

# PERFORMANCE EVALUATION OF A $2/2 \times m/n$ LOGIC FOR TRACK FORMATION IN CLUTTER USING A BI-BAND IMAGING SENSOR

J. Dezert<sup>1</sup> and T. Kirubarajan<sup>2</sup>

## 1 Introduction

Surveillance of targets from infrared (IR) satellite observations is a major concern for modern defense systems, especially for the detection and tracking of dim ballistic missiles. In such systems, track formation (or track initialization) is a crucial phase. The evaluation of the reliability of the track formation process (TFP) is important because of the disastrous effects the threat can have if it is not detected and a corresponding track not initiated (miss detection of a true target). Similarly, formation of false tracks when there is no target is also undesirable (false track initiation in the absence of a target). Thus, a tool for evaluating the true track detection and false track initiation characteristics of track initiation techniques is needed. The difficulty in developing such a tool comes from the limited resolution of the IR imaging sensor, possibility of miss detection of target-originated measurements, presence of false alarms due to environmental conditions (mainly due to cloud borders) and the uncertainty about target characteristics (number of targets, their dynamics, target types, etc.). Several approaches to developing such an analysis tool have been proposed in the literature [1],[3] for single-imaging (i.e., mono-band) sensors. Recent technological advances allow us to have several IR imaging sensors on the same satellite with different spectral bands in order to obtain better target detection. For such multi-band sensors no tools to evaluate the TFP exist. In this paper we develop a procedure for the evaluation of the performance of a track formation logic for bi-band sensors.

The next section briefly describes the basics of the classic performance evaluation tool developed in [3]. Extension of this method to bi-band imaging sensors is then presented in the Section 3. Comparison of track initiation performances for mono-band and bi-band systems is also presented.

## 2 The mono-band case

Here, we briefly summarize the basics of the performance evaluation using the classic cascaded logic track initiation presented in [3] for mono-band sensors. This TFP is based on the following two-stage cascaded logic ( $2/2 \times m/n$ ) consisting of the following stages :

1. Stage 1 : After the first detection at a sampling time (or in a frame), a validation gate [2] is set up for the next sampling time (or frame of data) and a detection has to be made within this validation gate. This is the  $2/2$  requirement.
2. Stage 2 : If the requirements for Stage 1 are satisfied, then gates are set up for the subsequent sampling times based on the assumed dynamic model. Detections have to be made in  $m$  out of the next  $n$  sampling times in the corresponding gates in order to initiate a new track. This is the  $m/n$  requirement.

The basic idea for the analysis of this TFP is to take into account both the probability of detection,  $P_d$ , of target-originated measurements, the false alarm probability,  $P_{fa}$ , (defined per resolution cell) as well validation gate sizes (or equivalently, the gating probability  $P_g$ ).

Table 1 presents the resulting false track probability, the average length (in scans) of a false track, and the expected number of false tracks at a given time for various false alarm densities and logic parameters  $m/n$  for a  $1000 \times 1000$  imaging sensor.

---

<sup>1</sup>J. Dezert is with ONERA, French National Establishment for Aerospace Research, 29 Av. Division Leclerc, 92320 Châtillon, France. Email : dezert@onera.fr.

<sup>2</sup>T. Kirubarajan is with the Electrical & Systems Engineering Department, University of Connecticut, Storrs, CT 06269-2157, USA. Email : kiruba@ee.uconn.edu.

