New Distance Measures of Evidence Based on Belief Intervals

Deqiang Han[1] and Jean Dezert[2] Yi Yang[3]

[1] Center for Information Engineering Science Research, Xi'an Jiaotong University, No. 28 West XianNing Road, Xi'an, China 710049. deqhan@gmail.com
[2] The French Aerospace Lab, Chemin de la Hunière, F-91761 Palaiseau, France. jean.dezert@onera.fr
[3] SKLSVMS, School of Aerospace, Xi'an Jiaotong University, No. 28 West XianNing Road, Xi'an, China 710049. jiafeiyy@mail.xjtu.edu.cn

Abstract. A distance or dissimilarity of evidence represents the degree of dissimilarity between bodies of evidence, which has been widely used in the applications based on belief functions theory. In this paper, new distance measures are proposed based on belief intervals $[Bel, Pl]$. For a basic belief assignment (BBA), the belief intervals of different focal elements are first calculated, respectively, which can be considered as interval numbers. Then, according to the distance of interval numbers, we can calculate the distance values between the corresponding belief intervals of the same focal elements in two given BBAs. Based on these distance values of belief intervals, new distance measures of evidence can be obtained using Euclidean and Chebyshev approaches, respectively. Some experiments and related analyses are provided to show the rationality and efficiency of the proposed measures.

Keywords: distance of evidence, dissimilarity, belief function theory, evidence theory

1 Introduction

The theory of belief functions [1], also called Dempster-Shafer evidence theory (DST), proposes a mathematical model to represent sources of evidences and to deal with uncertainty reasoning. DST has been used with some success in different civilian and military applications, especially in information fusion, pattern recognition and decision making. However, some limitations and flaws have been put in light by different researchers, see for example [2, 3], and references therein. With the development of DST, some refined or extended evidence theories have emerged, e.g., the transferable belief model (TBM) [4] and DSmT [5].

A distance or dissimilarity measure of evidence [6] can describe the degree of dissimilarity or similarity between bodies of evidence (BOEs), which has attracted more and more research interest recently and has been widely used in applications such as algorithm evaluation [7, 8] or optimization, clustering analysis, etc. Among the different measures proposed in the literature, Jousselme's distance of evidence [9] and Tessem's distance [10] (also called the betting commitment distance or the pignistic probability distance) are most frequently used. The conflict coefficient in Dempster's rule can also be considered as a generalized dissimilarity (not so strict). In our previous work [11], we have also proposed the dissimilarity of evidence based on fuzzy sets theory. Most available definitions on distance or dissimilarity measures of evidence can be found in an excellent and detailed survey [6].

In this paper, we propose new ways to define distances of evidence. For each piece of evidence, we calculate the belief interval of each focal element, respectively. Then, a basic belief assignment (BBA) is represented by a set of belief intervals, which can also be considered as a set of interval numbers or data. For two different BBAs, we calculate the distance between their corresponding focal element's belief intervals using the distance of interval numbers [12]. Based on the interval distance values corresponding to different focal elements, we propose an Euclidean-family distance based on sum of squares, and a Chebyshev-family distance based on the maximum selection, respectively. Actually, the distance between BBAs is represented by the combination or selection of the distance values between belief intervals corresponding to different focal elements. Some experiments and related analyses are provided to show the effectiveness and rationality of these new distances of evidence.

2 Basics of Belief Function Theory

In Dempster-Shafer evidence theory (DST) [1], the elements in frame of discernment (FOD) Θ are mutually exclusive and exhaustive. Define $m : 2^{\Theta} \rightarrow [0, 1]$ as a basic belief assignment (BBA, also called mass function) which satisfies:

$$\sum\nolimits_{A \subseteq \Theta} m(A) = 1, \quad m(\emptyset) = 0 \tag{1}$$

When $m(A) > 0$, A is called a focal element. The belief function and plausibility function are defined respectively as follows.

$$Bel(A) = \sum\nolimits_{B \subseteq A} m(B); \quad Pl(A) = \sum\nolimits_{A \cap B \neq \emptyset} m(B) \tag{2}$$

The belief interval $[Bel(A), Pl(A)]$ represents the imprecision or uncertainty degree of the proposition or focal element A.

