Theoretical development of an Integrated JPDAF for multitarget tracking in clutter

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Abstract

An improved version of Integrated Probabilistic Data Association Filter (IPDAF) based on a new concept of probability of target perceivability was introduced recently in [13] for tracking a single target in clutter. In this paper, we extend the previous theoretical results to the multitarget tracking case to come up with a new integrated version of the Joint Probabilistic Data Association Filter called IJPDAF. Such algorithm provides a new basis of an integrated approach to multiple track initiation, confirmation, termination and maintenance.

1 Introduction

The purpose of multitarget tracking is to estimate the state of several targets based on a set of measurements provided by a sensor. For tracking in a clutter-free environment with perfect data association, targets are always assumed perceivable and measurements are assumed to be available, unique and to arise from a known target at every scan. In such ideal case, multitarget tracking will follow conventional recursive filtering. In practice however the perfect data association assumption is never fulfilled and conventional filtering techniques cannot be used because several measurements are available at every scan and the origin of measurements is uncertain. Moreover tracking targets in clutter involves tracks initiation, confirmation, maintenance and termination. Tracks initiation, confirmation and termination are basically decision problems whereas tracks maintenance is an estimation problem compounded with measurement uncertainty. This paper is only focused on tracks maintenance. Tracks confirmation and termination processes were discussed in [23, 13]. As already pointed out in previous works [8, 27, 19, 20, 13], a fundamental limitation of the Probabilistic Data Association Filter (PDAF) [4, 6] and Joint PDAF [3] is the implicit strong assumption that targets are always perceivable. Of course in many real situations, this is not the case. To remove the implicit target perceivability assumption made by Bar-Shalom,Tse and Fortmann, a new formulation of IPDAF based on a new concept of target perceivability has been recently proposed for tracking a single target in clutter. We propose now to extend this formulation for the multitarget tracking applications. This new algorithm will be called Integrated JPDAF (IJPDAF) hereafter. A recent Viterbi Data Association (VDA) algorithm [17] has been developed with including target perceivability state within a multitarget tracker. However this method does not take fully into account crossing targets.

2 Brief Integrated PDAF review

At each time step, the sensor provides a set of candidate measurements to be associated with a target t (which may be potentially perceivable or unperceivable) by using a validation gate [4] around the predicted measurement $\hat{\mathbf{y}}^t(k|k-1)$ of the target. There are many different approaches to associate candidate measurements with predicted one. Here we succintly present the Integrated Probabilistic Data Association (IPDA) method [7, 8, 27, 20, 13] for tracking a single target in clutter. IPDAF takes into account the target perceivability presented in [24].

The set of m_k^t candidate measurements $\mathbf{y}_i^t(k)$, $i = 1, \ldots, m_k^t$ at time k is denoted $\mathbf{Y}^t(k)$. The set of all validated measurements for target t up to time k is denoted

$$\mathbf{Y}^{t,k} \triangleq \mathbf{Y}^t(k) \cup \mathbf{Y}^{t,k-1} \tag{1}$$

The corresponding innovations are for $i = 1, \ldots, m_k^t$

$$\tilde{\mathbf{y}}_i^t(k) \triangleq \mathbf{y}_i^t(k) - \hat{\mathbf{y}}^t(k|k-1) \tag{2}$$

If we consider only one target t (we don't care about some other existing targets in the environment) we can introduce the following integrated data association events

$$\begin{aligned}
\mathcal{E}_{-i}^t(k) &\triangleq \bar{O}_k^t \cap \theta_i^t(k) \qquad i = 1, \ldots, m_k^t \\
\mathcal{E}_{\bar{0}}^t(k) &\triangleq \bar{O}_k^t \cap \theta_0^t(k) \\
\mathcal{E}_0^t(k) &\triangleq O_k^t \cap \theta_0^t(k) \\
\mathcal{E}_i^t(k) &\triangleq O_k^t \cap \theta_i^t(k) \qquad i = 1, \ldots, m_k^t
\end{aligned}$$

where O_k^t, $\bar{O}_k^t$, $\theta_i^t(k)$ $(i = 0, \ldots, m_k^t)$ correspond to the following exclusive and exhaustive events

$$\begin{aligned}
O_k^t &\triangleq \{\text{target } t \text{ is perceivable}\} \\
\bar{O}_k^t &\triangleq \{\text{target } t \text{ is unperceivable}\}
\end{aligned}$$

and

$$\begin{aligned}\theta_i^t(k) &\triangleq \{y_i^t(k) \text{ comes from target } t\}\\ \theta_0^t(k) &\triangleq \{\text{none of } y_i^t(k) \text{ comes from target } t\}\end{aligned}$$