	$P_{fa} = 10^{-5}$			$P_{fa} = 10^{-4}$			$P_{fa} = 10^{-3}$		
	$P_{FT}$	$t_f$	$E[N_F]$	$P_{FT}$	$t_f$	$E[N_F]$	$P_{FT}$	$t_f$	$E[N_F]$
2/3	4.97e-12	1.51	7.55e-06	4.28e-08	1.67	7.16e-02	1.20e-04	2.40	289.84
3/3	6.99e-16	1.00	7.00e-10	6.59e-11	1.01	6.70e-05	3.75e-06	1.24	4.67
2/4	2.61e-11	2.04	5.33e-05	1.95e-07	2.30	0.45	2.84e-04	2.87	820.05
3/4	1.18e-14	1.51	1.80e-08	1.02e-09	1.66	1.70e-03	2.93e-05	2.52	74.17
4/4	1.18e-18	1.00	1.18e-12	1.11e-12	1.01	1.12e-06	5.87e-07	1.22	0.72
2/5	2.61e-11	2.57	6.72e-05	1.95e-07	2.87	0.56	2.84e-04	3	854.85
3/5	8.12e-14	2.04	1.65e-07	6.32e-09	2.31	1.46e-02	9.78e-05	3.67	359.48
4/5	2.24e-17	1.51	3.41e-11	1.94e-11	1.66	3.23e-05	5.61e-06	2.42	13.59
5/5	1.80e-21	1.00	1.80e-15	1.67e-14	1.01	1.70e-08	8.28e-08	1.22	0.10
2/6	2.61e-11	3	7.84e-05	1.95e-07	3	0.58	2.84e-04	3	854.85
3/6	3.75e-13	2.57	9.65e-07	2.54e-08	2.96	7.56e-02	2.01e-04	4.39	887.12
4/6	1.89e-16	2.04	3.86e-10	1.49e-10	2.30	3.46e-04	2.46e-05	3.96	97.90
5/6	3.86e-20	1.51	5.87e-14	3.35e-13	1.66	5.57e-07	9.55e-07	2.31	2.21
6/6	2.63e-24	1.00	2.63e-18	2.43e-16	1.01	2.47e-10	1.12e-08	1.22	0.013
2/7	2.61e-11	3	7.84e-05	1.95e-07	3	0.58	2.84e-04	3	854.85
3/7	1.36e-12	3.11	4.29e-06	7.88e-08	3.61	0.28	3.04e-04	4.80	1460.85
4/7	1.08e-15	2.57	2.80e-09	7.73e-10	2.96	2.29e-03	6.96e-05	5.61	391.05
5/7	3.89e-19	2.04	7.94e-13	3.11e-12	2.30	7.20e-06	5.28e-06	3.80	20.11
6/7	6.39e-23	1.51	9.70e-17	5.53e-15	1.66	9.19e-09	1.54e-07	2.27	0.35
7/7	3.78e-27	1.00	3.79e-21	3.47e-18	1.01	3.53e-12	1.51e-09	1.22	1.84e-03

TAB. 1 – False track probabilities and average false track lengths mono-band TFP

### 3 The bi-band case

In this section, we extend the analysis developed in [3] to the bi-band case and show how fusion of bi-band data can be used efficiently in the TFP. The bi-band sensor provides two images in two different spectral bands of the same observed scene with the same resolution (i.e., the same number of pixels for the same image). There is no alignment/registration errors between the two images.

Basically, the following two approaches can be used to solve the bi-band track formation problem with fusion :

– Approach 1 : Image fusion

Two techniques could be developed within this image fusion approach. The first (post-detection) technique is to merge the two primary spectral band signals into a single signal to form a new image. Then, signal thresholding techniques can be applied on this fused image for target detection. The second (pre-detection) technique consists of applying detection thresholding techniques on each spectral image and merge the thresholded images into a single image using classical AND/OR logic-based pixel fusion. Once the single fused image is available, the classical mono-band technique described in [3] can be used for TFP. With these two image fusion options, one does not have to develop a new TFP specific to bi-band data at all. That is, the bi-band problem is reduced to a mono-band one by the pixel-based fusion. The main difficulty in this approach is the development of efficient and optimal image fusion techniques.

– Approach 2 : Local track fusion

An alternative to the image fusion described above is local track fusion. It consists of using separately the two thresholded images to develop a new bi-band TFP based on optimal track fusion and AND/OR fusion rule for gating. There is no need to use image/pixel fusion in this approach, which greatly simplifies its implementation. In this paper we focus on local track fusion and develop a TFP specific to bi-band imaging sensors.

