Application of probabilistic PCR5 Fusion Rule for Multisensor Target Tracking

Aloïs Kirchner[a], Frédéric Dambreville[a], Francis Celeste[a], Jean Dezert[b], Florentin Smarandache[c]

general@fredericdambreville.com

[a]Délégation Générale pour l'Armement, DGA/CEP/GIP

[b]ONERA,

[c]University of New Mexico

Introduction

- non-Bayesian fusion rule for combining densities of probabilities
- Inspiration: PCR5 for fusing evidence with high conflict
- $\rightarrow$ probabilistic PCR5: p-PCR5, implemented by particle clouds
- $\rightarrow$ p-PCR5-based filter able to maintain filtering hypotheses (modes)
- Application example: distributed filtering for tracking
- Robustness in regard to the bad initialization or cinematic

PCR5 and p-PCR5

PCR5 redistribute the conflict proportionally:

$$m_{\text{PCR5}}(X) = \sum_{\substack{X_1,X_2\in\mathcal{P}(\Theta)\\ X_1\cap X_2=X}} m_1(X_1)m_2(X_2)$$

$$+ \sum_{\substack{Y\in\mathcal{P}(\Theta)\\ X\cap Y=\emptyset}} \left[\frac{m_1(X)^2 m_2(Y)}{m_1(X)+m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_2(X)+m_1(Y)}\right]$$

Corollary. Probabilistic PCR5 (p-PCR5):

$$p_{12}(x) \triangleq p_{\text{PCR5}}(x) = p_1(x)\int_\Theta \frac{p_1(x)p_2(y)}{p_1(x)+p_2(y)}dy + p_2(x)\int_\Theta \frac{p_2(x)p_1(y)}{p_2(x)+p_1(y)}dy$$

Properties

Proposition 1

$$\int_{\Theta} p_{12}(y) f(y,z)\,dy = \iint_{\Theta^2} p_1(y_1) p_2(y_2) \times \frac{p_1(y_1) f(y_1,z) + p_2(y_2) f(y_2,z)}{p_1(y_1) + p_2(y_2)} dy_1 dy_2$$

Proposition 2

$$p_{12}(z) = \iint_{\Theta^2} p_1(y_1) p_2(y_2) \pi(z|y_1,y_2)\,dy_1 dy_2\ ,$$

where $\quad \pi(z|y_1,y_2) = \dfrac{p_1(y_1)\delta[y_1 = z] + p_2(y_2)\delta[y_2 = z]}{p_1(y_1) + p_2(y_2)}$

