

Comparison of the expressiveness of Arc, Place and Transition Time Petri Nets

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Outline

- ▶ **Motivation**
- ▶ **Model definitions**
- ▶ **Comparing model expressiveness**
- ▶ **Conclusion**

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- ▶ Comparing model expressiveness
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Why this study?

In 30 years, several models developed:

- intervals on transitions [Mer74, BD91], places [KD96] or arcs [Han93, AN01, dFRA00]
- each model has its own application domain
- people are attached to *their* model
- when failing to model: user skill or model power?

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Previous studies

- several studies on power of some models: decidability of reachability, covering
- some comparisons have been done: [CA99], [BV00]
- *but*

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- some comparisons have been done: [CA99], [BV00]
- *but some results are contradictory*

Comparing what ? With which criterion ?

Comparing: Petri nets

- with time constraint with intervals on transitions [Mer74, BD91], places [KDCCD96] or arcs [Han93, AN01, dFRA00]
- with weak *and* strong semantics
- one bounded (\iff k-bounded with mono-server semantics)
- with large and strict intervals bounds ($[a, b]$, $[a, b[$, $]a, b]$, $]a, b[$), cf [BV00]

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Criterion: weak timed bisimulation

- “weak bisimulation” is “the same visible actions are possible in both models”

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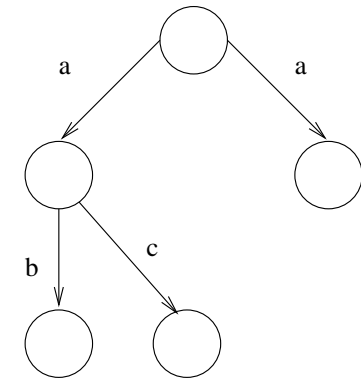
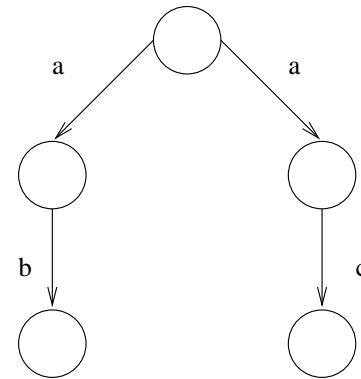
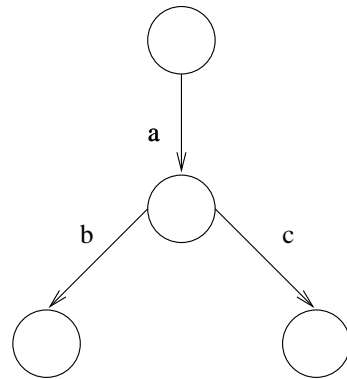
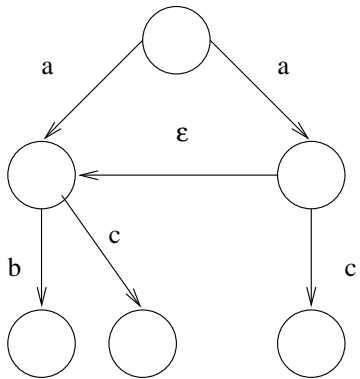
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- “weak bisimulation” is “the same visible actions are possible in both models”
- “weak *timed* bisimulation” is “the same visible actions are possible in both models *at the same time*”
- bisimulation implies language equality

Bisimulation



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▶ **Model definitions**

- T-time Petri net
- Strong vs Weak Semantics
- P-time Petri net
- A-time Petri net

▶ Comparing model expressiveness

- Introduction, previous works
- Results from [BCH⁺05]
- Weak with strong for P-TPN and A-TPN
- Something that P-TPN can't do, and T-TPN does
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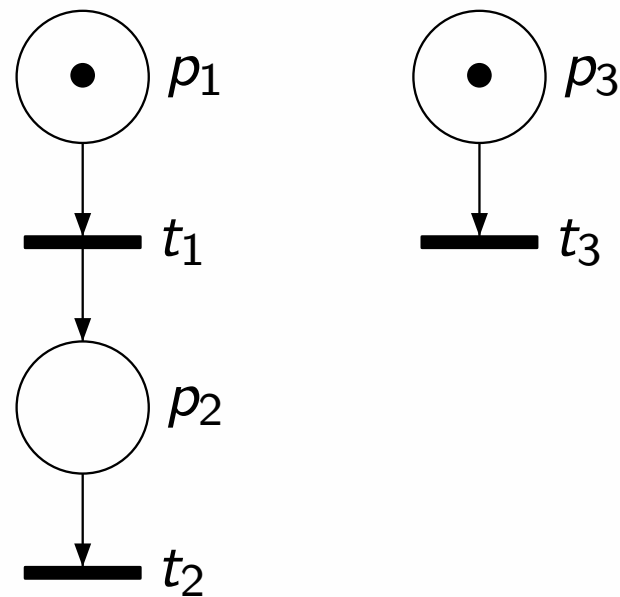
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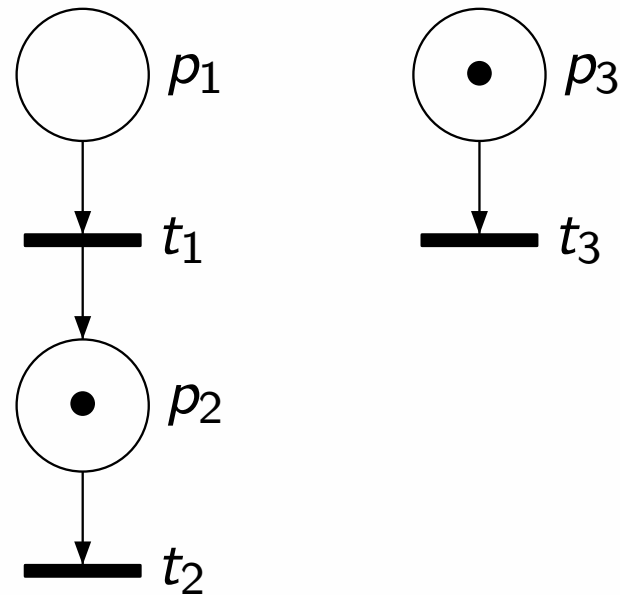
T-time Petri net (T -TPN): basis

Example (Basic: net from L. Gallon)



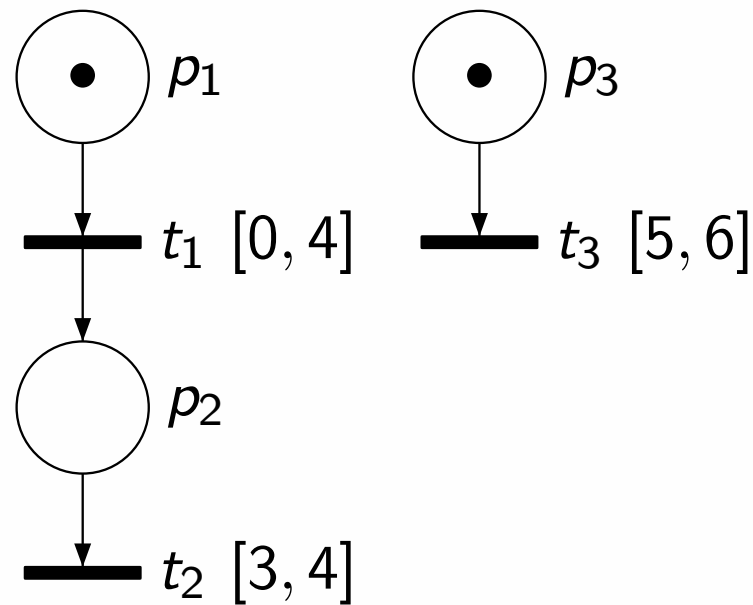
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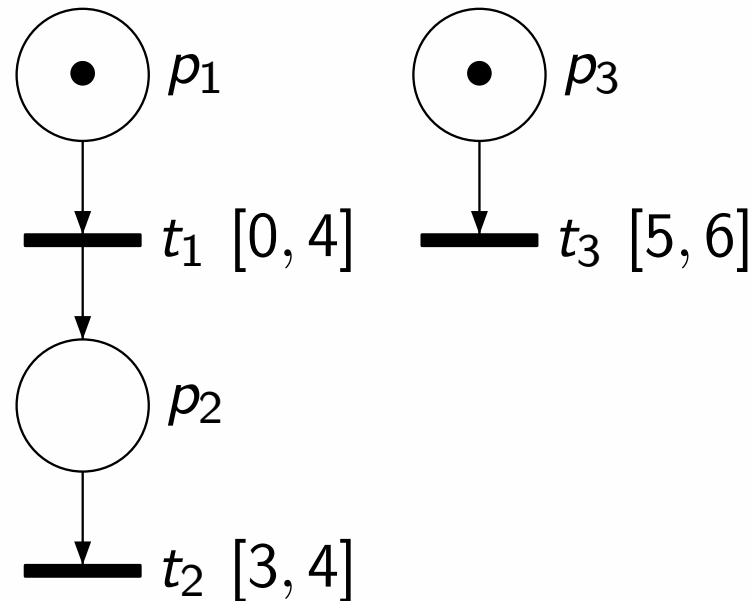
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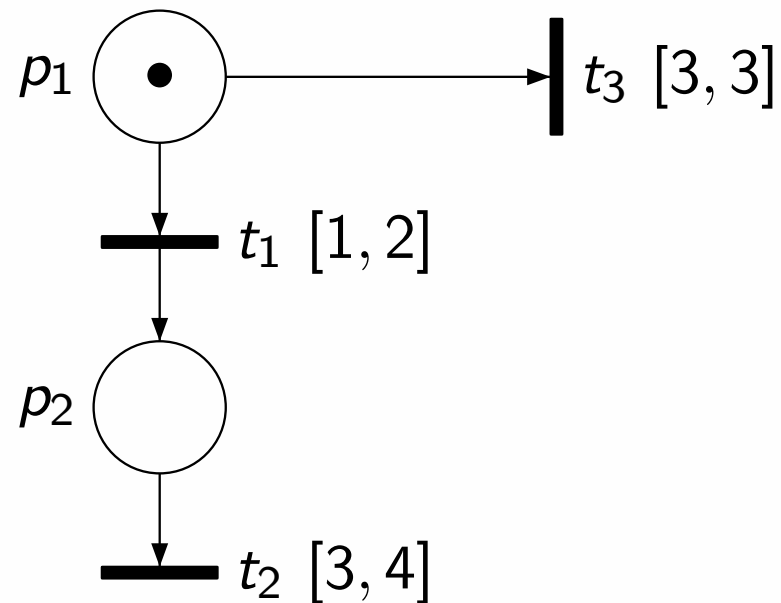
Example (Basic: net from L. Gallon)



$$\begin{array}{ccccccc}
 \{p_1, p_3\} & & \{p_1, p_3\} & & \{p_2, p_3\} & & \{p_2, p_3\} \\
 \nu(t_1) = 0 & \xrightarrow{4} & \nu(t_1) = 4 & \xrightarrow{t_1} & \nu(t_2) = 0 & \xrightarrow{1} & \nu(t_2) = 1 \xrightarrow{t_3} \dots \\
 \nu(t_3) = 0 & & \nu(t_3) = 4 & & \nu(t_3) = 4 & & \nu(t_3) = 5
 \end{array}$$

