Tracking maneuvering and bending extended target in cluttered environment

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ABSTRACT

There has been a growing interest in tracking maneuvering point-targets since the last decade. Up to now however, no algorithm has been proposed for tracking maneuvering and bending extended target in clutter. We develop in this paper a new approach based on Interacting Multiple Model (IMM) combined with a new Fast Point Pattern Matching (FPPM) algorithm to solve this new tracking problem. Simulation results for a bending target having four dynamic models and possibly two point-patterns are given to illustrate the ability of this new approach to track a maneuvering and bending extended target in 2D space.

Keywords: Target tracking, Maneuvering target, Extended target, Bending target, Cluttered environment, Pattern Recognition, Data association, Interacting multiple model, Point pattern matching, Inexact matching, Affine transformation, Registration, Maximum matching pairs support.

1. INTRODUCTION

Most of modern tracking algorithms described in the literature[1–3] concern mainly *point-target* tracking. Usually tracking systems track only the center of mass (COM) of targets because it is enough for common applications but mostly because of the limited resolution of sensors. The dynamic models involved in such algorithms are therefore only relative to the evolution of the COM of targets expressed in a relative (platform) or an absolute frame. Since the attitude of the target is not observable using point-target modeling, it cannot be taken into account in classical filters even if it could be a very helpful information for threat assessment for the defense system. With the improvement of sensor technology and digital signal processing (development of advanced hyper-resolution algorithms for radar signals for example) the point-target assumption usually adopted becomes less and less valid. For the next generation of tracking systems, targets should be considered not as point-targets but surely as extended targets having high maneuvering and intelligent hiding abilities. We will talk about bending extended targets (BET) to point out such kind of targets. To the knowledge of the author, only few algorithms for bending extended target tracking (BETT) are available in the literature. Moreover algorithms developed are fully based on digital image processing techniques which requires imaging sensors with a high computational load. In a general framework, extended target can also be observed by not only imaging sensors. In such cases, we cannot track extended target directly by existing image processing approaches. We propose in this paper a new approach for BETT problem in cluttered environment based on classical estimation theory combined with an improvement of a recent point pattern matching (PPM) algorithm to tackle data association problem. Some tentative for including PPM within multitarget tracking algorithms have already been presented in [4,5] but they do not concern directly BETT problem. To simplify our analysis, we consider in this work only the mono-sensor case with only one maneuvering and bending extended target evolving in 2D cluttered space with a detection probability less than one.

2. PROBLEM FORMULATION

We model the evolution and the observation of the extended target we want to track by the following jump linear fixed-structure hybrid system with mode transition modeled by an homogeneous Markov chain[2,6,7] :

$$\mathbf{x}(k+1) = \mathbf{F}[k, \mathbf{m}(k+1)]\mathbf{x}(k) + \mathbf{G}[k, \mathbf{m}(k+1)]\mathbf{v}[k, \mathbf{m}(k+1)] \tag{1}$$

$$\mathbf{z}(k) = \mathbf{H}[k, \mathbf{m}(k)]\mathbf{x}(k) + \mathbf{w}[k, \mathbf{m}(k)] \tag{2}$$

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$\mathbf{x}(k)$ is the **global** base state vector of the extended target which has the following structure $\mathbf{x}(k) = [\mathbf{x}_g(k)\mathbf{x}_p(k)]'$ where $\mathbf{x}_g(k)$ represents the **principal** state of the target relative only its center of mass (COM) ($\mathbf{x}_g(k)$ is the common state used in classical descriptions for dynamic systems). $\mathbf{x}_p(k)$ is called the **proper** state of the target and its evolution describes the proper dynamic of the target around its COM. The previous global dynamic system equation (1) must be view as the stacked version of the two following dynamic equations :

$$\mathbf{x}_g(k+1) = \mathbf{F}_g[k, \mathbf{m}(k+1)]\mathbf{x}(k) + \mathbf{G}_g\left[k, \mathbf{m}(k+1)]\mathbf{v}_g[k, \mathbf{m}(k+1)]\right] \tag{3}$$

$$\mathbf{x}_p(k+1) = \mathbf{F}_p[k, \mathbf{m}(k+1)]\mathbf{x}(k) + \mathbf{G}_p\left[k, \mathbf{m}(k+1)]\mathbf{v}_p[k, \mathbf{m}(k+1)]\right] \tag{4}$$

$\mathbf{m}(k) = [m_g(k), m_p(k), m_s(k)]'$ is the discrete-valued **modal state vector** (or global system mode index) of the system at time k. $m_g(k)$ and $m_p(k)$ denote respectively the *principal* and *proper* modes in effect during the sampling period ending at time k; $m_s(k)$ denotes the **shape-mode** valid during the sampling period ending at time k. The sequence of system modes is assumed a homogeneous Markov chain with known transition probabilities such that $\forall m_g^i, m_g^j, m_p^r, m_p^s, m_s^l, m_s^m \in M_S$

$$P\{\mathbf{m}_J(k+1)|\mathbf{m}_I(k)\} \equiv P\{\mathbf{m}_{j,s,m}(k+1)|\mathbf{m}_{i,r,l}(k)\} = \pi_{IJ} \tag{5}$$

$\mathbf{m}_I \equiv \mathbf{m}_{i,r,l}(k) = [m_g^i(k), m_p^r(k), m_s^l(k)]'$ and $\mathbf{m}_J \equiv \mathbf{m}_{j,s,m}(k+1) = [m_g^j(k+1), m_p^s(k+1), m_s^m(k+1)]'$ denote particular values of the modal state vector $\mathbf{m}$ at time k and $k+1$. M_S is the set of all modal states in all times. More precisely, we will assume :

$$\begin{array}{l} P\{m_g^j(k+1)|m_g^i(k), m_p(k), m_s(k)\} = \pi_g^{ij}[m_p(k), m_s(k)] \\ P\{m_p^s(k+1)|m_p^r(k), m_g(k), m_s(k)\} = \pi_p^{rs}[m_g(k), m_s(k)] \\ P\{m_s^m(k+1)|m_s^l(k), m_g(k), m_p(k)\} = \pi_s^{lm}[m_g(k), m_p(k)] \end{array} \tag{6}$$

$\mathbf{F}[k, \mathbf{m}(k+1)] = diag[\mathbf{F}_g[k, \mathbf{m}(k+1)], \mathbf{F}_p[k, \mathbf{m}(k+1)]]$, $\mathbf{G}[k, \mathbf{m}(k+1)] = diag[\mathbf{G}_g[k, \mathbf{m}(k+1)], \mathbf{G}_p[k, \mathbf{m}(k+1)]]$ are assumed known. $\mathbf{v}[k, \mathbf{m}(k+1)] = [\mathbf{v}_g[k, \mathbf{m}(k+1)]\ \mathbf{v}_p[k, \mathbf{m}(k+1)]]'$ is the gaussian mode-dependent process noise with mean $\bar{\mathbf{v}}[k, \mathbf{m}(k+1)]$ and covariance $\mathbf{Q}[k, \mathbf{m}(k+1)]$. The measurement vector $\mathbf{z}(k)$ is the stacked vector of n_k reflection points $\mathbf{z}_i(k)$, $(i = 1, \ldots, n_k)$ of the extended target. Since the probability of detection of the target can be less than one and the shape of the target can change during tracking, the number n_k of reflection points coming from the the extended target is a discrete random variable. Therefore, measurement equation (2) is actually a **random dimensional** mode-dependent observation equation. Eq.(2) implies that the global base state observations are mode-dependent and the mode information is embedded in the measurement sequence [6]. The system mode sequence $\mathbf{m}(k)$ is then an indirectly observed (or hidden) Markov chain. $\mathbf{w}[.]$ is the stacked vector $\left[\mathbf{w}_1[.], \ldots, \mathbf{w}_{n_k}[.]\right]'$ of gaussian measurement noise $\mathbf{w}_i[k, \mathbf{m}(k)]$ with mean $\bar{\mathbf{w}}_i[k, \mathbf{m}(k)]$ and covariance $\mathbf{R}_i[k, \mathbf{m}(k)]$. It is assumed that $\mathbf{v}$, $\mathbf{w}$, uncorrelated with $\mathbf{x}(0)$, are mutually uncorrelated. $\mathbf{x}(0)$ is assumed to be gaussian with mean $\hat{\mathbf{x}}(0|0)$ and covariance $\mathbf{P}(0|0)$. $\mathbf{H}[k, \mathbf{m}(k)]$ is the known - when data association is solved (see later) - stacked matrix $[\mathbf{H}_1[k, \mathbf{m}(k)], \ldots, \mathbf{H}_{n_k}[k, \mathbf{m}(k)]]'$. All vectors and matrices are assumed to have appropriate dimensions. The problem we are faced now consists to estimate the global base state (and also the hidden modal-state) of the hybrid system with all available information (prior information and past and present observations $\mathbf{Z}^k = \{\mathbf{z}(1), \ldots, \mathbf{z}(k)\}$.