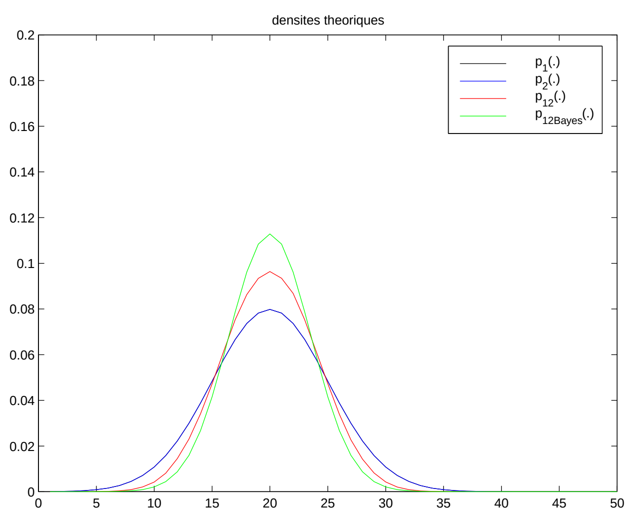
Theoretical p-PCR5

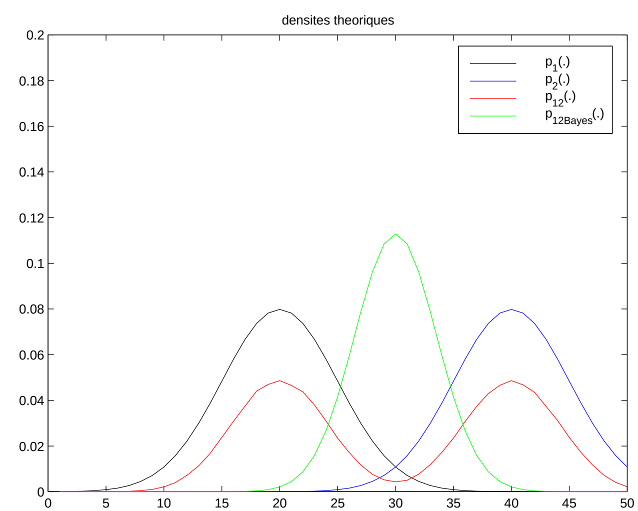
Comparison with Bayesian fusion: $p(x|z_1,z_2) \propto \frac{p(x|z_1)p(x|z_2)}{p(x)}$

Hypothesis: $p(x)$ uniform $\rightarrow p(x|z_1,z_2) \propto p(x|z_1)p(x|z_2)$

Computation of PCR5:

- Compute $I_i(x) = \int \frac{p_i(x)p_j(y)}{p_i(x)+p_j(y)}dy$, for $\{i,j\} = \{1,2\}$
- Then compute $p_{12}(x) = p_1(x)I_1(x) + p_2(x)I_2(x)$