T-time Petri net ($T\text{-TPN}$): properties

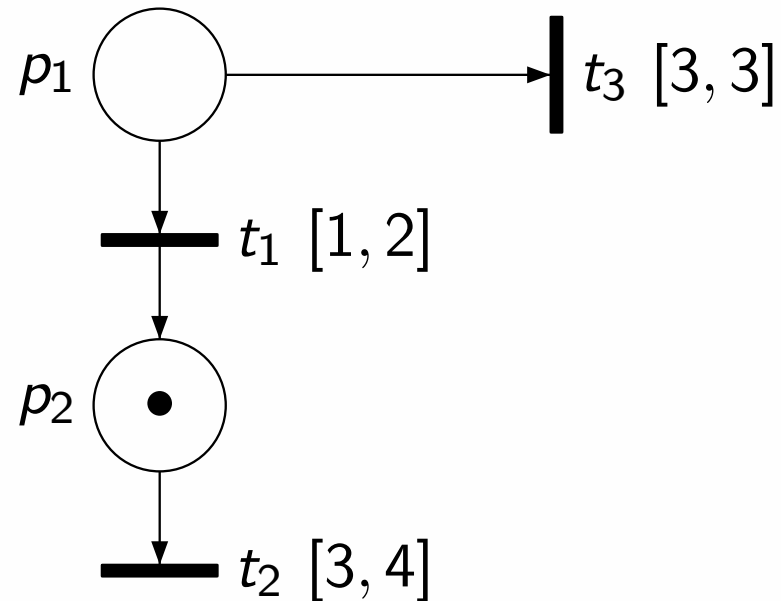
Example (Priority)



- Strong semantics ($\overline{T\text{-TPN}}$): t_3 is never fired, t_1 and t_2 always are
- Weak semantics ($\underline{T\text{-TPN}}$): all t_1 , t_2 and t_3 can be fired, or not

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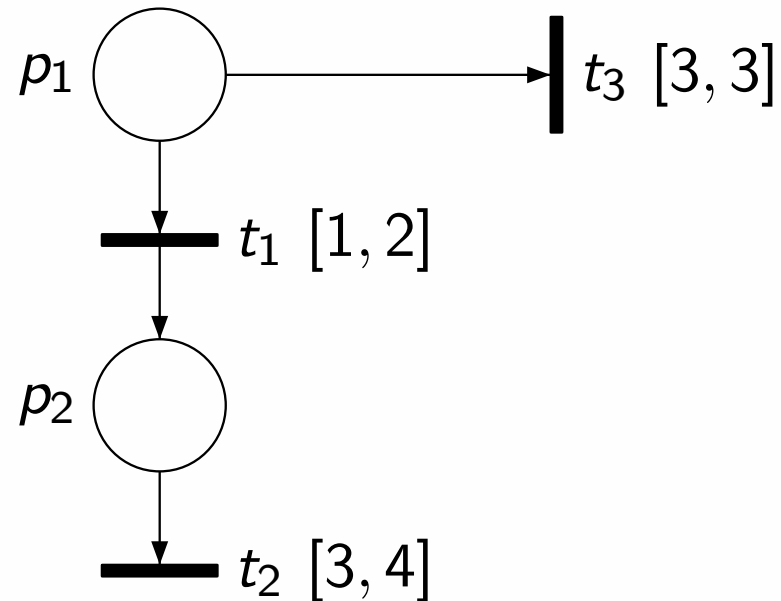
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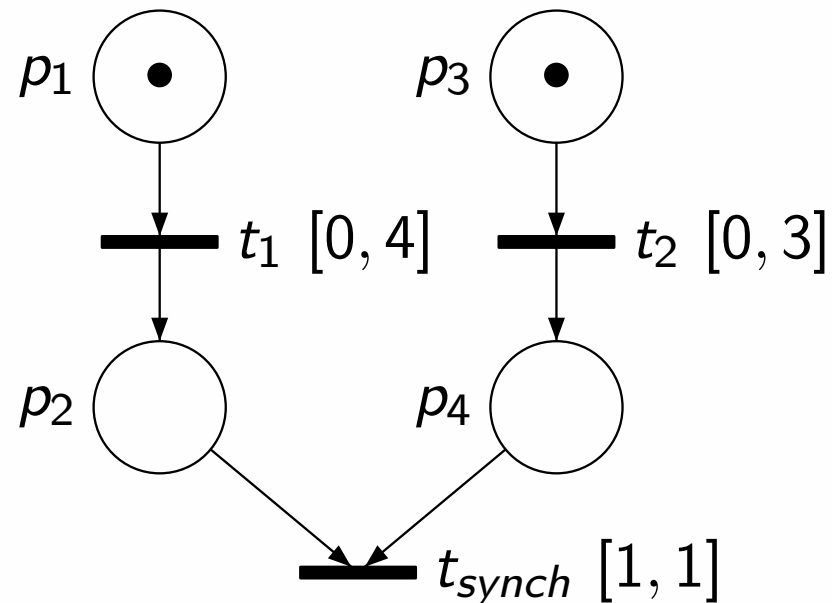
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T-time Petri net (T -TPN): properties

Example (Synchronisation)



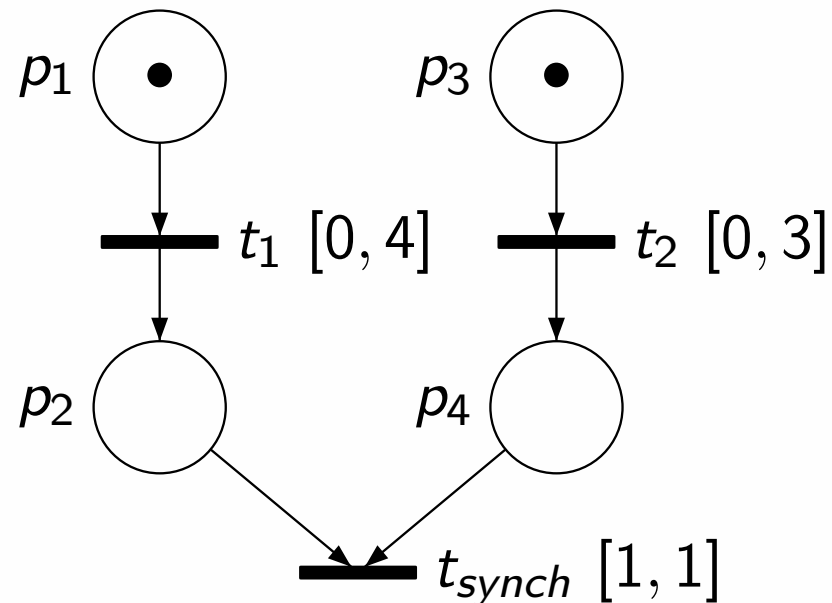
$$\{p_1, p_3\}$$

$$\nu(t_1) = 0$$

$$\nu(t_2) = 0$$

T-time Petri net (*T-TPN*): properties

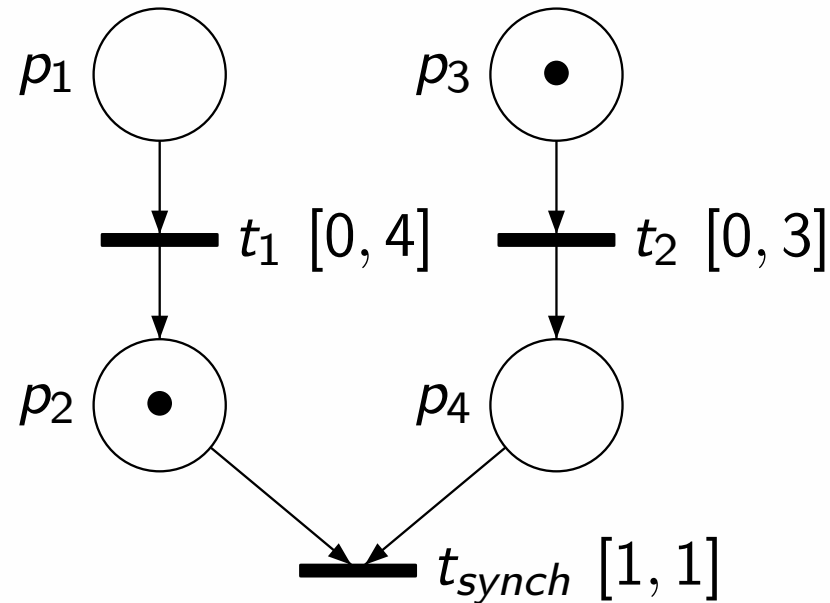
Example (Synchronisation)



$$\begin{array}{l}
 \{p_1, p_3\} \\
 \nu(t_1) = 0 \\
 \nu(t_2) = 0
 \end{array}
 \xrightarrow[\theta_1 \leq 3]{\theta_1}
 \begin{array}{l}
 \{p_1, p_3\} \\
 \nu(t_1) = \theta_1 \\
 \nu(t_2) = \theta_1
 \end{array}$$