Dempster's rule of combination is as follows. $\forall A \in 2^{\Theta}$:

$$m(A) = \begin{cases} 0, \quad \text{if} \quad A = \emptyset \\ \frac{\sum_{A_i \cap B_j = A} m_1(A_i) m_2(B_j)}{1 - K}, \quad \text{if} \quad A \neq \emptyset \end{cases} \tag{3}$$

where

$$K = \sum\nolimits_{A_i \cap B_j = \emptyset} m_1(A_i) m_2(B_j) \tag{4}$$

is the conflict coefficient representing the total degree of conflict between evidence sources. It is widely accepted that the combination should better not be normalized. Many alternative rules were proposed to redistribute the conflict [5].

3 Traditional Distances of Evidence

A distance or dissimilarity between BBAs can represent the degree of dissimilarity between different BOEs. As we can find in [6], there are various types of distance or dissimilarity definitions in evidence theory. Some are defined by directly using the BBAs under the framework of geometrical interpretation of evidence theory [13]. Jousselme's distance d_J is a representative one [9].

1) Jousselme's Distance

$$d_J(m_1, m_2) = \sqrt{0.5 \cdot (m_1 - m_2)^T \mathbf{Jac}\ (m_1 - m_2)} \tag{5}$$

where the elements $Jac(A, B)$ of Jaccard's weighting matrix **Jac** are defined as

$$\mathbf{Jac}(A, B) = |A \cap B| / |A \cup B| \tag{6}$$

Jousselme's distance is in fact an L_2 Euclidean distance with weighting matrix **Jac**. It has been proved to be a strict distance metric in [14]; however, it might cause some unreasonable results in some cases as shown in Exmaples 2 and 3 listed in section 5 of this paper.

Some other distances are defined using a transformation of BBAs at first, e.g., Tessem's distance and the fuzzy membership function (FMF)-based dissimilarity.

2) Tessem's Betting Commitment Distance

The pignistic probability corresponding to a BBA $m(\cdot)$ is defined by [4]

$$\mathrm{BetP}_m(A) = \sum\nolimits_{B \subseteq \Theta} \frac{|A \cap B|}{|B|} m(B) \tag{7}$$

The betting commitment distance (or Tessem's distance) d_T is computed by [10]

$$d_T(m_1, m_2) = \max_{A \subseteq \Theta} \{|\mathrm{BetP}_1(A) - \mathrm{BetP}_2(A)|\} \tag{8}$$

d_T is a Chebyshev L_∞ alike distance. It is actually not a strict distance metric [15].

3) FMF-based Dissimilarity

First transform BBAs $m_1(\cdot)$ and $m_2(\cdot)$ into FMFs: $\mu^{(1)}$ and $\mu^{(2)}$ as for $i = 1, 2$

$$\mu^{(i)} = \left[\mu^{(i)}(\theta_1), \mu^{(i)}(\theta_2), \cdots \mu^{(i)}(\theta_n)\right] = \left[Pl^{(i)}(\theta_1), Pl^{(i)}(\theta_2), \cdots, Pl^{(i)}(\theta_n)\right] \tag{9}$$

According to the dissimilarity definition between FMFs, d_F is defined as [11]

$$d_F(m_1, m_2) = 1 - \frac{\sum_{i=1}^{n} (\mu^{(1)}(\theta_i) \wedge \mu^{(2)}(\theta_i))}{\sum_{i=1}^{n} (\mu^{(1)}(\theta_i) \vee \mu^{(2)}(\theta_i))} \tag{10}$$

In (10), the operator $\wedge$ represents conjunction (min) and $\vee$ represents the disjunction (max). d_F in fact indirectly represents the dissimilarity between two BBAs using the dissimilarity between their corresponding FMFs.

Since the available definitions have some limitations, we attempt to propose new distances of evidence with desired properties, which are based on the distances between belief intervals as described in the next section.

