skew) μ upper-bound over a frequency range. The main difference in this third routine is that the frequency is now treated as an additional real parametric uncertainty. This method will then typically give good results on low-order plants, but will no longer be applicable on large systems.

- **robust.m**: the advanced user may be interested in reading this routine, which is essentially an interactive interface to the other main routines of the toolbox, to be able to directly use these main routines.

- **square_m.m**: this routine is to preliminarily used in order to transform possibly non-square systems (in feed-back loop with compatible non-square Δ blocks) into square ones.

- **worst_case_margin.m**: this routine generalizes the standard notions of gain, phase and delay margins to the class of parametrically uncertain plants. The issue is to compute an estimate of the worst-case value of the MIMO gain, phase and delay margins when classical model uncertainties are maintained inside the unit ball.

Some routines may also be directly called by advanced users:

- **mixed_mu_lb.m**: computation of the (skew) mixed-μ lower-bound at a single frequency-point with a power algorithm.

- **mixed_mu_ub.m**: computation of the mixed (skew) μ upper-bound at a frequency point.

- **mu_dailey.m**: computation of a μ lower bound at a single frequency point using the Dailey's algorithm for Δ blocks which only contain non-repeated real uncertainties. The method is exponential-time, and thus limited to about 10 or 12 uncertain parameters.

- **mu_locus.m**: plots the root-locus associated to the interconnection $M(s) - \alpha\Delta^*$ as a function of $\alpha \in [0\ 1]$

- **mu_ub.m**: LMI-based computation of a mixed-μ upper bound, which is simultaneously valid at several frequency points.

- **mu_zd.m**: computation of a μ upper-bound at a single frequency point using Zadeh and Desoer's algorithm for Δ blocks which only contain non-repeated real uncertainties. The method is exponential-time, and thus limited to about 10 or 12 uncertain parameters.

- **plot_bar.m**: This routine is used to display guaranteed mu-upper bounds as a function of frequency segments.

- **skew_mu_ub.m**: this routine computes a mixed (skew) μ upper-bound which is simultaneously valid at several frequency points.

Help of the main routines

mixed_mu_lb_freq.m

```
function [tab_mu,tab_pert]=mixed_mu_lb_freq(sys,blk,puls,method,alfa,cxi)
%
% call:  [tab_mu,tab_pert]=mixed_mu_lb_freq(sys,blk,puls,method,alfa,cxi)
%
% Robustness analysis of the standard interconnection structure M(s) - Delta.
%
% Computation of a mixed (skew) mu lower bound at each point of a
% frequency gridding "puls".
%
% sys = system matrix (ss format) of M(s), a continuous-time plant.
%
% The structure of Delta is described by blk (musyn format).
% If blk has only 2 columns, a classical mu problem is considered.
% Otherwise the third column describes the potentially skew nature of the
% uncertainty.
%
% Delta and M(s) must be square.
%
% See mu_frequency_gridding.m for the meaning of alfa and cxi.
%
% Internal use of mixed_mu_lb.m with different options.
%
% method = 1 --> the fastest method (default value).
%          2 --> a slower, but potentially more accurate method.
%          3 --> an even slower, but potentially even more accurate method.
%
% N.B.: method = 2 or 3 seems to provide in many cases the same result as method = 1.
```

