Enrichment of qualitative beliefs for reasoning under uncertainty

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Purpose & Outline

Purpose: We propose a kind of "second order" qualitative belief functions for taking into account explicitely the possible uncertainty on linguistic labels.

Outline:

Dedekind's lattice, quantitative and qualitative beliefs.

Fusion of Quantitative or Qualitative beliefs with Proportional Conflict Redistribution (PCR) rule

Arithmetic to compute with equidistant and non-equidistant enriched linguistic labels.

Simple examples

Dedekind's lattice (Hyper-power set)

Frame of the problem $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$. Finite set of exhaustive elements (discrete/continuous/fuzzy/relative concepts)

Hyper-power set: $|D^\Theta| > |2^\Theta|$

1) $\emptyset, \theta_1, \ldots, \theta_n \in D^\Theta$.

2) If $X, Y \in D^\Theta$, then $X \cap Y$ and $X \cup Y$ belong to D^Θ.

3) No other elements belong to D^Θ, except those obtained by using rules 1) or 2).

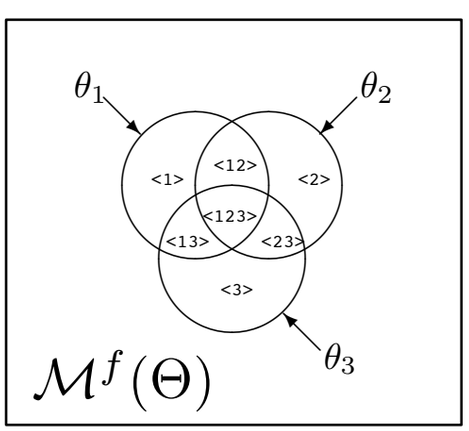
Example for n=3

$$|D^\Theta| = 19$$

$\alpha_0 \triangleq \emptyset$
$\alpha_1 \triangleq \theta_1 \cap \theta_2 \cap \theta_3$
$\alpha_2 \triangleq \theta_1 \cap \theta_2$
$\alpha_3 \triangleq \theta_1 \cap \theta_3$
$\alpha_4 \triangleq \theta_2 \cap \theta_3$
$\alpha_5 \triangleq (\theta_1 \cup \theta_2) \cap \theta_3$
$\alpha_6 \triangleq (\theta_1 \cup \theta_3) \cap \theta_2$
$\alpha_7 \triangleq (\theta_2 \cup \theta_3) \cap \theta_1$
$\alpha_8 \triangleq (\theta_1 \cap \theta_2) \cup (\theta_1 \cap \theta_3) \cup (\theta_2 \cap \theta_3)$
$\alpha_9 \triangleq \theta_1$
$\alpha_{10} \triangleq \theta_2$
$\alpha_{11} \triangleq \theta_3$
$\alpha_{12} \triangleq (\theta_1 \cap \theta_2) \cup \theta_3$
$\alpha_{13} \triangleq (\theta_1 \cap \theta_3) \cup \theta_2$
$\alpha_{14} \triangleq (\theta_2 \cap \theta_3) \cup \theta_1$
$\alpha_{15} \triangleq \theta_1 \cup \theta_2$
$\alpha_{16} \triangleq \theta_1 \cup \theta_3$
$\alpha_{17} \triangleq \theta_2 \cup \theta_3$
$\alpha_{18} \triangleq \theta_1 \cup \theta_2 \cup \theta_3$

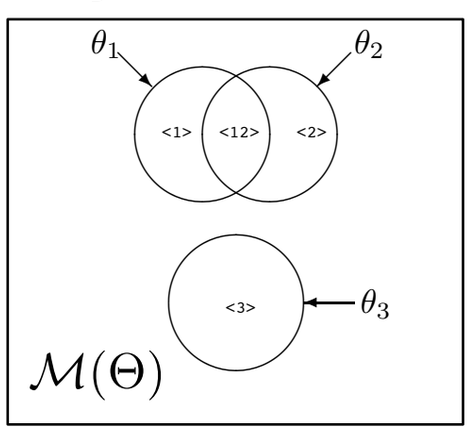
DSm models :