T-time Petri net (*T-TPN*): properties

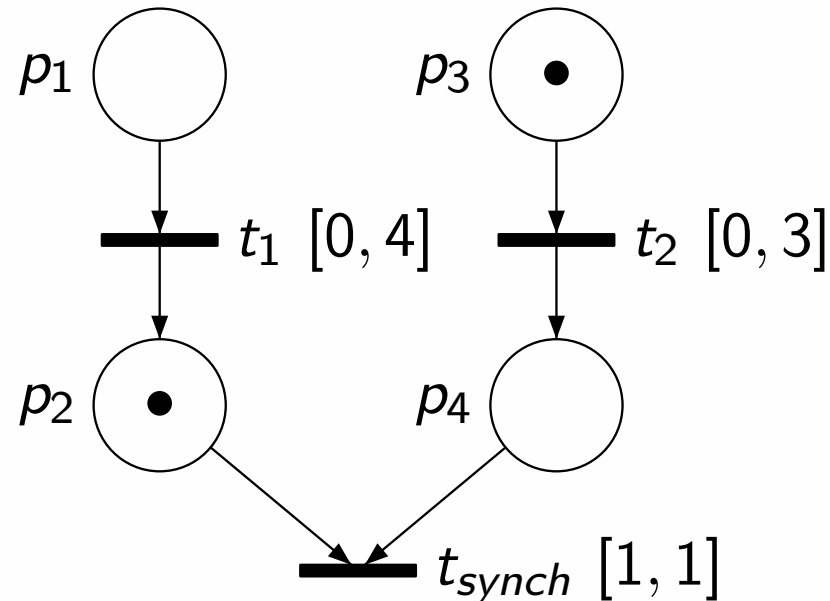
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 \nu(t_2) = \theta_1
 \end{array}
 \xrightarrow{t_1}
 \begin{array}{l}
 \{p_2, p_3\} \\
 \nu(t_2) = \theta_1
 \end{array}$$

T-time Petri net (*T-TPN*): properties

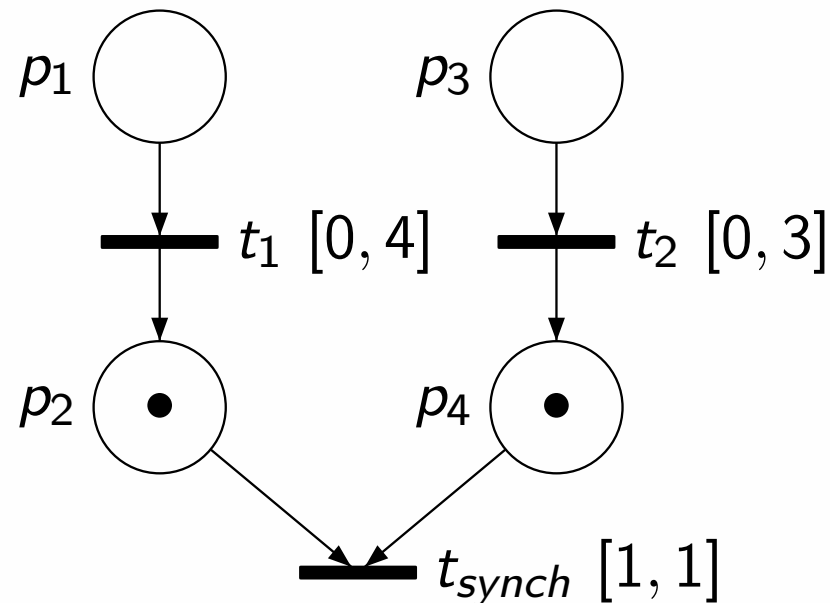
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$$\begin{array}{c}
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 \begin{array}{c}
 \{p_2, p_3\} \\
 \nu(t_2) = \theta_1
 \end{array}
 \xrightarrow[\theta_1 + \theta_2 \leq 3]{\theta_2}
 \begin{array}{c}
 \{p_2, p_3\} \\
 \nu(t_2) = \theta_1 + \theta_2
 \end{array}$$

T-time Petri net (*T-TPN*): properties

Example (Synchronisation)

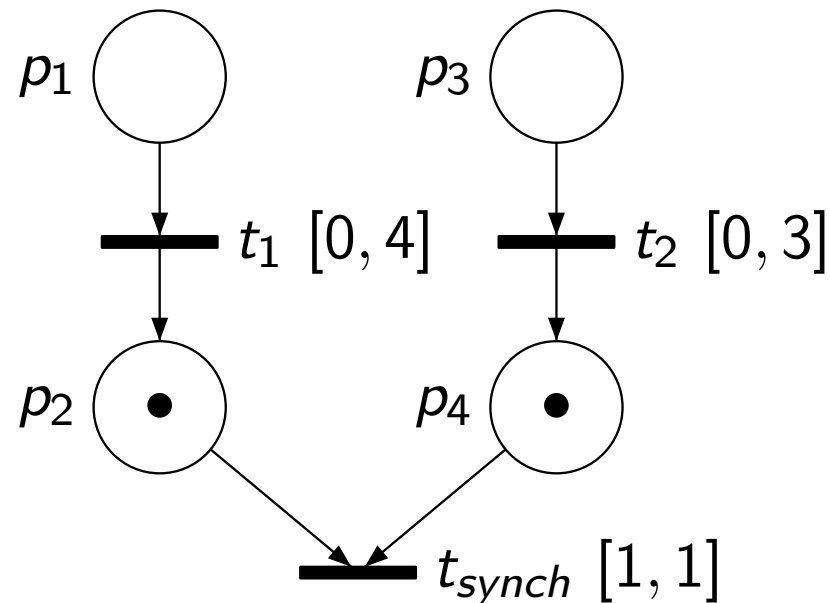


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$$\xrightarrow{t_2}
 \begin{array}{l}
 \{p_2, p_4\} \\
 \nu(t_{synch}) = 0
 \end{array}$$

T-time Petri net (*T-TPN*): properties

Example (Synchronisation)

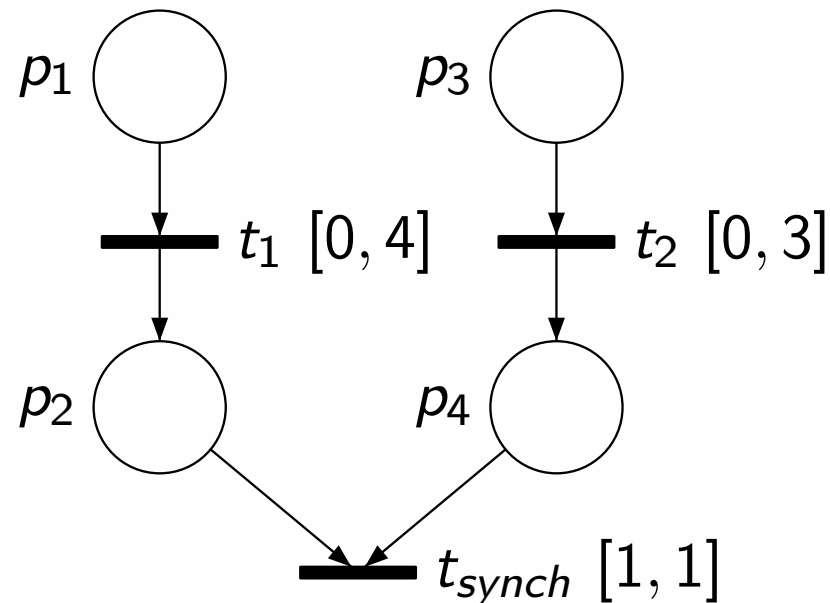


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 \nu(t_2) = \theta_1 + \theta_2
 \end{array}$$

$$\xrightarrow{t_2}
 \begin{array}{l}
 \{p_2, p_4\} \\
 \nu(t_{synch}) = 0
 \end{array}
 \xrightarrow{1}
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 \{p_2, p_4\} \\
 \nu(t_{synch}) = 1
 \end{array}$$

T-time Petri net (*T-TPN*): properties

Example (Synchronisation)

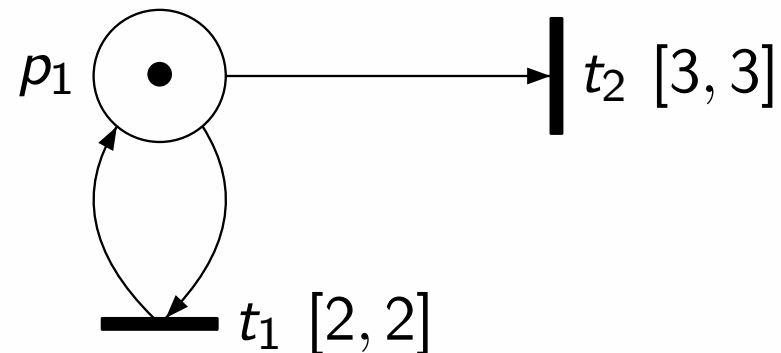


$$\begin{array}{l}
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 \end{array}
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 \begin{array}{l}
 \{p_2, p_3\} \\
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 \end{array}
 \xrightarrow[\theta_1 + \theta_2 \leq 3]{\theta_2}
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 \end{array}$$

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 \{p_2, p_4\} \\
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 \end{array}
 \xrightarrow{1}
 \begin{array}{l}
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 \nu(t_{synch}) = 1
 \end{array}
 \xrightarrow{t_{synch}} \emptyset$$

T-time Petri net (T -TPN): properties

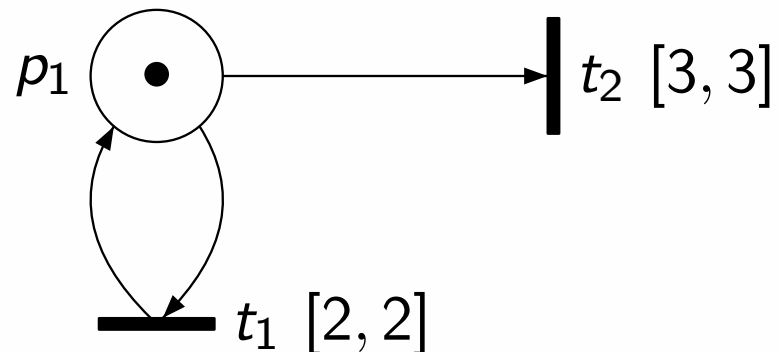
Example (Continuous enabling)



Transition t_2 can never be fired, in strong semantics.