mu_lb_with_freq.m

```
function [tab_mu,puls_add,tab_pert]=mu_lb_with_freq(sys,blk,puls,method,alfa,cxi,epsilon,...
verbatim,target)
%
% call :
%
% [tab_mu,puls_add,tab_pert]=mu_lb_with_freq(sys,blk,puls,method,alfa,cxi,...
% epsilon,verbatim,target)
%
% choose [] to use the default value, e.g.:
% [tab_mu,puls_add,tab_pert]=mu_lb_with_freq(sys,blk,puls,method,[],[],epsilon);
%
% the first four arguments are necessary, i.e. the minimal call is:
% [tab_mu,puls_add,tab_pert]=mu_lb_with_freq(sys,blk,puls,method);
```

```
%
% Computation of a mu lower bound for a transfer matrix M(s)
% (defined by system matrix sys, with ss format)
% and a model perturbation, whose structure is defined by "blk".
% A frequency gridding "puls" is used as an initial guess.
%
% THE AIM IS NOT TO COMPUTE A MU LOWER BOUND AS A FUNCTION OF FREQUENCY,
% BUT TO COMPUTE A LOWER BOUND OF THE MAXIMAL VALUE OF MU OVER THE FREQUENCY RANGE.
%
% NO SKEW UNCERTAINTY IS ALLOWED. If any, the 3d column of blk is not accounted for.
%
% ONLY THE CASE OF A REAL MODEL PERTURBATION IS TREATED.
%
% Instead of considering the left half plane, a truncated sector can be
% used: cxi is the minimal damping ratio, alfa the minimal
% degree of stability (the degree of stability of a matrix A is the
% maximal real part of the eigenvalues of A - alfa is thus negative if A
% is stable). alfa is required to be negative, cxi to be positive.
%
% verbatim = 0 --> no display
%          = 1 --> display of information at each frequency
%          = 2 --> minimal display
%
% tab_mu(i) corresponds to a lower bound of the real mu problem at puls_add(i),
% which is most generally different from puls(i).
% More precisely tab_mu(i) corresponds to the inverse of the norm
% of the destabilizing model perturbation Delta =
% tab_pert(:,:,i), i.e. a pole of  A + B Delta C is on the imaginary axis
% at point j*puls_add(i) (with some numerical tolerance). NOTE THAT
% puls_add IS NOT SORTED, NEITHER IN ASCENDING NOR DESCENDING ORDER.
%
% If a mu lower bound is found, that is greater than the last input argument "target",
% the algorithm stops !
%
% *** For a basic use of the routine just choose method=4 or 2, epsilon=[].
% Otherwise, the use of these 2 arguments is explained in the following. ***
%
% A lower bound of a regularized mu problem is
% first computed at frequency puls(i), i.e. a small amount of complex
% uncertainties is added to the real model perturbation, to improve the
% convergence properties of the power algorithm which computes a mu lower
% bound (in the case of a purely real model perturbation, this algorithm
% often does not converge, so that no mu lower bound is computed).
%
% A destabilizing model perturbation Delta is provided with the mu
% lower bound. Nevertheless, this value is destabilizing for the regularized
% mu problem, not for the original one, since this model perturbation
% contains real and complex parts : Delta = diag(Delta_R,Delta_C). The
% real part Delta_R of the model perturbation is extracted and applied to
```

```
% the original real mu problem. Assume for the sake of simplicity that M(s) =
% (A,B,C,0), i.e. M(s) is a strictly proper transfer matrix (noting that
% the routine treats the general case of a simply proper transfer
% matrix). A pole of A + B Delta_R C is expected to be located near the point
% j*puls(i) of the imaginary axis (remember puls(i) is the frequency used
% for the regularised mu problem). Thus Delta_R is a good initial guess
% for a real destabilizing model perturbation. Four methods can be used now:
%
% method = 1 : the eigenvalue of A + B Delta_R C, which is the closest
% to the point j*puls(i) of the imaginary axis, is identified. If this
% eigenvalue is stable, the size of Delta = alfa * Delta_R is
% increased, until the corresponding eigenvalue of A + alfa B Delta_R C
% crosses the imaginary axis (a priori for alfa slightly greater than 1).
% Otherwise, the size of Delta is decreased, here again until the
% eigenvalue crosses the imaginary axis.
%
% method = 2 or 3 : the eigenvalue of A + B Delta_R C, which is the closest
% to the point j*puls(i) of the imaginary axis, is identified. When
% shifting this eigenvalue toward the imaginary axis, the norm of the model
% perturbation is minimized. This can be the infinity (method = 2)
% or two norm (method = 3).
%
% method = 4 : all eigenvalues of A + B Delta_R C are inspected. If one
% of them is already unstable, the size of Delta = alfa * Delta_R is
% decreased, until  A + alfa B Delta_R C becomes marginally stable (all
% eigenvalues in the left half plane or on the imaginary axis).
% Otherwise, the size of Delta = alfa * Delta_R is increased, until  A +
% alfa B Delta_R C becomes marginally stable.
%
% In all these methods, a regularized mu problem is computed at
% frequency puls(i), and the imaginary axis is crossed at puls_add(i).
% THIS FREQUENCY CAN BE FAR FROM puls(i).
%
% N.B. (1) : for a given frequency puls(i), the method may fail during the
% first or second step. If the mu problem is not regularized enough, the
% power algorithm may not converge. The migration toward the
% imaginary axis may also fail for numerical reasons. Remember that the aim is
% not to compute a mu lower bound as a function of frequency, but a lower
% bound of the maximal value of mu over the frequency range.
% Consequently, it is not especially troublesome if the method fails at a
% given frequency puls(i).
%
% N.B. (2) : even if method=3, tab_mu(i) is the inverse of the
% infinity norm of the destabilizing model perturbation Delta =
% tab_pert(:,:,i).
%
% epsilon is the amount of complex uncertainties, which are added to
% regularize the mu problem. epsilon = 5e-2 (the default value) seems to be enough
% in most situations.
```

