

Land cover change prediction with a new theory of plausible and paradoxical reasoning

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Abstract - *The spatial prediction of land cover at the field scale in winter appears useful for the issue of bare soils reduction in agricultural intensive regions. High variability of the factors that motivate the land cover changes between each winter involves integration of uncertainty in the modelling process. Fusion process with Dempster-Shafer Theory (DST) presents some limits in generating errors in decision making when the degree of conflict, between the sources of evidence that support land cover hypotheses, becomes important. This paper focuses on the application of Dezert-Smarandache Theory (DSmT) method to the fusion of multiple land-use attributes for land cover prediction purpose. Results are discussed and compared with prediction levels achieved with DST. Through this first application of the Dezert-Smarandache Theory, we show an example of this new approach ability to solve some of practical problems where the Dempster-Shafer Theory usually fails.*

Keywords: Fusion, conflict analysis, Dempster-Shafer theory, Dezert-Smarandache theory, uncertainty, remote sensing data.

1 Introduction

In intensive agricultural regions, land cover during winter has an important impact on the water quality, and the identification and monitoring of vegetation covering dynamics at high spatial scales constitute a prior approach for the restoration of water resources. The spatial prediction modelling of land cover at the field scale in winter that appears useful for land management and helping local decision making, is specially complex because of the high variability of the factors that motivate the land cover changes between each winter. Thus, uncertainty in the data and the results has to be integrated in the modelling process for better decision making.

Dempster-Shafer Theory (DST) is considered as an interesting formalism to fusion uncertain data, essentially

because it is a flexible way to represent incertitude, be it total ignorance or any form of partial or total knowledge, and the Dempster's rule appears as an excellent tool for data aggregation (Smets, 1990a). Moreover, number of practical applications show that the fusion of uncertain data supporting different hypotheses is well achieved by Dempster's rule of combination. However, it also has been demonstrated that DST presents some limits in generating errors in decision making, when the level of conflict between the sources of evidence that support each of the considered hypotheses becomes important.

To solve this recurrent problem, we applied the Dezert-Smarandache Theory (DSmT), which can be considered as a generalization of the DST. In this new theory, the rule of combination takes into account both uncertain and paradoxical information. The DSmT deals directly with paradoxical/conflicting sources of information into this new formalism and proposes a new and very simple (associative and commutative) rule of combination for conflicting sources of information (corpus/bodies of evidence).

Before applying and evaluating DSmT comparatively to DST, it is necessary to briefly present first the issue of conflicts with DST and then the foundations of the DSmT for a better understanding of the differences between these two theories in managing conflict between hypotheses. More details on DST formalism can be found in Dezert, (2003).

2 The conflict management

The idea of introducing paradoxical information before data fusion in building the frame of discernment is related with the fact that sources of evidence are not always concordant, but often conflicting, even in some cases contradictory.

2.1 Conflicts between hypotheses with DST

The issue of the conflict between the sources of evidence results from its inappropriate managing with the DST. The DST foundation is based on the Shafer's model about the frame of discernment Θ , i.e. the exhaustivity and exclusivity of all elements belonging to Θ . A complete presentation of DST can be found in (Shafer, 1976).

Dempster's rule of combination for DST

Shafer has proposed the Dempster's rule of combination, symbolized by the operator \oplus , to fusion two distinct bodies of evidence β_1 and β_2 over the same frame of discernment Θ . Let $Bel_1(\cdot)$ and $Bel_2(\cdot)$, the two belief functions over the same frame of discernment Θ and m_1 and m_2 their corresponding bba (basic belief assignment) masses. The combined global belief function $Bel(\cdot) = Bel_1 \oplus Bel_2(\cdot)$ is obtained from the combination of the information granules m_1 and m_2 as follows:

$$m(\emptyset) = 0 \text{ and } \forall C \neq \emptyset \subseteq \Theta,$$

$$m(C) \equiv [m_1 \oplus m_2](C) = \frac{\sum_{A \cap B = C} m_1(A) m_2(B)}{1 - \sum_{A \cap B = \emptyset} m_1(A) m_2(B)}$$