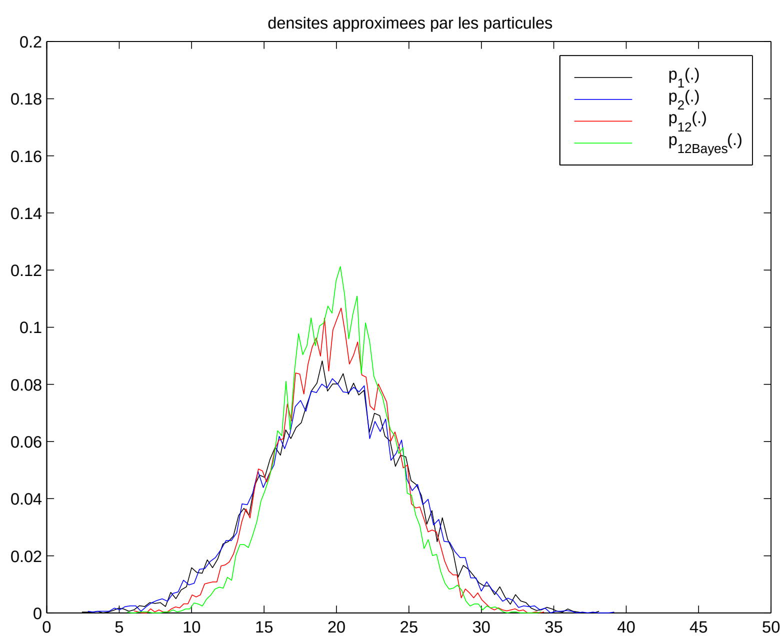


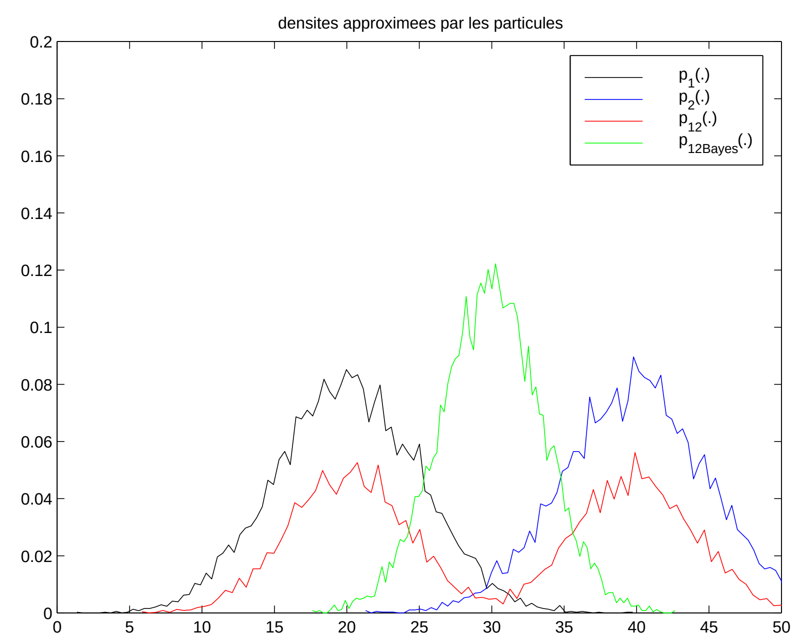
p-PCR5 maintains separated modes

Sampled p-PCR5

Corollary of Proposition 2:

- Generate y_i according to p_i ; compute $p_i(y_i)$
- Generate $\theta \in [0,1]$ *uniformely*
- If $\theta < \frac{p_1(y_1)}{p_1(y_1)+p_2(y_2)}$, set $z = y_1$, else set $z = y_2$.





Whitened p-PCR5

p-PCR5 on $p_1 = p_2$ amplifies p_{12}
Non-applicable $\leftarrow p_1$ and p_2 related to correlated variables

Whitened p-PCR5 decorrelates the data:

$$p_{\text{whitePCR5}}(y) = \iint_{\Theta^2} p_1(y_1) p_2(y_2) \pi(y|y_1, y_2)\, dy_1 dy_2 \ ,$$

$$\text{where } \pi(y|y_1, y_2) = \frac{\frac{p(y_1|z^1)}{p(y_1)} \delta[y_1 = y] + \frac{p(y_2|z^2)}{p(y_2)} \delta[y_2 = y]}{\frac{p(y_1|z^1)}{p(y_1)} + \frac{p(y_2|z^2)}{p(y_2)}}$$

Principle. $\frac{p(y|z^i)}{p(y)}$: intrinsical information of i

wp-PCR5 makes invariant the correlated information

Distributed filters

Bayesian fusion

$$p(y_{t:t+1}|z_{1:t}^{1:S}) = p(y_{t+1}|y_t)p(y_t|z_{1:t}^{1:S})$$

$$p(y_{t:t+1}|z_{1:t}^{1:S}, z_{t+1}^s) \propto p(z_{t+1}^s|y_{t+1})p(y_{t:t+1}|z_{1:t}^{1:S})$$

$$p(y_{t:t+1}|z_{1:t+1}^{1:S}) \propto \left(\prod_{s=1}^{S} \frac{p(y_{t+1}|z_{1:t}^{1:S}, z_{t+1}^s)}{p(y_{t+1}|z_{1:t}^{1:S})}\right)p(y_{t:t+1}|z_{1:t}^{1:S})$$

Distributed filters

PCR5 fusion

$$p(y_{t:t+1}|z_{1:t}^{1:S}) = p(y_{t+1}|y_t)p(y_t|z_{1:t}^{1:S})$$

$$p(y_{t:t+1}|z_{1:t}^{1:S}, z_{t+1}^s) \propto p(z_{t+1}^s|y_{t+1})p(y_{t:t+1}|z_{1:t}^{1:S})$$

$$p(y_{t+1}|z_{1:t+1}^{1:S}) = \int_{y_{t+1}^{1:S}} \left(\prod_{s=1}^{S} p(y_{t+1}^s|z_{1:t}^{1:S}, z_{t+1}^s) \right) \pi(y_{t+1}|y_{t+1}^{1:S}) dy_{t+1}^{1:S}$$

where $\pi(y_{t+1}|y_{t+1}^{1:S}) = \dfrac{\sum_{s=1}^{S} p(y_{t+1}^s|z_{1:t}^{1:S}, z_{t+1}^s)\delta[y_{t+1} = y_{t+1}^s]}{\sum_{s=1}^{S} p(y_{t+1}^s|z_{1:t}^{1:S}, z_{t+1}^s)}$

and $p(y_{t+1}^s|z_{1:t}^{1:S}, z_{t+1}^s)$ is an instance of $p(y_{t+1}|z_{1:t}^{1:S}, z_{t+1}^s)$

