A NEW FORMULATION OF IPDAF FOR TRACKING IN CLUTTER

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Abstract

An improved version of Integrated Probabilistic Data Association Filter (IPDAF) based on a new concept of probability of target perceivability was introduced recently. This paper presents theoretical results of a new formulation for IPDAF. Validity and comparison of this new IPDAF with its previous version is discussed through simulations results presented at the end of the paper. This provides a new basis of an integrated approach to track initiation, confirmation, termination and maintenance.

1 Introduction

The purpose of tracking is to estimate the state of a target based on a set of measurements provided by a sensor. For tracking in a clutter-free environment, the target is always assumed perceivable and the measurement is assumed to be available, unique and to arise from the target at every scan. In such a case, tracking follows conventional recursive filtering. For tracking in clutter, conventional filtering techniques cannot be used because several measurements are available at every scan and at most one measurement can arise from target when the target is perceivable by the sensor. Moreover tracking target in clutter involves track initiation, confirmation, maintenance and termination. Track initiation, confirmation and termination are basically decision problems whereas track maintenance is an estimation problem compounded with measurement uncertainty. We focuse our presentation here mainly on the track maintenance problem. Track confirmation and termination criterion will be shortly discussed at the end of this paper before simulation results analysis. Many probabilistic descriptions of decisions on track initiation, confirmation and termination have been developed in the past [10]. A fundamental limitation of the Probabilistic Data Association Filter (PDAF) [1, 3] is the implicit strong assumption that the target is always perceivable. Of course in many real situations, this is not the case. A new algorithm, called now Integrated Probabilistic Data Association Filter (IPDAF), was first developed by Colegrove and al. [4] and then by Musicki and al. [13, 14] in order to remove the implicit strong assumption on target perceivability done by Bar-Shalom and Tse. Up to now the most appealing algorithm is the latest version of the IPDAF developed recently in [8, 11] which includes a more rigourous concept of target perceivability into its formalism than before. The development of a new IPDAF version presented here follows the approach of Colegrove rather than the Musicki's. Simulations results show that this new IPDAF formulation performs a little bit better than the previous one [11] and has also a theoretical justification.

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2 Target perceivability

At time k, the target state of perceivability and its complement is represented by the exhaustive and exclusive events

$$
\begin{aligned}
O_k &\triangleq \{\text{target is perceivable}\} \\
\bar{O}_k &\triangleq \{\text{target is unperceivable}\}
\end{aligned}
$$

O_k will denote both the fact that target is perceivable and the random event. When there are validated measurements at time k, the intersection of these events with the classical data association (DA) events involved in PDAF formalism [1]

$$
\begin{aligned}
\theta_i(k) &\triangleq \{y_i(k) \text{ comes from target}\} \\
\theta_0(k) &\triangleq \{\text{none of } y_i(k) \text{ comes from target}\}
\end{aligned}
$$

defines a new set of existence events :

$$
\begin{aligned}
\mathcal{E}_{-i}(k) &\triangleq \bar{O}_k \cap \theta_i(k) \qquad i = 1, \ldots, m_k \\
\mathcal{E}_{\bar{0}}(k) &\triangleq \bar{O}_k \cap \theta_0(k) \\
\mathcal{E}_0(k) &\triangleq O_k \cap \theta_0(k) \\
\mathcal{E}_i(k) &\triangleq O_k \cap \theta_i(k) \qquad i = 1, \ldots, m_k
\end{aligned}
$$

Since target measurement cannot arise without target perceivability, events $\mathcal{E}_{-i}(k), i = 1, \ldots, m_k$ are actually impossible. Therefore, we have $\mathcal{E}_{-i}(k) \equiv \emptyset$ and $P\{\mathcal{E}_{-i}(k)|\mathbf{Y}^k\} = P\{\mathcal{E}_{-i}(k)|\mathbf{Y}^{k-1}\} = P\{\mathcal{E}_{-i}(k)\} = 0$ for $i = 1, \ldots, m_k$. Only events $\mathcal{E}_{\bar{0}}(k)$, $\mathcal{E}_0(k)$ and $\mathcal{E}_i(k)$ ($i = 1, \ldots, m_k$) may have a non null probability to occur.