The IPDA approach [13] is built with the assumptions that the estimation errors have Gaussian densities at each step and the perceivability state can be modeled as a first-order homogeneous Markov-chain. It is also assumed that the target measurement is detected with probability P_d^t and the number of false measurements follows a given distribution μ_F. Moreover false measurements are assumed to be uniformly distributed in measurement space. In the IPDA filtering approach, when $m_k^t > 0$, the conditional mean estimate $\hat{\mathbf{x}}^t(k|k)$ is obtained by

$$\hat{\mathbf{x}}^t(k|k) = \hat{\mathbf{x}}^t(k|k-1) + \mathbf{K}^t(k)\tilde{\mathbf{y}}^t(k) \tag{3}$$

where the combined innovation $\tilde{\mathbf{y}}^t(k)$ is given by

$$\tilde{\mathbf{y}}^t(k) \triangleq \sum_{i=1}^{m_k^t} \beta_i^t \tilde{\mathbf{y}}_i^t(k) \tag{4}$$

where $\beta_i^t(k)$ $(i = \bar{0}, 0, 1 \ldots m_k^t)$ are the posterior integrated association probabilities defined as

$$\beta_i^t(k) \triangleq P\{\mathcal{E}_i^t(k)|\mathbf{Y}^{t,k}\} \tag{5}$$

Expressions for these probabilities can be found in [13]. Probabilities $\beta_{-i}^t(k) = P\{\mathcal{E}_{-i}^t(k)|\mathbf{Y}^{t,k}\}$ are all zeroes since $\mathcal{E}_{-i}^t(k)$ are all empty.

The update of the covariance equation is given by

$$\begin{aligned}\mathbf{P}^t(k|k) &= \beta_{\bar{0}}^t(k)\mathbf{P}^t(k|k-1)\\ &\quad +\beta_0^t(k)[\mathbf{I} + q_0^t\mathbf{K}^t(k)\mathbf{H}^t(k)]\mathbf{P}^t(k|k-1)\\ &\quad +(1-\beta_{\bar{0}}^t(k) - \beta_0^t(k))\mathbf{P}^{t,c}(k|k) + \tilde{\mathbf{P}}^t(k)\end{aligned} \tag{6}$$

with $\mathbf{P}^{t,c}(k|k) = [\mathbf{I} - \mathbf{K}^t(k)\mathbf{H}^t(k)]\mathbf{P}^t(k|k-1)$ and the semi-definite positive stochastic matrix $\tilde{\mathbf{P}}^t(k)$ has the same expression as in standard PDAF [4]

$$\tilde{\mathbf{P}}^t(k) = \mathbf{K}^t(k)\Big[\sum_{i=1}^{m_k^t} \beta_i^t \tilde{\mathbf{y}}_i^t(k)\tilde{\mathbf{y}}_i^{t'}(k) - \tilde{\mathbf{y}}^t(k)\tilde{\mathbf{y}}^{t'}(k)\Big]\mathbf{K}^{t'}(k) \tag{7}$$

q_0^t is a weighting factor given by [16, 21, 20]

$$q_0^t \triangleq \frac{P_d^t(P_g - P_{gg})}{1 - P_d^t P_g} \tag{8}$$

where the gate probability P_g and probability P_{gg} are given by

$$P_g \triangleq P\{\chi^2_{n_y} \leq \gamma\} \tag{9}$$

$$P_{gg} \triangleq P\{\chi^2_{n_y+2} \leq \gamma\} \tag{10}$$

When $m_k^t = 0$, updating equations of the IPDAF are

$$\hat{\mathbf{x}}^t(k|k) = \hat{\mathbf{x}}^t(k|k-1) \tag{11}$$

$$\mathbf{P}^t(k|k) = [\mathbf{I} + q_0^t P^{O^t}_{k|k-1,0}\mathbf{K}^t(k)\mathbf{H}^t(k)]\mathbf{P}^t(k|k-1) \tag{12}$$

where $P^{O^t}_{k|k-1,0}$ is the predicted probability of target perceivability. Its expression can be found in [13]. Prediction of the target state and measurement to time $k+1$ are computed as in the standard Kalman filter [4].

3 Multiple interfering targets

The equations above define the IPDA filter for a single target. Several targets could be handled with multiple copies of the IPDAF. However, with respect to any given target, measurements from interfering targets do not behave at all like the random clutter assumed above. Rather, the probability density of each candidate measurement must be computed based upon the densities of all targets that are close enough to interfere and upon the perceivability of each interfering target.

In order to account for this interdependence, consider now a cluster of targets (established tracks) numbered $t = 1, \ldots, T$ at a given time k. The set of m_k candidate measurements associated with this cluster is denoted

$$\mathbf{Y}(k) = \{\mathbf{Y}^1(k) \cup \ldots \mathbf{Y}^T(k)\} \tag{13}$$

Each measurement $\mathbf{y}_i(k)$ of such cluster belongs either to one perceivable target of the set $t = 1, \ldots, T$ or belongs to the set of false measurements, which will be indexed by $t = 0$ in the sequel.