Free model



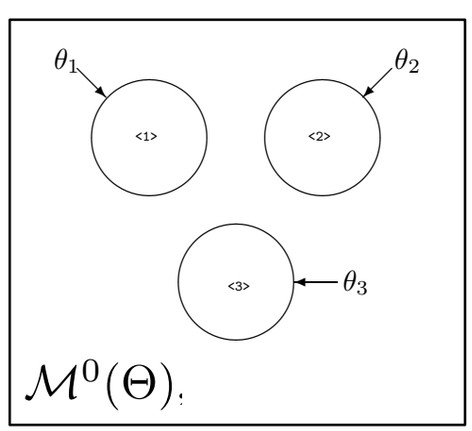
D^Θ

Hybrid model



G^Θ

Shafer's model



2^Θ

Quantitative & Qualitative belief masses

Quantitative belief mass:

$$m(.) : G^{\Theta} \rightarrow [0, 1]$$

$$m(\emptyset) = 0 \qquad \text{and} \qquad \sum_{A \in G^{\Theta}} m(A) = 1$$

Ordered Linguistic labels/mass: $L = \{L_0, L_1, L_2, \ldots, L_n, L_{n+1}\}$

very low, low, medium, etc

Linguistic operators (with equidistant labels) $L_i \mapsto i/(n+1)$

$$L = \{L_0, L_1, L_2, \ldots, L_n, L_{n+1}\} \longmapsto \{0, 1/(n+1), 2/(n+1), \ldots, n/(n+1), 1\}$$

$$L_i + L_j = \frac{i}{n+1} + \frac{j}{n+1} = \frac{i+j}{n+1} = L_{i+j}$$

$$L_i - L_j = \frac{i}{n+1} - \frac{j}{n+1} = \frac{i-j}{n+1}$$

$$L_i \times L_j = \frac{i}{n+1} \cdot \frac{j}{n+1} = \frac{(i \cdot j)/(n+1)}{n+1} \approx L_{[(i \cdot j)/(n+1)]}$$

$$L_i / L_j = \frac{i/(n+1)}{j/(n+1)} = \frac{(i/j) \cdot (n+1)}{n+1} \approx L_{[(i/j) \cdot (n+1)]}$$

Linguistic operators (by direct extrapolation)

$$L_i + L_j = \begin{cases} L_{i+j}, & \text{if } i + j < n + 1, \\ L_{n+1}, & \text{if } i + j \geq n + 1. \end{cases}$$

$$L_i - L_j = \begin{cases} L_{i-j} & \text{if} \quad i \geq j, \\ -L_{j-i} & \text{if} \quad i < j. \end{cases}$$

$$L_i \times L_j = L_{[(i \cdot j)/(n+1)]}$$

$$L_i / L_j = \begin{cases} L_{[(i/j) \cdot (n+1)]} & \text{if} [(i/j) \cdot (n+1)] < n + 1, \\ L_{n+1} & \text{otherwise.} \end{cases}$$

(Quasi-normalized) Qualitative belief mass:

$$qm(.) : G^{\Theta} \rightarrow L$$

$$qm(\emptyset) = L_0 \qquad \text{and} \qquad \sum_{A \in G^{\Theta}} qm(A) = L_{n+1}$$

Fusion of Quantitative belief masses with PCR5

PCR5 transfers the partial conflicting masses to the elements involved in the partial conflict proportionally to mass $m_1(.)$ and $m_2(.)$ of elements involved in the partial conflict ONLY.

$$\forall X \in G^\Theta \setminus \{\emptyset\}$$

$$m_{PCR5}(X) = m_{12}(X) + \sum_{\substack{Y \in G^\Theta \setminus \{X\} \\ X \cap Y = \emptyset}} \left[\frac{m_1(X)^2 m_2(Y)}{m_1(X) + m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_2(X) + m_1(Y)} \right] \quad \textbf{(PCR5)}$$

Extension possible for N>2 sources

PCR5 works on any model and preserves the neutrality of VBA.
PCR5 for imprecise bba is possible [see DSmT Book Vol 2]

(PCR6) for combining s>2 sources is usually preferred.