3 Target state estimation

3.1 Case 1 : $m_k \neq 0$

Using the total probability theorem, one has

$$\hat{\mathbf{x}}(k|k) = \beta_{\bar{0}}(k)\hat{\mathbf{x}}_{\bar{0}}(k|k) + \sum_{i=0}^{m_k} \beta_i(k)\hat{\mathbf{x}}_i(k|k)$$

where $\hat{\mathbf{x}}_i(k|k) \triangleq E[\mathbf{x}(k)|\mathcal{E}_i(k), \mathbf{Y}^k]$ is the updated state estimate conditioned on the event $\mathcal{E}_i(k)$ that the target is perceivable and the ith validated measurement is correct. $\hat{\mathbf{x}}_{\bar{0}}(k|k)$ is state estimate conditioned on the event $\mathcal{E}_{\bar{0}}(k)$, that the target is unperceivable and none of the measurements are target-originated. $\beta_i(k) \triangleq P\{\mathcal{E}_i(k)|\mathbf{Y}^k\}$ are called the *integrated* a posteriori data association probabilities because they take into account (integrate) both data association and target perceivability events. Conditional state estimates are given by the classical PDAF formalism

$$\hat{\mathbf{x}}_i(k|k) = \hat{\mathbf{x}}(k|k-1) + \mathbf{K}(k)[\mathbf{y}_i(k) - \hat{\mathbf{y}}(k|k-1)]$$

The gain $\mathbf{K}(k)$ is the same as in the standard Kalman filter [2] since conditioned on $\mathcal{E}_i(k)$ there is no measurement origin uncertainty. For $i = \bar{0}$ and $i = 0$, if none of the measurements is correct (whatever the perceivability of target is), the estimates are :

$$\hat{\mathbf{x}}_{\bar{0}}(k|k) = \hat{\mathbf{x}}_0(k|k) = \hat{\mathbf{x}}(k|k-1)$$

Combining these conditional estimates yields the state update equation

$$\hat{\mathbf{x}}(k|k) = \hat{\mathbf{x}}(k|k-1) + \mathbf{K}(k) \sum_{i=1}^{m_k} \beta_i[\mathbf{y}_i(k) - \hat{\mathbf{y}}(k|k-1)]$$

The covariance $\mathbf{P}(k|k)$ of estimation error is given by

$$\mathbf{P}(k|k) = \mathbf{P}^1 - \hat{\mathbf{x}}(k|k)\hat{\mathbf{x}}'(k|k) \tag{1}$$

with

$$\begin{aligned} \mathbf{P}^1 &= \sum_{i=\bar{0},0,\ldots}^{m_k} \beta_i(k)[\hat{\mathbf{x}}_i(k|k)\hat{\mathbf{x}}_i'(k|k) + \mathbf{P}^c(k|k)] \\ \mathbf{P}^c(k|k) &= [\mathbf{I} - \mathbf{K}(k)\mathbf{H}(k)]\mathbf{P}(k|k-1) \end{aligned}$$

When the target is present in the environment but is not perceivable (hypothesis $\mathcal{E}_{\bar{0}}(k)$) by the sensor at time k for some reason (i.e. sensor momentary down, target occasionally hidden, etc), the best we can do is to use the predicted value of the state estimation error covariance as the updated one, that is

$$\mathbf{P}_{\bar{0}}(k|k) = \mathbf{P}(k|k-1)$$

Under hypothesis $\mathcal{E}_0(k)$, the covariance $\mathbf{P}_0(k|k)$ is given by [7, 8, 9]

$$\mathbf{P}_0(k|k) = [\mathbf{I} + q_0\mathbf{K}(k)\mathbf{H}(k)]\mathbf{P}(k|k-1) \tag{2}$$

where q_0 is a weighting factor given by [7, 8, 9]

$$q_0 \triangleq \frac{P_d(P_g - P_{gg})}{1 - P_dP_g} \equiv \frac{P_dP_g(1 - c_T)}{1 - P_dP_g} \geq 0$$