Denoting $\hat{\mathbf{y}}^t(k|k-1)$ the predicted measurement for target t, the innovation corresponding to measurement i becomes

$$\tilde{\mathbf{y}}_i^t(k) \triangleq \mathbf{y}_i(k) - \hat{\mathbf{y}}^t(k|k-1) \qquad i = 1, \ldots, m_k \tag{14}$$

and the combined innovation becomes

$$\tilde{\mathbf{y}}^t(k) = \sum_{i=1}^{m_k} \beta_i^t(k)\tilde{\mathbf{y}}_i^t(k) \tag{15}$$

where $\beta_i^t(k)$ is the integrated posterior probability that measurement i originated from perceivable target t. $\beta_0^t(k)$ is the probability that none of measurements originated from perceivable target t and $\beta_{\bar{0}}^t(k)$ is the probability that target t is unperceivable at time k by the sensor. This is used in target t's state estimation equation to update the estimate $\hat{\mathbf{x}}^t(k|k)$.

In other words, the integrated Joint Probabilistic Data Association (IJPDA) and IPDA approaches utilize the same estimation equations; the difference is in the way the integrated association probabilities $\beta_i^t(k)$ will be computed. Wheras the IPDA algorithm computes $\beta_i^t(k)$, $i = \bar{0}, 0, \ldots, m_k$ separately for each target t, under the assumption that *all measurements not associated with target t are false* with taking into account the perceibability of the target, the IJPDA algorithm computes $\beta_i^t(k)$ *jointly* across the set of T targets and clutter. From the point of view of any target, this accounts for false measurements arising from both discrete interfering targets and random clutter. Derivation of thes probabilities are given in the next section.

4 Joint association probabilities

The key to the **standard JPDA algorithm** [3, 4] is based on the evaluation of the conditional probabilities of all the following feasible joint events :

$$\Theta(k) = \bigcap_{i=1}^{m_k} \Theta_i^{t_i}(k) \tag{16}$$

where $\Theta_i^{t_i}(k)$ is the event that the measurement i originated from origin t_i, $0 \leq t_i \leq T$. $t_i > 0$ is the index of the target to which measurement i is associated at time k. $t_i = 0$ means that measurement i is a false measurement. The feasible events are those joint events in which no more than one measurement originates from each target.

Actually, another better equivalent expression for a (classical) feasible joint association event $\Theta(k)$ is

$$\Theta(k) = \Big[\bigcap_{i=1}^{m_k} \mathcal{O}_i(k)\Big] \bigcap \Big[\bigcap_{t=1}^{T} \mathcal{P}_t(k) \equiv O_k^t\Big] \tag{17}$$

where $\mathcal{O}_i(k)$ represents the origin (clutter, target 1, ... or target T) of measurement i and $\mathcal{P}_t(k)$ is the perceivability state for target t. In the **standard JPDAF development**, all target are implicitly assumed to be perceivable (i.e. $P\{\mathcal{P}_t(k) = O_k^t\} \equiv 1$). This notation will make more sense for the definition of integrated feasible joint association events in the sequel. Thus, in the standard JPDAF, the previous expression reduces to

$$\Theta(k) = \Big[\bigcap_{i=1}^{m_k} \mathcal{O}_i(k)\Big] \tag{18}$$

The probabilities $\beta_i^t(k)$ that measurement i belongs to target t (implicitly assumed to be perceivable by the sensor) is obtained by summing over all feasible events $\Theta(k)$ for which this condition is true, that is

$$\beta_i^t(k) = \sum_{\Theta(k)} P\{\Theta|\mathbf{Y}^k\}\hat{\omega}_{it}(\Theta) \qquad i = 1, \ldots, m_k \tag{19}$$

$$\beta_0^t(k) = 1 - \sum_{i=1}^{m_k} \beta_i^t(k) \tag{20}$$

where time index k has been dropped for notation convenience and $\hat{\omega}_{it}(\Theta)$ is the corresponding component of the feasible association matrix

$$\hat{\Omega}(\Theta) = [\hat{\omega}_{it}(\Theta)] \tag{21}$$

This event matrix represents a feasible joint association event Θ whenever the following conditions are fulfilled

- any event matrix $\hat{\Omega}(\Theta)$ must be compatible with validation matrix Ω

$$\hat{\omega}_{it}(\Theta) = \begin{cases} 1 & \text{if } \Theta_i^t \in \Theta \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

- a measurement has only one origin

$$\sum_{t=0}^{T} \hat{\omega}_{it}(\Theta) = 1 \qquad \forall i \tag{23}$$

- at most one measurement arises from a target

$$\sum_{i=1}^{m_k} \hat{\omega}_{it}(\Theta) \leq 1 \qquad t = 1, \ldots, T \tag{24}$$

Example

Consider the following validation matrix for two targets

$$\Omega = \begin{array}{cc} & t \quad 0 \quad 1 \quad 2 \\ j & \\ 1 & \quad\; 1 \quad 1 \quad 0 \\ 2 & \quad\; 1 \quad 1 \quad 1 \\ 3 & \quad\; 1 \quad 0 \quad 1 \end{array} \tag{25}$$

Then the set of feasible joint association matrices is

$$\hat{\Omega}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \hat{\Omega}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\hat{\Omega}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \hat{\Omega}_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\hat{\Omega}_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \hat{\Omega}_6 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\hat{\Omega}_7 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \hat{\Omega}_8 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Generation of such feasible event matrices $\hat{\Omega}$ can be done from the initial validation matrix [6] Ω by using different kind of fast algorithms [14, 11, 29, 10].