$\sum_{A \cap B = C}$ represents the sum over all $A, B \subseteq \Theta$ such that $A \cap B = C$ (the interpretation for the other summation notations follows directly by analogy). The orthogonal sum $m(\cdot)$ is a proper bba if $k \triangleq 1 - k = 1 - \sum_{A \cap B = C = \emptyset} m_1(A) m_2(B) \neq 0$. If $k = 0$, which means $\sum_{A \cap B = C = \emptyset} m_1(A) m_2(B) = 1$ then orthogonal sum $m(\cdot)$ does not exist and the bodies of evidence β_1 and β_2 are said to be totally contradictory or in full contradiction. Such case arises whenever the nodes of $Bel_1(\cdot)$ and $Bel_2(\cdot)$ are disjoint or equivalently when there exists $A \subseteq \Theta$ such that $Bel_1(A) = 1$ and $Bel_2(A^c) = 1$.

The quantity k is called the weight of conflict between the bodies of evidence β_1 and β_2 . It is easy to show that the Dempster's rule of combination is commutative ($m_1 \oplus m_2 = m_2 \oplus m_1$) and associative ($[(m_1 \oplus m_2) \oplus m_3 = m_1 \oplus (m_2 \oplus m_3)]$). The vacuous belief function such that $m_v(A) = 0$ and $m_v(\Theta) = 1$ for $A \neq \Theta$ is the identity element for \oplus function operator, i.e. $m_v \oplus m = m \oplus m_v \equiv m$. If $Bel_1(\cdot)$ and $Bel_2(\cdot)$ are two combinable functions and if $Bel_1(\cdot)$ is Bayesian, then $Bel_1(\cdot)$ and $Bel_2(\cdot)$ is a bayesian belief function.

Conflict managing with Dempster's rule

Despite of the well-known advantages of Dempster's rule, DST presents several limitations (Dezert, 2003).

Among them, the issue concerning the managing of the information sources is not the least, as the following example illustrates it.

In 1982, Zadeh has given to Smet the example of a use of the Dempster's rule which shows an unexpected result drawn from the DST (Zadeh, 1979). Two doctors examine a patient and agree that he suffers from either meningitis (M) concussion (C) or brain tumor (T). Thus $\Theta = \{M, C, T\}$. Assume that the doctors agree in their low expectation of a tumor, but disagree in likely cause and provide the following diagnosis:

$$m_1(M) = 0.99 \quad m_1(T) = 0.01$$

$$m_2(C) = 0.99 \quad m_2(T) = 0.01$$

If we now combine belief functions using Dempster's rule of combination, one gets the unexpected final conclusion $m(T) = 0.0001 / (1 - 0.0099 - 0.0099 - 0.9801) = 1$ which means that the patient suffers from brain tumor! This unexpected result arises from the fact that the two bodies of evidence (doctors) agree that patient does not suffer from tumor but are in almost full contradiction for the other causes of disease. Although it is an extreme example with an almost maximal conflict of sources of evidence, this very simple but disturbing example shows the limitations of practical use of the DST for automated reasoning. A justification of non effectiveness of the Dempster's rule in such kind of example based on an information entropy argument has already been reported in Sun et al. (1999). Therefore, a special caution on the degree of conflict of the sources must always be taken before taking a final decision based on the Dempster's rule to reduce it at maximum and that way minimize the decision errors generated by the fusion process. In a practical way, it is not easy to achieve, and more often DST users apply different threshold techniques on the degree of conflict between sources, in order to let the choice to accept or refuse decision delivered by Dempster's fusion rule.