T-time Petri net (*T-TPN*): properties

Example (Continuous enabling)



Transition t_2 can never be fired, in strong semantics.

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 \{p_1\} & & \{p_1\} & & \{p_1\} & & \{p_1\} \\
 \nu(t_1) = 0 & \xrightarrow{2} & \nu(t_1) = 2 & \xrightarrow{t_1} & \nu(t_1) = 0 & \xrightarrow{2} & \nu(t_1) = 2 \xrightarrow{t_1} \dots \\
 \nu(t_2) = 0 & & \nu(t_2) = 2 & & \nu(t_2) = 0 & & \nu(t_2) = 2
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▶ **Model definitions**

- T-time Petri net
- **Strong vs Weak Semantics**
- P-time Petri net
- A-time Petri net

▶ Comparing model expressiveness

- Introduction, previous works
- Results from [BCH⁺05]
- Weak with strong for P-TPN and A-TPN
- Something that P-TPN can't do, and T-TPN does
- Emulating T-TPN rule with A-TPN

▶ Conclusion

Strong vs Weak Semantics

Strong and weak semantics description

- *Weak* semantics: a transition is never forced to be fired, no urgency
- *Strong* semantics: when the deadline is reached, the transition *must* be fired; time can not disable firing of a transition

Strong semantics definition

$$t \notin \text{firable}(S + d) \Rightarrow \forall d' \in [0, d] : t \notin \text{firable}(S + d')$$

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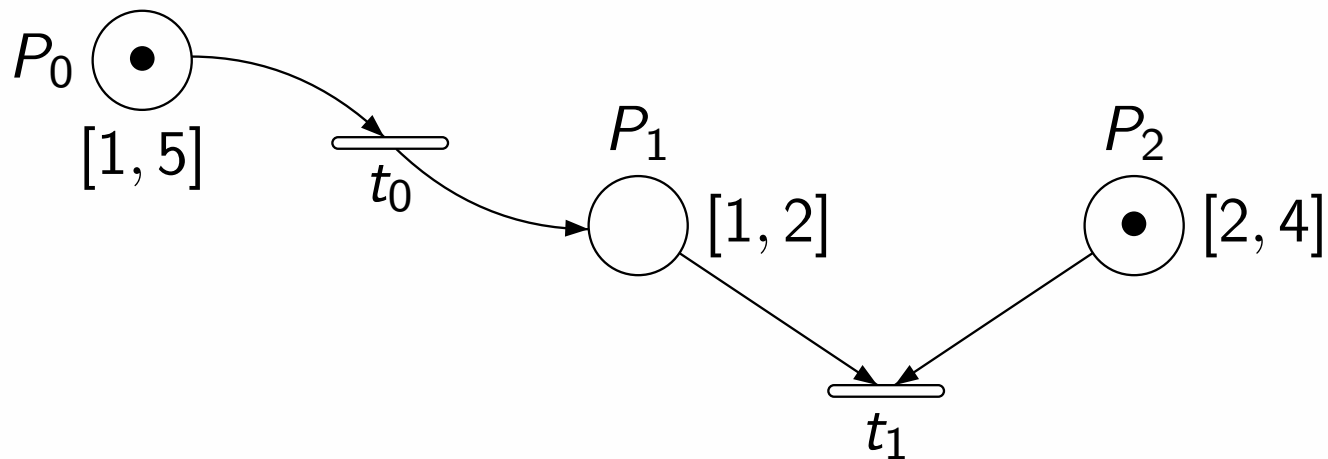
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P-time Petri net

P-TPN: time interval are associated to places

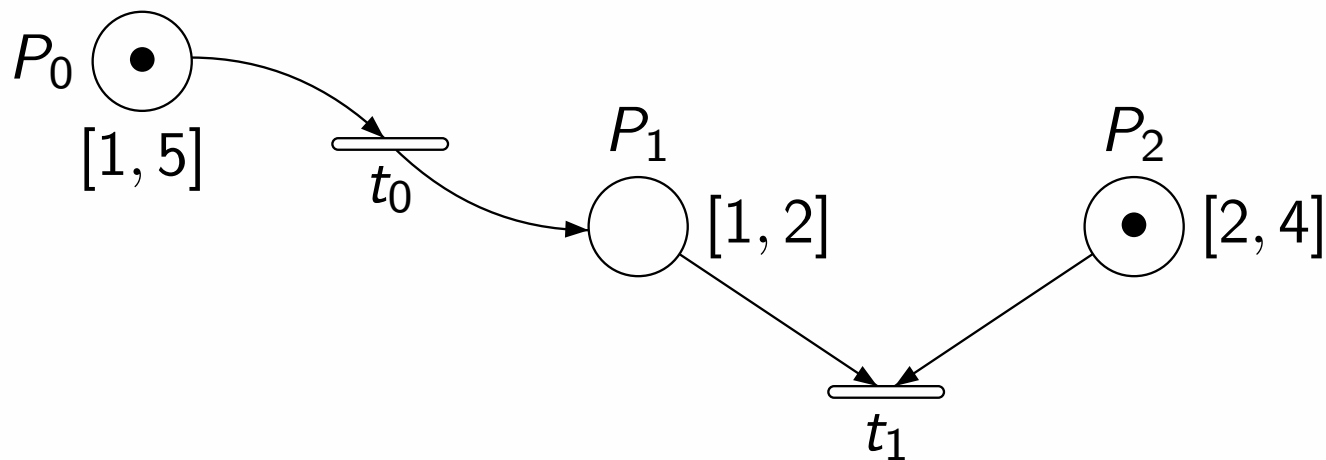
Example (A TPN $\in P\text{-TPN}$)



P-time Petri net

P-TPN: time interval are associated to places

Example (A TPN \in *P*-TPN)

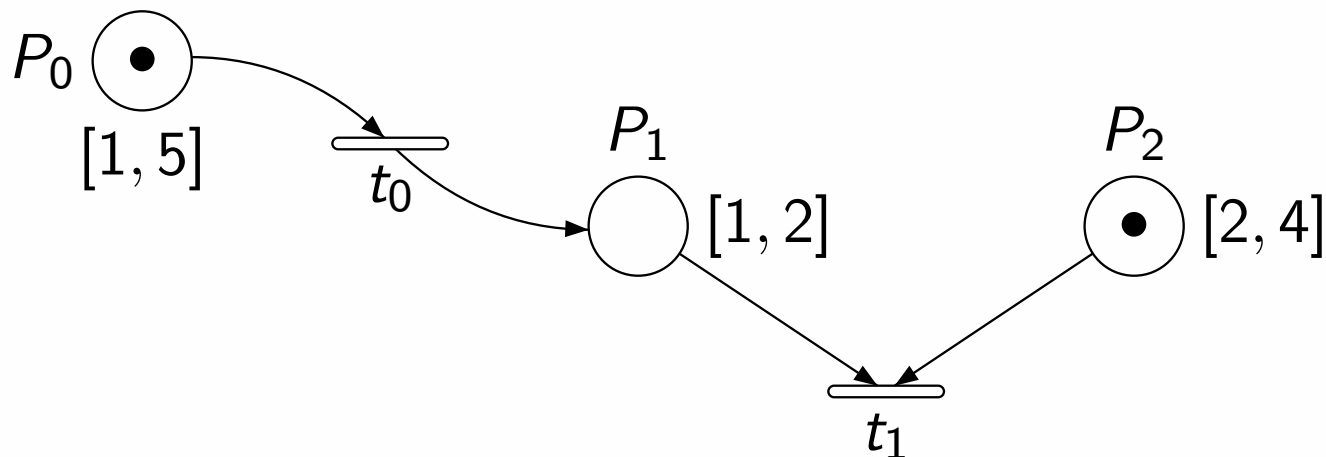


$$\begin{array}{ccccccc}
 \{p_0, p_2\} & & \{p_0, p_2\} & & \{p_1, p_2\} & & \{p_1, p_2\} \\
 \nu(p_0) = 0 & \xrightarrow{3} & \nu(p_0) = 3 & \xrightarrow{t_0} & \nu(p_1) = 0 & \xrightarrow{1} & \nu(p_1) = 1 \\
 \nu(p_2) = 0 & & \nu(p_2) = 3 & & \nu(p_2) = 3 & & \nu(p_2) = 4 \\
 & & & & & & \xrightarrow{t_1}
 \end{array}$$

P-time Petri net

P-TPN: time interval are associated to places

Example (A TPN $\in P\text{-TPN}$)