Assuming the states of the targets conditioned on the past observations mutually independent, the posterior probability of a feasible joint association event is given by [6]

$$P\{\Theta|\mathbf{Y}^k\} = \frac{1}{c}\frac{\Phi(\Theta)!}{m_k!}\frac{\mu_F(\Phi(\Theta))}{V^{\Phi(\Theta)}}\prod_{i=1}^{m_k}[e_{t_i}(\mathbf{y}_i(k))]^{\tau_i(\Theta)}\prod_{t=1}^{T}[P_d^t]^{\delta_t(\Theta)}[1-P_d^t]^{1-\delta_t(\Theta)} \tag{26}$$

where c is a normalization constant and $\mu_F(\Phi)$ is the probability mass function (pmf) of the number of false measurements and V the volume of the surveillance region.

$e_{t_i}(\mathbf{y}_i(k)) \triangleq \mathcal{N}[\mathbf{y}_i(k); \hat{\mathbf{y}}^{t_i}(k|k-1), \mathbf{S}^{t_i}(k)]$ is the likelihood function of the measurement $\mathbf{y}_i(k)$ associated with target t_i. $\hat{\mathbf{y}}^{t_i}(k|k-1)$ is the predicted measurement for target t_i with associated innovation covariance $\mathbf{S}^{t_i}(k)$.

$\delta_t(\Theta)$, $\tau_i(\Theta)$ and $\Phi(\Theta)$ are respectively the target detection, measurement association and false measurement indicators of the event Θ_k under consideration. These indicators are defined as

$$\delta_t(\Theta) \triangleq \sum_{i=1}^{m_k}\hat{\omega}_{it}(\Theta) \leq 1 \qquad t = 1, \ldots, T \tag{27}$$

$$\tau_i(\Theta) \triangleq \sum_{t=1}^{T}\hat{\omega}_{it}(\Theta) \tag{28}$$

$$\Phi(\Theta) \triangleq \sum_{i=1}^{m_k}[1-\tau_i(\Theta)] \tag{29}$$

According to the model used for the pmf $\mu_F(\Phi)$ of the number of false measurements, two versions of JPDA have been proposed in [3, 4]

- **Parametric JPDA :** If we assume a Poisson pmf for $\mu_F(\Phi)$ which requires the spatial density λ of the false measurements,

$$\mu_F(\Phi) = \frac{(\lambda V)^\Phi}{\Phi!}e^{-\lambda V} \tag{30}$$

Thus the joint association probabilities are given by

$$P\{\Theta|\mathbf{Y}^k\} = \frac{1}{c}\prod_{i=1}^{m_k}[\lambda^{-1}e_{t_i}(\mathbf{y}_i(k))]^{\tau_i(\Theta)}\prod_{t=1}^{T}[P_d^t]^{\delta_t(\Theta)}[1-P_d^t]^{1-\delta_t(\Theta)} \tag{31}$$

where c is a new normalization constant.

- **Non parametric JPDA :** If we assume a diffuse prior pmf for $\mu_F(\Phi)$,

$$\mu_F(\Phi) = \epsilon \tag{32}$$

the joint association probabilities are now given by

$$P\{\Theta|\mathbf{Y}^k\} = \frac{\Phi!}{c}\prod_{i=1}^{m_k}[Ve_{t_i}(\mathbf{y}_i(k))]^{\tau_i(\Theta)}\prod_{t=1}^{T}[P_d^t]^{\delta_t(\Theta)}[1-P_d^t]^{1-\delta_t(\Theta)} \tag{33}$$

where c is a new normalization constant.

5 Integrated joint association probabilities

The derivation of the integrated joint posterior probabilities is based on the evaluation of the conditional probabilities of all the feasible integrated joint events which take into account the perceivability of targets involved in the data association process. To clarify this, we give first a simple example of the IJPDA process.

5.1 Example

Consider as previously the following validation matrix for a two targets case

$$\Omega = \begin{array}{cccc} & t & 0 & 1 & 2 \\ j & & & & \\ 1 & & 1 & 1 & 0 \\ 2 & & 1 & 1 & 1 \\ 3 & & 1 & 0 & 1 \end{array} \tag{34}$$