Distributed filters

Whitened PCR5 fusion

$$p(y_{t:t+1}|z^{1:S}_{1:t}) = p(y_{t+1}|y_t)p(y_t|z^{1:S}_{1:t})$$

$$p(y_{t:t+1}|z^{1:S}_{1:t}, z^s_{t+1}) \propto p(z^s_{t+1}|y_{t+1})p(y_{t:t+1}|z^{1:S}_{1:t})$$

$$p(y_{t+1}|z^{1:S}_{1:t+1}) = \int_{y^{1:S}_{t+1}} \left(\prod_{s=1}^{S} p(y^s_{t+1}|z^{1:S}_{1:t}, z^s_{t+1}) \right) \pi(y_{t+1}|y^{1:S}_{t+1}) dy^{1:S}_{t+1}$$

where $\pi(y_{t+1}|y^{1:S}_{t+1}) = \dfrac{\sum_{s=1}^{S} \frac{p(y^s_{t+1}|z^{1:S}_{1:t}, z^s_{t+1})}{p(y^s_{t+1}|z^{1:S}_{1:t})} \delta[y_{t+1} = y^s_{t+1}]}{\sum_{s=1}^{S} \frac{p(y^s_{t+1}|z^{1:S}_{1:t}, z^s_{t+1})}{p(y^s_{t+1}|z^{1:S}_{1:t})}}$

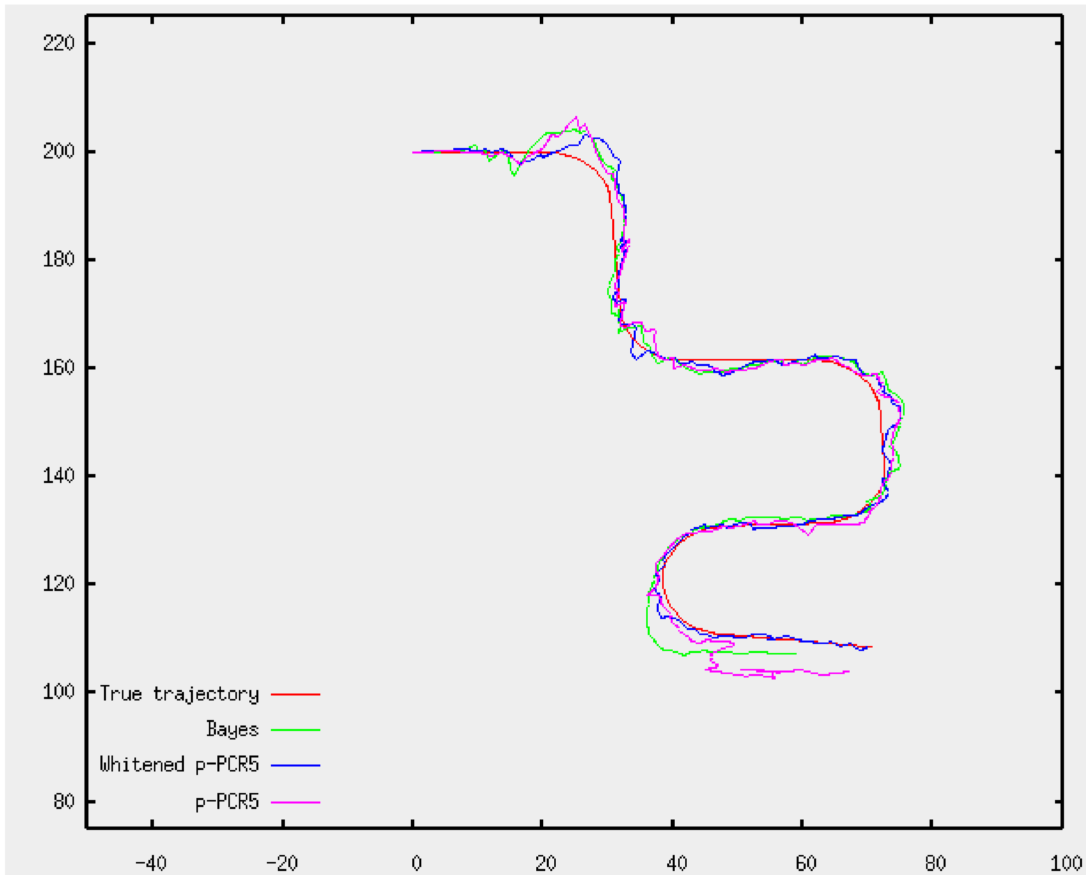
and y^s_{t+1} is an instance of y_{t+1} for sensor s