[see Martin & Osswald 2006 for justification and examples]

Example of Fusion of Quantitative belief masses with PCR5

$\Theta = \{A, B\}$

Shafer's model

Inputs

	A	B	$A \cup B$
$m_1(.)$	0.6	0.3	0.1
$m_2(.)$	0.2	0.3	0.5
$m_{12}(.)$	0.44	0.27	0.05

$$k_{12} = m_{12}(A \cap B) = m_1(A)m_2(B) + m_1(B)m_2(A) = 0.18 + 0.06 = 0.24$$

Let x_1 be the conflicting mass to be redistributed to A, and y_1 the conflicting mass redistributed to B from the first partial conflicting mass 0.18, and similarly for x_2 and y_2 with partial conflict 0.06; one has:

$$x_1/0.6 = y_1/0.3 = (x_1 + y_1)/(0.6 + 0.3) = 0.18/0.9 = 0.2 \longrightarrow \begin{cases} x_1 = 0.6 \cdot 0.2 = 0.12 \\ y_1 = 0.3 \cdot 0.2 = 0.06 \end{cases}$$

$$x_2/0.2 = y_2/0.3 = (x_2 + y_2)/(0.2 + 0.3) = 0.06/0.5 = 0.12 \longrightarrow \begin{cases} x_2 = 0.2 \cdot 0.12 = 0.024 \\ y_2 = 0.3 \cdot 0.12 = 0.036 \end{cases}$$

Fusion results

With PCR5
$m_{PCR5}(A) = 0.44 + 0.12 + 0.024 = 0.584$
$m_{PCR5}(B) = 0.27 + 0.06 + 0.036 = 0.366$
$m_{PCR5}(A \cup B) = 0.05 + 0 = 0.05$

Fusion of Qualitative belief masses with PCR5

Qualitative conjunctive rule:

$$qm_{qCR}(X) = \sum_{\substack{X_1,\ldots,X_s \in 2^\Theta \\ X_1 \cap \ldots \cap X_s = X}} \prod_{i=1}^{s} qm_i(X_i)$$

Linguistic addition & *product*

Linguistic "masses"

Total qualitative conflict:

$$K_{1\ldots s} = \sum_{\substack{X_1,\ldots,X_s \in 2^\Theta \\ X_1 \cap \ldots \cap X_s = \emptyset}} \prod_{i=1}^{s} qm_i(X_i)$$

Qualitative (PCR5): $X \in G^\Theta \setminus \{\emptyset\}$

$$qm_{PCR5}(X) = qm_{12}(X) + \sum_{\substack{Y \in G^\Theta \setminus \{X\} \\ X \cap Y = \emptyset}} \left[\frac{qm_1(X)^2 qm_2(Y)}{qm_1(X) + qm_2(Y)} + \frac{qm_2(X)^2 qm_1(Y)}{qm_2(X) + qm_1(Y)} \right]$$

Linguistic addition & *product* & *division*

Example of Fusion of Qualitative belief masses with PCR5

$\Theta = \{A, B\}$

Shafer's model

$L = \{L_0, L_1, L_2, L_3, L_4, L_5, L_6\}, n = 5,$

Inputs

	A	B	$A \cup B$	$A \cap B$
$qm_1(\cdot)$	L_1	L_2	L_3	
$qm_2(\cdot)$	L_4	L_2	L_0	
$qm_{12}(\cdot)$	L_3	L_2	L_0	L_1
$qm_{PCR5}(\cdot)$	L_4	L_2	L_0	L_0

$$qm_{12}(A) = qm_1(A)qm_2(A) + qm_1(A)qm_2(A \cup B) + qm_2(A)qm_1(A \cup B)$$
$$= L_1L_4 + L_1L_0 + L_4L_3 = L_{[(1\cdot4)/6]} + L_{[(0\cdot1)/6]} + L_{[(4\cdot3)/6]} = L_1 + L_0 + L_2 = L_{1+0+2} = L_3$$

$$qm_{12}(B) = qm_1(B)qm_2(B) + qm_1(B)qm_2(A \cup B) + qm_2(B)qm_1(A \cup B)$$
$$= L_2L_2 + L_2L_0(1) + L_2L_3 = L_{[(2\cdot2)/6]} + L_{[(2\cdot0)/6]} + L_{[(2\cdot3)/6]} = L_1 + L_0 + L_1 = L_{1+0+1} = L_2$$