P_g, P_{gg} and c_T are given by

$$\begin{aligned} P_g &\triangleq P\{\chi^2_{n_y} \leq \gamma\} \\ P_{gg} &\triangleq P\{\chi^2_{n_y+2} \leq \gamma\} \\ c_T &\triangleq \frac{\Gamma_{\gamma/2}(1 + n_y/2)}{(n_y/2)\Gamma(n_y/2)} \end{aligned}$$

c_T is a ratio of incomplete Gamma function defined by

$$\Gamma_\alpha(x) \triangleq \int_0^\alpha u^{x-1}e^{-u}du$$

From equation (1) and previous expressions, we get

$$\begin{aligned} \mathbf{P}(k|k) = \; & \beta_{\bar{0}}(k)\mathbf{P}(k|k-1) \\ & + \beta_0(k)[\mathbf{I} + q_0\mathbf{K}(k)\mathbf{H}(k)]\mathbf{P}(k|k-1) \\ & + (1 - \beta_{\bar{0}}(k) - \beta_0(k))\mathbf{P}^c(k|k) + \tilde{\mathbf{P}}(k) \end{aligned} \tag{3}$$

The stochastic matrix $\tilde{\mathbf{P}}(k)$ has the same expression as in the standard PDAF [1] and will not be given here due to space limitation.

3.2 Case 2 : $m_k = 0$

When there is no validated measurement in the gate $\mathbf{Y}^k = \{\mathbf{Y}(k) = \emptyset, m_k = 0, \mathbf{Y}^{k-1}\}$, we theoretically have using theorem of total probability

$$\hat{\mathbf{x}}(k|k) = P^O_{k|k-1,0}\hat{\mathbf{x}}^O(k|k) + (1 - P^O_{k|k-1,0})\hat{\mathbf{x}}^{\bar{O}}(k|k)$$

where $P^O_{k|k-1,0}$ is given by (see section 4.1)

$$P^O_{k|k-1,0} = \frac{(1 - P_dP_g)P^O_{k|k-1}}{1 - P_dP_gP^O_{k|k-1}} \tag{4}$$

$P^O_{k|k-1} \triangleq P\{O_k|\mathbf{Y}^{k-1}\}$ is given in section 4.2 and

$$\begin{aligned} \hat{\mathbf{x}}^O(k|k) &\triangleq E[\mathbf{x}(k)|O_k, m_k = 0, \mathbf{Y}^{k-1}] \\ \hat{\mathbf{x}}^{\bar{O}}(k|k) &\triangleq E[\mathbf{x}(k)|\bar{O}_k, m_k = 0, \mathbf{Y}^{k-1}] \end{aligned}$$

But actually, when there is no measurement, there is no way to improve state estimate whatever the perceivability of the target is; thus we have

$$\hat{\mathbf{x}}^{\bar{O}}(k|k) = \hat{\mathbf{x}}^{O}(k|k) = \hat{\mathbf{x}}(k|k-1) \tag{5}$$

and therefore

$$\hat{\mathbf{x}}(k|k) = \hat{\mathbf{x}}(k|k-1) \tag{6}$$

The covariance $\mathbf{P}(k|k)$ associated with the estimation error is then given by

$$\mathbf{P}(k|k) = P^O_{k|k-1,0}\mathbf{P}^O(k|k) + (1 - P^O_{k|k-1,0})\mathbf{P}^{\bar{O}}(k|k)$$

with

$$\begin{aligned} \mathbf{P}^O(k|k) &= [\mathbf{I} + q_0\mathbf{K}(k)\mathbf{H}(k)]\mathbf{P}(k|k-1) \\ \mathbf{P}^{\bar{O}}(k|k) &= \mathbf{P}(k|k-1) \end{aligned}$$

Hence we get

$$\mathbf{P}(k|k) = [\mathbf{I} + q_0 P^O_{k|k-1,0}\mathbf{K}(k)\mathbf{H}(k)]\mathbf{P}(k|k-1) \tag{7}$$

3.3 State prediction

Prediction of the state and measurement is done using classical Kalman filter equations. The predicted covariance is given by

$$\mathbf{P}(k+1|k) = \mathbf{F}(k)\mathbf{P}(k|k)\mathbf{F}'(k) + \mathbf{Q}(k) \tag{8}$$

where $\mathbf{P}(k|k)$ is given by (3) or (7) depending on m_k.