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%
% epsilon must be strictly positive.
%
% A full complex block is added to regularize the problem.
%
% When considering a truncated sector instead of the left half plane,
% the regularized mu problem is computed on the border of the
% sector, instead of the imaginary axis.
%
% When considering a robust stability problem inside the left half
% plane, the frequency omega corresponds to the point j*omega of the
% imaginary axis. In the case of a truncated sector, the frequency omega
% corresponds to the point xxx+j*omega of the border of the truncated
% sector, i.e. omega is the imaginary part of the point on the border.
%
% See for further details:
% G. Ferreres and J.M. Biannic
% "Reliable computation of the robustness margin for a flexible transport aircraft"
% Proceedings of the AIAA GNC 2000
% and Control Engineering Practice, pp. 1267-1278, vol. 9, 2001.
%
```

mu_elim.m

```
function [list_intervals,list_intervals_OK,critical_pts,tab_mu,list_intervals_bis]=...
 mu_elim(sys,blk,gamma,list_intervals,nbr_iter_max,opt,possible_stop,epsilon,verbatim)
%
% call:  [list_intervals,list_intervals_OK,critical_pts,tab_mu,list_intervals_bis]=...
% mu_elim(sys,blk,gamma,list_intervals,nbr_iter_max,opt,possible_stop,epsilon,verbatim)
%
% Robustness analysis of the interconnection structure M(s) - Delta.
%
% sys = system matrix of M(s) (ss format)
%
% blk describes the structure of the model perturbation Delta.
% blk must have three columns.
%
% M(s) and Delta must be square.
%
% Elimination of frequency intervals inside a specified frequency interval [wmin, wmax]
% which satisfy (skew) mu_ub(M(jw)) <= gamma.
%
% To this aim, [wmin, wmax] is partitioned at each iteration into smaller intervals.
% nbr_iter_max represents the maximal number of iterations.
%
% If a skew mu problem is considered, the problem is reduced to a
% classical mu test: mu_ub(N(jw)) <= 1.
%
```

```
% The mu upper bound mu_ub(M(jw)) or mu_ub(N(jw)) is computed with mu.m.
% opt is the option of mu.m (e.g. opt = 'f' or opt = 'u').
% 'w' will automatically be added. See the help of mu.m for more details.
%
% epsilon is used to regularize the mu problem, in order to render the
% scalings suboptimal.
%
% The input argument list_intervals is typically [wmin wmax],
% but it can also be the output argument list_intervals_bis (see below).
%
% If possible_stop is non-zero, the algorithm stops if a frequency w is found,
% for which (skew) mu_ub(M(jw)) >= gamma.
%
% There are two main situations:
% 1/ [wmin wmax] is a large frequency interval: nbr_iter_max is low (about 5)
%    and possible_stop=0. The issue is to eliminate frequency intervals
%    inside [wmin wmax], which satisfy (skew) mu_ub(M(jw)) <= gamma.
% 2/ [wmin wmax] is a small frequency interval: nbr_iter_max can be higher
%    but possible_stop=1. The issue is to check if (skew) mu_ub(M(jw)) <= gamma
%    on [wmin,wmax].
%
% verbatim = 0 --> minimal display
%          = 1 --> display of intermediate results
%          = 2 --> display of intermediate results and pause at the end
%                  of each iteration
%
% At the output, list_intervals contains frequency intervals,
% which do not satisy (skew) mu_ub(M(jw)) <= gamma.
% list_intervals is sorted and compacted.
% list_intervals = [] means that all frequencies were eliminated.
%
% Conversely, list_intervals_OK contains frequency intervals,
% which satisy (skew) mu_ub(M(jw)) <= gamma. list_intervals_OK is sorted and compacted.
%
% critical_pts contains frequencies where (skew) mu_ub(M(jw)) >= gamma
% (with opt used as the option for mu.m). tab_mu contains corresponding values of the
% mu lower and upper bounds.
%
% CAUTION ! If a skew mu problem is considered tab_mu does not correspond to
% skew mu upper and lower bounds !!!
%
% list_intervals_bis is not sorted, nore compacted. It is possible to use it
% as an input argument:
%
% [list_intervals,list_intervals_OK,critical_pts,list_intervals_bis]=...
% mu_elim_freq(sys,blk,gamma,[wmin wmax],nbr_iter_max,opt,possible_stop,epsilon);
%
% [list_intervals,list_intervals_OK,critical_pts,list_intervals_bis]=...
% mu_elim_freq(sys,blk,gamma,list_intervals_bis,1,opt,possible_stop,epsilon);
```