$$qm_{12}(A \cup B) = qm_1(A \cup B)qm_2(A \cup B) = L_3L_0 = L_{[(3\cdot0)/6]} = L_0$$

$$qm_{12}(A \cap B) = qm_1(A)qm_2(B) + qm_1(B)qm_2(A) = L_1L_2 + L_2L_4 = L_{[(1\cdot2)/6]} + L_{[(2\cdot4)/6]} = L_0 + L_1 = L_1$$

qPCR5 rule

$$qm_1(A)qm_2(B) = L_1L_2 = L_0$$
$$qm_2(A)qm_1(B) = L_4L_2 = L_1$$

"nul" linguistic mass => no need of redistribution is redistributed back to A and B

$$\frac{x_A}{L_4} = \frac{y_B}{L_2} = \frac{L_1}{L_4 + L_2} = \frac{L_1}{L_6} = L_{[(1/6)\cdot6]} = L_1$$

$$x_A = L_4 \cdot L_1 = L_{[(4\cdot1)/6]} = L_1$$
$$y_B = L_2 \cdot L_1 = L_{[(2\cdot1)/6]} = L_0$$

$$qm_{PCR5}(A) = L_3 + L_0 + x_A = L_3 + L_0 + L_1 = L_{3+0+1} = L_4$$

$$qm_{PCR5}(B) = L_2 + L_0 + y_B = L_2 + L_0 + L_0 = L_{2+0+0} = L_2$$

Enriched Linguistic labels

Purpose: We enrich the linguistic labels L_i by a quantitative or qualitative confidence factor.

$$L_i \longrightarrow L_i(\epsilon_i)$$

ϵ_i characterizes the weight of reinforcing or discounting expressed by the source when using label L_i for committing its qualitative belief to a given proposition $A \in G^\Theta$.

Enriched Linguistic labels of Type 1 (quantitative)

When $\epsilon_i \in [0, \infty)$, $L_i(\epsilon_i)$ is called an enriched label of Type 1.

If $L_1 \triangleq \mathit{Good}$ and $\epsilon_1 = 1.2$, then $L_1(1.2)$ represents a 20% reinforcement of its nominal value.

Enriched Linguistic labels of Type 2 (qualitative)

When $\epsilon_i \in X$ with $X = \{\text{(ordered)linguistic labels}\}$, $L_i(\epsilon_i)$ is called an enriched label of Type 2.

$PS \triangleq$ "positive small" $PM \triangleq$ "positive medium" $PB \triangleq$ "positive big"

Typically $X = \{NB, NM, NS, O, PS, PM, PB\}$

$NB \triangleq$ "negative big" $NM \triangleq$ "negative medium" $NS \triangleq$ "negative small" $O \triangleq$ "neutral"

Extension of operators on linguistic labels

Purpose: One wants to compute with $L_i(\epsilon_i)$ and $L_j(\epsilon_j)$

We propose 3 possible solutions for dealing with confidence factors

a) $c = (\epsilon_i + \epsilon_j)/2;$

b) $c = \min\{\epsilon_i, \epsilon_j\};$

c) $[\epsilon_{\min}, \epsilon_{\max})]$ where $\epsilon_{\min} \triangleq \min\{\epsilon_i, \epsilon_j\}$, $\epsilon_{\max} \triangleq \max\{\epsilon_i, \epsilon_j\}$

The averaging operator (case a)) is not associative.

The most prudent and easier to use, is the min operator (case b)).

The confidence interval operator (case c)) provides both a lower and a upper confidence level.

Extension of operators on linguistic labels (cont'd)

- *qe*-addition of enriched labels

$$L_i(\epsilon_i) + L_j(\epsilon_j) = \begin{cases} L_{i+j}(c) & \text{if} \quad i+j < n+1, \\ L_{n+1}(c) & \text{otherwise.} \end{cases}$$

- *qe*-subtraction of enriched labels

$$L_i(\epsilon_i) - L_j(\epsilon_j) = \begin{cases} L_{i-j}(c) & \text{if} \quad i \geq j, \\ -L_{j-i}(c) & \text{if} \quad i < j. \end{cases}$$