4 Association probabilities

When $m_k \neq 0$, one has to evaluate for $i = \bar{0}, 0, 1 \dots m_k$

$$\beta_i(k) \triangleq P\{\mathcal{E}_i(k)|\mathbf{Y}(k), m_k, \mathbf{Y}^{k-1}\} \tag{9}$$

Using Bayes' rule, one gets

$$\begin{aligned} \beta_i(k) &= \frac{1}{c}P^1_i P^2_i P\{O_k|m_k, \mathbf{Y}^{k-1}\} \\ \beta_0(k) &= \frac{1}{c}P^1_0 P^2_0 P\{O_k|m_k, \mathbf{Y}^{k-1}\} \\ \beta_{\bar{0}}(k) &= \frac{1}{c}P^1_{\bar{0}} P^2_{\bar{0}} P\{\bar{O}_k|m_k, \mathbf{Y}^{k-1}\} \end{aligned}$$

where c is a normalization constant such that $\sum_{i=\bar{0},0,\dots}^{m_k} \beta_i(k) = 1$.

- for $i = 1 \dots m_k$ and assuming the true measurement residual Gaussian distributed in the validation gate of volume V_k and false alarms uniformly and independently distributed in V_k with μ_F, we have [1, 2, 8]

$$\begin{cases} P^1_i \triangleq p[\mathbf{Y}(k)|\mathcal{E}_i(k), m_k, \mathbf{Y}^{k-1}] = V_k^{-m_k+1} e_i(k) \\ P^2_i \triangleq P\{\theta_i(k)|O_k, m_k, \mathbf{Y}^{k-1}\} = \frac{1}{c_1}\frac{P_d P_g}{m_k} \\ P\{O_k|m_k, \mathbf{Y}^{k-1}\} \triangleq P^O_{k|k-1,m_k} \end{cases}$$

$$\begin{aligned} e_i(k) &\triangleq P_g^{-1}\mathcal{N}[(\mathbf{y}_i(k) - \hat{\mathbf{y}}(k|k-1)); 0; \mathbf{S}(k)] \\ c_1 &\triangleq P_d P_g + (1 - P_d P_g)\xi_k \\ \xi_k &\triangleq \frac{\mu_F(m_k)}{\mu_F(m_k - 1)} \end{aligned}$$

- for $i = 0$, we have

$$\begin{cases} P^1_0 \triangleq p[\mathbf{Y}(k)|\mathcal{E}_0(k), m_k, \mathbf{Y}^{k-1}] = V_k^{-m_k} \\ P^2_0 \triangleq P\{\theta_0(k)|O_k, m_k, \mathbf{Y}^{k-1}\} = \frac{\xi_k}{c_1}(1 - P_d P_g) \\ P\{O_k|m_k, \mathbf{Y}^{k-1}\} \triangleq P^O_{k|k-1,m_k} \end{cases}$$

- for $i = \bar{0}$, we have

$$\begin{cases} P^1_{\bar{0}} \triangleq p[\mathbf{Y}(k)|\mathcal{E}_{\bar{0}}(k), m_k, \mathbf{Y}^{k-1}] = V_k^{-m_k} \\ P^2_{\bar{0}} \triangleq P\{\theta_0(k)|\bar{O}_k, m_k, \mathbf{Y}^{k-1}\} = 1 \\ P\{\bar{O}_k|m_k, \mathbf{Y}^{k-1}\} = 1 - P^O_{k|k-1,m_k} \end{cases}$$

The conditional predicted target perceivability probability $P^O_{k|k-1,m_k}$ will be explicited in the next section. Combining previous equations yields following final expressions

$$\begin{aligned} \beta_i(k) &= \frac{1}{c}e_i(k)P^O_{k|k-1,m_k} && (10) \\ \beta_0(k) &= \frac{1}{c}b_0(k)P^O_{k|k-1,m_k} && (11) \\ \beta_{\bar{0}}(k) &= \frac{1}{c}b_{\bar{0}(k)}(1 - P^O_{k|k-1,m_k}) && (12) \end{aligned}$$

with

$$\begin{aligned} b_0(k) &\triangleq \frac{m_k}{V_k}\frac{1 - P_d P_g}{P_d P_g}\xi_k && (13) \\ b_{\bar{0}}(k) &\triangleq \frac{m_k}{V_k}\frac{1}{P_d P_g}\left[P_d P_g + (1 - P_d P_g)\xi_k\right] && (14) \end{aligned}$$

Remarks :