3. HYBRID STATE ESTIMATION

The Interacting Multiple Model (IMM) has been shown to be one of the most cost-effective and simple schemes for the estimation in hybrid systems[2,3,6–8] and specially for tracking maneuvering targets. Therefore it is suitable to be used for tracking maneuvering and bending extended targets as well. We recall that IMM is a recursive, modular algorithm and has fixed computational requirements per cycle. One typical IMM cycle for BETT application will consist of the following steps :

1. **Interaction**
   Calculation of the mixing probabilities

$$\begin{array}{ll} \mu_J^-(k) & = P\{\mathbf{m}_J(k)|\mathbf{Z}^{k-1}\} = \sum_I \mu_I(k-1)\pi_{IJ} \\ \mu_{I|J}(k-1|k-1) & = P\{\mathbf{m}_I(k-1)|\mathbf{m}_J(k), \mathbf{Z}^{k-1}\} = \mu_I(k-1)\pi_{IJ}/\mu_J^-(k) \end{array} \tag{7}$$

Mode-conditional filtering initialization (mixing)

$$\begin{aligned}\hat{\mathbf{x}}_J^0(k-1|k-1) &= \sum_I \mu_{I|J}(k-1|k-1)\hat{\mathbf{x}}_I(k-1|k-1)\\ \mathbf{P}_J^0(k-1|k-1) &= \sum_I \mu_{I|J}(k-1|k-1)\big[\mathbf{P}_I(k-1|k-1)+\\ &\quad [\hat{\mathbf{x}}_I(k-1|k-1)-\hat{\mathbf{x}}_J^0(k-1|k-1)][\hat{\mathbf{x}}_I(k-1|k-1)-\hat{\mathbf{x}}_J^0(k-1|k-1)]'\big]\end{aligned} \tag{8}$$

2. **Mode-conditional measurement prediction**

$$\begin{aligned}\hat{\mathbf{x}}_J(k|k-1) &= \mathbf{F}_J(k-1)\hat{\mathbf{x}}_J^0(k-1|k-1)+\mathbf{G}_J(k-1)\bar{\mathbf{v}}_J(k-1)\\ \mathbf{P}_J(k|k-1) &= \mathbf{F}_J(k-1)\mathbf{P}_J^0(k-1|k-1)\mathbf{F}_J(k-1)'+\mathbf{G}_J(k-1)\mathbf{Q}_J(k-1)\mathbf{G}_J(k-1)'\\ \hat{\mathbf{z}}_J^i(k|k-1) &= \mathbf{h}_J^i[(k,\hat{\mathbf{x}}_J(k|k-1),\mathbf{x}_J^i]+\bar{\mathbf{w}}_J(k) \qquad i=1,\ldots,\hat{n}_J(k|k-1)\end{aligned} \tag{9}$$

3. **Measurement validation and data association via PPM**
For every possible shape-mode m_s of the extended target :
   - Validate measurements using a statistical gate[1,9] which takes into account (mode-dependent) target size.
   - Solve the Point Pattern Matching (PPM) problem using Fast PPM algorithm to get the (mode-dependent) matched measurement vector $\mathbf{z}_J(k)$.

4. **Mode-matched filtering**

$$\begin{aligned}\hat{\mathbf{x}}_J(k|k-1) &= \mathbf{F}_J(k-1)\hat{\mathbf{x}}_J^0(k-1|k-1)+\mathbf{G}_J(k-1)\bar{\mathbf{v}}_J(k-1)\\ \mathbf{P}_J(k|k-1) &= \mathbf{F}_J(k-1)\mathbf{P}_J^0(k-1|k-1)\mathbf{F}_J(k-1)'+\mathbf{G}_J(k-1)\mathbf{Q}_J(k-1)\mathbf{G}_J(k-1)'\\ \hat{\mathbf{z}}_J(k|k-1) &= \mathbf{H}_J(k)\hat{\mathbf{x}}_J(k|k-1)+\bar{\mathbf{w}}_J(k)\\ \tilde{\mathbf{z}}_J(k) &= \mathbf{z}_J(k)-\hat{\mathbf{z}}_J(k|k-1)\\ \mathbf{S}_J(k) &= \mathbf{H}_J(k)\mathbf{P}_J(k|k-1)\mathbf{H}_J(k)'+\mathbf{R}_J(k)\\ \mathbf{K}_J(k) &= \mathbf{P}_J(k|k-1)\mathbf{H}_J(k)'\mathbf{S}_J^{-1}(k)\\ \hat{\mathbf{x}}_J(k|k) &= \hat{\mathbf{x}}_J(k|k-1)+\mathbf{K}_J(k)\tilde{\mathbf{z}}_J(k)\\ \mathbf{P}_J(k|k) &= \mathbf{P}_J(k|k-1)-\mathbf{K}_J(k)\mathbf{S}_J(k)\mathbf{K}_J(k)'\end{aligned} \tag{10}$$

5. **Mode probability update**

$$\begin{aligned}\Lambda_J(k) &= \mathcal{N}\big[\tilde{\mathbf{z}}_J(k);0,\mathbf{S}_J(k)\big]\\ \mu_J(k) &= \Lambda_J(k)\mu_J^-(k)/\sum_j \Lambda_j(k)\mu_j^-(k)\end{aligned} \tag{11}$$

6. **Estimate and covariance combination**

$$\begin{aligned}\hat{\mathbf{x}}(k|k) &= \sum_J \mu_J(k)\hat{\mathbf{x}}_J(k|k)\\ \hat{\mathbf{P}}(k|k) &= \sum_J \mu_J(k)\big[\mathbf{P}_J(k|k)+[\hat{\mathbf{x}}(k|k)-\hat{\mathbf{x}}_J(k|k)][\hat{\mathbf{x}}(k|k)-\hat{\mathbf{x}}_J(k|k)]'\big]\end{aligned} \tag{12}$$

$\mu_I(k-1)$ is the mode probability at time $k-1$ ($\mu_I, I=1,\ldots,N$ is usually chosen equi-distributed for initial time $k=0$ if we have no prior information on initial mode); $\mu_{I|J}(k-1|k-1)$ is the mixing probability at time $k-1$ (the weights with which the estimates from the previous cycle are given to each filter at the beginning of the current cycle); $\Lambda_J(k)$ is the likelihood function of the mode-matched filter J. $\hat{\mathbf{x}}_J^0(k-1|k-1)$ and $\mathbf{P}_J^0(k-1|k-1)$ are the mixed initial condition for mode-matched filter J at time $k-1$. $\hat{\mathbf{x}}_J(k|k)$ and $\mathbf{P}_J(k|k)$ are the global base state estimate and its covariance in mode-matched filter J at time k (Kalman filter). $\hat{\mathbf{x}}(k|k)$ and $\mathbf{P}(k|k)$ are the combined global base state estimate and its covariance at time k. We will present in next section details about the new PPM algorithm for solving 2D data association problem involved in step 3 of the BETT algorithm.

4. DATA ASSOCIATION PROBLEM

4.1. Previous works

ETT comes up against the following difficult data association problem : find an optimal point pattern matching (PPM) between a unknown subset of given point pattern (i.e. the reference target pattern) and another unknown subset of point pattern (the set of measurements given by a sensor). Many approaches have already been proposed for solving different PPM problems involved for instance in pattern recognition[10,11] , computer vision[12–14] , autonomous navigation[15] , and astronautics[16–18] . General papers and recent surveys on PPM techniques can be found in [19–22] . Some of them are based on Delaunay triangulation, genetic algorithms, relaxation[23,24] , fuzzy relaxation[25] , invariant features[26] , inter-point distances[27] , convex hull[28] , neighbor relations[27] or neural networks[29] . To the knowledge of the author, only recent works [29,26,24,11] present a PPM algorithm which takes into account the following five defective conditions : additing or suppressing points, location distortion, rotation, scaling and translation. The approaches proposed by of L. Spirkovska,M.B. Reid and S. Li are actually impractical for real-time ETT because of their computational load. The modified relaxation method proposed by Cheng based on an arbitrary choice of a compatibility function does not give good performance for the detection of extended target in clutter. Actually, only recent work published by S.H. Chang and al. is useful for ETT because it is fast and presents low computational burden. However this algorithm (based on an arbitrary vote procedure) can fail when there are many false alarms and a small number of target reflection points. We propose in the sequel and improvement of the Chang's PPM algorithm based on an optimal statistical testing procedure to compute the conditional matching pairs supports.