- *qe*-multiplication of enriched labels

$$L_i(\epsilon_i) \times L_j(\epsilon_j) = L_{[(i \cdot j)/(n+1)]}(c)$$

- *qe*-division of enriched labels

$$\frac{L_i(\epsilon_i)}{L_j(\epsilon_j)} = \begin{cases} L_{[(i/j)\cdot(n+1)]}(c) & \text{if } [(i/j) \cdot (n+1)] < n+1, \\ L_{n+1}(c), & \text{otherwise.} \end{cases}$$

These *qe*-operators are consistent with the classical qualitative operators when $e_i = e_j = 1$ (neutral quantitative confidence degree) and/or when $e_i = e_j = "O"$ (neutral qualitative confidence degree).

Example of fusion of qualitative enriched beliefs (Type 1)

$\Theta = \{A, B\}$ with Shafer's model. $L = \{L_0, L_1, L_2, L_3, L_4, L_5, L_6\}$ $(n = 5)$

Inputs {

	A	B	$A \cup B$	$A \cap B$
$qm_1(\cdot)$	$L_1(0.3)$	$L_2(1.1)$	$L_3(0.8)$	
$qm_2(\cdot)$	$L_4(0.6)$	$L_2(0.7)$	$L_0(1)$	
$qm_{12}(\cdot)$	$L_3(0.3)$	$L_2(0.7)$	$L_0(0.8)$	$L_1(0.3)$

$$
\begin{aligned}
qm_{12}(A) &= qm_1(A)qm_2(A) + qm_1(A)qm_2(A \cup B) + qm_2(A)qm_1(A \cup B) \\
&= L_1(0.3)L_4(0.6) + L_1(0.3)L_0(1) + L_4(0.6)L_3(0.8) \\
&= L_{[(1\cdot 4)/6]}(\min\{0.3, 0.6\}) + L_{[(0\cdot 1)/6]}(\min\{0.3, 1\}) + L_{[(4\cdot 3)/6]}(\min\{0.6, 0.8\}) \\
&= L_1(0.3) + L_0(0.3) + L_2(0.6) = L_{1+0+2}(\min\{0.3, 0.3, 0.6\}) = L_3(0.3)
\end{aligned}
$$

we use min operator here (case b))

Similarly for computing $qm_{12}(B)$ and $qm_{12}(A \cup B)$.

The conflict is decomposed as follows:

$$
\begin{aligned}
qm_{12}(\emptyset) &= qm_1(A)qm_2(B) + qm_2(A)qm_1(B) = L_1(0.3)L_2(0.7) + L_4(0.6)L_2(1.1) \\
&= L_{[(1\cdot 2)/6]}(\min\{0.3, 0.7\}) + L_{[(4\cdot 2)/6]}(\min\{0.6, 1.1\}) \\
&= L_0(0.3) + L_1(0.6) = L_{0+1}(\min\{0.3, 0.6\}) = L_1(0.3)
\end{aligned}
$$

Example of fusion of qualitative enriched beliefs (Type 1)

Inputs {

	A	B	$A \cup B$	$A \cap B$
$qm_1(\cdot)$	$L_1(0.3)$	$L_2(1.1)$	$L_3(0.8)$	
$qm_2(\cdot)$	$L_4(0.6)$	$L_2(0.7)$	$L_0(1)$	
$qm_{12}(\cdot)$	$L_3(0.3)$	$L_2(0.7)$	$L_0(0.8)$	$L_1(0.3)$

Applying PCR5:

$qm_1(A)qm_2(B) = L_1(0.3)L_2(0.7) = L_0(0.3)$ is redistributed back to A and B proportionally with respect to $qm_1(A) = L_1(0.3)$ and $qm_2(B) = L_2(0.7)$. Since $L_0(0.3)$ is the null qualitative label, both A and B get $L_0(0.3)$.