4 Distance of Evidence Using Belief Intervals

Suppose that two BBAs $m_1(\cdot)$ and $m_2(\cdot)$ are defined on $\Theta = \{\theta_1, \theta_2, ..., \theta_n\}$. For each focal element $A_i \subseteq \Theta$ $(i = 1, ..., 2^n-1)$, we can calculate belief intervals of A_i for $m_1(\cdot)$ and $m_2(\cdot)$, respectively, which are denoted by $[Bel_1(A_i), Pl_1(A_i)]$ and $[Bel_2(A_i), Pl_2(A_i)]$. A belief interval is nothing but a classical interval number included in $[0, 1]$. The strict distance between interval numbers $[a_1, b_1]$ and $[a_2, b_2]$ $(b_i \geq a_i, i = 1, 2)$ is defined[4] by [12]

$$d^I\left([a_1, b_1], [a_2, b_2]\right) = \sqrt{\left[\frac{a_1 + b_1}{2} - \frac{a_2 + b_2}{2}\right]^2 + \frac{1}{3}\left[\frac{b_1 - a_1}{2} - \frac{b_2 - a_2}{2}\right]^2} \tag{11}$$

Therefore, we can calculate the distance between $BI_1(A_i) : [Bel_1(A_i), Pl_1(A_i)]$ and $BI_2(A_i) : [Bel_2(A_i), Pl_2(A_i)]$ according to Eq. (11). $d^I\left(BI_1(A_i), BI_2(A_i)\right)$ can be regarded as the dissimilarity between $m_1(\cdot)$ and $m_2(\cdot)$ when considering the focal element A_i. We can obtain totally $2^n - 1$ belief interval distance values for all $A_i \subseteq \Theta$. Using all the $2^n - 1$ distance values, we propose two different distances of evidence based on two commonly used distance types [6], i.e., the Euclidean family and the Chebyshev family.

1) Euclidean-family Belief Interval-based Distance d_{BI}^E

$$d_{BI}^E(m_1, m_2) = \sqrt{N_c \cdot \sum\nolimits_{i=1}^{2^n-1} \left[d^I(BI_1(A_i), BI_2(A_i))\right]^2} \tag{12}$$

Here $N_c = 1/2^{n-1}$ is the normalization factor. Eq. (12) can be re-written as

$$d_{BI}^E(m_1, m_2) = \sqrt{N_c \cdot \mathbf{d}_I \cdot \mathbf{I}^{(2^n-1)} \cdot \mathbf{d}_I^T} = \sqrt{N_c \cdot \mathbf{d}_I \cdot \mathbf{d}_I^T} \tag{13}$$

where T denotes transpose, $\mathbf{I}^{(2^n-1)}$ is an identity matrix with rank $2^n - 1$, and $\mathbf{d}_I = \left[d^I(BI_1(A_1), BI_2(A_1)), \cdots, d^I(BI_1(A_{2^n-1}), BI_2(A_{2^n-1}))\right]$. The proof for the normalization factor N_c is as follows.

Proof. Suppose that the FOD is $\{\theta_1, \theta_2, ..., \theta_n\}$. $m_1(\cdot)$ and $m_2(\cdot)$ are two BOEs. The maximum distance value is reached when

$$m_1(\{\theta_i\}) = 1, m_2(\{\theta_j\}) = 1, \forall i \neq j. \tag{14}$$

When the focal element $|A| = 1$, there are only two belief intervals with distance value d^I of 1 (i.e., $d^I(BI_1(\theta_i), BI_2(\theta_i)) = 1$ and $d^I(BI_1(\theta_j), BI_2(\theta_j)) = 1$). The other values are 0.

When the focal element $|A| > 1$, d^I values of those focal elements including θ_i or θ_j (but not both including θ_i and θ_j) are 1. The other values are 0.

To be specific,

[4] It corresponds to Mallows' distance between two distributions when we assume that each interval is the support of a uniform distribution. It should be noted that there are also other types of distance between interval numbers [12]. We use the definition in (11), because it is a strict distance metric, which is very crucial for defining distances of evidence.

when $|A| = 2$, d^I values of $2 \times C_{n-2}^1$ focal elements are 1; [5]
when $|A| = 3$, d^I values of $2 \times C_{n-2}^2$ focal elements are 1;
$\vdots$
when $|A| = n-1$, d^I values of $2 \times C_{n-2}^{n-2}$ focal elements are 1;
when $|A| = n$, the d^I value of unique focal element, i.e., total set (Θ) is 0.