4.2. Preliminaries

We are given two point patterns $\mathcal{P} = \{\mathbf{x}_i, i = 1, \ldots, m\}$ and $\mathcal{Q} = \{\mathbf{y}_j, j = 1, \ldots, n\}$ (sets of points in N-dimensional space). In the following $\mathcal{P}$ is referred as the reference pattern we want to detect (a given point-target shape model) and $\mathcal{Q}$ is the set of observations available from the sensor at a given time. In cluttered environments with target detection probability less than one, we usually have $m \leq n$ but it is not a necessary condition. The general PPM problem consists to find the optimal matching $\mathcal{M}^\star$ between some points $\mathbf{x}_i \in \mathcal{P}$ and their mapping $\mathbf{y}_j = G_r(\mathbf{x}_i) \in \mathcal{Q}$ under an unknown affine registration $G_r(.)$ characterized by the parameters $\mathbf{R}$ (rotation matrix satisfying $\mathbf{R}^{-1} = \mathbf{R}'$ and $\det(\mathbf{R}) = 1$), $\mathbf{T}$ (translation vector) and $s > 0$ (scaling factor). Any affine registration will be represented by $r = (\mathbf{R}, \mathbf{T}, s)$ for short in the following. A given matching $\mathcal{M}_p$ of size p is a one-to-one mapping from a subset of $\mathcal{P}$ of size p into a subset of $\mathcal{Q}$ of size p. An affine registration[30,19] is a one-to-one mapping from the Euclidean space onto itself, consisting of a rotation, translation and scaling. The affine registration of $\mathbf{x}_i$ under $G_r(.)$ is defined as

$$\mathbf{y}_j = G_r(\mathbf{x}_i) = \mathbf{T} + s\mathbf{R}\mathbf{x}_i \tag{13}$$

In practical situation, the sensor gives only a noisy measurement of $G_r(\mathbf{x}_i)$ and we have

$$\mathbf{y}_j = G_r(\mathbf{x}_i) + \mathbf{b}_i \tag{14}$$

where $\mathbf{b}_i$ is a zero-mean white gaussian noise with known covariance matrix assumed to be $\mathbf{P}_{bb} = \sigma^2\mathbf{I}$ ($\mathbf{I}$ is the identity matrix of the appropriate dimension). In 2D space, the translation vector will be denoted $\mathbf{T} = [t_x t_y]'$ and the rotation matrix will be given by :

$$\mathbf{R} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

The matching pair error between two any points under a given affine registration G_r is defined as :

$$\epsilon^2[\mathbf{x}_i, \mathbf{y}_j|(\mathbf{R}, \mathbf{T}, s)] = (\mathbf{y}_j - G_r(\mathbf{x}_i))'\mathbf{P}_{bb}^{-1}(\mathbf{y}_j - G_r(\mathbf{x}_i)) = \mathbf{e}_j'\mathbf{P}_{bb}^{-1}\mathbf{e}_j \tag{15}$$

Consider now two subsets of points patterns $\mathcal{X} \in \mathcal{P}$ and $\mathcal{Y} \in \mathcal{Q}$ of same size p which match under some $\mathcal{M}_p$. We denote by m_i the corresponding index of point $\mathbf{x}_i \in \mathcal{X}$ under $\mathcal{M}_p$ (i.e $\mathbf{y}_{m_i} = G_r(\mathbf{x}_i) + \mathbf{b}_i \in \mathcal{Y}$). Shinji Umeyama proves in his theorem[30] , that it is always possible (as soon as p is greater than the number of independent parameters involved in the registration) to compute the optimal affine registration $G_{\hat{r}}$ which minimizes the classical (i.e. with $\mathbf{P}_{bb} = \mathbf{I}$) mean square error of the two point patterns $\mathcal{X}$ and $\mathcal{Y}$:

$$\bar{\epsilon}^2[\mathbf{R}, \mathbf{T}, s] = \frac{1}{p}\sum_{i=1}^{p} \epsilon^2[\mathbf{x}_i, \mathbf{y}_{m_i}|(\mathbf{R}, \mathbf{T}, s)] \tag{16}$$

His closed-form solution of the standard least-squares problem of the similarity transformation parameter estimation is based on the singular value decomposition (SVD) of the sample covariance matrix of the data. However it is also possible to estimate the parameters of affine registration via the generalized least square estimator. We present succinctly the method for the 2D case (more details can be found in [11]). From eq.(14) and eq.(15), we can write

$$\mathbf{e}_j = \mathbf{y}_j - G_r(\mathbf{x}_i) = \mathbf{y}_j - \begin{bmatrix} 1 & 0 & x_i^1 & -x_i^2 \\ 0 & 1 & x_i^2 & x_i^1 \end{bmatrix} \begin{bmatrix} t_x \\ t_y \\ s\cos(\theta) \\ s\sin(\theta) \end{bmatrix} = \mathbf{y}_j - \mathbf{C}_j \begin{bmatrix} \mathbf{T} \\ \mathbf{a} \end{bmatrix} = \mathbf{y}_j - \mathbf{C}_j\mathbf{r} \tag{17}$$

where $\mathbf{a} = [s\cos(\theta)\ s\sin(\theta)]'$, $\mathbf{T} = [t_x\ t_y]'$ and $\mathbf{x}_i = [x_i^1\ x_i^2]'$. Stacking these p differences, we have $\mathbf{E} = \mathbf{Y} - \mathbf{Cr}$ where $\mathbf{E} = [\mathbf{e}_1', \ldots, \mathbf{e}_p']'$ $\mathbf{Y} = [\mathbf{y}_1', \ldots, \mathbf{y}_p']'$ and $\mathbf{C} = [\mathbf{C}_1', \ldots, \mathbf{C}_p']'$. The generalized least square criteria to minimize is given by

$$J = \mathbf{E}'\mathbf{P}^{-1}\mathbf{E} = (\mathbf{Y} - \mathbf{Cr})'\mathbf{P}^{-1}(\mathbf{Y} - \mathbf{Cr}) \tag{18}$$

with $\mathbf{P}^{-1} = diag[\mathbf{P}_{bb}^{-1}, \ldots, \mathbf{P}_{bb}^{-1}]$. The well known solution of this minimization problem is given by the generalized least square estimator (GLSE)[1] which has the following form

$$\hat{\mathbf{r}} = \left(\mathbf{C}'\mathbf{P}^{-1}\mathbf{C}\right)^{-1}\mathbf{C}'\mathbf{P}^{-1}\mathbf{Y} \tag{19}$$

4.3. Point pattern matching in two dimensional space

In 2D space, there are four unknown parameters (s, θ, t_x, t_y) in the 2D affine registration equation (14). Therefore there are infinite solutions satisfying eq.(14). So, for a given matching pair $(\mathbf{x}_i \leftrightarrow \mathbf{y}_j)$ we can never determine an unique affine registration $\hat{r}$. But now, if we take at least a couple (i.e $p \geq 2$) of different matching pairs of points $((\mathbf{x}_i, \mathbf{x}_h) \leftrightarrow (\mathbf{y}_j, \mathbf{y}_k))$ then an unique affine registration $\hat{r} = (\hat{\mathbf{R}}, \hat{\mathbf{T}}, \hat{s})$ exists and can be computed using GLSE as previously. Corollary, if $(\mathbf{x}_i, \mathbf{x}_h)$ and $(\mathbf{y}_j, \mathbf{y}_k)$ are two matching pairs under $(\hat{\mathbf{R}}, \hat{\mathbf{T}}, \hat{s})$, then vector $\overrightarrow{\mathbf{x}_i\mathbf{x}_h}$ matches with $\overrightarrow{\mathbf{y}_j\mathbf{y}_k}$ under the reduced affine registration $(\hat{\mathbf{R}}, \hat{s})$ since one has :

$$\overrightarrow{\mathbf{y}_j\mathbf{y}_k} = G_{\hat{r}}(\mathbf{x}_h) + \mathbf{b}_h - G_{\hat{r}}(\mathbf{x}_i) - \mathbf{b}_i = (\hat{\mathbf{T}} + \hat{s}\hat{\mathbf{R}}\mathbf{x}_h) - (\hat{\mathbf{T}} + \hat{s}\hat{\mathbf{R}}\mathbf{x}_i) + (\mathbf{b}_h - \mathbf{b}_i) = \hat{s}\hat{\mathbf{R}}\overrightarrow{\mathbf{x}_i\mathbf{x}_h} + \overrightarrow{\mathbf{b}_i\mathbf{b}_h} \tag{20}$$