```
%
% The second call enables to further eliminate frequencies.
%
```

mu_frequency_gridding.m

```
function [tab_mu,tab_pert]=mu_frequency_gridding(sys,blk,puls,method,alfa,cxi)
%
% call:  [tab_mu,tab_pert]=mu_frequency_gridding(sys,blk,puls,method,alfa,cxi)
%
% Robustness analysis of the standard interconnection structure M(s) - Delta.
%
% sys = system matrix (ss format) of M(s), a continuous-time plant.
%
% The structure of Delta is described by blk (musyn format).
% If blk has only 2 columns, a classical mu problem is considered.
% Otherwise the third column describes the potentially skew nature of the
% uncertainty.
%
% Delta and M(s) must be square.
%
% Computation of bounds of the (skew) s.s.v. at each point of a
% frequency gridding "puls".
%
% method = 1 --> computation of the mixed mu upper bound.
% tab_mu(iw) is the mu upper bound at frequency puls(iw). tab_pert=[].
%
% method = 2 --> computation of the mixed mu lower bound.
% tab_mu(iw) is the mu lower bound at frequency puls(iw).
% tab_pert(:,:,iw) is the structured model perturbation that destabilizes
% the closed loop at frequency puls(iw).
% In case of skew uncertainties (or if mu.m is not available)
% an SMT version of the power-algorithm is used.
% Otherwise, the power-algorithm by musyn is used.
%
% method = 3 --> computation of a mu upper bound with Zadeh and
% Desoer's result. See the help of mu_zd.m inside this toolbox for
% further details. Delta must only contain non-repeated real
% uncertainties. The method is
% exponential-time, and thus limited to about 10 or 12 uncertain parameters.
%
% method = 4 --> computation of a mu lower bound with Dailey's method.
% See the help of mu_dailey.m inside this toolbox for further details.
% Delta must only contain non-repeated real uncertainties.
% The method is exponential-time, and thus limited
% to about 10 or 12 uncertain parameters.
%
% Robust stability is studied either inside the left half plane, or inside a
```

```
% truncated sector. alfa is the minimal degree of stability (alfa < 0 for
% a stable plant) and cxi is the minimal damping ratio. alfa=0 and cxi=0
% correspond to the left half plane. Only real uncertainties are allowed
% for robust stability inside a truncated sector, complex uncertainties
% are defined only on the imaginary axis.
%
% sys, blk, puls and method are mandatory. Other input arguments are
% optional ([] may also be used to specify them). Default values are
% alfa=0 and cxi=0.
```

mu_margin.m

```
function [muub,wc,bnds,tab] = mu_margin(sys,blk,opt);
%
% MU_MARGIN    -  Computes a guaranteed stab/perf robustness margin
%-------------------------------------------------------------------
% PURPOSE
% This routine computes a (skew) mu upper-bound on a frequency seg-
% ment either on the imaginary axis or on the boundary of a trunca-
% ted sector.
%
% SYNOPSIS
% [muub,wc,bnds,tab] = mu_margin(sys,blk[,opt]);
%
% INPUT ARGUMENTS
% sys   SS-object defining the interconnection structure
% blk   nblk-by-2 or nblk-by-3 matrix defining the structure of the
%       uncertainty block DELTA = diag(DELTA_1,...,DELTA_nblk) :
%          blk(i,:) = [-n 0] --> DELTA_i = di*eye(n),    di is real
%          blk(i,:) = [n 0]  --> DELTA_i = di*eye(n), di is complex
%          blk(i,:) = [n n]  --> DELTA_i  full n-by-n complex block
%       A third column can be used to specify skew uncertainties :
%          blk(i,3) = 0 :  sig_max(DELTA_i) <= 1
%          blk(i,3) = 1 :  sig_max(DELTA_i) <= 1/muub
% opt   optional strutured variable with fields :
%         * trace   : trace of execution
%         * itermax : maximum number of iterations
%         * muT     : initial guess for mu (lower-bound)
%         * w0      : starting frequency point
%         * puls    : frequency segment [wmin wmax] or gridding
%         * sector  : truncated sector [alpha xi]
%         * tol     : tolerance on mu  (muub=(1+tol)*muopt)
%         * lmi     : switch to LMI  (if lmi=1) for better accuracy
%         * reg     : regularization parameter for real-mu problems
%                     by default 1% of complex uncertainty is added
%                     in case of convergence problems (reg=0.01)
%         * elim    : activation/desactivation  of  the pawl effect
%                     on mu which is used by default  to fasten the
```