So, the summation S_c of all the $(d^I)^2$ value is

$$\begin{aligned} S_c &= 2 \times 1 + 2 \times C_{n-2}^1 + 2 \times C_{n-2}^2 + ... + 2 \times C_{n-2}^{n-2} + 0 \\ &= 2 \times (C_{n-2}^0 + C_{n-2}^1 + C_{n-2}^2 + ... + C_{n-2}^{n-2}) \\ &= 2 \times 2^{n-2} \\ &= 2^{n-1} \end{aligned} \tag{15}$$

So, the normalization factor $N_c = 1/S_c = 1/2^{n-1}$ □

2) Chebyshev-family Belief Interval-based Distance d_{BI}^C

$$d_{BI}^C(m_1, m_2) = \max_{A_i \subseteq \Theta, i=1,...,2^n-1} \left\{ d^I(BI_1(A_i), BI_2(A_i)) \right\} \tag{16}$$

Actually, we use the distance of belief intervals for focal elements instead of their mass assignments to define the distances of evidence when compared with the traditional definitions. A strict distance metric defined on the set ε $d : \varepsilon \times \varepsilon \to \Re$, $(x, y) \mapsto d(x, y)$ should satisfy that [9]

1) Nonnegativity: $d(x, y) \geq 0$;
2) Nondegeneracy: $d(x, y) = 0 \Leftrightarrow x = y$;
3) Symmetry: $d(x, y) = d(y, x)$;
4) Triangle inequality: $d(x, y) + d(y, z) \geq d(x, z), \forall z \in \varepsilon$.

It can be proved that our new definitions are strict distance metric. The proof is as follows.

Proof. d_{BI}^E and d_{BI}^C are defined over belief intervals. Given a BBA $(m(A_i), i = 1, ..., 2^n - 1)$, we can generate a set of belief intervals $([Bel(A_i), Pl(A_i)])$. On the other hand, given a set of belief intervals $([Bel(A_i), Pl(A_i)])$, according to the Möbius transformation [1], we can generate a unique BBA $(m(A_i), i = 1, ..., 2^n - 1)$ from $Pl(A_i), i = 1, ..., 2^n - 1$ or $Bel(A_i), i = 1, ..., 2^n - 1$. So, there is a one-to-one mapping between a set of belief intervals $([Bel(A_i), Pl(A_i)])$ and a BBA $(m(A_i), i = 1, ..., 2^n - 1)$.

According to the Eq. (12-13, 16), it is easy to find that d_{BI}^E and d_{BI}^C satisfy nonnegativity, nondegeneracy and symmetry of are satisfied. Then we prove the property of triangle inequality of d_{BI}^E.

Suppose that there are 3 BBAs $m_1(\cdot), m_2(\cdot), m_3(\cdot)$ defined over the same FOD with size of n. Because d^I defined in Eq. (11) is a strict distance metric [12], so, for each A_i $(i = 1, ..., s, s = 2^n - 1)$ there exists

[5] Choose 1 element θ_k out of the $\Theta' = \Theta - \{\theta_i, \theta_j\}(|\Theta'| = n-2)$. Then, together with θ_i and θ_j, respectively, to constitute focal element $\{\theta_k, \theta_i\}$ and $\{\theta_k, \theta_j\}$, respectively. So, the number of focal elements with d^I values of 1 is $2 \times C_{n-2}^1$ It is same way to obtain the values in other cases for $A > 1$.