Several methods that attempt to make the fusion operator more reliable in considering the different causes of the conflict are available (Yager, 1983; Smets, 1990b; Cholvy, 2002; Dubois and Prade, 1998). No optimal technique exists yet, even if an approximate adjustment of the fusion threshold can be sufficient for some applications. This threshold is generally chosen according to past-experiences when databases are available or to expert knowledge. Though, what about a fusion result achieved from a threshold of 0.8 when the conflict reaches 0.79? Does it make sense to reject definitively this result? *A contrario* is it reasonable to accept a result produced with a conflict of 0.8?

Actually, DST can be used without any restriction until the conflict between the sources of evidence remains low

and the consequences of decision errors are not disastrous. As the degree of conflict increases, the question of the use of DST has to be set; the choice of the fusion process depends of the errors rate and the nature of the consequences that the user is ready to accept. However, when the conflict becomes high or very high another way is not appropriate to produce performant decisions making.

Finally, DST does not consider paradoxical nature of information in its formalism. This fusion approach rather attempts to avoid it through mass normalization process. Another way is proposed through the DSMT approach.

2.2. The Dezert-Smarandache Theory

The Dezert-Smarandache Theory (DSMT), which can be considered as a generalization of the Dempster-Shafer Theory (DST), is likely to keep and handle all existing conflicts between information sources.

In this new theory, the rule of combination takes into account both uncertain and paradoxical information (Dezert, 2003). This method offers a specific framework for a wide class of fusion problems because unlike the DST, the frame of discernment is exhaustive but not necessarily exclusive due to the intrinsic nature of its elements (like, by example, the notions of smallness/tallness, beauty/ugliness, pleasure/pain, heat/coldness, even the notion of colors - due to the continuous spectrum of the light, etc; none of these notions or concepts can be clearly refined/separated in an absolute manner so that they cannot be considered as exclusive and one cannot also define precisely what their conjunctions are.). The interpretation of Θ through the bba mechanism given by each source is, in general, built only from its own limited knowledge/experience and senses.

The Shafer's model considers as basis that the frame Θ of the problem under consideration is a set of finite exhaustive and exclusive elements Θ and requires in some way a refinement in order to choose/select Θ as exclusive. With the DSMT model, the framework is set up with the dealing of paradoxical information for all sources of evidence, through an hyper-powerset created with \cup and \cap operators (Figure 1) (Dezert and Smarandache, 2003) and the use of the DSMT rule of combination. Thus, any source of information that can be rational, uncertain or paradoxical can be combined in problems where the DSMT model holds. The DSMT model can be view actually as the model opposite to the Shafer's model where none of the Θ are considered exclusive.

Let be the simplest frame of discernment $\Theta = \{\theta_1, \theta_2\}$ involving only two elementary hypotheses with no more additional assumptions on θ_1 et θ_2 , then DSMT deals with

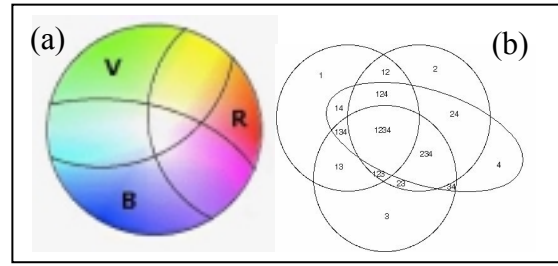


Figure 1. Extended frame of discernment for Theta = 3 (a) and for Theta = 4 (b)

new basic belief assignments $m(.) \in [0,1]$ in accepting the possibility for paradoxical information such that :

$$m(\theta_1) + m(\theta_2) + m(\theta_1 \cup \theta_2) + m(\theta_1 \cap \theta_2) = 1$$