The previous feasible event matrices $\hat{\Omega}(\Theta)$ must now been modified to take into account the perceivability or unperceivability of each target involved in a feasible joint data association event. This can be done by adding a row corresponding to a dummy measurement with indice $j = 0$. This row will describe the perceivability state of each target t. Every binary component $\hat{\omega}_{0t}$ $(t > 0)$ will characterize the perceivability of target t when $\hat{\omega}_{0t} = 1$ and unperceivability of t when $\hat{\omega}_{0t} = 0$. The dummy component $\hat{\omega}_{00}$ can take any arbitrary value. By convention we will always set $\hat{\omega}_{00} = 0$ in the following. Now, the set of integrated feasible event matrices $\hat{\Omega}^I$ can be obtained from the set of (unintegrated) feasible event matrices $\hat{\Omega}$ as follows :

$$\hat{\Omega}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \rightarrow \hat{\Omega}_1^I = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \hat{\Omega}_2^I = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\hat{\Omega}_3^I = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \hat{\Omega}_4^I = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\hat{\Omega}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \hat{\Omega}_5^I = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \hat{\Omega}_6^I = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\hat{\Omega}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \rightarrow \hat{\Omega}_7^I = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \hat{\Omega}_8^I = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\hat{\Omega}_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \hat{\Omega}_9^I = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\hat{\Omega}_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \rightarrow \hat{\Omega}_{10}^I = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \hat{\Omega}_{11}^I = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\hat{\Omega}_6 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \rightarrow \hat{\Omega}_{12}^I = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \hat{\Omega}_{13}^I = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\hat{\Omega}_7 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \hat{\Omega}_{14}^I = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\hat{\Omega}_8 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \rightarrow \hat{\Omega}_{15}^I = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

where the additional following **feasibility constraint** :

(C1) : Any detected target is necessarily perceivable has been used for generating $\hat{\Omega}^I$ from $\hat{\Omega}$.

Every feasible integrated joint association matrix $\hat{\Omega}^I_j, j = 1 \ldots, 15$ charaterizes an **integrated joint association event** $\mathcal{E}_j$. Let $P(\mathcal{E}_j|\mathbf{Y}^k)$ be the integrated joint posterior probability of $\mathcal{E}_j$. Since events $\mathcal{E}_j$ are exhaustive and mutually exclusive, we always have

$$\sum_j P(\mathcal{E}_j|\mathbf{Y}^k) = 1 \tag{35}$$

When $P(\mathcal{E}_j|\mathbf{Y}^k)$ are known (see next section for details), the marginal integrated association probabilities $\beta^t_i(k), i = \bar{0}, 0, \ldots, m_k$ will be obtained from the integrated joint probabilities by summing over all the integrated joint events $\mathcal{E}_j$ in which the marginal event of interest occurs. In our example, the marginal probabilities for target $t = 1$ and $t = 2$ will be obtained as follows

Marginal integrated association probabilities for target t_1

$$\beta^1_{\bar{0}}(k) = P\{\bar{O}^1_k \cap \theta^1_0(k)|\mathbf{Y}^k\} = P(\mathcal{E}_1|\mathbf{Y}^k) + P(\mathcal{E}_4|\mathbf{Y}^k) + P(\mathcal{E}_5|\mathbf{Y}^k) + P(\mathcal{E}_{11}|\mathbf{Y}^k)$$
$$\beta^1_0(k) = P\{O^1_k \cap \theta^1_0(k)|\mathbf{Y}^k\} = P(\mathcal{E}_2|\mathbf{Y}^k) + P(\mathcal{E}_3|\mathbf{Y}^k) + P(\mathcal{E}_6|\mathbf{Y}^k) + P(\mathcal{E}_{10}|\mathbf{Y}^k)$$
$$\beta^1_1(k) = P\{O^1_k \cap \theta^1_1(k)|\mathbf{Y}^k\} = P(\mathcal{E}_{12}|\mathbf{Y}^k) + P(\mathcal{E}_{13}|\mathbf{Y}^k) + P(\mathcal{E}_{14}|\mathbf{Y}^k) + P(\mathcal{E}_{15}|\mathbf{Y}^k)$$
$$\beta^1_2(k) = P\{O^1_k \cap \theta^1_2(k)|\mathbf{Y}^k\} = P(\mathcal{E}_7|\mathbf{Y}^k) + P(\mathcal{E}_8|\mathbf{Y}^k) + P(\mathcal{E}_9|\mathbf{Y}^k)$$
$$\beta^1_3(k) = P\{O^1_k \cap \theta^1_3(k)|\mathbf{Y}^k\} = 0$$

Marginal integrated association probabilities for target t_2

$$\beta^2_{\bar{0}}(k) = P\{\bar{O}^2_k \cap \theta^2_0(k)|\mathbf{Y}^k\} = P(\mathcal{E}_1|\mathbf{Y}^k) + P(\mathcal{E}_2|\mathbf{Y}^k) + P(\mathcal{E}_8|\mathbf{Y}^k) + P(\mathcal{E}_{13}|\mathbf{Y}^k)$$
$$\beta^2_0(k) = P\{O^2_k \cap \theta^2_0(k)|\mathbf{Y}^k\} = P(\mathcal{E}_3|\mathbf{Y}^k) + P(\mathcal{E}_4|\mathbf{Y}^k) + P(\mathcal{E}_7|\mathbf{Y}^k) + P(\mathcal{E}_{12}|\mathbf{Y}^k)$$
$$\beta^2_1(k) = P\{O^2_k \cap \theta^2_1(k)|\mathbf{Y}^k\} = 0$$
$$\beta^2_2(k) = P\{O^2_k \cap \theta^2_2(k)|\mathbf{Y}^k\} = P(\mathcal{E}_{10}|\mathbf{Y}^k) + P(\mathcal{E}_{11}|\mathbf{Y}^k) + P(\mathcal{E}_{15}|\mathbf{Y}^k)$$
$$\beta^2_3(k) = P\{O^2_k \cap \theta^2_3(k)|\mathbf{Y}^k\} = P(\mathcal{E}_5|\mathbf{Y}^k) + P(\mathcal{E}_6|\mathbf{Y}^k) + P(\mathcal{E}_9|\mathbf{Y}^k) + P(\mathcal{E}_{14}|\mathbf{Y}^k)$$