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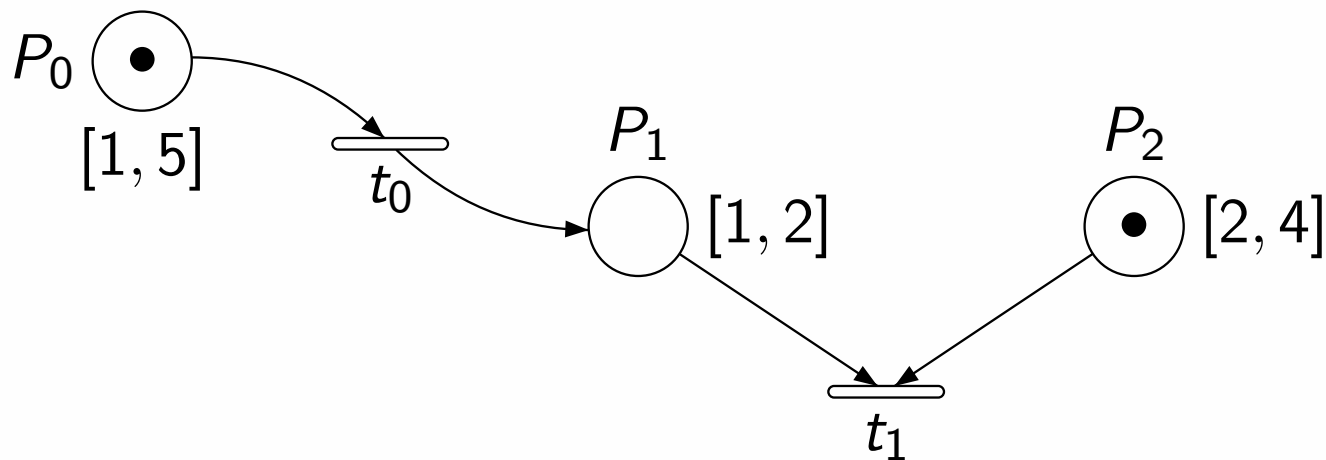
Weak semantics :

$$\begin{array}{ccccccc}
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 \nu(p_2) = 0 & & \nu(p_2) = 3 & & \nu(p_2) = 3 & & \nu(p_2) = 5 &
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P-time Petri net

P-TPN: time interval are associated to places

Example (A TPN $\in P\text{-TPN}$)



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 \nu(p_2) = 0 & & \nu(p_2) = 3 & & \nu(p_2) = 3 & & \nu(p_2) = 4 \\
 & & & & & & \xrightarrow{t_1}
 \end{array}$$

Strong semantics :

$$\begin{array}{ccccccc}
 \{p_0, p_2\} & & \{p_0, \hat{p}_2\} & & \{p_1, \hat{p}_2\} & & \\
 \nu(p_0) = 0 & \xrightarrow{5} & \nu(p_0) = 5 & \xrightarrow{t_0} & \nu(p_1) = 0 & \xrightarrow{\dots} & \\
 \nu(p_2) = 0 & & \nu(p_2) = 5 & & \nu(p_2) = 5 & &
 \end{array}$$

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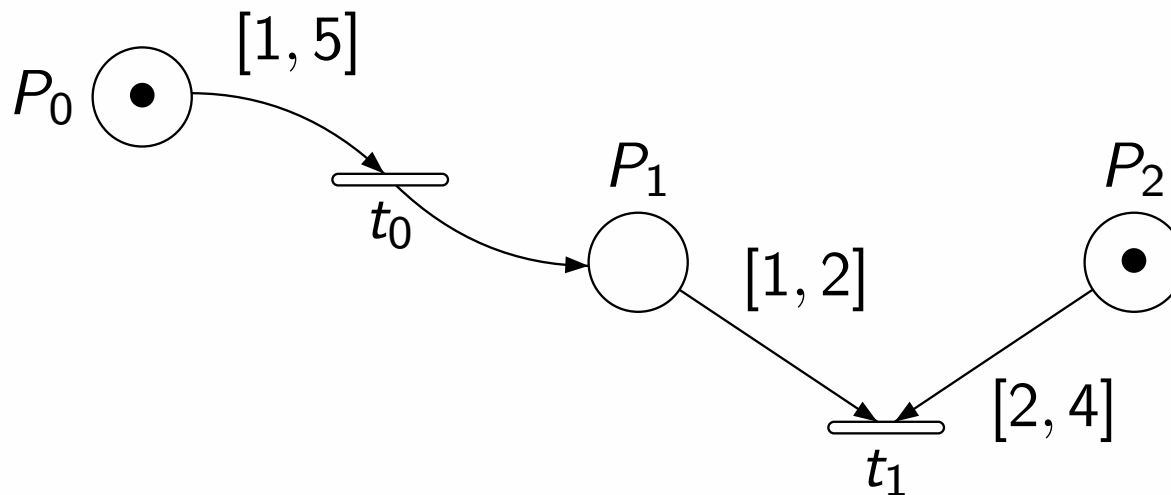
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A-time Petri net (*A-TPN*)

P-TPN: time interval are associated to arcs (p,t)

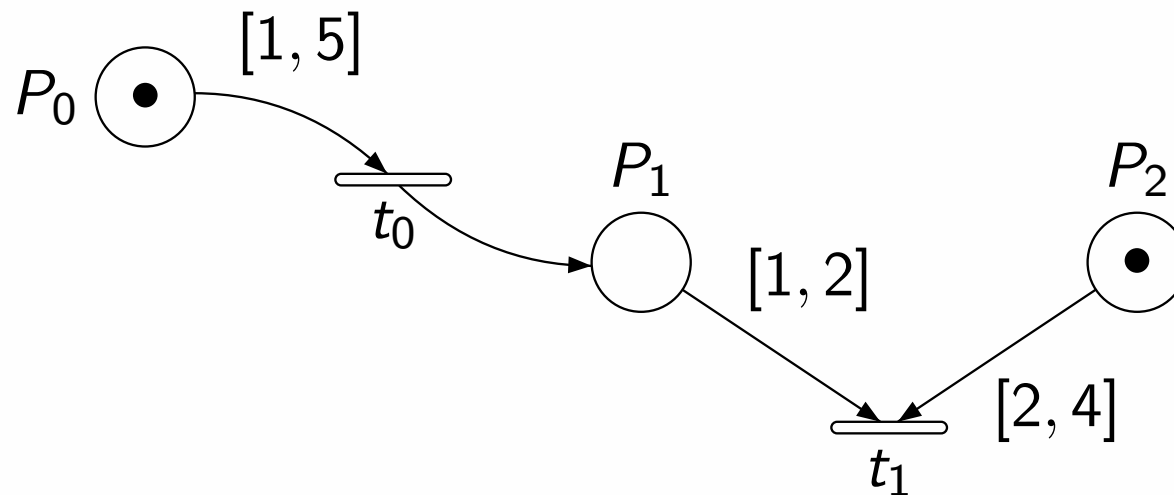
Example (A TPN \in *A-TPN* without choice)



A-time Petri net (*A-TPN*)

P-TPN: time interval are associated to arcs (p,t)

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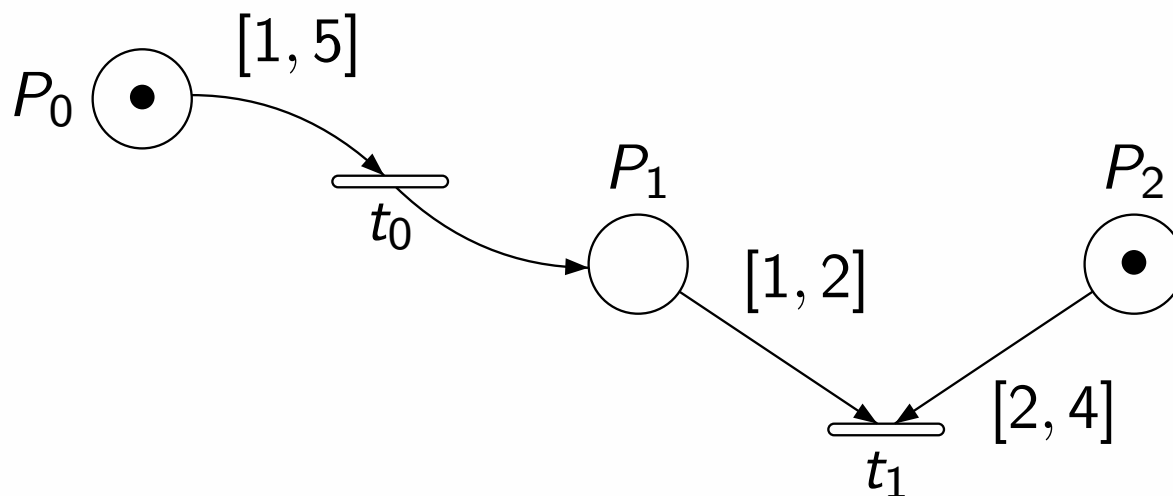


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 & & & & & & \xrightarrow{t_1}
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A-time Petri net (*A-TPN*)

P-TPN: time interval are associated to arcs (p,t)

Example (A *TPN* \in *A-TPN* without choice)



$$\begin{array}{ccccccc}
 \{p_0, p_2\} & & \{p_0, p_2\} & & \{p_1, p_2\} & & \{p_1, p_2\} \\
 \nu(p_0) = 0 & \xrightarrow{3} & \nu(p_0) = 3 & \xrightarrow{t_0} & \nu(p_1) = 0 & \xrightarrow{1} & \nu(p_1) = 1 \\
 \nu(p_2) = 0 & & \nu(p_2) = 3 & & \nu(p_2) = 3 & & \nu(p_2) = 4 \\
 & & & & & & \xrightarrow{t_1}
 \end{array}$$