The DSMT framework reflects better application conditions for a wide class of fusion problems, where nature of the elements of Θ can be incomplete, vague, imprecise, paradoxical and therefore not refinable at all into exclusive and precise subsets. A complete presentation of the DSMT can be found in Dezert, (2003) and is not reported here due to space limitation constraints. The DSMT can then deal directly with elements/concepts which have possibly (but not necessary) continuous and/or relative interpretation to the corpus of evidences. In this study, information sources which support the hypothesis defined to predict land cover vegetation presence in the fields are not refined/separated in an absolute manner so that they cannot be considered as exclusive and we cannot also define precisely what their conjunctions are. Their interpretations/estimations through the bba mechanism given by any corpus of evidence are always built from its own (limited) knowledge/experience and senses. Some subjectivity on the information provided by a source is almost unavoidable and may be interpreted differently by the distinct sources of evidence. In most of cases, the sources of evidence provide their beliefs about some hypotheses only with respect to their own worlds of knowledge, experiences, feelings, senses without reference to the (inaccessible) absolute truth of the space of possibilities and without out any probabilistic background argumentations.

3 The land cover prediction problem

3.1 The case study

The study area is a catchment area of 61 km² localised on the western coast of Brittany (France). The watershed of the Yar is characterised by a relatively intensive farming combined with wet and warm autumns that produce significant amounts of nitrogen before winter infiltration of water. For several years high nitrogen rates in rivers largely due to excessive fertilization are observed