One can easily check that

$$\sum_{i=\bar{0},0,1,\ldots,m_k} \beta^t_i(k) = 1 \qquad \forall t = 1, 2 \tag{36}$$

The state estimation equations will be exactly the same as in the IPDAF presented in the first section.

5.2 Derivation of integrated joint probabilities

We define an **integrated joint association event** $\mathcal{E}$ pertaining to the current time k as

$$\mathcal{E}(k) = \Big[\bigcap_{i=1}^{m_k} \mathcal{O}_i(k)\Big] \bigcap \Big[\bigcap_{t=1}^{T} \mathcal{P}_t(k)\Big] \tag{37}$$

where $\mathcal{O}_i(k)$ represents the source (clutter, target 1, ... or target T) of measurement i and $\mathcal{P}_t(k)$ represents the perceivability state for target t ($\mathcal{P}_t(k) = O_k^t$ when target t is perceivable or $\mathcal{P}_t(k) = \bar{O}_k^t$ otherwise). Such event characterizes both the origin of all measurements and the perceivability state of all targets.

Every integrated joint association event $\mathcal{E}(k)$ can be represented by an **integrated event matrix** of size $(m_k + 1) \times (T + 1)$

$$\hat{\Omega}^I(\mathcal{E}) = [\hat{\omega}_{it}(\mathcal{E})] \tag{38}$$

consisting of the units in the validation matrix Ω corresponding to the integrated association in $\mathcal{E}$, i.e. for $t = 0, 1, \dots, T$ and $i = 1, \dots, m_k$

$$\hat{\omega}_{it}(\mathcal{E}) = \begin{cases} 1 & \text{if } (\mathcal{O}_i(k) = t) \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases} \tag{39}$$

and for $t = 1, \dots, T$

$$\hat{\omega}_{0t}(\mathcal{E}) = \begin{cases} 1 & \text{if } (\mathcal{P}_t(k) = O_k^t) \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases} \tag{40}$$

A **feasible integrated association event** is one satisfying the following constraints

(1) a measurement has only one origin, i.e.,

$$\sum_{t=0}^{T} \hat{\omega}_{it}(\mathcal{E}) = 1 \qquad \forall i > 0 \tag{41}$$

(2) at most one measurement originates from a (perceivable) target

$$\delta_t(\mathcal{E}) \triangleq \sum_{i=1}^{m_k} \hat{\omega}_{it}(\mathcal{E}) \leq 1 \qquad t = 1, \dots, T \tag{42}$$

(3) any detected target is necessarily perceivable

$$\hat{\omega}_{0t}(\mathcal{E}) - \delta_t(\mathcal{E}) \geq 0 \qquad t = 1, \dots, T \tag{43}$$

The binary variable $\delta_t(\mathcal{E})$ is called the **target detection indicator** since it indicates wether target t has been detected under $\mathcal{E}$. The **measurement association indicator** $\tau_i(\mathcal{E})$ and **false measurement indicator** $\Phi(\mathcal{E})$ are defined in the same way as in the JPDA approach, i.e.

$$\tau_i(\mathcal{E}) \triangleq \sum_{t=1}^{T} \hat{\omega}_{it}(\mathcal{E}) \tag{44}$$

$$\Phi(\mathcal{E}) \triangleq \sum_{i=1}^{m_k} [1 - \tau_i(\mathcal{E})] \tag{45}$$

The binary component $\pi_t(\mathcal{E}) \triangleq \hat{\omega}_{0t}(\mathcal{E})$ is called the **target perceivability indicator** since it indicates whether the target is perceivable in the integrated joint event $\mathcal{E}$.

The generation of the integrated event matrices $\hat{\Omega}^I(\mathcal{E})$ can be obtained from every (non integrated) feasible event matrice $\hat{\Omega}(\Theta)$ by adding any row $i = 0$ which characterizes the feasible perceivability state for all targets. Hence from every given event matrix $\hat{\Omega}(\Theta)$, we will have to generate N_Θ integrated event matrices $\hat{\Omega}^I(\mathcal{E})$ where

$$N_\Theta = \prod_{t=1}^{T} 2^{1-\delta_t(\Theta)} \tag{46}$$

As in the classical JPDA approach, the evaluation of the integrated joint association event probabilities are obtained with Bayes' formula as follows

$$\begin{aligned} P\{\mathcal{E}|\mathbf{Y}^k\} &= P\{\mathcal{E}|\mathbf{Y}(k), m_k, \mathbf{Y}^{k-1}\} \\ &= \frac{1}{c} p[\mathbf{Y}(k)|\mathcal{E}, m_k, \mathbf{Y}^{k-1}] P\{\mathcal{E}|m_k\} \end{aligned}$$

where c is a normalization constant.