Now, if $\mathcal{X} = \{\mathbf{x}_i, i = 1, \ldots, p\} \in \mathcal{P}$ and $\mathcal{Y} = \{\mathbf{y}_i, i = 1, \ldots, p\} \in \mathcal{Q}$ are two sets of matching points under a given registration $\hat{r}$, then for each matching pair $(\mathbf{x}_i \leftrightarrow \mathbf{y}_i)$, the sets of vectors $\mathcal{X}_i = \{\overrightarrow{\mathbf{x}_i\mathbf{x}_1}, \ldots, \vec{0}, \ldots, \overrightarrow{\mathbf{x}_i\mathbf{x}_p}\}$ and $\mathcal{Y}_i = \{\overrightarrow{\mathbf{y}_i\mathbf{y}_1}, \ldots, \vec{0}, \ldots, \overrightarrow{\mathbf{y}_i\mathbf{y}_p}\}$ match under the reduced affine registration $(\hat{\mathbf{R}}, \hat{s})$. Since $\overrightarrow{\mathbf{x}_i\mathbf{x}_j}$ matches $\overrightarrow{\mathbf{y}_i\mathbf{y}_j}$ for $j = 1, \ldots, p$ $(j \neq i)$, there are necessary $p - 1$ matching vectors under the reduced registration $(\hat{\mathbf{R}}, \hat{s})$ for each matching pair $(\mathbf{x}_i \leftrightarrow \mathbf{y}_i)$. More generally, suppose that $p \leq \min(m, n)$ is the maximum number of matched pairs between $\mathcal{P}$ and $\mathcal{Q}$ under some unknown registration $r = (\mathbf{R}, \mathbf{T}, s)$. If $\mathbf{x}_i \in \mathcal{P}$ matches with $\mathbf{y}_j \in \mathcal{Q}$, then there exists $p - 1$ other pairs of matching points. Therefore, there are exactly $p - 1$ corresponding vectors between $\overrightarrow{\mathbf{x}_i\mathbf{x}_h}$ under $\overrightarrow{\mathbf{y}_j\mathbf{y}_k}$ under the reduced registration $(\mathbf{R}, s)$ or in a same way (s, θ). Shih-Hsu Chang and al. in their recent paper[11] discover that if $\mathbf{x}_i$ matches with $\mathbf{y}_j$ then (s, θ) corresponds to the peak value w_{ij}^* of the accumulator array $M_{i,j}(s, \theta)$ (where $h = 1, \ldots, m$ and $k = 1, \ldots, n$ with $h \neq i$ and $k \neq j$). Moreover the peak value w_{ij}^* called the **support** of matching $\mathbf{x}_i$ with $\mathbf{y}_j$ is exactly equal to $p - 1$. $M_{i,j}(s, \theta)$ is an array used to accumulate (s, θ) determined by matching $\overrightarrow{\mathbf{x}_i\mathbf{x}_h}$ with $\overrightarrow{\mathbf{y}_j\mathbf{y}_k}$ for all $h \neq i$ and $k \neq j$. When $\mathbf{x}_i$ does not match with $\mathbf{y}_j$ one has $w_{ij}^* < p - 1$. They first develop a matching pairs support algorithm (MPSF) based on the construction of the accumulator $M_{i,j}(s, \theta)$ using Hough Transform. Then they develop a fast procedure to find the maximal pair support $w^\star = \max(w_{ij}^*)$ in order to estimate the scaling and rotation parameters. However, the practical construction of the accumulators $M_{i,j}(s, \theta)$ $(i = 1, \ldots, m$ and $j = 1, \ldots, m)$ is not explicitly given. Actually, they use a discretization of the (s, θ)-space and also an *arbitrary* vote procedure to increment the accumulator. The major problem being to decide if two pairs of assumed matching vectors (say $(\overrightarrow{\mathbf{x}_i\mathbf{x}_h}, \overrightarrow{\mathbf{y}_j\mathbf{y}_k})$ and $(\overrightarrow{\mathbf{x}_i\mathbf{x}_{h'}}, \overrightarrow{\mathbf{y}_j\mathbf{y}_{k'}})$) support the same underlying registration. Their attractive (but heuristic) MPSF approach can lead however to severe mismatching for detection of small extended target when noise observation (distortion) becomes high with respect to the target size and when the clutter density increases. We develop in the next section a new MPSF algorithm based only on statistical consideration.

4.3.1. Maximum likelihood estimation of scaling factor and rotation angle for a given pair of vectors

First a presentation of the maximum likelihood estimation (MLE) of reduced registration parameters $(\mathbf{R}, s)$ for a pair of matching vectors $\mathbf{x}$ and $\mathbf{y}$ is required. This can be view as a simplified version of Umeyama's theorem. The objective function J we want to minimize is

$$J = (\mathbf{y} - s\mathbf{R}\mathbf{x})'\mathbf{P}^{-1}(\mathbf{y} - s\mathbf{R}\mathbf{x}) + Tr(\mathbf{L}(\mathbf{R}'\mathbf{R} - \mathbf{I})) + g\{\det(\mathbf{R}) - 1\} \tag{21}$$

where $Tr(.)$ and $\det(.)$ are the trace and determinant operators; g is a Lagrange multiplier and $\mathbf{L}$ is a symmetric matrix of Lagrange multipliers. The two last terms of J represent conditions for $\mathbf{R}$ to be a proper rotation matrix. $\mathbf{P}$ is the known covariance of observation noise assumed to be gaussian and zero-mean in the sequel. Partial differentiations of J with respect to $\mathbf{R}$ and s lead to the following system of equations :

$$\frac{\partial J}{\partial s} = -\mathbf{x}'\mathbf{R}'\mathbf{P}^{-1}\mathbf{y} - \mathbf{y}'\mathbf{P}^{-1}\mathbf{R}\mathbf{x} + 2s\mathbf{x}'\mathbf{R}'\mathbf{P}^{-1}\mathbf{R}\mathbf{x} = 0 \tag{22}$$

$$\frac{\partial J}{\partial \mathbf{R}} = -2s\mathbf{P}^{-1}\mathbf{y}\mathbf{x}' + 2s^2\mathbf{P}^{-1}\mathbf{R}\mathbf{x}\mathbf{x}' + \mathbf{R}(2\mathbf{L} + g\mathbf{I}) = \mathbf{O} \tag{23}$$

From eq.(22) we directly get

$$\hat{s} = \frac{\mathbf{x}'\mathbf{R}'\mathbf{P}^{-1}\mathbf{y}}{\mathbf{x}'\mathbf{R}'\mathbf{P}^{-1}\mathbf{R}\mathbf{x}} \tag{24}$$

The MLE of $\mathbf{R}$ seems difficult to obtain from eq.(23). Fortunately, a very simple analytical expression can be obtained when assuming $\mathbf{P} = \Psi^2\mathbf{I}$ which is a quiet reasonable assumption for most applications. Hence the expression of $\hat{s}$ becomes

$$\hat{s} = \frac{\mathbf{x}'\mathbf{R}'\mathbf{y}}{\mathbf{x}'\mathbf{x}} = \frac{\mathbf{x}'\mathbf{R}^{-1}\mathbf{y}}{\|\mathbf{x}\|^2} \tag{25}$$

Equation (23) with $\mathbf{P}^{-1} = \frac{1}{\Psi^2}\mathbf{I}$ gives $\mathbf{R}\mathbf{N} = s\mathbf{y}\mathbf{x}'$ where $\mathbf{N}$ is a symmetric matrix given by $\mathbf{N} = s^2\mathbf{x}\mathbf{x}' + \Psi^2(\mathbf{L} + \frac{1}{2}g\mathbf{I})$. Now let $\mathbf{U}\mathbf{D}\mathbf{V}'$ a singular value decomposition with denoting $\mathbf{x}^\perp = [-x_2\ x_1]'$ and $\mathbf{y}^\perp = [-y_2\ y_1]'$ one has

$$\mathbf{y}\mathbf{x}' = \underbrace{\frac{1}{\|\mathbf{y}\|}[\mathbf{y}\mathbf{y}^\perp]}_{\mathbf{U}} \underbrace{\begin{bmatrix} \|\mathbf{x}\|\|\mathbf{y}\| & 0 \\ 0 & 0 \end{bmatrix}}_{\mathbf{D}} \underbrace{[\mathbf{x}\mathbf{x}^\perp]'\frac{1}{\|\mathbf{x}\|}}_{\mathbf{V}'} \tag{26}$$