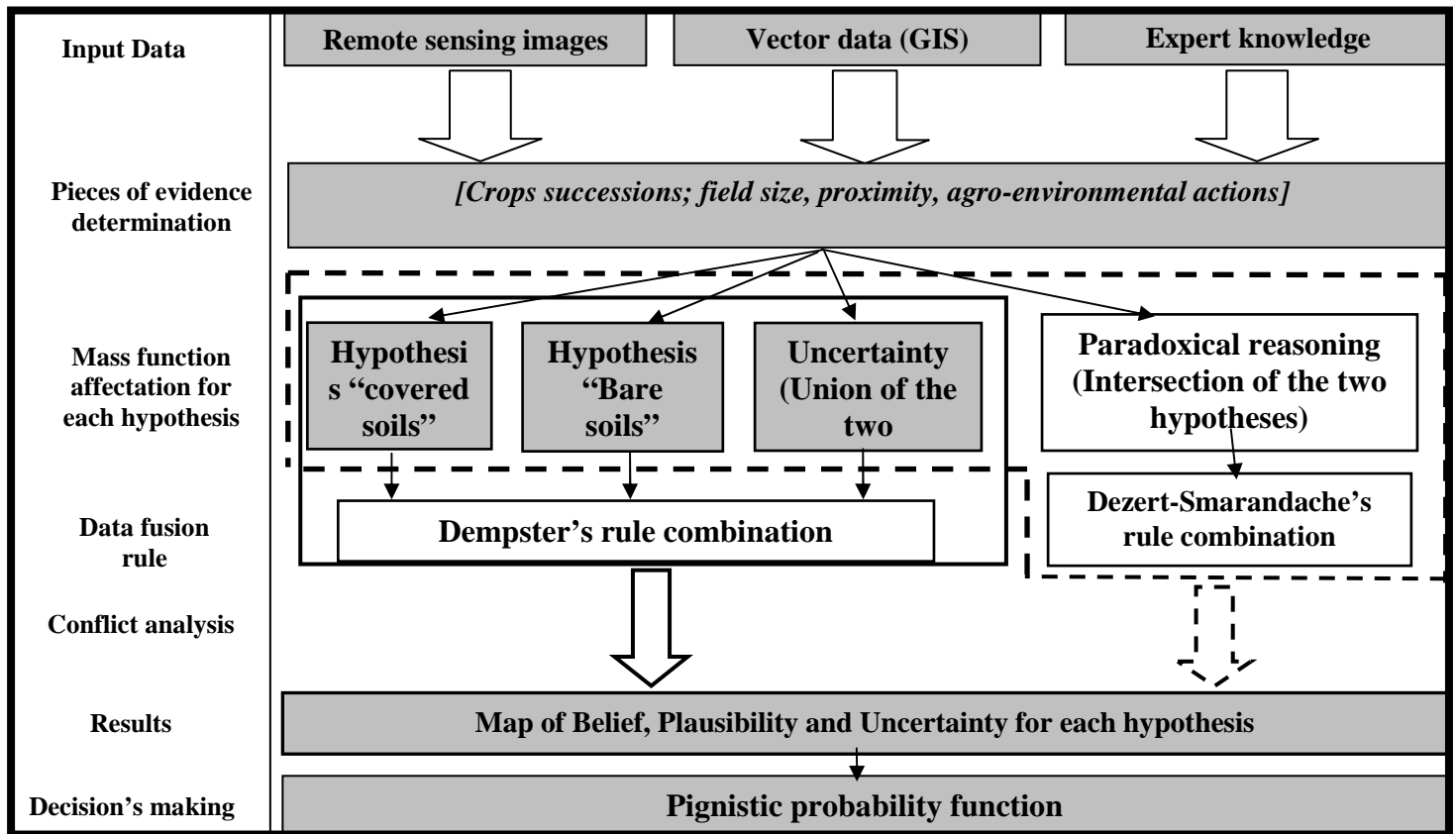


Figure 2. Application of the Dempster-Shafer and Dezert-Smarandache theories for the land cover prediction

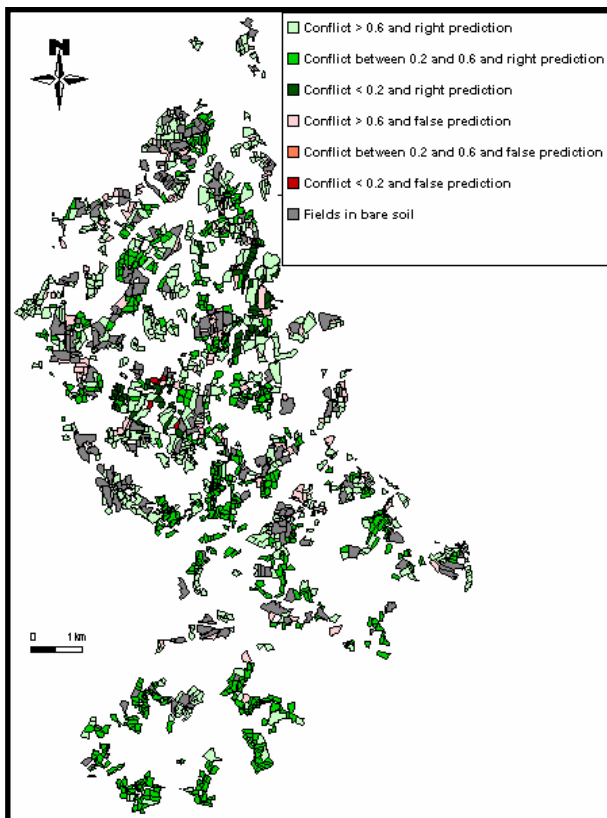


Figure 4. Prediction performance according to the degree of conflict on the watershed of the Yar, for the hypothesis "Soils covered with vegetation"

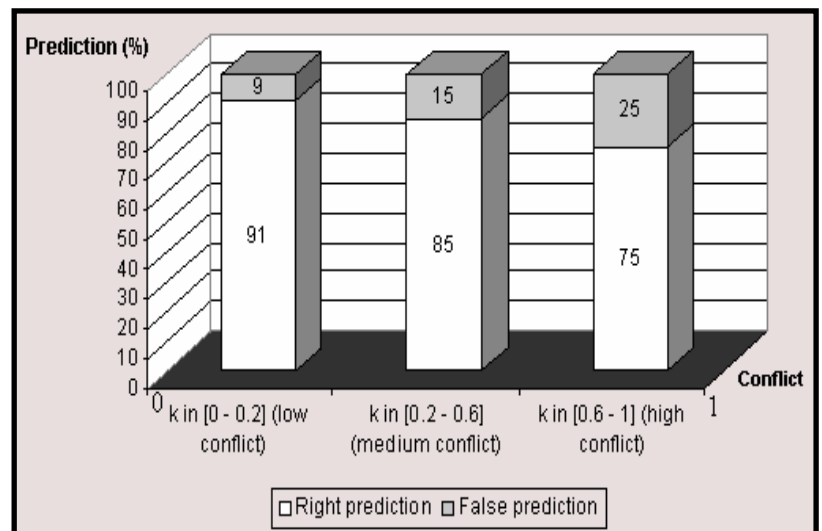


Figure 3. Relation between level of conflict for the two hypotheses 'Bare soils' and "Soils with vegetation" and prediction performance with the Dempster-Shafer Theory