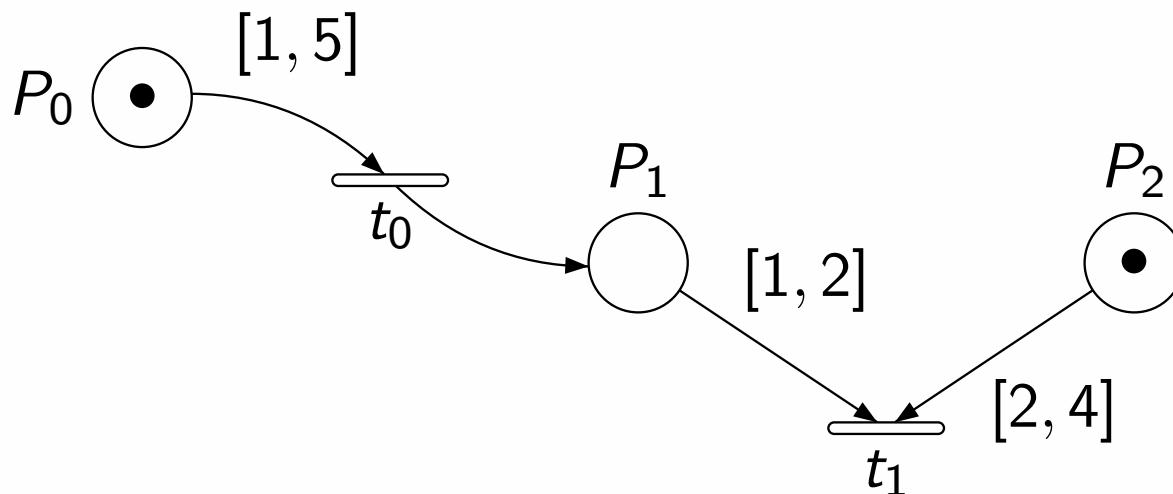
Weak semantics :

$$\begin{array}{ccccccc}
 \{p_0, p_2\} & & \{p_0, p_2\} & & \{p_1, p_2\} & & \{p_1, \hat{p}_2\} \\
 \nu(p_0) = 0 & \xrightarrow{3} & \nu(p_0) = 3 & \xrightarrow{t_0} & \nu(p_1) = 0 & \xrightarrow{2} & \nu(p_1) = 2 \\
 \nu(p_2) = 0 & & \nu(p_2) = 3 & & \nu(p_2) = 3 & & \nu(p_2) = 5 \\
 & & & & & & \xrightarrow{\dots}
 \end{array}$$

A-time Petri net (*A-TPN*)

P-TPN: time interval are associated to arcs (p,t)

Example (A $TPN \in A-TPN$ without choice)



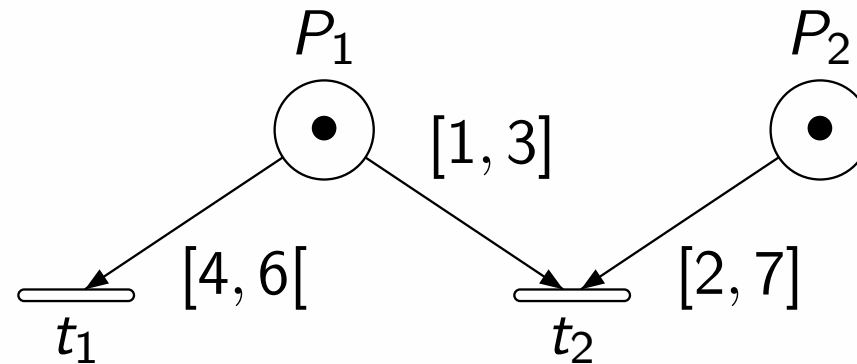
$$\begin{array}{ccccccc}
 \{p_0, p_2\} & & \{p_0, p_2\} & & \{p_1, p_2\} & & \{p_1, p_2\} \\
 \nu(p_0) = 0 & \xrightarrow{3} & \nu(p_0) = 3 & \xrightarrow{t_0} & \nu(p_1) = 0 & \xrightarrow{1} & \nu(p_1) = 1 \\
 \nu(p_2) = 0 & & \nu(p_2) = 3 & & \nu(p_2) = 3 & & \nu(p_2) = 4
 \end{array}$$

Strong semantics :

$$\begin{array}{ccccccc}
 \{p_0, p_2\} & & \{p_0, \hat{p}_2\} & & \{p_1, \hat{p}_2\} & & \\
 \nu(p_0) = 0 & \xrightarrow{5} & \nu(p_0) = 5 & \xrightarrow{t_0} & \nu(p_1) = 0 & \xrightarrow{\dots} & \\
 \nu(p_2) = 0 & & \nu(p_2) = 5 & & \nu(p_2) = 5 & &
 \end{array}$$

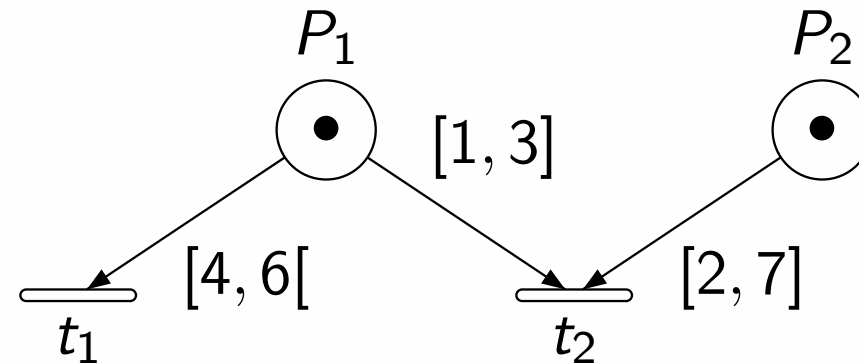
A-time Petri net (*A-TPN*)

Example (A TPN \in *A-TPN*, with choice)



A-time Petri net (*A-TPN*)

Example ($A \text{ TPN} \in A\text{-TPN}$, with choice)

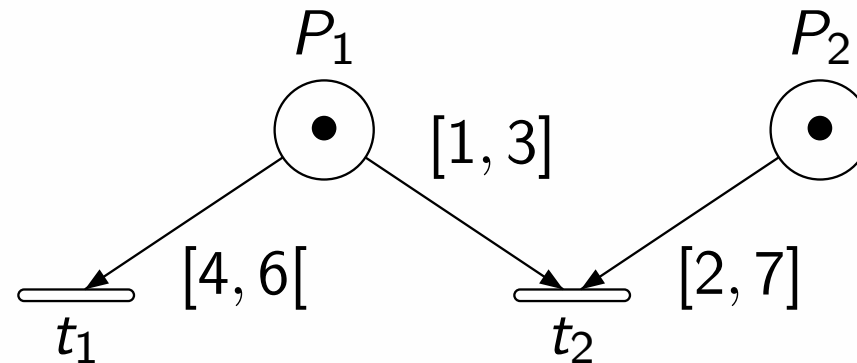


Strong semantics :

$$\begin{array}{l}
 \{p_1, p_2\} \\
 \nu(p_1) = 0 \\
 \nu(p_2) = 0
 \end{array}
 \xrightarrow{3}
 \begin{array}{l}
 \{p_1, p_2\} \\
 \nu(p_1) = 3 \\
 \nu(p_2) = 3
 \end{array}
 \xrightarrow{t_2}
 \begin{array}{l}
 \{\} \\
 . \\
 .
 \end{array}
 \xrightarrow{\dots}$$

A-time Petri net (*A-TPN*)

Example ($A \text{ TPN} \in A\text{-TPN}$, with choice)



Strong semantics :

$$\begin{array}{l}
 \{p_1, p_2\} \\
 \nu(p_1) = 0 \\
 \nu(p_2) = 0
 \end{array}
 \xrightarrow{3}
 \begin{array}{l}
 \{p_1, p_2\} \\
 \nu(p_1) = 3 \\
 \nu(p_2) = 3
 \end{array}
 \xrightarrow{t_2}
 \begin{array}{l}
 \{\} \\
 . \\
 .
 \end{array}
 \xrightarrow{\dots}$$

Weak semantics :

$$\begin{array}{l}
 \{p_1, p_2\} \\
 \nu(p_1) = 0 \\
 \nu(p_2) = 0
 \end{array}
 \xrightarrow{5}
 \begin{array}{l}
 \{p_1, p_2\} \\
 \nu(p_1) = 5 \\
 \nu(p_2) = 5
 \end{array}
 \xrightarrow{t_1}
 \begin{array}{l}
 \{p_2\} \\
 \nu(p_2) = 5 \\
 .
 \end{array}
 \xrightarrow{3}
 \begin{array}{l}
 \{\hat{p}_2\} \\
 \nu(p_2) = 5 \\
 .
 \end{array}
 \xrightarrow{\dots}$$

Outline

- ▶ Motivation
- ▶ Model definitions
- ▶ **Comparing model expressiveness**
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Previous works

w.r.t. language acceptance

From [CA99] :

$$\overline{P\text{-TPN}} =_{\mathcal{L}} \overline{T\text{-TPN}} =_{\mathcal{L}} \overline{A\text{-TPN}}$$

$$\underline{P\text{-TPN}} =_{\mathcal{L}} \underline{T\text{-TPN}} =_{\mathcal{L}} \underline{A\text{-TPN}}$$

From [BV00] :

$$\overline{T\text{-TPN}} \subset_{\mathcal{L}} \overline{A\text{-TPN}} \text{ and } \overline{P\text{-TPN}} \not\subset_{\mathcal{L}} \overline{T\text{-TPN}} \text{ (with } \leq \text{ constraints only).}$$

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w.r.t. bisimulation

Dans [BV00] :

$$\overline{T\text{-TPN}} \subset_{\approx} \overline{A\text{-TPN}}, \overline{P\text{-TPN}} \subseteq_{\approx} \overline{A\text{-TPN}}$$

$$\overline{P\text{-TPN}} \not\subseteq_{\approx} \overline{T\text{-TPN}} \text{ (with } \leq \text{ constraints only).}$$

In [Kha92] :

$$\overline{P\text{-TPN}} \text{ and } \overline{T\text{-TPN}} \text{ are declared incomparable (without proof).}$$

Another result [BCH⁺05]

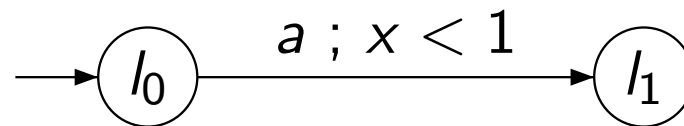


Figure: The Timed Automaton \mathcal{A}_0

Theorem ([BCH⁺05])

There is no TPN $\in \overline{T\text{-TPN}}$ weakly timed bisimilar to $\mathcal{A}_0 \in \mathcal{TA}$.