We can easily check that $\mathbf{U}\mathbf{U}' = \mathbf{I}$ and $\mathbf{V}\mathbf{V}' = \mathbf{I}$ and we also have with this SVD $\det(\mathbf{U}) = 1$ and $\det(\mathbf{V}) = 1$ which will be helpful in the sequel. Replacing $\mathbf{y}\mathbf{x}'$ by its expression in $\mathbf{R}\mathbf{N}$ gives $\mathbf{R}\mathbf{N} = s\mathbf{U}\mathbf{D}\mathbf{V}'$. By multiplying $\mathbf{R}\mathbf{N}$ on the left by its transpose, and using the fact that $\mathbf{R}'\mathbf{R} = \mathbf{I}$ and $\mathbf{N}' = \mathbf{N}$, we deduce $\mathbf{N} = s\mathbf{V}\mathbf{D}\mathbf{V}'$. Therefore, after a simplification by s, we get $\mathbf{R}(\mathbf{V}\mathbf{D}\mathbf{V}') = \mathbf{U}\mathbf{D}\mathbf{V}'$. Multiplying this equation on the left by $\mathbf{U}'$ and on the right by $\mathbf{V}$ gives $(\mathbf{U}'\mathbf{R}\mathbf{V})\mathbf{D} = \mathbf{D}$ and by identification we get $\mathbf{U}'\mathbf{R}\mathbf{V} = \mathbf{I}$. Since $\det(\mathbf{U}) = 1$ and $\det(\mathbf{V}) = 1$, we obtain the main result :

$$\hat{\mathbf{R}} = \mathbf{U}\mathbf{V}' = \frac{1}{\|\mathbf{y}\|}[\mathbf{y}\mathbf{y}^\perp][\mathbf{x}\mathbf{x}^\perp]'\frac{1}{\|\mathbf{x}\|} = \begin{bmatrix} x_1y_1 + x_2y_2 & x_2y_1 - x_1y_2 \\ x_1y_2 - x_2y_1 & x_1y_1 + x_2y_2 \end{bmatrix} \frac{1}{\|\mathbf{x}\|\|\mathbf{y}\|} \tag{27}$$

Let $\theta_\mathbf{x}$ and $\theta_\mathbf{y}$ be the two angles of vectors and $\mathbf{x}$ and $\mathbf{y}$. From eq.(27) we deduce

$$\hat{\mathbf{R}} = \begin{bmatrix} \cos(\theta_\mathbf{x})\cos(\theta_\mathbf{y}) + \sin(\theta_\mathbf{x})\sin(\theta_\mathbf{y}) & \sin(\theta_\mathbf{x})\cos(\theta_\mathbf{y}) - \cos(\theta_\mathbf{x})\sin(\theta_\mathbf{y}) \\ \cos(\theta_\mathbf{x})\sin(\theta_\mathbf{y}) - \sin(\theta_\mathbf{x})\cos(\theta_\mathbf{y}) & \cos(\theta_\mathbf{x})\cos(\theta_\mathbf{y}) + \sin(\theta_\mathbf{x})\sin(\theta_\mathbf{y}) \end{bmatrix} = \begin{bmatrix} \cos(\theta_\mathbf{y} - \theta_\mathbf{x}) & -\sin(\theta_\mathbf{y} - \theta_\mathbf{x}) \\ \sin(\theta_\mathbf{y} - \theta_\mathbf{x}) & \cos(\theta_\mathbf{y} - \theta_\mathbf{x}) \end{bmatrix}$$

The MLE of rotation angle is then given by

$$\hat{\theta} = \theta_\mathbf{y} - \theta_\mathbf{x} \tag{28}$$

If we now replace $\hat{\mathbf{R}}'$ by its analytical expression in eq.(25), we get with elementary algebra

$$\hat{s} = \frac{\|\mathbf{y}\|}{\|\mathbf{x}\|} \tag{29}$$

Note that the MLE of s and θ coincide exactly with the intuitive (but unjustified) expressions already used by Chang and al. in reference [11]. The density of $\hat{s}$ is a Rayleigh probability density function (pdf). The density of $\hat{\theta}$ is more difficult to obtained without an extensive use of symbolic language (like Mathematica or Mapple). But actually, as we will prove in the sequel, the knowledge of these pdfs is useless because of the following lemma :

Lemma 1 :

The pair of independent variables $z = \mathbf{x}'\mathbf{y}$ and $z^\perp = \mathbf{x}^{\perp\prime}\mathbf{y}$ contains exactly the same information as $(\hat{s}, \hat{\theta})$.

Proof : This comes from the fact there exists the following mapping between $(z, z^\perp)$ and $(\hat{s}, \hat{\theta})$

$$\hat{s} = \sqrt{\left(\frac{z}{\mathbf{x}'\mathbf{x}}\right)^2 + \left(\frac{z^\perp}{\mathbf{x}^{\perp\prime}\mathbf{x}^\perp}\right)^2} \qquad \hat{\theta} = arctg\left(\frac{z^\perp}{z}\right) \tag{30}$$

Moreover by hypothesis, we have $\mathbf{y} = s\mathbf{R}\mathbf{x} + \mathbf{b}$ where $\mathbf{b}$ is the observation noise assumed to be gaussian zero-mean with covariance $2\sigma^2\mathbf{I}$. The pdf of $\mathbf{y}$ is therefore normal with mean $\bar{\mathbf{y}} = s\mathbf{R}\mathbf{x}$ and covariance matrix $\mathbf{P}_{\mathbf{yy}} = 2\sigma^2\mathbf{I}$. Since one has

$$\mathbf{z} = \begin{bmatrix} z \\ z^{\perp} \end{bmatrix} = \begin{bmatrix} \mathbf{x}' \\ \mathbf{x}^{\perp\prime} \end{bmatrix} \mathbf{y} = \underbrace{\begin{bmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{bmatrix}}_{\mathbf{A}_\mathbf{x}} \mathbf{y} \tag{31}$$

the random vector $\mathbf{z}$ is gaussian and has the density

$$\mathcal{N}(\mathbf{z}; \bar{\mathbf{z}}, \mathbf{P}_{\mathbf{zz}}) = \det(2\pi\mathbf{P}_{\mathbf{zz}})^{-1/2} \exp(-(\mathbf{z} - \bar{\mathbf{z}})'\mathbf{P}_{\mathbf{zz}}{}^{-1}(\mathbf{z} - \bar{\mathbf{z}})/2) \tag{32}$$

with

$$\bar{\mathbf{z}} = \mathbf{A}_\mathbf{x}\bar{\mathbf{y}} = \begin{bmatrix} \|\mathbf{x}\|^2 s\cos(\theta) \\ \|\mathbf{x}\|^2 s\sin(\theta) \end{bmatrix} = \|\mathbf{x}\|^2\mathbf{a} \qquad \mathbf{P}_{\mathbf{zz}} = \mathbf{A}_\mathbf{x}\mathbf{P}_{\mathbf{yy}}\mathbf{A}'_\mathbf{x} = 2\sigma^2\|\mathbf{x}\|^2\mathbf{I} \tag{33}$$

Components z and $z^{\perp}$ of the gaussian vector $\mathbf{z}$ being uncorrelated because $\mathbf{P}_{\mathbf{zz}}$ is diagonal, they are also independent, which completes the proof.

4.3.2. Similarity test for underlying affine registration between a set of pairs of N vectors

First we recall that for the general case, if $\mathbf{z}_i$ are independent with $\mathbf{z}_i \sim \mathcal{N}(\alpha_i\mathbf{a}, \mathbf{P}_i)$ (for $i = 1\ldots, N-1$ and where α_i is a non singular matrix with appropriate dimension) then

$$\mathbf{P}_{1,N-1}^{-1}[\sum_{i=1}^{N-1} \alpha_i'\mathbf{P}_i^{-1}\mathbf{z}_i] \sim \mathcal{N}(\mathbf{a}, \mathbf{P}_{1,N-1}^{-1}) \tag{34}$$

where $\mathbf{P}_{1,N-1} = \sum_{i=1}^{N-1} \alpha_i'\mathbf{P}_i^{-1}\alpha_i$. If now a new independent random vector $\mathbf{z}_N \sim \mathcal{N}(\alpha_N\mathbf{a}, \mathbf{P}_N)$ is available then

$$(\alpha_N^{-1}\mathbf{z}_N - \mathbf{P}_{1,N-1}^{-1}[\sum_{i=1}^{N-1} \alpha_i'\mathbf{P}_i^{-1}\mathbf{z}_i]) \sim \mathcal{N}(\vec{0}, \mathbf{P}_{1,N-1}^{-1} + \alpha_N^{-1}\mathbf{P}_N\alpha_N^{-1\prime}) \tag{35}$$

and the quantity

$$\epsilon = (\alpha_N^{-1}\mathbf{z}_N - \mathbf{P}_{1,N-1}^{-1}[\sum_{i=1}^{N-1} \alpha_i'\mathbf{P}_i^{-1}\mathbf{z}_i])'\left[\mathbf{P}_{1,N-1}^{-1} + \alpha_N^{-1}\mathbf{P}_N\alpha_N^{-1\prime}\right]^{-1}(\alpha_N^{-1}\mathbf{z}_N - \mathbf{P}_{1,N-1}^{-1}[\sum_{i=1}^{N-1} \alpha_i'\mathbf{P}_i^{-1}\mathbf{z}_i]) \tag{36}$$

is chi-square distributed with n_z degrees of freedom. We can now use this property for our basic testing problem. We assume we already have a cluster of $N-1$ pairs of matching vectors $\mathcal{C}_{N-1} = \{(\mathbf{x}_i, \mathbf{y}_i), i = 1\ldots, N-1\}$ or more precisely a set of gaussian random vectors $\{\mathbf{z}_i, \ldots, \mathbf{z}_{N-1}\}$ with $\mathbf{z}_i = \mathbf{A}_{\mathbf{x}_i}\mathbf{y}_i$. A new matching pair $(\mathbf{x}_N, \mathbf{y}_N)$ is then given from which we build a new $\mathbf{z}_N$. We want to know if $\mathbf{z}_N$ belongs to the cluster $\mathcal{C}_{N-1}$ or not. Only two cases are invested here (actually only the result of the second case is useful for the development of the fast PPM algorithm).