- These new expressions for $\beta_i(k)$ are coherent with Bar-Shalom derivation (taking into account Li and Guézengar correction term) for which perfect perceivability of the target was assumed (replace $P^O_{k|k-1,m_k}$ by 1 into (10)-(12)).
- Following [5, 3, 8, 11] additional amplitude and/or recognition information can easily be taken into account within $\beta_i(k)$ by replacing $e_i(k)$ terms by $e_i(k)\rho_i(k)$. $\rho_i(k)$ being the ratio of the pdfs of the amplitude feature/or another recognition feature of the true to the false measurements.
- Since the true clutter density is never known in most of practical applications, it must be estimated on line. The simpliest estimator $\hat{\lambda}_k = \frac{m_k}{V_k}$ is mostly used. The more appealing estimator suggested in [12] $\hat{\lambda}_k = 0$ (when $m_k = 0$) and when $m_k \neq 0$

$$\hat{\lambda}_k = \frac{1}{V_k}(m_k - P_d P_g P^O_{k|k-1,m_k}) \tag{15}$$

should provide theoretically better results than the previous one. However since $P^O_{k|k-1,m_k}$ is itself a function of the unknown clutter density λ (as we will see in (16) and (18)), this estimator cannot be used directly without a good model for the prediction of λ. A better issue is to use one of the three classes of theoretically solid estimators of the clutter density based on the Bayesian (conditional mean) estimation, the maximum likelihood method and the least squares method developed in [8, 12].

4.1 Derivation of $P^O_{k|k-1,m_k}$

The evaluation of $\beta_i(k)$ $(i = \bar{0}, 0, \ldots m_k)$ requires the computation of **conditional predicted target perceivability probability** $P^O_{k|k-1,m_k} \triangleq P\{O_k|m_k, \mathbf{Y}^{k-1}\}$. Due to space limitation, we only give here final expression for $P^O_{k|k-1,m_k}$ (see [8] for details). After elementary algebra, it can be shown (using Poisson prior for μ_F)

$$P^O_{k|k-1,m_k} = \frac{(1-\epsilon_k)P^O_{k|k-1}}{1-\epsilon_k P^O_{k|k-1}} \tag{16}$$

with

$$P^O_{k|k-1} \triangleq P\{O_k|\mathbf{Y}^{k-1}\} \tag{17}$$

$$\epsilon_k \triangleq \begin{cases} P_d P_g & m_k = 0 \\ P_d P_g(1-\frac{m_k}{\lambda V_k}) & m_k \neq 0 \end{cases} \tag{18}$$

The derivation of **unconditional predicted target perceivability probability** $P^O_{k|k-1}$ will now be presented.

4.2 Derivation of $P^O_{k|k-1}$ and $P^O_{k|k}$

$P^O_{k|k-1}$ and $P^O_{k|k}$ can be obtained from Bayes' rule and take the following concise forms [8, 12, 11]

$$P^O_{k|k-1} = \pi_{11}P^O_{k-1|k-1} + \pi_{21}(1-P^O_{k-1|k-1}) \tag{19}$$

$$P^O_{k|k} = \frac{(1-\phi_k)P^O_{k|k-1}}{1-\phi_k P^O_{k|k-1}} \tag{20}$$

where

$$\phi_k \triangleq \begin{cases} P_d P_g & m_k = 0 \\ P_d P_g(1-\frac{1}{\lambda}\sum_{i=1}^{m_k} e_i) & m_k \neq 0 \end{cases} \tag{21}$$

Thus, unconditional perceivability probabilities $P^O_{k|k-1}$ and $P^O_{k|k}$ can be computed on-line recursively by (19) and (20) as soon as the **design parameters** π_{11}, π_{21} and $P^O_{1|0}$ have been set. First theoretical investigation on design of trackers for perceivability probability enhancement can be found in [10]. In this reference (and in our simulations), authors assume that the sequence of perceivability state for the target $\{O_k\}$ can be modeled as a first-order homogeneous Markov-chain, i.e.

$$\pi_{11} \triangleq P\{O_k|O_{k-1}\}$$
$$\pi_{21} \triangleq P\{O_k|\bar{O}_{k-1}\}$$