#### 3.1 Bi-band track formation cascaded logic based on AND fusion rule

The true TFP for the bi-band imaging sensor follows exactly the true TFP described for the mono-band case. We have to consider the same pixel, say pixel  $p_{ij}$ , from both images in spectral bands  $B_1$  and  $B_2$ . We denote by  $D_1$  the target detection for  $p_{ij}$  in band  $B_1$  and by  $D_2$  the target detection for  $p_{ij}$  in band  $B_2$ . In others words, if the values of pixels  $p_{ij}$  in both images are one (i.e., one has  $D_1D_2$ ), then we will say that there is a target detection at this pixel. On the other hand, if one or both pixel values are zero (i.e., one has  $\bar{D}_1D_2$ ,  $D_1\bar{D}_2$  or  $\bar{D}_1\bar{D}_2$ ) we will assume that there is no bi-band target detection. This is the AND fusion rule. Hence, the bi-band target detection indicator at time  $k$ , denoted by  $\delta(k)$ , can take the following values depending on the target detection in bands  $B_1$  and  $B_2$  :

$$\begin{aligned}
\delta(k) &= [1 \ 1]' & \text{if } D_1 D_2 & \text{ (bi-band AND detection)} \\
\delta(k) &= [0 \ 1]' & \text{if } \bar{D}_1 D_2 & \text{ (no bi-band AND detection)} \\
\delta(k) &= [1 \ 0]' & \text{if } D_1 \bar{D}_2 & \text{ (no bi-band AND detection)} \\
\delta(k) &= [0 \ 0]' & \text{if } \bar{D}_1 \bar{D}_2 & \text{ (no bi-band AND detection)}
\end{aligned}$$

The transition  $\pi_{ij}$  between states  $s_i$  and  $s_j$  of the Markov chain of the bi-band TFP is governed by the occurrence or non-occurrence of the bi-band detection event. Hence the state transition probability will be

$$\begin{aligned}
&P_{d1}P_{g1}P_{d2}P_{g2} & \text{if } D_1 D_2 \\
&(1 - P_{d1}P_{g1})P_{d2}P_{g2} & \text{if } \bar{D}_1 D_2 \\
&P_{d1}P_{g1}(1 - P_{d2}P_{g2}) & \text{if } D_1 \bar{D}_2 \\
&(1 - P_{d1}P_{g1})(1 - P_{d2}P_{g2}) & \text{if } \bar{D}_1 \bar{D}_2
\end{aligned}$$

where  $P_{d1}, P_{d2}$  are the target detection probabilities for spectral bands  $B_1$  and  $B_2$ , respectively, and  $P_{g1}$  and  $P_{g2}$  are the corresponding gating probabilities.

As for the mono-band case, the probability of confirmation of a **bi-band target-originated sequence** (true track formation) is obtained by propagating the state probability vector  $\mu$ . The evaluation of the bi-band false track acceptance probability of a **false-alarm-originated sequence** (false track formation) is more complex because it requires, for each time  $k$ , the computation of the volumes of the gates  $V_1(k)$  and  $V_2(k)$  for spectral bands  $B_1$  and  $B_2$  of the TFP. The gate volume (expressed in number of resolution cells) associated with a given state highly depends on the previous states in the Markov chain and the sequence of detection indicators  $\delta$ . The steps required for the evaluation of gate volumes are given below :

– For  $t = 1$  to current time  $k$  do :

$$\begin{aligned}
\mathbf{P}(t|t-1) &= \mathbf{F}'\mathbf{P}(t-1|t-1)\mathbf{F} + \mathbf{Q} \\
\mathbf{S}_1(t) &= \mathbf{H}_1\mathbf{P}(t|t-1)\mathbf{H}_1' + \mathbf{R}_1 \\
\mathbf{P}_1(t|t) &= \mathbf{P}(k|k-1) - \delta_1(t)\mathbf{P}(t|t-1)\mathbf{H}_1'\mathbf{S}_1^{-1}(t)\mathbf{H}_1\mathbf{P}(t|t-1) \\
\mathbf{S}_2(t) &= \mathbf{H}_2\mathbf{P}(t|t-1)\mathbf{H}_2' + \mathbf{R}_2 \\
\mathbf{P}_2(t|t) &= \mathbf{P}(t|t-1) - \delta_2(t)\mathbf{P}(t|t-1)\mathbf{H}_2'\mathbf{S}_2^{-1}(t)\mathbf{H}_2\mathbf{P}(t|t-1) \\
\mathbf{P}(t|t) &= [\mathbf{P}_1^{-1}(t|t) + \mathbf{P}_2^{-1}(t|t) - \mathbf{P}^{-1}(t|t-1)]^{-1} \quad (\text{Chong's optimal fusion equation [4]})
\end{aligned}$$