$d_{BI}^E(m_1(A_i), m_2(A_i)) + d_{BI}^E(m_2(A_i), m_3(A_i)) \geq d_{BI}^E(m_1(A_i), m_3(A_i))$.
Suppose that
$d_{BI}^E(m_1(A_i), m_2(A_i)) = a_i$; $d_{BI}^E(m_2(A_i), m_3(A_i)) = b_i$;
$d_{BI}^E(m_1(A_i), m_3(A_i)) = c_i$.
There exists

$$\begin{aligned}
&a_i + b_i \geq c_i \\
&\Rightarrow (a_i + b_i)^2 \geq c_i^2 \\
&\Rightarrow a_i^2 + b_i^2 + 2a_ib_i \geq c_i^2 \\
&\Rightarrow \sum_{i=1}^{s} a_i^2 + \sum_{i=1}^{s} b_i^2 + 2\sum_{i=1}^{s} a_ib_i \geq \sum_{i=1}^{s} c_i^2
\end{aligned} \tag{17}$$

According to the famous Cauchy-Schwarz inequality, there exists

$$\sqrt{\sum_{i=1}^{s} a_i^2 \sum_{i=1}^{s} b_i^2} \geq \sum_{i=1}^{s} a_ib_i \tag{18}$$

So,

$$\begin{aligned}
&\sum_{i=1}^{s} a_i^2 + \sum_{i=1}^{s} b_i^2 + 2\sqrt{\sum_{i=1}^{s} a_i^2 \sum_{i=1}^{s} b_i^2} \geq \sum_{i=1}^{s} a_i^2 + \sum_{i=1}^{s} b_i^2 + 2\sum_{i=1}^{s} a_ib_i \geq \sum_{i=1}^{s} c_i^2 \\
&\Rightarrow \sum_{i=1}^{s} a_i^2 + \sum_{i=1}^{s} b_i^2 + 2\sqrt{\sum_{i=1}^{s} a_i^2 \sum_{i=1}^{s} b_i^2} \geq \sum_{i=1}^{s} c_i^2
\end{aligned} \tag{19}$$

Then we have

$$\begin{aligned}
&\sum_{i=1}^{s} a_i^2 + \sum_{i=1}^{s} b_i^2 + 2\sqrt{\sum_{i=1}^{s} a_i^2 \sum_{i=1}^{s} b_i^2} \\
&= \left(\sqrt{\sum_{i=1}^{s} a_i^2} + \sqrt{\sum_{i=1}^{s} b_i^2}\right)^2 \\
&= \left(d_{BI}^E(m_1, m_2) + d_{BI}^E(m_2, m_3)\right)^2 \\
&\Rightarrow \left(d_{BI}^E(m_1, m_2) + d_{BI}^E(m_2, m_3)\right)^2 \geq \left(d_{BI}^E(m_1, m_3)\right)^2 \\
&\Rightarrow d_{BI}^E(m_1, m_2) + d_{BI}^E(m_2, m_3) \geq d_{BI}^E(m_1, m_3)
\end{aligned} \tag{20}$$

So, the triangle inequality of d_{BI}^E is satisfied.
For d_{BI}^C, we have

$$\begin{aligned}
&d_{BI}^C(m_1, m_2) + d_{BI}^C(m_2, m_3) = \max_{i=1,\ldots,s} a_i + \max_{i=1,\ldots,s} b_i \\
&d_{BI}^C(m_1, m_3) = \max_{i=1,\ldots,s} c_i = a_k + b_k, \quad k = \arg\max_{i=1,\ldots,s} c_i
\end{aligned} \tag{21}$$

There exists

$$a_k + b_k \leq \max_{i=1,\ldots,s} a_i + \max_{i=1,\ldots,s} b_i = d_{BI}^C(m_1, m_2) + d_{BI}^C(m_2, m_3) \tag{22}$$

i.e., $d_{BI}^C(m_1, m_2) + d_{BI}^C(m_2, m_3) \geq d_{BI}^C(m_1, m_3)$. d_{BI}^C satisfies triangle inequality.
In summary, d_{BI}^E and d_{BI}^C are strict distance metrics. □

5 Simulation Results

To verify the rationality of the proposed distances, numerical examples are provided. In each example, d_J, d_T, d_F, d_C[6], d_{BI}^E and d_{BI}^C are compared.