Because: time can not disable an action (transition) in strong semantics, it can in automata.

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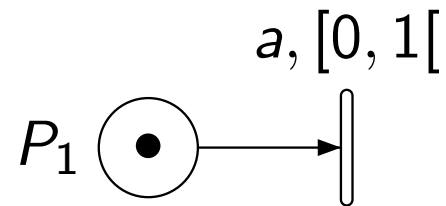
T -TPN and $\overline{T$ -TPN

Figure: The TPN $\mathcal{N}_{T_0} \in \underline{T$ -TPN bisimilar to \mathcal{A}_0

Corollary

There is no TPN $\in \overline{T$ -TPN weakly timed bisimilar to $\mathcal{N}_{T_0} \in \underline{T$ -TPN.

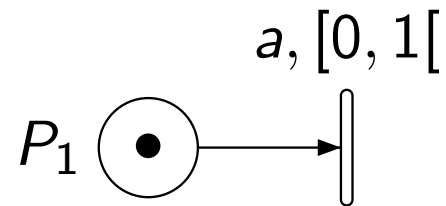
T-TPN and $\overline{T-TPN}$ 

Figure: The TPN $\mathcal{N}_{T0} \in \underline{T-TPN}$ bisimilar to \mathcal{A}_0

Corollary

There is no TPN $\in \overline{T-TPN}$ weakly timed bisimilar to $\mathcal{N}_{T0} \in \underline{T-TPN}$.

$\underline{T-TPN} \not\approx \overline{T-TPN}$

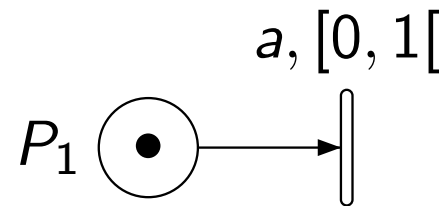
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$\underline{T-TPN} \not\approx \overline{T-TPN}$

$\underline{T-TPN}$ and $\overline{T-TPN}$ are incomparable.

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P-TPN and $\overline{P-TPN}$

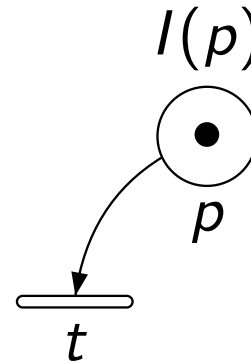


Figure: The translation from P-TPN into $\overline{P-TPN}$

P -TPN and $\overline{P$ -TPN

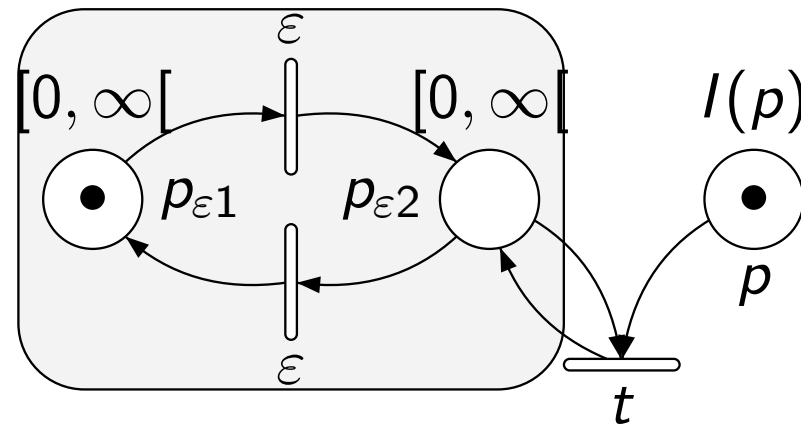


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P-TPN and $\overline{P-TPN}$

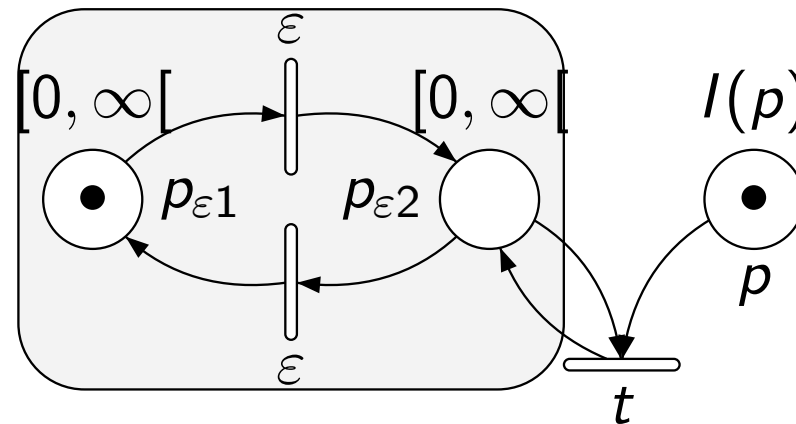


Figure: The translation from P-TPN into $\overline{P-TPN}$

$\mathcal{N} \in \underline{P-TPN}$ and its translation $\overline{\mathcal{N}} \in \overline{P-TPN}$ are weakly timely bisimilar.

P-TPN and $\overline{P-TPN}$

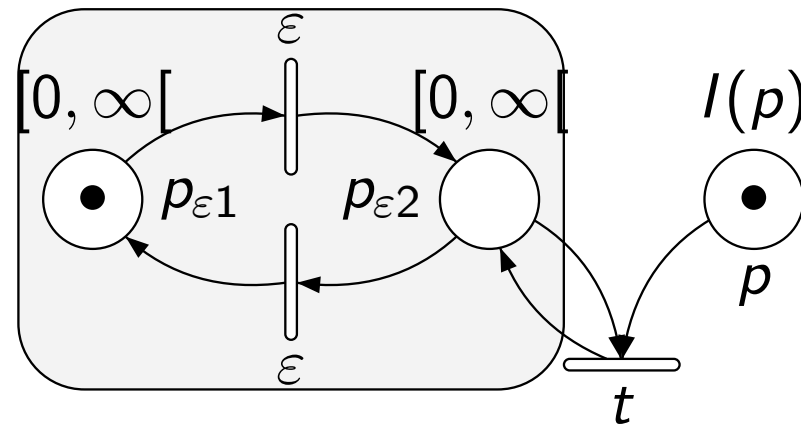


Figure: The translation from P-TPN into $\overline{P-TPN}$

$\mathcal{N} \in \underline{P-TPN}$ and its translation $\overline{\mathcal{N}} \in \overline{P-TPN}$ are weakly timely bisimilar.

$$\underline{P-TPN} \subset_{\approx} \overline{P-TPN}$$

Corollaries and similar results

$$\overline{P\text{-TPN}} \not\approx \overline{T\text{-TPN}}$$

We are able to build a strong $P\text{-TPN}$ equivalent to the weak $P\text{-TPN}$ of the theorem $\underline{P\text{-TPN}} \approx \overline{T\text{-TPN}}$.

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$$\text{The same way } \underline{A\text{-TPN}} \subseteq_{\approx} \overline{A\text{-TPN}}$$

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$$\overline{T\text{-TPN}} \not\approx \overline{P\text{-TPN}}$$

Lemma

In $P\text{-TPN}$ ($\overline{P\text{-TPN}}$ or $\underline{P\text{-TPN}}$) a clock (of a token/place p) can be reset only when the value is in the firing interval ($v(p) \in I(p)$).

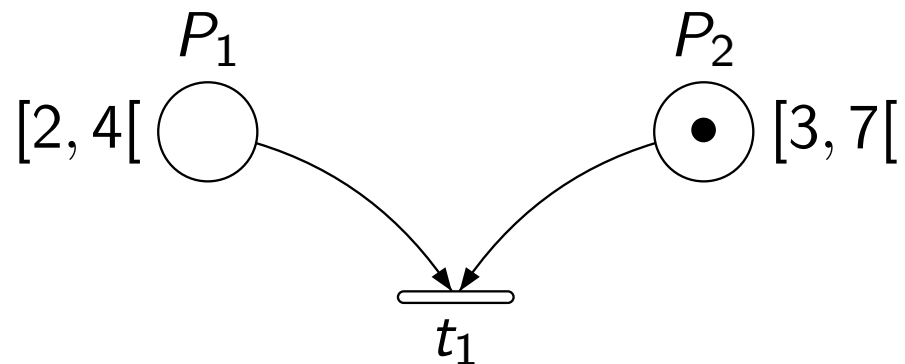


Figure: A $\text{TPN} \in P\text{-TPN}$

$$\overline{T\text{-TPN}} \not\approx \overline{P\text{-TPN}}$$

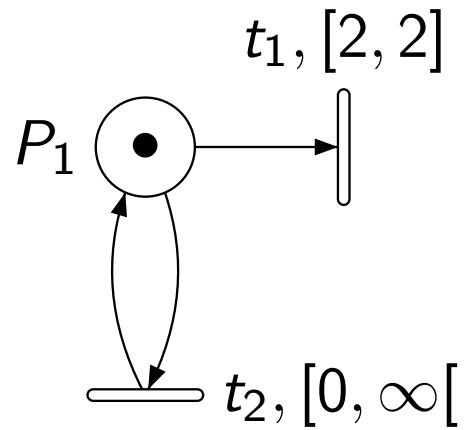


Figure: The TPN $\mathcal{N}_{T1} \in \overline{T\text{-TPN}}$

$$\overline{T\text{-TPN}} \not\approx \overline{P\text{-TPN}}$$

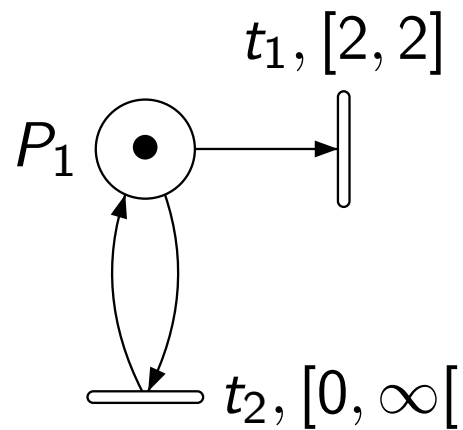


Figure: The TPN $\mathcal{N}_{T1} \in \overline{T\text{-TPN}}$

Theorem

There is no TPN $\in \overline{P\text{-TPN}}$ weakly timed bisimilar to $\mathcal{N}_{T1} \in \overline{T\text{-TPN}}$.