Case 1 : No correlation

If random gaussian vectors $\mathbf{y}_1, \ldots, \mathbf{y}_N$ (build from the set of validated observations points) do not share point in common then the covariance matrix of $\mathbf{y} = [\mathbf{y}_1, \ldots, \mathbf{y}_N]'$ is given by $\mathbf{P}_{\mathbf{yy}} = 2\sigma^2\mathbf{I}_N \otimes \mathbf{I}$. $\mathbf{I}_N$ denotes the identity matrix of size N and $\mathbf{I}$ by convention the identity matrix of size 2. $\otimes$ is the Kronecker product. The covariance of $\mathbf{z} = [\mathbf{z}_1, \ldots, \mathbf{z}_N]' = \mathbf{A}\mathbf{y} = diag[\mathbf{A}_{\mathbf{x}_1}, \ldots, \mathbf{A}_{\mathbf{x}_N}]\mathbf{y}$ is $\mathbf{P}_{\mathbf{zz}} = \mathbf{A}\mathbf{P}_{\mathbf{yy}}\mathbf{A}' = 2\sigma^2 diag[\|\mathbf{x}_1\|^2\mathbf{I}, \ldots, \|\mathbf{x}_N\|^2\mathbf{I}]$. Moreover we have $\bar{\mathbf{z}} = \mathbf{A}\bar{\mathbf{y}} = [\|\mathbf{x}_1\|^2\mathbf{a}, \ldots, \|\mathbf{x}_N\|^2\mathbf{a}]'$. Since random vectors $\mathbf{z}_1, \ldots, \mathbf{z}_N$ are independent we can directly perform the chi-square test[2] on ϵ given in eq.(36) with $\alpha_i = \|\mathbf{x}_i\|^2\mathbf{I}$ and $\mathbf{P}_i = 2\sigma^2\|\mathbf{x}_i\|^2\mathbf{I}$ for $i = 1, \ldots, N$ in order to accept or reject H_0 : "$\mathbf{z}_N$ and $\mathcal{C}_{N-1} = \{\mathbf{z}_1, \ldots, \mathbf{z}_{N-1}\}$ are statistically identical". In other words, we can perform the following test :

Similarity test : Accept H_0 if $\epsilon \in [r_1, r_2]$

where the acceptance interval $[r_1, r_2]$ is determined such that $Pr\{\epsilon \in [r_1, r_2] | H_0\} = 1 - \alpha$. For example, with $\alpha = 0.05$ we have from the chi-square tables, for a two-sided interval $r_1 = 0.0506$ and $r_2 = 7.377$. $[r_1, r_2]$ is the (two-sided) 95% probability region for ϵ.

Case 2 : Correlation of type 1

If vectors $\mathbf{y}_1, \ldots \mathbf{y}_N$ share their beginning points (as it will be in the CMPSF algorithm described in next section) or their ending points then the covariance of the gaussian stacked vector $\mathbf{z} = \mathbf{A}\mathbf{y}$ is

$$\mathbf{P_{zz}} = \mathbf{A}\mathbf{P_{yy}}\mathbf{A}' = \mathbf{A} \begin{bmatrix} 2\sigma^2\mathbf{I} & \sigma^2\mathbf{I} & \ldots & \sigma^2\mathbf{I} \\ \sigma^2\mathbf{I} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \sigma^2\mathbf{I} \\ \sigma^2\mathbf{I} & \ldots & \sigma^2\mathbf{I} & 2\sigma^2\mathbf{I} \end{bmatrix} \mathbf{A}' \tag{37}$$

The decorrelation of $\mathbf{z}_1, \ldots, \mathbf{z}_N$ is obtained by the following triangularization[31] of $\mathbf{P_{yy}}$

$$\mathbf{P_{yy}} = \underbrace{\begin{bmatrix} \mathbf{I} & \mathbf{O} & \ldots & \ldots & \mathbf{O} \\ \frac{1}{2}\mathbf{I} & \mathbf{I} & \ddots & & \vdots \\ \vdots & \frac{1}{3}\mathbf{I} & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \mathbf{O} \\ \frac{1}{2}\mathbf{I} & \frac{1}{3}\mathbf{I} & \ldots & \frac{1}{N}\mathbf{I} & \mathbf{I} \end{bmatrix}}_{\mathbf{L}} \underbrace{\begin{bmatrix} 2\sigma^2\mathbf{I} & \mathbf{O} & \ldots & \ldots & \mathbf{O} \\ \mathbf{O} & \frac{3}{2}\sigma^2\mathbf{I} & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \mathbf{O} \\ \mathbf{O} & \ldots & \ldots & \mathbf{O} & \frac{N+1}{N}\sigma^2\mathbf{I} \end{bmatrix}}_{\mathbf{D}} \underbrace{\begin{bmatrix} \mathbf{I} & \frac{1}{2}\mathbf{I} & \ldots & \ldots & \frac{1}{2}\mathbf{I} \\ \mathbf{O} & \mathbf{I} & \frac{1}{3}\mathbf{I} & \ldots & \frac{1}{3}\mathbf{I} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \frac{1}{N}\mathbf{I} \\ \mathbf{O} & \ldots & \ldots & \mathbf{O} & \mathbf{I} \end{bmatrix}}_{\mathbf{L}'} \tag{38}$$

Now by applying the linear transformation $\mathbf{M}^{-1} = \mathbf{L}^{-1}\mathbf{A}^{-1}$ on $\mathbf{z}$, we get $\tilde{\mathbf{z}} = \mathbf{M}^{-1}\mathbf{z}$ having its components $\tilde{\mathbf{z}}_1, \ldots, \tilde{\mathbf{z}}_N$ gaussian and uncorrelated (and so independent). The covariance matrix of $\tilde{\mathbf{z}}$ is $\mathbf{P}_{\tilde{\mathbf{z}}\tilde{\mathbf{z}}} = \mathbf{D}$ and the mean $\bar{\tilde{\mathbf{z}}} = E[\tilde{\mathbf{z}}]$ is

$$\bar{\tilde{\mathbf{z}}} = \mathbf{M}^{-1}\bar{\mathbf{z}} = \mathbf{M}^{-1} \begin{bmatrix} \|\mathbf{x}_1\|^2\mathbf{a} \\ \vdots \\ \|\mathbf{x}_N\|^2\mathbf{a} \end{bmatrix} = \begin{bmatrix} \mathbf{T}_1{}^{-1}\mathbf{a} \\ \vdots \\ \mathbf{T}_N{}^{-1}\mathbf{a} \end{bmatrix} \tag{39}$$

where $\mathbf{T}_i{}^{-1} = \mathbf{A}'_{\mathbf{x}_i} - \sum_{k=1}^{i-1} \frac{1}{k+1}\mathbf{A}'_{\mathbf{x}_k}$ $(i = 1, \ldots, N)$. We have then $\tilde{\mathbf{z}}_i \sim \mathcal{N}(\mathbf{T}_i{}^{-1}\mathbf{a}, \frac{i+1}{i}\sigma^2\mathbf{I})$ for $i = 1, \ldots, N$. Since random vectors $\tilde{\mathbf{z}}_1, \ldots, \tilde{\mathbf{z}}_N$ are independent we can directly perform the chi-square test on ϵ given in eq.(36) by replacing $\mathbf{z}_i$ by $\tilde{\mathbf{z}}_i$ and taking $\alpha_i = \mathbf{T}_i{}^{-1}$ and $\mathbf{P}_i = \frac{i+1}{i}\sigma^2\mathbf{I}$ for $i = 1, \ldots, N$ in order to accept or reject H_0.