1) Example 1 The size of FOD is 3. We calculated the dissimilarities between $m_1(\cdot)$ and $m_i(\cdot)$, $i = 2, ..., 7$ as illustrated in Fig. 1. $m_1(\cdot)$ has relatively large mass assignment value for $\{\theta_2\}$. Therefore, intuitively, for $m_i(\cdot)$, $i = 2, ..., 7$ listed in Table 2, if the mass assignment for $\{\theta_2\}$ is relative large, the distance between $m_1(\cdot)$ and $m_i(\cdot)$ should be relatively small. As illustrated in Fig. 1, all the dissimilarities perform similarly in all seven cases, which show that they are all rational in this example. For m_5 and m_6, the mass of focal elements containing θ_2 (i.e., $\theta_1 \cup \theta_2$ and $\theta_2 \cup \theta_3$) is 0.8, it should be more rational if the distance values with respect to $m_5(\cdot)$ and $m_6(\cdot)$ decrease.

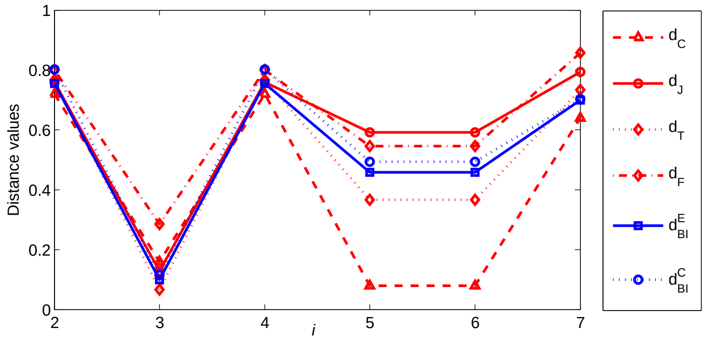

Fig. 1. Dissimilarities between m_1 and $m_i, i = 2, ..., 7$.

2) Example 2 [16]
Let us define three BBAs on the FOD $\Theta = \{\theta_1, ..., \theta_n\}$ as follows:

$$m_1(\{\theta_1\}) = m_1(\{\theta_2\}) = \cdots = m_1(\{\theta_n\}) = 1/n;$$
$$m_2(\Theta) = 1;$$
$$m_3(\{\theta_k\}) = 1, \text{ for some } k \in \{1, ..., n\}.$$

Table 1. BBA $m_1(\cdot)$

Focal element	θ_1	θ_2	θ_3	$\theta_1 \cup \theta_2$	$\theta_2 \cup \theta_3$	$\theta_1 \cup \theta_3$	$\theta_1 \cup \theta_2 \cup \theta_3$
Mass assignment	0.1	0.8	0.1	0	0	0	0

[6] d_C corresponds to the conflict coefficient K given by (4).

Table 2. BBAs $m_i(\cdot)$, $i = 2, \ldots, 7$

Focal el.\ BBAs	$m_2(\cdot)$	$m_3(\cdot)$	$m_4(\cdot)$	$m_5(\cdot)$	$m_6(\cdot)$	$m_7(\cdot)$
θ_1	0.8	0	0	0	0	0
θ_2	0	0.8	0	0	0	0
θ_3	0	0	0.8	0	0	0
$\theta_1 \cup \theta_2$	0	0	0	0.8	0	0
$\theta_2 \cup \theta_3$	0	0	0	0	0.8	0
$\theta_1 \cup \theta_3$	0	0	0	0	0	0.8
$\theta_1 \cup \theta_2 \cup \theta_3$	0.2	0.2	0.2	0.2	0.2	0.2

In this example, $m_3(\cdot)$ is absolutely confident in θ_k and it is significantly different from both $m_1(\cdot)$ and $m_2(\cdot)$. $m_1(\cdot)$ is rather different from $m_2(\cdot)$ even if they represent both two different uncertain sources. $m_2(\cdot)$ is actually a vacuous belief assignment representing the full ignorance. $m_1(\cdot)$ is much more specific than $m_2(\cdot)$ since it is a Bayesian belief assignment. As one sees in Fig 2, Jousselme's distance cannot discriminate well the difference between these two very different cases for dealing efficiently with the specificity of the information because $d_J(m_1, m_2) = d_J(m_1, m_3) = \sqrt{\frac{1}{2}(1 - \frac{1}{n})}$. For d_F, $d_F(m_1, m_2) = d_F(m_2, m_3)$. The discriminating ability is not so well. For Tessem's distance, one gets $d_T(m_1, m_2) = 0$ thus it cannot discriminate $m_1(\cdot)$ and $m_2(\cdot)$. d_C cannot discriminate $m_1(\cdot)$ and $m_2(\cdot)$, and also $m_2(\cdot)$ and $m_3(\cdot)$. For the new defined belief intervals-based distance of evidences can discriminate all the three BOE's pretty well as shown in Fig. 2.