If we assume the states of targets given the past observations mutually independent, the **likelihood function of the measurements** $p[\mathbf{Y}(k)|\mathcal{E}, m_k, \mathbf{Y}^{k-1}]$ is exactly the same as the one derived in the stantard JPDA, i.e.

$$p[\mathbf{Y}(k)|\mathcal{E}, m_k, \mathbf{Y}^{k-1}] = V^{-\Phi(\mathcal{E})} \prod_{i=1}^{m_k} [e_{t_i}(\mathbf{y}_i(k))]^{\tau_i(\mathcal{E})} \tag{47}$$

where $e_{t_i}(\mathbf{y}_i(k)) \triangleq \mathcal{N}[\mathbf{y}_i(k); \hat{\mathbf{y}}^{t_i}(k|k-1), \mathbf{S}^{t_i}(k)]$ is the likelihood function of the measurement $\mathbf{y}_i(k)$ associated with target $t_i = \mathcal{O}_i(\mathcal{E})$. $\hat{\mathbf{y}}^{t_i}(k|k-1)$ is the predicted measurement for target t_i with associated innovation covariance $\mathbf{S}^{t_i}(k)$. V is the volume of the surveillance region.

The **prior probability** of an integrated joint association event is given now by

$$\begin{aligned} P\{\mathcal{E}|m_k\} = \frac{\Phi(\mathcal{E})!}{m_k!} \mu_F(\Phi(\mathcal{E})) \prod_{t=1}^{T} [P_d^t]^{\delta_t(\mathcal{E})} [1 - P_d^t]^{1-\delta_t(\mathcal{E})} \\ \times \prod_{t=1}^{T} [P_{k|k-1,m_k}^{O^t}]^{\pi_t(\mathcal{E})} [1 - P_{k|k-1,m_k}^{O^t}]^{1-\pi_t(\mathcal{E})} \end{aligned} \tag{48}$$

where $\mu_F(\Phi)$ is the probability mass function (pmf) of the number of false measurements. $P^{O^t}_{k|k-1,m_k}$ is the conditional predicted target perceivability which can be easily computed on line (see in [13] for details).

The posterior probability $P\{\mathcal{E}|\mathbf{Y}^k\}$ of an integrated joint association event is thus given by

$$P\{\mathcal{E}|\mathbf{Y}^k\} = \frac{1}{c}\frac{\Phi(\mathcal{E})!}{m_k!}\frac{\mu_F(\Phi(\mathcal{E}))}{V^{\Phi(\mathcal{E})}}\prod_{i=1}^{m_k}[e_{t_i}(\mathbf{y}_i(k))]^{\tau_i(\mathcal{E})} \times \prod_{t=1}^{T}[P_d^t]^{\delta_t(\mathcal{E})}[1-P_d^t]^{1-\delta_t(\mathcal{E})} \times \prod_{t=1}^{T}[P^{O^t}_{k|k-1,m_k}]^{\pi_t(\mathcal{E})}[1-P^{O^t}_{k|k-1,m_k}]^{1-\pi_t(\mathcal{E})} \quad (49)$$

Depending on the model used for the pmf $\mu_F(\Phi)$, two versions of IJPDA can be used

- **Parametric IJPDA :** If we assume a Poisson pmf for $\mu_F(\Phi)$ (which requires the spatial density λ of the false measurements), the integrated joint association probabilities become

$$P\{\mathcal{E}|\mathbf{Y}^k\} = \frac{1}{c}\prod_{i=1}^{m_k}[\lambda^{-1}e_{t_i}(\mathbf{y}_i(k))]^{\tau_i(\mathcal{E})} \times \prod_{t=1}^{T}[P_d^t]^{\delta_t(\mathcal{E})}[1-P_d^t]^{1-\delta_t(\mathcal{E})} \times \prod_{t=1}^{T}[P^{O^t}_{k|k-1,m_k}]^{\pi_t(\mathcal{E})}[1-P^{O^t}_{k|k-1,m_k}]^{1-\pi_t(\mathcal{E})} \quad (50)$$

  where c is a new normalization constant.

- **Non parametric IJPDA :** If we assume a diffuse prior pmf for $\mu_F(\Phi)$, the integrated joint association probabilities become

$$P\{\mathcal{E}|\mathbf{Y}^k\} = \frac{\Phi(\mathcal{E})!}{c}\prod_{i=1}^{m_k}[Ve_{t_i}(\mathbf{y}_i(k))]^{\tau_i(\mathcal{E})} \times \prod_{t=1}^{T}[P_d^t]^{\delta_t(\mathcal{E})}[1-P_d^t]^{1-\delta_t(\mathcal{E})} \times \prod_{t=1}^{T}[P^{O^t}_{k|k-1,m_k}]^{\pi_t(\mathcal{E})}[1-P^{O^t}_{k|k-1,m_k}]^{1-\pi_t(\mathcal{E})} \quad (51)$$

  where c is a new normalization constant.