```
%                     elimination technique.  Set this parameter to
%                     0 to get better estimates of secondary peaks.
%         * smm     : skew-mu method
%                     1 : fixed-point type algorithm
%                     2 : dichotomy search
%
% OUTPUT ARGUMENTS
% muub   guaranteed mu upper-bound
% wc     critical frequency associated to the max. upper-bound
% bnds   structured variable with fields
%           bnds.w : list of frequency points
%           bnds.mu : associated upper-bounds
% tab   structured variable with fields
%           tab.seg : list of segments
%           tab.mu  : associated guaranteed upper-bounds
%
```

mu_max_1.m

```
function [tab_beta_visu,tab_puls_visu]=mu_max_1(a1,a2,a3,a4,a5,a6)
%
% call:  [tab_beta_visu,tab_puls_visu]=mu_max_1(sys,blk,puls)
%        [tab_beta_visu,tab_puls_visu]=mu_max_1(sys,blk,puls,num_figure)
%        [tab_beta_visu,tab_puls_visu]=mu_max_1(sys,blk,puls,alfa,cxi)
%        [tab_beta_visu,tab_puls_visu]=mu_max_1(sys,blk,puls,alfa,cxi,num_figure)
%
% Robust stability of the standard interconnection structure M(s) - Delta.
%
% sys = system matrix (ss format) of M(s), a continuous-time plant.
%
% The structure of Delta is described by blk (musyn format).
% The third column describes the potentially skew nature of
% the uncertainty:
%
% blk(i,3) = 0 --> norm of the uncertainty less than 1.
%          = 1 --> norm of the uncertainty less than k, where k is maximized.
%
% Delta and M(s) must be square.
%
% puls = initial frequency gridding.
%
% Computation of a guaranteed mu upper bound on a union of small frequency intervals.
% The computation is performed either on the imaginary axis or on the borderline
% of a truncated sector. In that case, only real uncertainties are allowed.
% alfa is the minimal degree of stability (alfa < 0 for a stable plant)
% and cxi is the minimal damping ratio. (alfa=0 and cxi=0 correspond to the LHP)
%
% Let [wmin, wmax] a small interval. D,G scaling matrices are computed, which are valid
```

```
% at wmin and wmax. It is then checked whether these scaling matrices are valid
% inside the associated interval. If yes, a guaranteed mu upper bound has been computed
% on the interval.
%
% tab_beta_visu(i) is a guaranteed mu upper bound over the
% frequency interval [tab_puls_visu(i,1), tab_puls_visu(i,2)]. Results can be visualized
% with plot_bar.m
%
% It may be useless to use a too fine initial gridding puls, this may even be
% time consuming. D,G scaling matrices are first computed at puls(i) and puls(i+1).
% If they are not valid inside the associated interval this one is split into
% [puls(i), (puls(i)+puls(i+1))/2] and [(puls(i)+puls(i+1))/2, puls(i+1)].
%
% Nevertheless, assume that a guaranteed mu upper bound was computed
% on a frequency interval. It may happen that when splitting this
% frequency interval into smaller ones, the maximal guaranteed mu
% upper-bound over these smaller intervals is less than the initial
% guaranteed mu upper bound.
%
```

mu_max_2.m