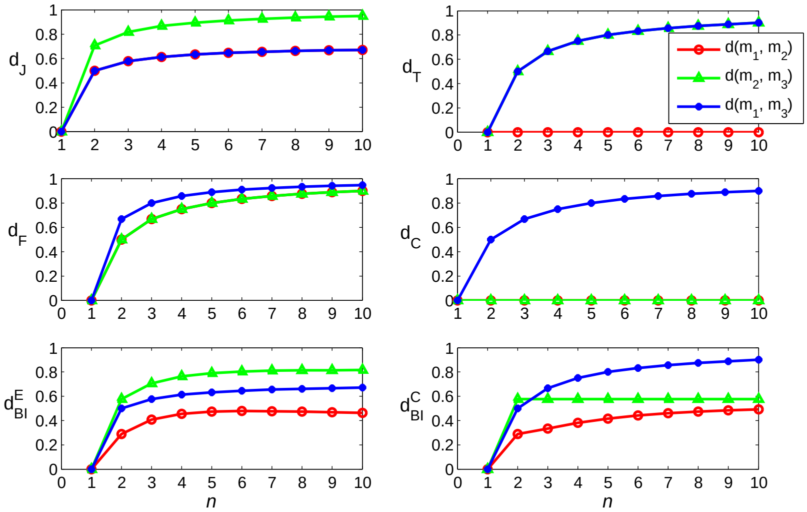

Fig. 2. Dissimilarities between $m_1(\cdot)$, $m_2(\cdot)$ and $m_3(\cdot)$ for Example 2.

3) Example 3 [16]
Let us define three BBAs on the FOD $\Theta = \{\theta_1, ..., \theta_n\}$ as follows:

$$m_1(\{\theta_1\}) = m_1(\{\theta_2\}) = m_1(\{\theta_3\}) = 1/3;$$
$$m_2(\{\theta_1\}) = m_2(\{\theta_2\}) = m_2(\{\theta_3\}) = 0.1, m_2(\Theta) = 0.7;$$
$$m_3(\{\theta_1\}) = m_3(\{\theta_2\}) = 0.1, m_2(\theta_3) = 0.8.$$

The values of the different dissimilarities between $m_1(\cdot)$ and $m_2(\cdot)$, and between $m_1(\cdot)$ and $m_3(\cdot)$ are given in Table 3.

Table 3. Example 3: Results based on different distances of evidence.

Distance types	d_J	d_T	d_F	d_C	d_{BI}^E	d_{BI}^C
$d(m_1, m_2)$	0.4041	0	0.5833	0.2000	0.2858	0.2333
$d(m_1, m_3)$	0.4041	0.4667	0.6364	0.6667	0.4041	0.4667

$m_1(\cdot)$ and $m_2(\cdot)$ correspond to two very different situations in term of the specificity of their informational content. $m_3(\cdot)$ assigns its largest mass assignment to θ_3. Intuitively, it seems reasonable to consider that $m_1(\cdot)$ and $m_2(\cdot)$ are closer than $m_1(\cdot)$ and $m_3(\cdot)$ since $m_1(\cdot)$ and $m_2(\cdot)$ yield the same indeterminate choice in decision-making because of the ambiguity in choice among the singletons in the FOD. Using Jousselme's distance, one obtains $d_J(m_1, m_2) = d_J(m_1, m_3) = 0.4041$ which is not very satisfactory for such a case. Based on the results of Table 3, one sees that when using d_T, d_F, d_{BI}^E and d_{BI}^C, one gets $d(m_1, m_2) < d(m_1, m_3)$ which is more reasonable. However, for Tessem's distance, one gets $d_T(m_1, m_2) = 0$ which is not rational (intuitively acceptable) or at least very questionable.