4.3 Closed form of β_i

The following relations can be obtained from (10)-(14) and (16) using elementary algebra

$$(1-\phi_k)P^O_{k|k-1} = \frac{1}{c_1}\frac{\lambda V_k}{m_k}(1-P_dP_g)(1-\epsilon_k)P^O_{k|k-1} + \frac{1}{c_1}\frac{P_dP_g}{m_k}V_k(1-\epsilon_k)P^O_{k|k-1}\sum_{i=1}^{m_k} e_i(k)$$

$$c = V^{-m_k}\frac{1-\phi_k P^O_{k|k-1}}{1-\epsilon_k P^O_{k|k-1}}$$

$$\beta_0(k) + \sum_{i=1}^{m_k}\beta_i(k) = \frac{(1-\phi_k)P^O_{k|k-1}}{1-\phi_k P^O_{k|k-1}} \equiv P\{O_k|\mathbf{Y}^k\}$$

$$\beta_{\bar{0}}(k) = \frac{1-P^O_{k|k-1}}{1-\phi_k P^O_{k|k-1}} = 1 - P^O_{k|k} = P\{\bar{O}_k|\mathbf{Y}^k\}$$

Rearranging expressions (10)-(14) yields the following useful concise forms for $\beta_i(k)$

$$\beta_i(k) = E_i(k)/C \quad \beta_0(k) = B_0(k)/C \quad \beta_{\bar{0}}(k) = B_{\bar{0}}(k)/C$$

where the normalization constant is $C = 1-\phi_k P^O_{k|k-1}$ and $E_i(k)$, $B_0(k)$, $B_{\bar{0}}(k)$ are defined as

$$E_i(k) \triangleq \frac{1}{c_1}\frac{P_dP_g}{m_k}V_k(1-\epsilon_k)P^O_{k|k-1} \times e_i(k)$$
$$B_0(k) \triangleq \frac{1}{c_1}\frac{\lambda V_k}{m_k}(1-P_dP_g)(1-\epsilon_k)P^O_{k|k-1}$$
$$B_{\bar{0}}(k) \triangleq 1 - P^O_{k|k-1}$$

5 Track confirmation

The track confirmation or termination can be done using different approaches. In [11] authors propose to compare the probability of perceivability with some given confirmation and termination thresholds P_c and P_t as follows

$$\text{if } P^O_{k|k} \geq P_c \Rightarrow \delta_L = H_c(k) \text{ (confirmation)}$$
$$\text{if } P^O_{k|k} \leq P_t \Rightarrow \delta_L = H_{\bar{c}}(k) \text{ (termination)}$$

Their choices for P_c and P_t for tracking probability enhancement can be found in [10]. We propose here another approach based on the Sequential Probability Ratio Test

(SPRT). To perform this test, one has to choose the probability of decision error of first and second kind defined as

$$P_F = P(\delta_W(k) = H_c(k) | H_{\bar{c}}(k))$$
$$P_M = P(\delta_W(k) = H_{\bar{c}}(k) | H_c(k))$$

where $\delta_W(k)$ corresponds here to the decision taken by the following decision rule (SPRT) at time k

$$\text{if } SPR(k) \geq A \Rightarrow \delta_W(k) = H_c(k) \text{ (confirmation)}$$
$$\text{if } SPR(k) \leq B \Rightarrow \delta_W(k) = H_{\bar{c}}(k) \text{ (termination)}$$

with

$$SPR(k) = \frac{P\{\mathbf{Y}^k, O_k\}}{P\{\mathbf{Y}^k, \bar{O}_k\}} = \frac{(1-\phi_k)P^O_{k|k-1}}{1-P^O_{k|k-1}} \quad (22)$$

The decision δ_W is postponed to next scan whenever $B < SPR(k) < A$. The SPRT bounds A and B are given by

$$A = \frac{1-P_M}{P_F} \qquad \text{and} \qquad B = \frac{P_M}{1-P_F}$$

Actually, both criteria presented here concern only instantaneous decision. However during tracking process, a track can either be declared confirmed during several scans, terminated or being a tentative track because of the possible fluctuations of $P^O_{k|k}$ computed by the IPDAF. This phenomenon generates difficulties to evaluate IPDAF performance in Monte Carlo simulations because we could average some tracks which correspond most of the time only to tentative tracks based on false measurements. To overcome this problem, two other procedures have been used in our simulations for track confirmation decision. These procedures are both based on the percentage of instantaneous confirmation decision over the time of interest. Thus in the following, a track will be deemed confirmed by the procedure 1 or by procedure 2 whenever at least 80% of time, one has $\delta_L = H_c(k)$ or $\delta_W = H_c(k)$ respectively.