```
function  [tab_beta_visu,tab_puls_visu]=mu_max_2(sys,blk,puls)
%
% call:   [tab_beta_visu,tab_puls_visu]=mu_max_2(sys,blk,puls)
%
% Robust stability of the standard interconnection structure M(s) - Delta.
%
% sys = system matrix (ss format) of M(s), a continuous-time plant.
%
% The structure of Delta is described by blk (musyn format).
% The third column describes the potentially skew nature of
% the uncertainty:
%
% blk(i,3) = 0 --> norm of the uncertainty less than 1.
%          = 1 --> norm of the uncertainty less than k, where k is maximized.
%
% Delta and M(s) must be square.
%
% puls = frequency gridding.
%
% Computation of a guaranteed mu upper bound on a union of frequency intervals.
% tab_beta_visu(i) is a guaranteed mu upper bound over the
% frequency interval [puls(i), puls(i+1)]. This frequency interval is renamed as
% [tab_puls_visu(i,1), tab_puls_visu(i,2)].
%
% The frequency gridding puls is not refined inside the routine, unlike in mu_max_1.m.
%
```

```
% Nevertheless, assume that a guaranteed mu upper bound was computed on a frequency interval.
% It may happen that when splitting this frequency interval into smaller ones,
% the maximal guaranteed mu upper bound over these smaller intervals is less than the initial
% guaranteed mu upper bound.
%
% Results can be visualized with plot_bar.m
%
% See for further details:
% G. Ferreres and V. Fromion,
% "Computation of the robustness margin with the skewed mu tool",
% Systems and Control Letters,
% Vol. 32, n. 4, pp. 193-202, December 1997.
%
```

worst_case_margin.m

```
function [puls,tab_delay,tab_phase,tab_gain]=worst_case_margin(sys_M1,sys_M2,blk,puls,...
          improved_delay_margin)
%
% call:
%
% [puls,tab_delay,tab_phase,tab_gain]=worst_case_margin(sys_M1,sys_M2,blk,puls,...
%                                                       improved_delay_margin)
%
% Computation of worst-case gain, phase and delay margins (with the small gain theorem)
% in the presence of "classical" model uncertainties (i.e. parametric
% uncertainties and neglected dynamics). The issue is to compute an estimate
% of the worst-case value of the MIMO gain, phase and delay margins
% when classical model uncertainties are maintained inside the unit ball.
%
% Uncertain gains, phases and delays are represented by additional direct
% or inverse neglected dynamics (typically at the plant inputs and/or outputs).
%
% Use of 2 standard interconnection structures M(s) - diag(Delta,Delta_c):
% Delta contains classical model uncertainties, whose structure is described by blk.
% Delta_c contains additional direct or inverse neglected dynamics
% (whose structure is not described by blk). Note that these neglected dynamics thus
% correspond to the last inputs and outputs of M(s).
%
% sys_M1 corresponds to the standard interconnection structure with direct neglected dynamics.
% sys_M2 corresponds to the standard interconnection structure with inverse ones.
% It is possible to choose sys_M1=[] or sys_M2=[].
%
% If a direct block Delta_c of neglected dynamics is used, I+Delta_c must
% correspond to the uncertain gain k, phase e^(j phi) or delay e^(-tau s).
% More precisely, if e.g. uncertain gains are considered:
% I+Delta_c = diag(k_i) for i between 1 and N.
% The same is true for uncertain phases and delays.
```

```
%
% If an inverse block Delta_c of neglected dynamics is used, (I-Delta_c)^(-1) must
% correspond to the uncertain gain k, phase e^(j phi) or delay e^(-tau s).
%
% blk can have 2 or 3 columns. But the value of the 3th column is not accounted for,
% since "classical" model uncertainties are maintained inside the unit ball.
%
% blk=[] means that no classical model uncertainty is accounted for in the problem,
% just additional neglected dynamics representing the uncertain gains, phases or delays.
%
% M(s) and diag(Delta,Delta_c) must be square.
%
% Basic computation of worst-case gain, phase and delay margins:
% the phase margin corresponds to [-A, A], the delay margin
% to [-A/omega, A/omega] where omega is the frequency.
% (remember phi = - omega * tau, where phi is the phase and tau the delay)
%
% if improved_delay_margin=1 the delay margin is improved, by considering
% an interval [0, tau_max] instead of [-tau_max tau_max].
%
% tab_delay, tab_gain and tab_phase represent conservative estimates of
% the margin (i.e. lower bounds) as a function of frequency puls(i).
%
% The initial frequency gridding puls is refined inside the routine.
%
% If the closed loop is not robustly stable in the presence of classical model uncertainties
% maintained inside their unit ball, gain, phase and delay margins are taken as 0.
%
```