– Derivation of gate volumes  $V_1(k)$  and  $V_2(k)$  :

$$V_1(k) = \gamma\pi\sqrt{|\mathbf{S}_1(k)|} \quad \text{and} \quad V_2(k) = \gamma\pi\sqrt{|\mathbf{S}_2(k)|}$$

where indices 1 and 2 correspond to bands  $B_1$  and  $B_2$ , respectively, and  $\gamma$  is the validation threshold associated with the gating probability  $P_g$  (here, we assume that  $P_{g1} = P_{g2} \equiv P_g$ ). The components of bi-band detection indicator  $\delta(t)$  are denoted by  $\delta_1(t)$  and  $\delta_2(t)$ .

At each time  $k$ , the transition matrix of the Markov chain must be reevaluated because of varying gate volumes. For a given logic, which generates the  $N$  steps, the initial state probability vector  $\mu(0) = [\mu_1(0) \dots \mu_N(0)]'$  must be set to  $\mu(0) = [1 - P_{fa1}P_{fa2} \ P_{fa1}P_{fa2} \ 0 \dots 0]'$ . The bi-band false track probability  $P_{FT}$  and the expected number of false tracks at a given time are given as in the mono-band case. The expected number of false tracks at a given time is also given as in the mono-band case.

Table 2 presents the resulting false track probability, the average length (in scans) of a false track, and the expected number of false tracks at a given time for various false alarm densities and logic parameters  $m/n$  for a bi-band  $1000 \times 1000$  imaging sensor. Comparing with Table 1, it can be seen that TFP with a bi-band sensor yields substantially better results.

### 3.2 Bi-band track formation cascaded logic based on OR fusion rule

As in the previous case, we consider the same pixel, say  $p_{ij}$ , from both images for spectral bands  $B_1$  and  $B_2$ . We denote by  $D_1$  the target detection for  $p_{ij}$  in band  $B_1$  and by  $D_2$  the target detection for  $p_{ij}$  in band  $B_2$ . Since we adopt the OR fusion rule here, we consider that there is a bi-band target detection if *at least* one detection occurs (i.e., one of the pixels  $p_{ij}$  in an image is 1) in one of the spectral bands. The bi-band target detection indicator  $\delta(k)$  at a given time  $k$  corresponding to the OR fusion rule can take the following possible values depending on the target detection in bands  $B_1$  and  $B_2$  :

	$P_{fa} = 10^{-5}$			$P_{fa} = 10^{-4}$			$P_{fa} = 10^{-3}$		
	$P_{FT}$	$t_f$	$E[N_F]$	$P_{FT}$	$t_f$	$E[N_F]$	$P_{FT}$	$t_f$	$E[N_F]$
2/3	3.76e-24	1.50	5.64e-18	2.84e-16	1.53	4.35e-10	2.66e-09	1.97	5.24e-03
3/3	2.89e-32	1.00	2.89e-26	2.64e-22	1.00	2.65e-16	1.12e-12	1.01	1.13e-06
2/4	6.02e-23	2.00	1.20e-16	4.01e-15	2.07	8.31e-09	1.84e-08	2.73	5.03e-02
3/4	6.01e-30	1.50	9.03e-24	4.52e-20	1.53	6.94e-14	4.28e-11	1.90	8.15e-05
4/4	3.62e-38	1.00	3.62e-32	3.28e-26	1.00	3.28e-20	1.26e-14	1.01	1.27e-08
2/5	5.613e-22	2.50	1.40e-15	3.0562e-14	2.64	8.07e-08	4.1551e-08	3.32	0.137
3/5	1.32e-28	2.00	1.40e-15	9.04e-19	2.07	8.07e-08	4.36e-10	2.65	0.137
4/5	7.55e-36	1.50	1.13e-29	5.66e-24	1.53	8.69e-18	5.33e-13	1.89	1.01e-06
5/5	4.07e-44	1.00	4.07e-38	3.65e-30	1.00	3.65e-24	1.28e-16	1.01	1.29e-10
2/6	3.54e-21	3.00	1.06e-14	1.32e-13	3.21	4.26e-07	5.63e-08	3.69	0.20
3/6	1.71e-27	2.50	4.29e-21	1.01e-17	2.63	2.67e-11	2.10e-09	3.62	7.62e-03
4/6	2.13e-34	2.00	4.27e-28	1.46e-22	2.07	3.03e-16	7.26e-12	2.53	1.83e-05
5/6	8.66e-42	1.50	1.30e-35	6.48e-28	1.53	9.93e-22	6.04e-15	1.89	1.14e-08
6/6	4.39e-50	1.00	4.39e-44	3.91e-34	1.00	3.91e-28	1.25e-18	1.01	1.26e-12