$qm_2(A)qm_1(B) = L_4(0.6)L_2(1.1) = L_1(0.6)$ is redistributed back to A and B proportionally with respect to $qm_2(A) = L_4(0.6)$ and $qm_1(B) = L_2(1.1)$, i.e.

$$\frac{x_A}{L_4(0.6)} = \frac{y_B}{L_2(1.1)} = \frac{L_1(0.6)}{L_4(0.6) + L_2(1.1)} = \frac{L_1(0.6)}{L_6(0.6)} = L_{[(1/6)\cdot 6]}(\min\{0.6, 0.6\}) = L_1(0.6)$$

whence $\begin{cases} x_A = L_4(0.6) \cdot L_1(0.6) = L_{[(4\cdot 1)/6]}(\min\{0.6, 0.6\}) = L_1(0.6) \\ y_B = L_2(1.1) \cdot L_1(0.6) = L_{[(2\cdot 1)/6]}(\min\{1.1, 0.6\}) = L_0(0.6) \end{cases}$

$$qm_{PCR5}(A) = L_3(0.3)+L_0(0.3)+x_A = L_3(0.3)+L_0(0.3)+L_1(0.6) = L_{3+0+1}(\min\{0.3, 0.3, 0.6\}) = L_4(0.3)$$

$$qm_{PCR5}(B) = L_2(0.7)+L_0(0.3)+y_B = L_2(0.7)+L_0(0.3)+L_0(0.6) = L_{2+0+0}(\min\{0.7, 0.3, 0.6\}) = L_2(0.3)$$

$$qm_{PCR5}(A \cup B) = L_0(0.8) \qquad qm_{PCR5}(A \cap B = \emptyset) = L_0 = L_0(1)$$

Example of fusion of qualitative enriched beliefs (Type 2)

$\Theta = \{A, B\}$ with Shafer's model. $L = \{L_0, L_1, L_2, L_3, L_4, L_5, L_6\}$ $(n = 5)$

Inputs

	A	B	$A \cup B$
$qm_1(\cdot)$	$L_1(NB)$	$L_2(PS)$	$L_3(NS)$
$qm_2(\cdot)$	$L_4(NM)$	$L_2(NS)$	$L_0(O)$

$$X = \{NB, NM, NS, O, PS, PM, PB\}$$

The minimum operator (case b)) on X (qualitative degrees) works similarly as on $\mathbb{R}^+$.

The qualitative conjunctive and PCR5 fusion rules are obtained with derivations identical to the previous ones, since $NB \prec NM \prec NS \prec O \prec PS \prec PM \prec PB$.

We just use linguistic operators for qualitative confidence degrees instead numerical operators as in previous example.

Final result with type 2:

	A	B	$A \cup B$	$A \cap B$
$qm_{12}(\cdot)$	$L_3(NB)$	$L_2(NS)$	$L_0(NS)$	$L_1(NB)$
$qm_{PCR5}(\cdot)$	$L_4(NB)$	$L_2(NB)$	$L_0(NS)$	$L_0(O)$

Coherence with type 1: $NB = 0.3$ or less, $NM = [0.5, 0.6]$, $NS = [0.7, 0.8]$, $O = 1$ and $PS = 1.1$.

$qm_{PCR5}(\cdot)$	$L_4(0.3)$	$L_2(0.3)$	$L_0(0.8)$	$L_0(1)$

Conclusions

In this work, we have proposed two possible enrichments (quantitative and/or qualitative) of qualitative belief masses.

A direct extension of the qualitative operators developed in DSmT framework has been done.

We have shown how to fuse qualitative-enriched belief masses expressed in natural language by human experts with Proportional Conflict Redistribution Rule # 5 (PCR5).

Very simple examples have been presented in details to explain how to use qualitative-enriched operators (qe-operators) and qualitative PCR5.

Note: Applications of this approach in robotics for autonomous mobile robot navigation map building are under progress and will be presented in a forthcoming publication.

Call for collaborations and contributions

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Call for contributions to Vol. 3 (Fall 2008)

A third volume on Advances and applications of DSmT for information fusion is planned for publication by Fall 2008. Any contribution on applications and comparison of DSmT with other approaches is very welcome as well as theoretical investigations on DSmT. If you want to contribute to this third forthcoming volume, please directly contact the editors F. Smarandache and J. Dezert at smarand@unm.edu and jean.dezert@onera.fr . Only 8 pages minimum (one column format) contribution will be considered. No limitation on the maximum numbers of pages. Inclusion of color figures is accepted for the final electronic version of the book. The book (paper version) however will be printed by publisher in B/W. LaTeX format is preferred.

Submission deadline: June 30th, 2008