If we assume the states of the targets given the past mutually independent, one needs the **integrated marginal association probabilities** which are obtained from the integrated joint probabilities by summing over all the integrated joint event $\mathcal{E}$ in which the integrated marginal event of interest occurs. Hence we will have for $t = 1, \dots, T$

$$\beta_i^t(k) \triangleq P\{O_k^t \cap \theta_i^t(k)\} = \sum_{\mathcal{E}} P\{\mathcal{E}|\mathbf{Y}^k\}\hat{\omega}_{it}(\mathcal{E}) \tag{52}$$

$$\beta_0^t(k) \triangleq P\{O_k^t \cap \theta_0^t(k)\} = \sum_{\mathcal{E}} P\{\mathcal{E}|\mathbf{Y}^k\}[1-\delta_t(\mathcal{E})]\pi_t(\mathcal{E}) \tag{53}$$

$$\beta_{\bar{0}}^t(k) \triangleq P\{\bar{O}_k^t \cap \theta_0^t(k)\} = \sum_{\mathcal{E}} P\{\mathcal{E}|\mathbf{Y}^k\}[1-\delta_t(\mathcal{E})][1-\pi_t(\mathcal{E})] \tag{54}$$

Once the integrated marginal probabilities $\beta_i^t(k)$ $(i = \bar{0}, 0, \dots, m_k)$ are computed, the state estimation equations similar to those in the IPDAF can be used for track maintenance, termination and confirmation.

5.3 Simplification for IJPDAF

For integrated joint probabilities $P\{\mathcal{E}|\mathbf{Y}^k\}$ evaluations, a huge number of integrated event matrices has to be generated by IJPDAF comparatively to the standard JPDA approach. This could become a severe drawback for IJPDAF specially for heavy clutter/multitarget tracking applications. But even if this remark is perfectly true, we must however point out the fact that only integrated marginal probabilities $\beta_i^t(k)$ are required for track maintenance. The good news is that evaluation of $\beta_i^t(k)$ does not require the generation of all integrated event matrices $\hat{\Omega}^I(\mathcal{E})$ at all but only unintegrated event matrices $\hat{\Omega}(\Theta)$. Actually, it can be shown from (52)-(54) with elementary algebra that $\beta_i^t(k)$ $(i = \bar{0}, 0, 1, \dots, m_k)$ can be finally expressed as

$$\beta_i^t(k) = \sum_{\Theta} P\{\Theta|\mathbf{Y}^k\} P_{k|k-1,m_k}^{O^t} \prod_{j \neq t} [P_{k|k-1,m_k}^{O^j}]^{\delta_j(\Theta)} \hat{\omega}_{it}(\Theta) \tag{55}$$

$$\beta_0^t(k) = \sum_{\Theta} P\{\Theta|\mathbf{Y}^k\} P_{k|k-1,m_k}^{O^t} \prod_{j \neq t} [P_{k|k-1,m_k}^{O^j}]^{\delta_j(\Theta)} [1-\delta_t(\Theta)] \tag{56}$$

$$\beta_{\bar{0}}^t(k) = \sum_{\Theta} P\{\Theta|\mathbf{Y}^k\} [1-P_{k|k-1,m_k}^{O^t}] \prod_{j \neq t} [P_{k|k-1,m_k}^{O^j}]^{\delta_j(\Theta)} [1-\delta_t(\Theta)] \tag{57}$$

Hence the cost of computation involved in the IJPDAF is almost the same than the cost required within the standard JPDAF. Only the cost of computation of predicted target perceivability probability (which is not computationally greedy) must be add to the computation cost of the standard JPDAF. Moreover expressions (55)-(57) become fully consistent with those of standard JPDAF as soon as target perceivability probabilities tend towards unity.

5.4 Remark

In the preceding, targets' states given the past were assumed mutually independent. Actually, we could also consider the targets' state, given the past, as correlated and perform a coupled estimation for the targets under consideration. This will yield the IJPDA coupled filter (IJPDACF). Details on this approach for JPDACF can be found in [6]. Furthermore amplitude information can also be included in the IJPDAF without any difficulty by the way already described in [18, 12, 20].

6 Conclusion

A new theoretical development of an integrated version of JPDAF has been given here. After a rigorous derivation of integrated joint association probabilities based on an enumeration of all feasible integrated event matrices, we have been able to express the integrated marginal association probabilities in a very simple form which requires only the generation of (unintegrated) event matrices as for the standard JPDAF. This important result shows that the cost of IJPDAF is comparable to JPDAF. Target state estimation is done by the IPDAF equations and track confirmation/termination can be performed using quite new procedures based on sequential probability ratio test (SPRT) or optimal design thresholdings. The IJPDAF formulation is fully consistent with the standard JPDAF when the perceivability probability of each target becomes one. Simulations results and tracking performance analysis of IJPDAF will be reported in forthcoming papers.

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