TAB. 2 – False track probabilities and average false track lengths for bi-band TFP with AND fusion

$$\begin{aligned}
\delta(k) &= 1 \quad \delta(k) = [1 \ 1]' && \text{if } D_1 D_2 \quad (\text{bi-band OR detection}) \\
\delta(k) &= [0 \ 1]' && \text{if } \bar{D}_1 D_2 \quad (\text{bi-band OR detection}) \\
\delta(k) &= [1 \ 0]' && \text{if } D_1 \bar{D}_2 \quad (\text{bi-band OR detection}) \\
\delta(k) &= [0 \ 0]' && \text{if } \bar{D}_1 \bar{D}_2 \quad (\text{no bi-band OR detection})
\end{aligned}$$

The transition  $\pi_{ij}$  between states  $s_i$  and  $s_j$  of the Markov chain of the bi-band TFP is governed by the occurrence or non-occurrence of the bi-band detection event. The state transition probabilities can be derived as in the AND case.

The probability of confirmation of a **bi-band target-originated sequence** (true track formation) is still obtained by the propagation of the state probability vector  $\mu$ . The evaluation of the bi-band false track acceptance probability of a **false-alarm-originated sequence** (false track formation) is done as in the AND fusion case. The bi-band false track probability  $P_{FT}$ , the expected number of false tracks at a given time and the expected number of false tracks are given as in the mono-band case.

Simulations show that the true track acceptance probability obtained by bi-band TFP based on OR fusion logic is significantly better than those obtained by the TFP for the mono-band case and the bi-band TFP with AND fusion. On the other hand, performance in terms of false track acceptance is worse.

## 4 Conclusions

In this paper, an extension of cascaded logic track formation for bi-band imaging sensors was presented. The proposed performance evaluation technique avoids the need for extensive simulations and is applicable to both AND/OR fusion rules on the two images. This new technique can be used to select logic parameters based on the desired true track detection and false track acceptance probabilities. It was shown that for a given bi-band  $2/2 \times m/n$  cascaded logic, the probability of true track acceptance can be improved by choosing the OR fusion rule whereas the probability of false track acceptance can be reduced by choosing the AND fusion rule. It is not possible to increase the true track probability and decrease the false track probability simultaneously by using the same fusion rule.

## References

- [1] Barniv Y., "Dynamic Programming Algorithm for Detecting Dim Moving Targets", in *Multitarget-Multisensor Tracking: Advanced Applications*, Artech House (Y. Bar-Shalom Ed., pp. 85-154, (1990).
- [2] Bar-Shalom Y., Fortmann T.E., "Tracking and Data Association", *Academic Press*, (1988).
- [3] Bar-Shalom Y., Chang K.C., Shertukde H.M. "Performance Evaluation of a Cascaded Logic for Track Formation in Clutter", *IEEE Trans. AES*, vol. 25, no 6, pp. 873-878, (1986).
- [4] Chong C.Y., "Hierarchical Estimation", *Proceedings of MIT/ONR C3 Workshop*, Monterey, CA, 1979