6 Simulation results

The purpose of our simulation was to compare the performance of this new IPDAF version with the previous one [8, 11]. The two-dimensional scenario of [14, 11] was simulated. 500 independent runs were used in the simulation; each with 21 scans. The number of false measurements satisfies a Poisson distribution with density $\lambda = 0.0001/scan/m^2$. A single target is moving with constant velocity with the following dynamic model

$$\mathbf{x}(k+1) = \mathbf{F}\mathbf{x}(k) + \mathbf{v}(k)$$

where $\mathbf{x}(k) = [x\ \dot{x}\ y\ \dot{y}]'$ is the target state vector at time k and $\mathbf{F}$ is the following transition matrix

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_1 & \mathbf{O} \\ \mathbf{O} & \mathbf{F}_1 \end{bmatrix} \qquad \mathbf{F}_1 = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$$

where $T = 1s$ is the sampling period. The process noise $\mathbf{v}(k)$ is a zero-mean white Gaussian noise with known covariance $\mathbf{Q}(k)$ with

$$\mathbf{Q} = 0.75 \begin{bmatrix} \mathbf{Q}_1 & \mathbf{O} \\ \mathbf{O} & \mathbf{Q}_1 \end{bmatrix} \qquad \mathbf{Q}_1 = \begin{bmatrix} T^4/4 & T^3/2 \\ T^3/2 & T^2 \end{bmatrix}$$

The sensor introduced independent errors in x and y with root mean square (RMS) value $r = 5m$. For each run, the true initial state for the target was $\mathbf{x}(0) = [-50m\ 35m/s\ 0m\ 0m/s]'$. To simplify simulations, initial state estimate $\hat{\mathbf{x}}(0|0)$ for each run follows $\mathcal{N}(\mathbf{x}(0), \mathbf{P}(0|0))$ with $\mathbf{P}(0|0)$ given by [1, 2]

$$\mathbf{P}(0|0) = \begin{bmatrix} \mathbf{P}_1 & \mathbf{O} \\ \mathbf{O} & \mathbf{P}_1 \end{bmatrix} \qquad \mathbf{P}_1 = \begin{bmatrix} r^2 & r^2/T \\ r^2/T & 2r^2/T \end{bmatrix}$$

The uniform initial predicted probability of perceivability $P^0_{1|0} = 0.5$ was used. To get a fair comparison between results, the true value of clutter density λ instead of $\hat{\lambda}$ was used in both IPDAF implemented versions. Both IPDAF versions used the same set of design parameters [10] ($P_g = 0.99$, $P_d = 0.9$, $\pi_{11} = 0.988$, $\pi_{21} = 0$). Two cases were simulated for performing confirmation procedures (case 1 with $P_t = 0.0983$, $P_c = 0.9$, $P_M = P_F = 0.1$ and case 2 with same P_t and P_c but with $P_M = P_F = 0.05$ for procedure 2). Figure 1 and 3 shows the percentage of confirmed tracks for both IPDAF versions based on procedure 1 and 2 for case 1 and case 2 respectively. Figures 2 and 4 shows the corresponding RMS position errors. Results obtained by procedure 1 and 2 for each IPDAF implementation for case 1 are very close and only 2 curves appear actually on figures 1 and 2. The comparison of RMS velocity errors are not given here due to space limitation and because there is no significant difference between plots. Results indicates that this new version of IPDAF gives on average as good performance as its previous version concerning the percentage of confirmed tracks. Concerning the RMS position errors, this new IPDAF version performs a little bit better than the previous one.

7 Conclusions

A new formulation of IPDAF based on the recently developed probability of perceivability has been presented with theoretical justification. Specially, this filter formulation is fully coherent and intuitively appealing with the PDAF formulation as soon as the probability of perceivability becomes unitary. A new approach for track confirmation and termination based on sequential probability ratio test has also been given. Performance of this new IPDAF is quite comparable to the previous version of IPDAF concerning the percentage of confirmed tracks but is a little bit better on average in terms of RMS estimation errors. An extension of this approach to mutitarget tracking can be found in [6].