Scenario and tests

- Passive sensors located at $(0, 100)$ and $(100, 0)$
- Noisy azimuth measurement (*0.01 rad.* normal noise)
- A tracking particle filter associated to each sensor (200 particles)
- Fusion of the local posterior densities + feedback loop (fusion process)
- Fusion processes: Bayesian, p-PCR5 and wp-PCR5 rules.

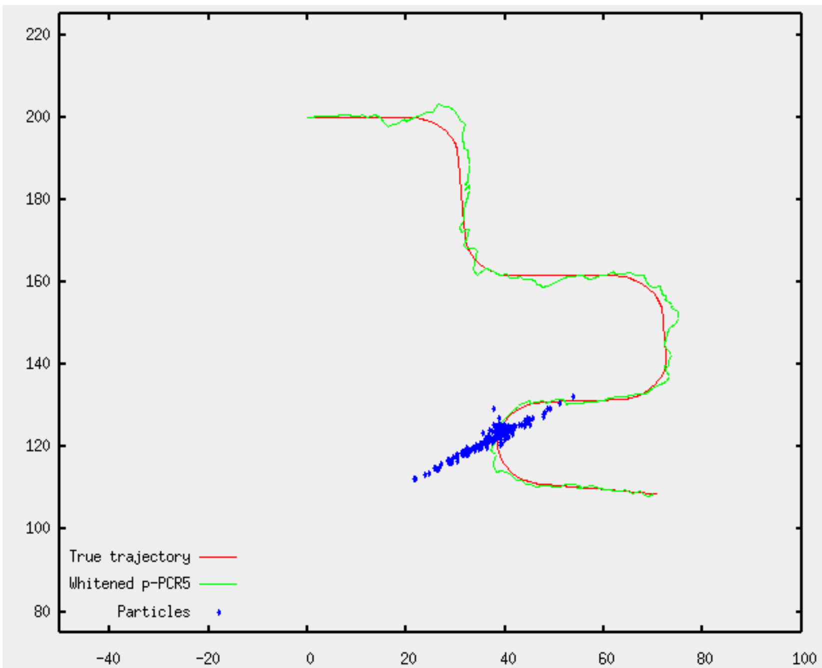
Filter comparison

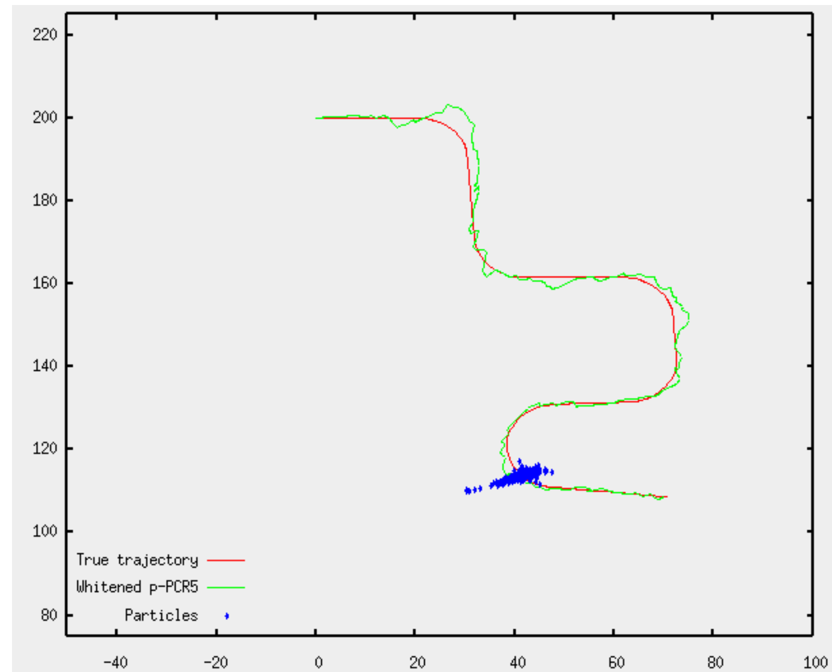
Comparative example. Good initialization



Details of the clouds

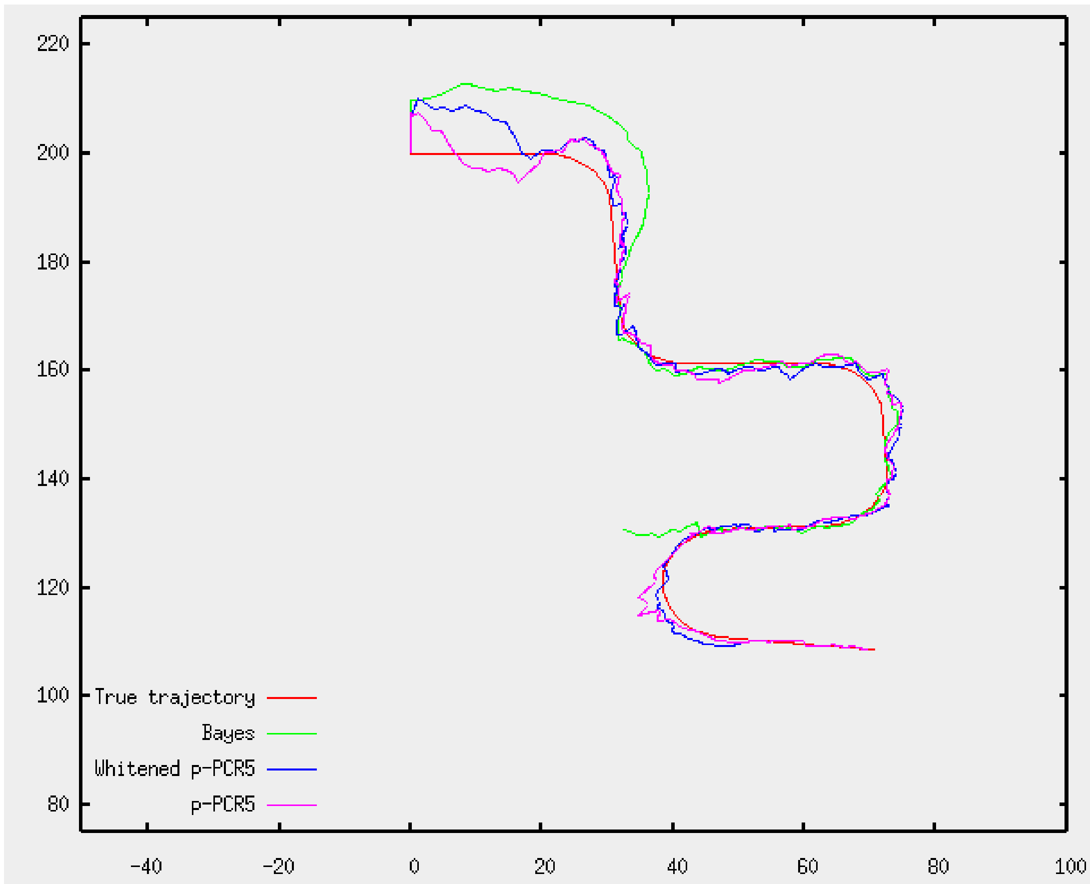
Cloud shape for successive steps (t, t+10)