4.3.3. A new conditional matching pairs support algorithm (CMPSF)

For a given pair $(\mathbf{x}_i \in \mathcal{P}, \mathbf{y}_j \in \mathcal{Q})$ assumed to match, we now present a *conditional* matching pairs support algorithm which gives the appropriate matching support w^*_{ij} and conditional PPM $\mathcal{M}^*_{ij}$.

MPSF algorithm for a given pair $(\mathbf{x}_i, \mathbf{y}_j)$

Step 1 : Initialization

- Choose an arbitrary pair of points $(\mathbf{x}_{h'}, \mathbf{y}_{k'}) \in \mathcal{P} \times \mathcal{Q}$ with $h' \neq i$ and $k' \neq j$
- Set $Cluster(1) = \{(\mathbf{x}_{h'}, \mathbf{y}_{k'})\}$ and $nc_{\max} = 1$

Step 2 : Data clustering (under same underlying affine registration parameters)

▷ if $Mflag(h, k) = 1$ (i.e. $(\mathbf{x}_h, \mathbf{y}_k)$ is a possible matching pair) do :

⋆ For $h = 1, \ldots, m$ $(h \neq h'$ and $h \neq i)$

⋆ For $k = 1, \ldots, n$ $(k \neq k'$ and $k \neq j)$

- Test the similarity between $(\overrightarrow{\mathbf{x}_i\mathbf{x}_h}, \overrightarrow{\mathbf{y}_j\mathbf{y}_k})$ and $Cluster(nc)$ for $nc = 1, \ldots, nc_{\max}$ using chi-square test previously described (for correlation of type 1)
- if there is a similarity with an existing cluster (say $nc^\star$) update corresponding cluster as $Cluster(nc^\star) = Cluster(nc^\star) \cup \{(\mathbf{x}_h, \mathbf{y}_k)\}$ otherwise create a new cluster by setting $nc_{\max} = nc_{\max} + 1$ and $Cluster(nc_{\max}) = \{(\mathbf{x}_h, \mathbf{y}_k)\}$

⋆ End loop on k

⋆ End loop on h

▷ End if

Step 3 : Search for the index nc^* of the cluster having a maximal cardinality

- $w^*_{ij} = Card[Cluster(nc^*)]$ is desired support of pair $(\mathbf{x}_i, \mathbf{y}_j)$
- $\mathcal{M}^*_{ij} = \{(\mathbf{x}_i, \mathbf{y}_j)\} \cup Cluster(nc^*)$ is the *conditional* PPM with respect to $(\mathbf{x}_i, \mathbf{y}_j)$.

Step 4 (if required) : Parameters estimation

- Estimate $(s^*_{ij}, \theta^*_{ij})$ corresponding to $\mathcal{M}^*_{ij}$ using GLSE or Umeyama's theorem.

If $w^*_{ij} = c$, then it means that there are other c pairs of corresponding points supporting $\mathbf{x}_i$ matched with $\mathbf{y}_j$; and therefore there are $c + 1$ pairs of corresponding points matching under the same underlying affine registration $(s^*_{ij}, \theta^*_{ij})$. The cardinality of $\mathcal{M}^*_{ij}$ is exactly equals to $c + 1$. In this MPSF algorithm we introduce the match flag array $Mflag$ to represent the matching condition between each point in $\mathcal{P}$ and each point in $\mathcal{Q}$. $Mflag(h, k) = 1$ means that $(\mathbf{x}_h, \mathbf{y}_k)$ is a possible point matching pair. $Mflag(h, k) = 0$ means that $\mathbf{x}_h$ cannot match with $\mathbf{y}_k$. This flag array will be useful in the sequel to save computation time as it will be shown in next section. If we have no prior information on possible point matching pairs, each element of $Mflag$ must be set to 1. In case there are several clusters (say for example $Cluster(nc)$ and $Cluster(nc')$) having the same maximal cardinality in step 3, we have to select the one giving the minimal mean square error (MSE) given in eq.(16). This requires first to use GLSE of eq.(19) to estimate the reduced affine registration parameters relative to $\mathcal{M}_{ij}$ and $\mathcal{M}_{i'j'}$.

4.3.4. Fast Point Pattern Matching Algorithm (FPPM)

We propose from now to follow exactly the FPPM algorithm proposed recently by Chang and al. but without their determination of matching pairs (DTM) step since it is useless with our approach. A detailed discussion of this algorithm is given in reference [11].

Fast Point Pattern Matching Algorithm

Step 1 : Initialization

- Set $w^* = 0$, $Mflag(i, j) = 1$ ($i = 1, \ldots, m$ and $j = 1, \ldots, n$) and set $\hat{p} = \min(m, n)$

Step 2 : Scanning of $\mathcal{P}$ and $\mathcal{Q}$ sets

⋆ For $i = 1, \ldots, m$

⋆ For $j = 1, \ldots, n$

(a) Determine w^*_{ij} and $\mathcal{M}^*_{ij}$ using CMPSF algorithm
(b) if $w^*_{ij} < w_{\min}$ then set $Mflag(i, j) = 0$
(c) if $w^*_{ij} > w^*$ then set $w^* = w^*_{ij}$, $\mathcal{M}^* = \mathcal{M}^*_{ij}$, $i^* = i$ and $j^* = j$
(d) if $w^* \geq w_{\max}$ then the support is found; go to step 3.
(e) if $w^* \geq \min(m, n) - i$ then the maximum support is found; go to step 3.

⋆ End loop on j

⋆ The maximum support is not found, decrease $\hat{p}$ by one $\Rightarrow \hat{p} = \hat{p} - 1$

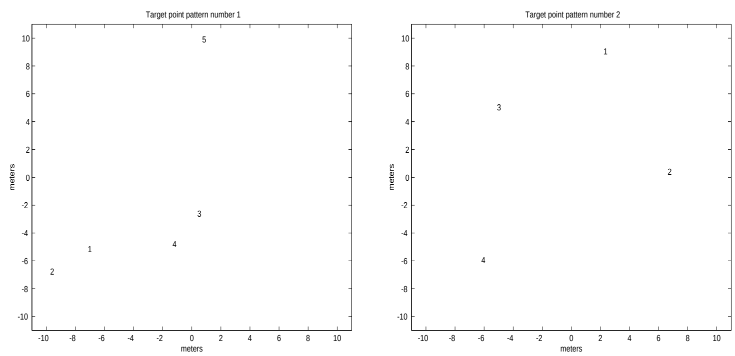
⋆ End loop on i

Step 3 : Return the PPM solution

- The pair $(\mathbf{x}_{i^*}, \mathbf{y}_{j^*})$ has the maximum matching pair support w^*. The PPM solution is given by $\mathcal{M}^*$. Using GLSE with $\mathcal{M}^*$ we get the estimate $(\hat{s}^*, \hat{\theta}^*)$ of underlying reduced affine registration parameters if needed. The number of detections (i.e. matching pairs) is given by $\hat{p}$.

5. SIMULATIONS RESULTS

A four-dynamic-model IMM algorithm for tracking 2D maneuvering and bending extended target was used in this simulation. The principal state $\mathbf{x}_g(k) = [x_g\ \dot{x}_g\ y_g\ \dot{y}_g]'$ follows either a constant velocity (CV) model, a clockwise nearly coordinated turn (CT) model with a standard turn rate $\omega_r = -3$ deg/s. These models can be found in [32,2]. As in [9], we assume that turn rate is known. The proper state $\mathbf{x}_p(k) = [\alpha\ \dot{\alpha}]'$ of the target follows either a null velocity (NV) model or a constant velocity (CV) model. α is the attitude of the extended target and $\dot{\alpha}$ is rotation rate of the target around its center of mass. The attitude is defined as the angle between a given axis in a target frame and a given axis in the absolute (or platform) frame. One uses a random 5-points pattern for *shape-mode* 1 and a random 4-points pattern for *shape-mode* 2 of the extended target (see figure below). These two patterns have been obtained by generating points uniformly distributed in a square $[-L; L] \times [-L; L]$ where maximal semi-extension of the extended target was $L = 10$ meters. For any shape-mode, the measurement equation associated with the i-th



point $\mathbf{z}^i_{ref} = [x_i\ y_i]'$ of a reference-point-pattern is given by :