and involve an increasing phenomenon of eutrophication on the coastal area. Consequences on environment and tourist activities have led local authorities to take action to restore water quality. In this area, crops cover approximately 60% of the total vegetated areas. Main crops are produced in relation to industrial breeding, principally corn, wheat and artificial meadows. During winter, which corresponds here to a rainy season, fields mostly remain without any vegetation cover after corn harvesting. Nitrogen inputs are still too high, especially on bare soils in winter, following and preceding corn sowing.

Therefore, it is useful to know the surface and the spatial distribution of bare fields during winter: From an historical point of view to assess farming practices change and in a predictive way to help decision-making to improve environmental conditions (Corgne *et al.*, 2002).

A previous study has demonstrated the interest of the application of DST for land cover prediction during winter on this area (Hubert-Moy *et al.*, 2001; Hubert-Moy *et al.*, 2002); the results issued of the Dempster-Shafer fusion's rule for the land use prediction were quite good, as the model predicted well 4/5 of the studied fields. Though, these fields totalised only 10% of the total area, and were dedicated only for milk products, one of the five different farming production types the farming production systems of this watershed, which were defined from expert knowledge. In this case, degree of conflict was mainly low between the defined sources of evidence.

This study focuses about the fields concerned with milk and meat production that represent 47% (1799 ha) of the total farming surface of the watershed. The land cover for this type of production is especially difficult to predict because of the variety of factors involved in the land use managing.

3.2 Land cover prediction with DST

The processing chain of fusion process used is presented on the **figure 2**. Firstly, some pieces of evidence are defined as the driven factors that motivate the land cover changes for each field. For this application, four driven factors determined by expert knowledge and statistical analysis are chosen to be integrated in the data fusion's rule: *the past-observed bare soils, field size, distance from farm buildings and agro-environmental actions*. The past-observed bare soils are determined with the processing of a series of 12 satellite images (SPOT XS, Xi and IRS-LISS III ; 2 per year since 1996). Thus, a winter land cover change trajectory is produced for each field on the watershed. The fields size and the distance from farm buildings are

obtained through spatial data integrated in a GIS (Geographical Information System), and the information about the fields concerned with agro-environmental actions is given by two public bodies, the "Chambre d'Agriculture des Côtes d'Armor" and the "Communauté de Communes de Lannion". For each piece of evidence, mass functions determined from statistical analysis and expert knowledge are defined to support the hypotheses "Soils with vegetation", "bare soils" and the union of the two hypotheses that represents the uncertainty in the DST.

The fusion results issued (realised with the pignistic probability function, Smets, 1990) of the Dempster's rule show the level of conflict between some sources of information that support the hypotheses "Soils covered with vegetation" and "Bare soils". The degree of conflict k in the DS data fusion's rule is determined as following:

$$k = \sum_{A \cap B = \emptyset} m_1(A)m_2(B)$$

The sources of information are contradictory, or at least highly conflicting. The **figure 3** presents the relation between the level of conflict and the performance of the Dempster-Shafer's results. When the conflict is high between the evidences $k > 0.6$, thus the prediction performance is lower. In this case, only 75% of the fields concerned by a high degree of conflict are correctly predicted. On the contrary, when the conflict is low $k < 0.2$ the results become clearly better, with 91% of right prediction. The **figure 4** represents the spatial distribution of this relation. We note for this application that a large number of fields (835) are characterised with a high level of conflict. They include the largest fields regularly covered with cereals crops, as well as other fields without specific characteristics.