Filter comparison (bad init)

Comparative example. Bad initialization

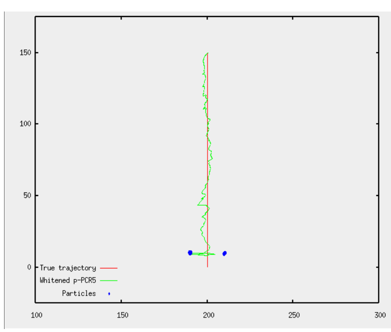


Robustness against initialization

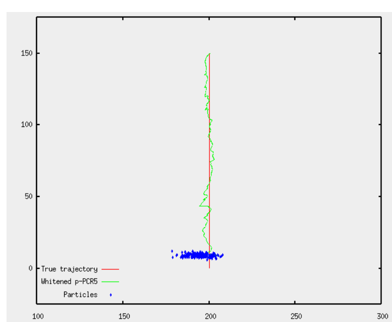
Target from $(200, 0)$ to $(200, 150)$ at speed $(0, 1)$

		x	y	x speed	y speed
First example	Filter 1	190	10	0	0
	Filter 2	210	10	0	0
Second example	Filter 1	190	10	0.1	-1
	Filter 2	210	10	0.5	1.5

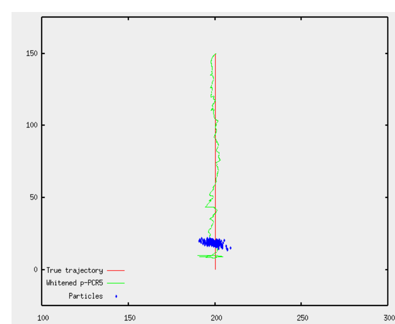
Poor initialization; :



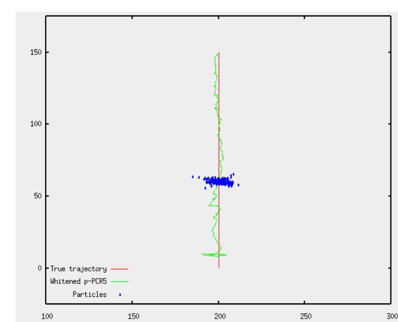
$t = 1$



$t = 10$



$t = 20$



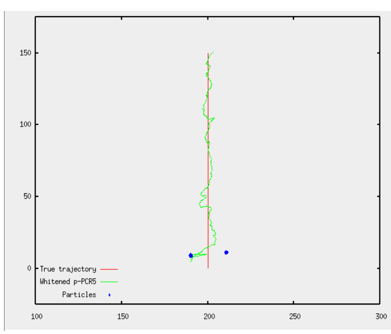
$t = 60$

Robustness against initialization

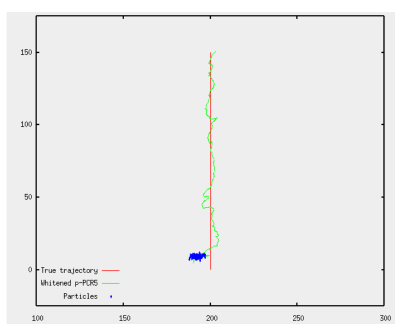
Target from $(200, 0)$ to $(200, 150)$ at speed $(0, 1)$

		x	y	x speed	y speed
First example	Filter 1	190	10	0	0
	Filter 2	210	10	0	0
Second example	Filter 1	190	10	0.1	-1
	Filter 2	210	10	0.5	1.5