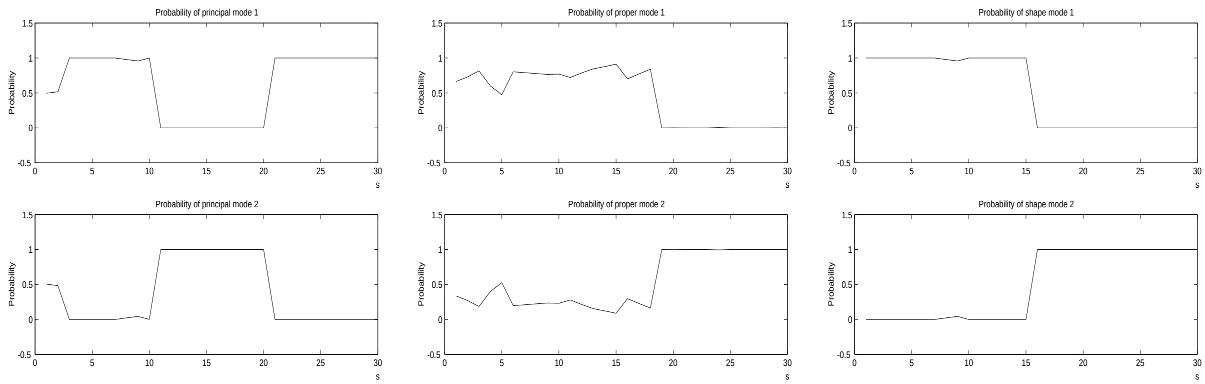
$$\mathbf{z}_i(k) = \mathbf{h}_i[k, \underbrace{\mathbf{x}_g(k), \mathbf{x}_p(k)}_{\mathbf{x}(k)}, \mathbf{z}^i_{ref}] + \mathbf{w}^i(k) = \begin{bmatrix} x_g(k) + x_i \cos\alpha(k) - y_i \sin\alpha(k) \\ y_g(k) + x_i \sin\alpha(k) + y_i \cos\alpha(k) \end{bmatrix} + \mathbf{w}^i(k) \tag{40}$$

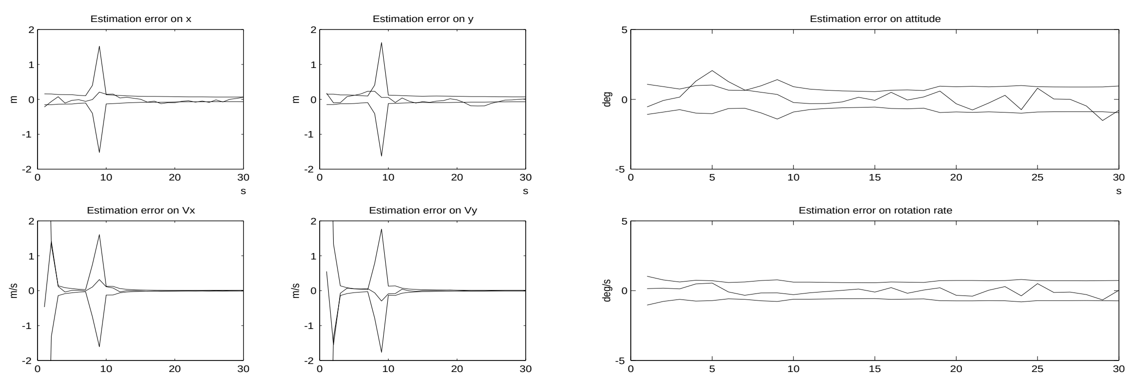
where $\mathbf{w}^i(k)$ is a zero-mean gaussian observation noise with covariance $\mathbf{R}^i(k)$. The observation matrix (relative to the i-th (shape-mode) referenced point) required in the Extended Kalman Filter implementation (EKF) for IMM/EBTT is given by :

$$\mathbf{H}_i(k) = \left[\nabla \mathbf{h}'_i[k, \mathbf{x}(k), \mathbf{z}^i_{ref}]\right]'_{\mathbf{x}=\hat{\mathbf{x}}(k|k-1)} = \begin{bmatrix} 1 & 0 & 0 & 0 & -x_i \sin\hat{\alpha}(k|k-1) - y_i \cos\hat{\alpha}(k|k-1) & 0 \\ 0 & 0 & 1 & 0 & x_i \cos\hat{\alpha}(k|k-1) - y_i \sin\hat{\alpha}(k|k-1) & 0 \end{bmatrix} \tag{41}$$

Note that at least two detections are necessary for PPM and observability of the system otherwise the proper state cannot be estimated via the filter. Thus the (shape-mode) measurement vector $\mathbf{z}(k)$ (assuming PPM solved) is actually a stacked vector with $\mathbf{z}_i(k)$ and observation matrix $\mathbf{H}(k)$ is a stacked matrix with $\mathbf{H}_i(k)$. The detection condition is fullfilled when the detection probability and/or the number of points for each reference pattern is big enough. We have taken $Pd = 0.9$ in this simulation. False alarms (FA) have been generated here at each sampling time ($T = 1$s) and uniformly distributed in a square having a volume $V = \{[x_g(k) - 3L/2; x_g(k) + 3L/2] \times [y_g(k) - 3L/2; y_g(k) + 3L/2]\}$. We used the Poisson model[1] with parameter $\lambda_{fa}V$ to get the number of FA to be generated in V at each observation step. The FA spatial density was set to $\lambda_{fa} = 0.005$ FA$/m^2$ to have (in average) almost the same number of true target detections and FA. Since 2 models for $\mathbf{x}_g(k)$, 2 models for $\mathbf{x}_p(k)$ and 2 shape patterns are possible, then we have 8 possible different global modes $\mathbf{m}(k)$ for the system. An 8-model IMM/BETT filter have been implemented in this simulation. Note that from the dynamic point of view, if we assume that target shape does not depend of its dynamic, there is actually only 4 different possible models for the global state $\mathbf{x}(k)$. Our simulation consists of 30 sampling periods. During the 1st *principal* segment for $k = 1\ldots, 10$s the (center of mass) target evolves in straight line ($m_g(k) = 1$); during the 2nd *principal* segment for $k = 11\ldots, 20$s, target makes a right coordinated turn ($m_g(k) = 2$) then goes back to straight flight during the 3rd *principal* segment for $k = 21\ldots, 30$s ($m_g(k) = 1$). During the 1st *proper* segment for $k = 1\ldots, 17$s the target does not move around its center of mass

($m_p(k) = 1$), then makes a constant rotation around its center of mass ($m_p(k) = 2$) with $\dot{\alpha} = 5$ deg/s (1g) up to $k = 30$s. From $k = 1 \ldots, 15$s (1st *shape* segment) target has the reference pattern 1 then switches to reference pattern 2 for $k = 16 \ldots, 30$s (2nd *shape* segment). The position measurement error for each cartesian coordinate (diagonal terms of $\mathbf{R}^i(k)$) is assumed to be 0.3 m. The covariance for the true trajectory is set to zero whereas for the filter the following $\mathbf{Q} = diag[\mathbf{Q}_g\ \mathbf{Q}_p]$ was used with $\mathbf{Q}_g^1 = (0.004)^2\mathbf{I}$, $\mathbf{Q}_g^2 = (0.001)^2\mathbf{I}$ and $\mathbf{Q}_p^2 = \mathbf{Q}_p^1 = (0.01)^2$ where $\mathbf{I}$ is the 2×2 identity matrix. True initial global state was $\mathbf{x}(0) = [0\ 100\ 0\ 100\ 45\ 0]'$. Initial covariance of the state was chosen as $\mathbf{P}(0|0) = diag[(100m)^2\ (10m/s)^2\ (100m)^2\ (10m/s)^2\ (2deg)^2\ (2deg/s)^2]$. The diagonal elements of 8×8 transition probability matrices π_{IJ} given in eq.(5) for IMM were set to 0.95. In our case, off-diagonal terms were set to 0.002, 0.004 or 0.012 depending on the number of elementary mode transitions between global mode index I and J. For example, we set $\pi_{IJ} = 0.012$ if $\mathbf{m}_I = [m_g = 1, m_p = 1, m_s = 2]$ and $\mathbf{m}_J = [m_g = 1, m_p = 1, m_s = 1]$ and 0.004 for the global transition from same $\mathbf{m}_I$ to $\mathbf{m}_J = [m_g = 1, m_p = 2, m_s = 1]$. Following figures show the estimated mode probabilities and state estimation errors (with +/- standard deviation) obtained from IMM/BETT filter. These results confirm ability of the algorithm to track maneuvering and bending extended target in 2D space.





6. CONCLUSIONS

In this paper, a new algorithm have been described to track maneuvering and bending extended target in cluttered environment in 2D space. The proposed approach is based on the Interacting Multiple Model Filter to estimate the global base state (and hidden modal-state) of the hybrid system combined with a new Point Pattern Matching algorithm to solve 2D data association problem. The tracking ability of this new BETT algorithm has been evaluated on a very simple scenario for an arbitrary two shape-mode bending target. Sensitivity analysis of the algorithm to shape distortion for more realistic scenarios and extension of BETT algorithm for 3D tracking are under investigations.

ACKNOWLEDGMENTS

The author wishes to express his thanks to Christian Musso for his helpful discussions about this work and also Fang-Hsuan Cheng and Shih-Hsu Chang for their valuable comments on their PPM algorithms.

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