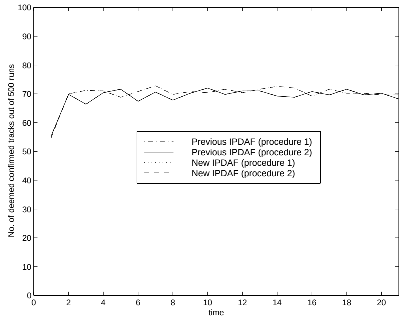

Figure 1: Percentage of confirmed tracks (case 1)

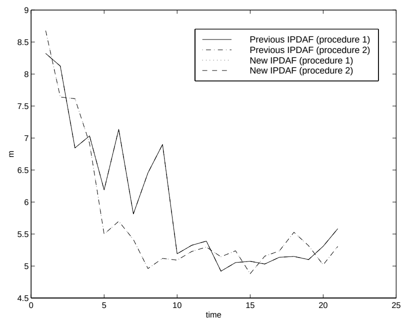

Figure 2: RMS position errors (case 1)

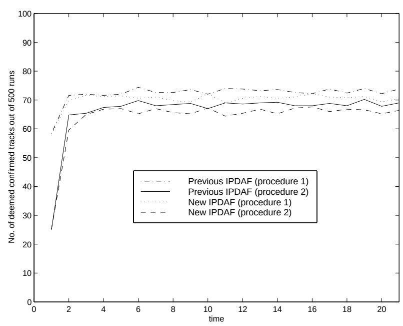

Figure 3: Percentage of confirmed tracks (case 2)

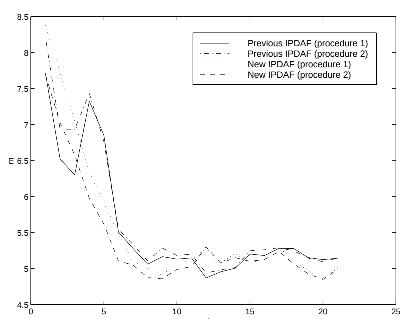

Figure 4: RMS position errors (case 2)

References

[1] Bar-Shalom Y., Fortmann T.E., "Tracking and Data Association", *Academic Press*, (1988).

[2] Bar-Shalom Y., Li X.R., "Estimation and Tracking: Principles,Techniques, and Software", *Artech House*, (1993).

[3] Bar-Shalom Y., Li X.R., "Multitarget-Multisensor Tracking: Principles and Techniques", *YBS Publishing*, (1995).

[4] Colegrove S.B., Ayliffe J.K., "An extension of Probabilistic Data Association to Include Track Initiation and Termination", *20th IREE International Convention,Melbourne*, pp. 853-856, (1985).

[5] Dezert J., "Autonomous Navigation with Uncertain Reference Points Using the PDAF", *in Multitarget-Multisensor Tracking : Applications and Advances, Y. Bar-Shalom Editor, Artech House*, **vol. 2**, (1992).

[6] Dezert J., Li N., Li X.R.,"Theoretical Development of an Integrated JPDAF for Multitarget Tracking in Clutter", *Proc. Workshop ISIS-GDR/NUWC, ENST, Paris*, (1998).

[7] Guézengar Y., "Radar Tracking in Cluttered Environment Applied to Adaptive Phased Array Radar", *Ph.D. Dissertation, Nantes Univ, France*, (1994).

[8] Li X.R., Li N., "Intelligent PDAF : Refinement of IPDAF for Tracking in Clutter", *Proc. 29th Southeastern Symp. on Syst. Theory*, pp. 133-137, (1997).

[9] Li X.R., "Tracking in Clutter with Strongest Neighbor Measurements - Part 1 : Theoretical Analysis", *IEEE Trans. AC*,**vol. 43**, pp. 1560-1578, (1998).

[10] Li N., Li X.R., "Theoretical Design of Trackers for Tracking Probability Enhancement", *Proc. of SPIE Conf., Orlando*,**vol. 3373**, (1998).

[11] Li N., Li X.R., "Target Perceivability : An Integrated Approach to Tracker Analysis and Design", *Proc. of Fusion 98 Intern. Conf., Las Vegas*, pp. 174-181, (1998).

[12] Li X.R., Li N., Zhu Y. , "Integrated Real-Time Estimation of Clutter Density for Tracking", *submitted to IEEE Trans. on Signal Processing.*, (1999).

[13] Musicki D., Evans R.J., Stankovic S., "Integrated Probabilistic Data Association (IPDA)", *Proc. 31st IEEE CDC, Tucson, AZ*, (1992).

[14] Musicki D., Evans R.J., Stankovic S., "Integrated Probabilistic Data Association", *IEEE Trans. AC*,**vol. 39**, pp. 1237-1241, (1994).