According to the above simple examples, we can see that the new defined belief intervals-based distances present an acceptable behavior with respect to other classical distances presented in this paper.

6 Conclusions

In this paper, two novel distances of evidence are proposed based on the distances between belief intervals. It is experimentally shown that our proposed distances can well describe the degree of dissimilarity between different BOEs. In future work, besides the simple examples in this paper, we will try to use a general formal property to show that our measures can satisfy a reasonable set of properties. Furthermore, we will use these new distances in different applications, like data clustering, target recognition, etc, to evaluate how they perform with respect to classical distances used so far.

Acknowledgement

This work was supported by the Grant for State Key Program for Basic Research of China (973) No. 2013CB329405, National Natural Science Foundation (No. 61104214, No. 61203222), Specialized Research Fund for the Doctoral Program of Higher Education (20120201120036), Shaanxi Province Science and Technology Research and Development Program (2013KJXX-46), and Fundamental Research Funds for the Central Universities.

References

1. Shafer, G.: A Mathematical Theory of Evidence. Princeton University Press, Princeton (1976)
2. Zadeh, L.A.: A Simple View of the Dempster-Shafer Theory of Evidence and Its Implication for the Rule of Combination. AI magazine. 7, 85–90 (1986)
3. Dezert J., Tchamova A.: On the Validity of Dempster's Fusion Rule and Its Interpretation as a Generalization of Bayesian Fusion Rule. Int. J. of Intelligent Systems. 29, 223–252 (2014)
4. Smets, P., Kennes, R.: The Transferable Belief Model. Artificial Intelligence. 66, 191–234 (1994)
5. Smarandache, F., Dezert J. (Editors): Applications and Advances of DSmT for Information Fusion, Vol III. American Research Press, Rehoboth (2009)
6. Jousselme, A.-L., Maupin, P.: Distances in Evidence Theory: Comprehensive Survey and Generalizations. Int. J. of Approximate Reasoning. 5, 118–145 (2012)
7. Cuzzolin, F.: Lp Consonant Approximations of Belief Functions. IEEE Trans. on Fuzzy Systems. 22, 420–436 (2014)
8. Cuzzolin, F.: Two New Bayesian Approximations of Belief Functions Based on Convex Geometry. IEEE Trans. on SMC - part B. 37, 993–1008 (2007)
9. Jousselme, A.-L., Grenier, D., Bossé, E.: A New Distance between Two Bodies of Evidence. Information Fusion. 2, 91–101 (2001)
10. Tessem, B.: Approximations for Efficient Computation in the Theory of Evidence. Artificial Intelligence. 61, 315–329 (1993)
11. Han, D.Q., Dezert, J., Han, C.Z., et al.: New Dissimilarity Measures in Evidence Theory. Proc. of the 14th Int. Conf. on Information Fusion, Chicago, USA, 483–489 (2011)
12. Irpino, A., Verde, R.: Dynamic Clustering of Interval Data Using a Wasserstein-based Distance. Pattern Recognition Letters. 29, 1648–1658 (2008)
13. Cuzzolin, F.: A Geometric Approach to the Theory of Evidence. IEEE Trans. on SMC - Part C. 38, 522–534 (2008)
14. Bouchard, M., Jousselme, A.-L., Doré, P.-E.: A Proof for the Positive Definiteness of the Jaccard Index Matrix. Int. J. of Approximate Reasoning. 54, 615–626 (2013)
15. Han, D.Q, Deng, Y., Han, C.Z., et al.: Some Notes on Betting Commitment Distance in Evidence Theory. Science China - Information Sciences. 55, 558–565 (2012)
16. Liu, Z.G., Dezert, J., Pan, Q.: A New Measure of Dissimilarity between Two Basic Belief Assignments. hal.archives-ouvertes.fr/docs/00/48/80/45/PDF/DissimilarityV8.pdf? (2010)