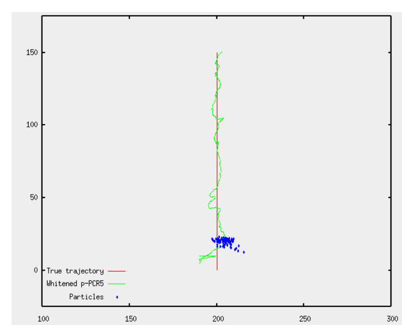
Bad initialization; :



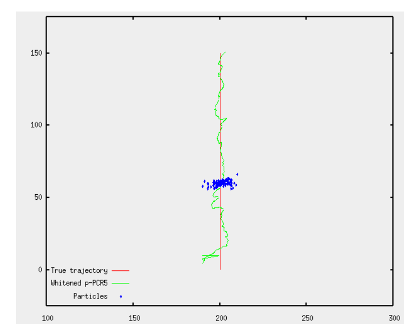
$t = 1$



$t = 10$



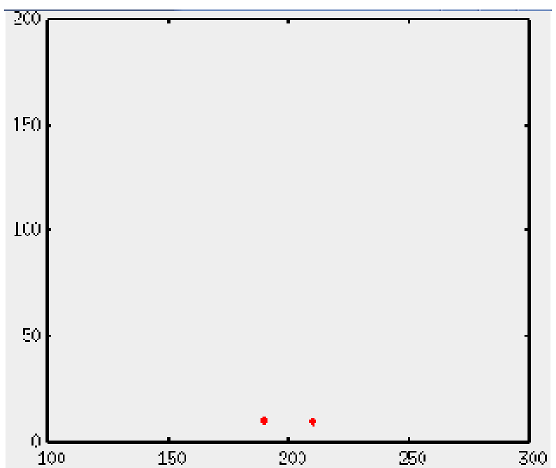
$t = 20$



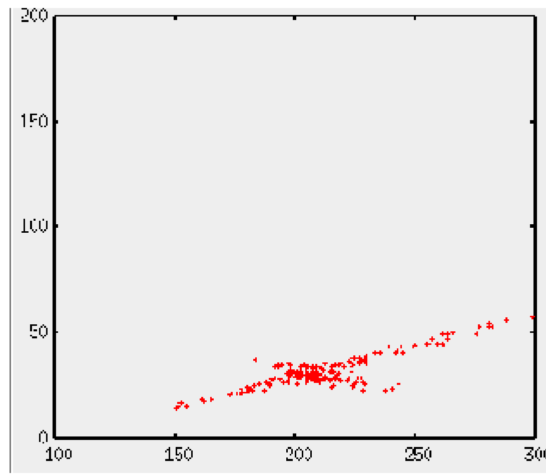
$t = 60$

Mean instead of p-PCR5

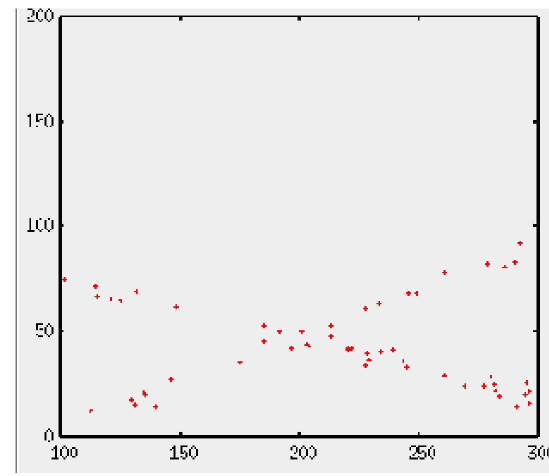
p-PCR5 is more that the mean
It concentrates the information



$t = 1$



$t = 30$



$t = 45$

Conclusion

- p-PCR5, PCR5 based method, for fusing probabilistic densities.
- Probabilistic interpretation and Monte-Carlo
- Comparison with Bayesian rule - pPCR5 is adaptive to the conflict
- Testing example on distributed target tracking (filter)
- Better robustness agianst the bad initialization