3.3 Introducing paradoxical information in the fusion process

The introduction of paradoxical information in the fusion process is thus justified by an high level of conflict between the information sources. The **right side of the figure 2** and **the table 1** illustrate this stage in including this time some paradoxical information through mass function affectation. For each evidence, some classes are defined in order to preferentially support one of the hypotheses. For example, the evidence "Distance from farm buildings" is divided in two classes: the fields located at a distance of less than 1 km from farm buildings that are generally covered with vegetation because the fields near the farm buildings are usually used as permanent meadows. On the contrary, the fields located at a distance of less than 1 km from farm buildings are often used for crops like corn or wheat liable to integrate the land cover category "Bare

	Classes	Hypothesis "Bare soils"	Hypothesis "Soils with vegetation"	Uncertainty (Union of the two hypotheses)	Paradoxical (Intersection of the two hypotheses)
Distance from farm buildings	1 {fields < 1 km from farm buildings}	0.3	0.5	0	0.2
	2 {fields > 1 km from farm buildings}	0.6	0.2	0	0.2
Agro- environmen- tal actions	1 {fields without environmental action}	0.6	0.3	0	0.1
	2 {fields with environmental action}	0.005	0.95	0	0.045
Field size	1 {fields < 2 ha}	0.2	0.5	0	0.3
	2 {fields > 2 ha}	0.65	0.2	0	0.15
Crop successions (1996-2002)	1 {soils covered during all winters}	0.005	0.95	0	0.045
	2 {bare soils during one winter}	0.01	0.9	0	0.09
	3 {bare soils during two winters}	0.25	0.7	0	0.05
	4 {bare soils during three winters}	0.45	0.4	0	0.15
	5 {bare soils during four winters}	0.65	0.3	0	0.05
	6 {bare soils during five winters}	0.85	0.1	0	0.05

Table 1. Mass function affectation including paradoxical information

	Land use for the winter 2001/02 (remote sensing data)	Dempster-Shafer Theory		Dezert-Smarandache Theory	
		Prediction	Right prediction	Prediction	Right prediction
Bare soils	319 ha [268 fields]	406 ha	114 ha (35.7%)	363 ha	121 ha (38%)
Soils with vegetation	1480 ha [1588 fields]	1393 ha	1193 ha (80.6%)	1436 ha	1244 ha (84.05%)
TOTAL	1799 ha [1856 fields]	1799 ha	1307 ha (72.65%)	1799 ha	1365 ha (75.9%)

Table 2. Performance of Dempster-Shafer and Dezert-Smarandache fusion rules

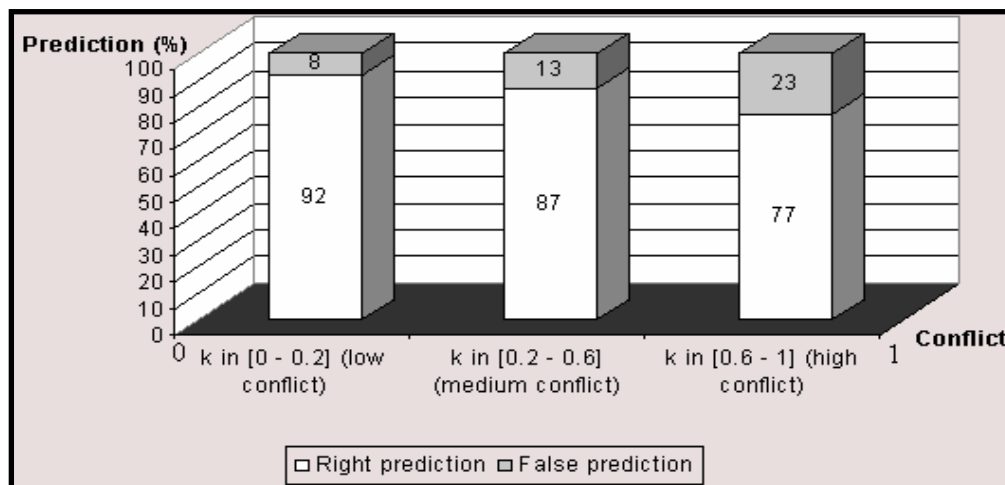


Figure 5. Relation between level of conflict for the two hypotheses 'Bare soils' and "Soils with vegetation" and prediction performance with the Dezert-Smarandache Theory