$$\overline{T\text{-TPN}} \not\approx \overline{P\text{-TPN}}$$

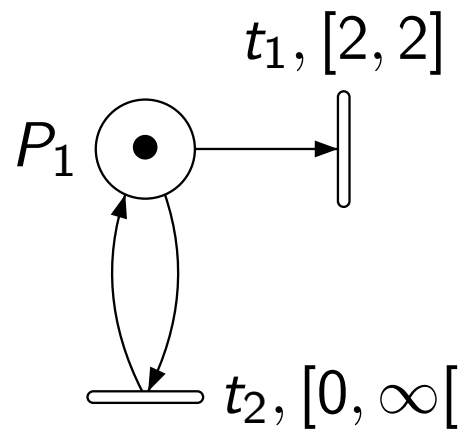


Figure: The TPN $\mathcal{N}_{T_1} \in \overline{T\text{-TPN}}$

Theorem

There is no TPN $\in \overline{P\text{-TPN}}$ weakly timed bisimilar to $\mathcal{N}_{T_1} \in \overline{T\text{-TPN}}$.

Corollary

$$\overline{T\text{-TPN}} \not\approx \overline{P\text{-TPN}}$$

Other corollaries

Corollary

$$\underline{T\text{-TPN}} \not\approx \overline{P\text{-TPN}}$$

$$\underline{T\text{-TPN}} \not\approx \underline{P\text{-TPN}}$$

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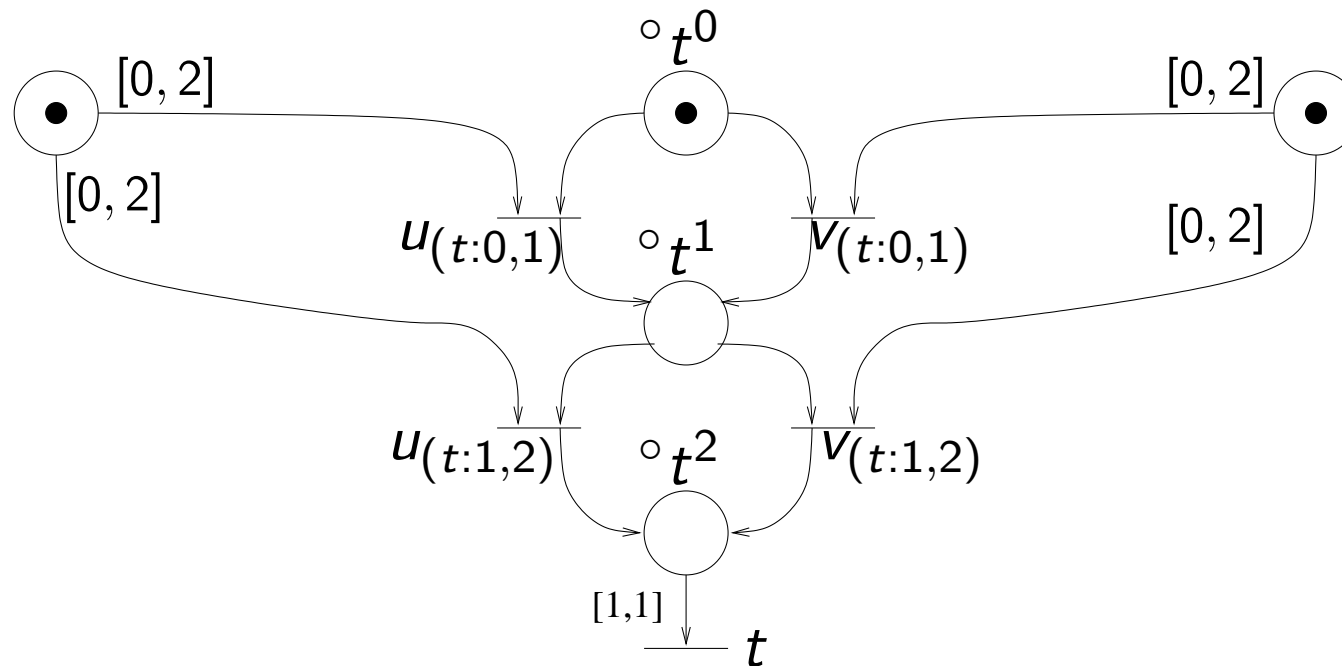
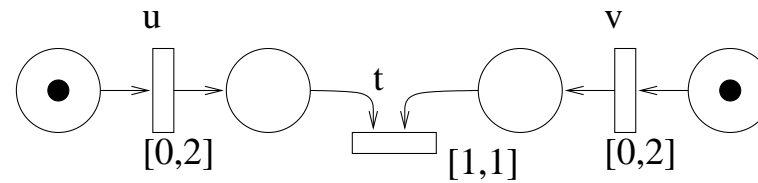
Theorem (Translation from T-TPN into A-TPN)

$$\overline{T-TPN} \subseteq_{\approx} \overline{A-TPN}$$

$$\underline{T-TPN} \subseteq_{\approx} \underline{A-TPN}$$

Pb: how to emulate the rule “start counting time when everybody is there” with the rule “everyone has its own clock”

Rep: count people, without time, start clock when everybody is there



Outline

- ▶ Motivation
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Overview [BR07]

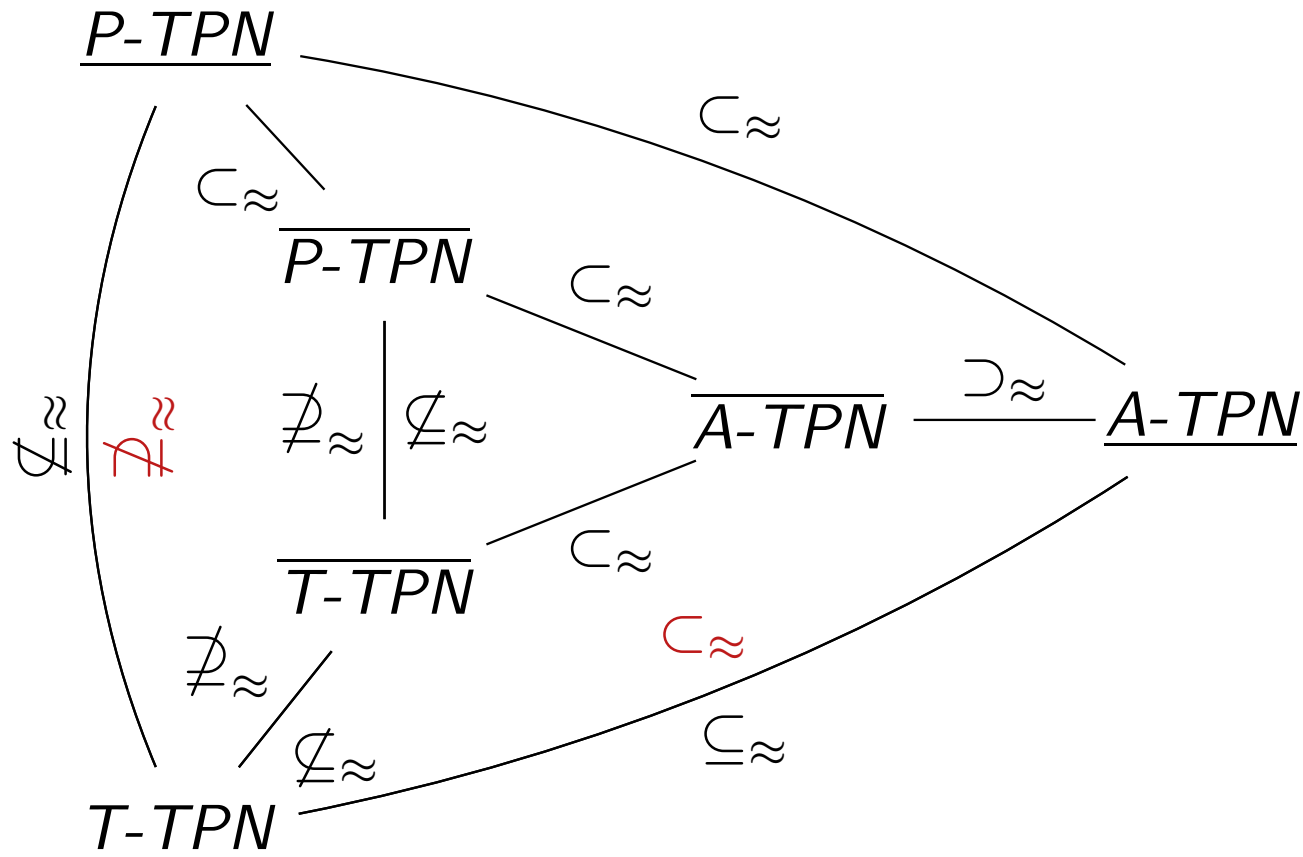


Figure: The classification explained

In a few words

Several theorems, but, what to keep in mind ?

In a few words

Several theorems, but, what to keep in mind ?

- A -TPN and P -TPN are very close from each other,
- A -TPN generalises T -TPN in theory, but not for human modelling,
- the strong semantics generalises the weak one for P -TPN and A -TPN, but not for T -TPN.



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