soils” in winter. With the DSMT approach, part of the mass functions that were previously affected to the uncertainty is now integrated on the intersection of the two hypotheses. The mass affectation is defined from validated past observation and assessed expert knowledge. Then, the decision’s making is performed with the generalized pignistic probability transform for both hypotheses (Dezert *et al.*, 2003).

Finally, a comparison is realized between the level of conflict and their consequences on the prediction for each theory.

3.4 First results

The comparison of the results issued of the two theories is summarized in **the table 2**. First, we note that the prediction performance with the DSMT is higher since the average of the right prediction for the 2 hypotheses is 75.9% for the DSMT against 72.6% for the DST. The main contribution of the DSMT is the improvement of the global surface prediction for each hypothesis. Thus, the overestimation for the hypothesis “Bare soils” (363 ha) is less important than for the DST (406 ha) and the precision performance is higher with a well-predicted surface of 121 ha (38%) against 114 ha for the DST. Several factors can explain the weak rate of right prediction for the hypothesis “Bare soils”. It is strongly linked with the high spatio-temporal variability of the studied land-use processes, i.e. an important number of fields covered with meadows since four or five years are ploughed in autumn and re-integrated in a cycle of crop successions. This type of change is especially difficult to model because it can result from unexpected individual human decisions and exceptional and isolated weather-events. For the hypothesis “Soils covered with vegetation”, the DSMT improves the results too, with a right prediction of 1436 ha on the 1480 ha detected (84%).

The analyze of the degrees of conflict issued of the DSMT (**Figure 5**) highlights the important reduction of the number of fields concerned with a high level of conflict ($K < 0.2$), 115 fields for the DSMT against 835 fields with the DST. They correspond quite exclusively to the largest fields regularly covered with cereals crops. On another hand, the majority of the fields that have a degree of conflict comprised between 0.2 and 0.6 are better predicted compared to DST results. Furthermore, the number of fields with a very small conflict ($K > 0.8$) increases and this category presents, like the DST, very good results (more than 90% of right prediction).

The management of the conflict between information sources proposed by the DSMT, which authorizes the paradoxical reasoning in the data fusion process, offers some results slightly better than performance rates

achieved with the DST. The use of a GIS allowed identifying the spatial distribution of the fields that are still concerned with an high level of conflict. For some of these fields, the determination of new pieces of evidence could increase the level of the prediction’s performance of land cover changes. For the others, prediction limits are probably reached...

4 Conclusions

This first application of the Dezert-Smarandache Theory has been performed to evaluate this new approach to fusion multiple land-use attributes for land cover prediction purpose. In intensive farming regions, uncertainty over land cover allocation for the next winter can be high and even very high. In this context, the degree of conflict between the hypotheses about the presence or the absence of vegetation cover becomes important or very important in many cases; therefore, the introduction of paradoxical information in the fusion process appears to be relevant in managing conflict between sources of evidence. In our case, the first results presented here point out that the new distribution of mass functions has improved land cover prediction accuracy for both hypotheses, in comparison with results achieved in applying Dempster-Shafer Theory. The analysis of the last prediction errors suggests that they correspond to fields where uncertainty is so high that the sources of evidence are definitely contradictory.

Finally, this fusion process leads to more relevant results to make a decision for the issue of bare soils reduction in agricultural intensive regions. Through this application of the Dezert-Smarandache Theory, we show an example of this new approach ability to solve some of the practical problems where the Dempster-Shafer usually fails.

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