A PCR-BIMM filter for maneuvering target tracking

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Abstract − *In this paper we show how to correct and improve the Belief Interacting Multiple Model filter (BIMM) proposed in 2009 by Nassreddine et al. for tracking maneuvering targets. Our improved algorithm, called PCR-BIMM is based on results developed in DSmT (Dezert-Smarandache Theory) framework and concerns two main steps of BIMM: 1) the update of the basic belief assignment of modes which is done by the Proportional Conflict Redistribution Rule no. 5 rather than Smets' rule (conjunctive rule); 2) the global target state estimation which is obtained from the DSmP probabilistic transformation rather than the commonly used Pignistic transformation. Monte-Carlo simulation results are presented to show the performances of this PCR-BIMM filter with respect to classical IMM and BIMM filters obtained on a very simple maneuvering target tracking scenario.*

Keywords: Tracking, Maneuvering target, IMM, BIMM, DSmT.

1 Introduction

In Fusion 2009 international conference, Nassreddine, Abdallah, and Denœux [13] have proposed an interesting idea to extend the classical Interacting Multiple Models (IMM) filter with belief function theory in order to deal with an unknown and variant motion models. Their algorithm is based on the classical/historical belief function theory developed by Shafer in 1976 [14], known as Dempster-Shafer Theory (DST) and requires both Smets' rule, i.e. the conjunctive fusion rule equivalent to the non normalized Dempster's rule, and the probabilistic pignistic transformation. This algorithm is called Belief Interacting Multiple Model algorithm (BIMM). According to authors results, BIMM algorithm outperforms classical IMM algorithm at least in the vehicle localization problem studied in their works. These appealing results and the possible extension of IMM in belief function theory framework motivates our interest to analyze and evaluate this new BIMM filter.

A deep analysis of the paper yields to the following comments:

1. The derivation of the predicted prior basic belief assignment of modes in Step 1 of BIMM algorithm was clearly wrong in [13] as proved in the sequel. This mistake implies a serious doubt on the validity of the results presented in [13].

2. The simulations results presented in [13] cannot be verified precisely, nor reproduced, because some settings parameters (like α_i discounting factors) required for the BIMM filter have not be provided by the authors and the essential step 9 of the algorithm was not detailed enough.

3. It is known (see Chapter 1 of [15] Vol. 3) that the conjunctive rule does not perform efficiently in a sequential fusion process because the empty set is an absorbing element for the conjunctive fusion rule. Therefore, in order to implement successfully the BIMM filter, some ad-hoc numerical techniques are necessary (or some extra normalization steps) in the BIMM algorithm in order to prevent the mass of belief committed to empty set to become close to one and make Smets' rule responding to new information. This serious problem has unfortunately not been discussed in [13].

From the theoretical point of view, it is quite surprising that one gets better performances with the BIMM (which proceeds with less specific information since it deals with non Bayesian basic belief assignments) than with the classical Bayesian IMM filter (which deals with more specific information, i.e. with Bayesian basic belief assignments). The first purpose of this work is to verify if the conclusions given in [13] are valid on a very simple reproducing maneuvering target tracking scenario. We want also to see if a more justified Belief-based IMM algorithm can be developed to improve the BIMM algorithm and to evaluate it to get a fair comparison of its performance with respect to

classical IMM filter. The improvement of the BIMM algorithm we propose in this paper is based on advanced theoretical results obtained in the development of Dezert-Smarandache Theory (DSmT) of information fusion [15]. This paper is organized as follows: After a brief recall of classical (fixed structure) IMM algorithm given in section 2, one presents in section 3 the Belief IMM algorithm and its flaws. Motivations for the improvement of the BIMM filter is presented in section 4 with the presentation of the main steps of our new algorithm called PCR-BIMM filter (Proportional Conflict Redistribution-based BIMM). In section 5, we examine the performances of the IMM, and PCR-BIMM on a very simple tracking scenario through Monte-Carlo results. Conclusions and perspectives for further investigations are given in section 6.

2   Classical IMM algorithm

The IMM filter is one of the most used algorithm for tracking maneuvering targets and was developed originally by Henk Blom in eighties [5, 6, 2]. The IMM filter is a recursive filter with a low complexity and has been proved very efficient in many real-data tracking applications [4] and many extensions of IMM have been developed since its original publication for dealing with multitarget-multisensor case, cluttered environments, etc, see [12] for a good survey of Multiple Models techniques. The classical IMM algorithm considers a hybrid Multiple Models (MM) system which obeys one of a finite number r of dynamic models M_i, $i = 1, \ldots, r$ and estimates the posterior mode probabilities from their prior probabilities and target measurements (Bayesian framework). Its specificity is that IMM mixes hypotheses with depth 1 only at the start of each cycle and thus has a low complexity of order $O(r)$, while providing same performances as the more effective Generalized Pseudo-Bayesian estimator of order 2. We briefly recall the principle of classical IMM filter, see [3, 4] for more details with examples. A hybrid MM system is characterized by two state variables: 1) the *base-state* variable $\mathbf{x}(k)$ of dimension n_x including the position, velocity, etc. of the target, and 2) a *modal-state* $M_j(k)$ belonging to a known finite set $\mathcal{M}_r(k) = \{M_i(k), i = 1, \ldots, r\}$ of r possible dynamic models for the target during its motion. For simplicity of presentation, we consider only a fixed-structure IMM, i.e. $\mathcal{M}_r(k) = \mathcal{M}_r$ is invariant with time. Variable-structure IMM is possible and has been introduced by Xiao-Rong Li in [10, 11]. The hybrid system is described by the equations[1]

$$\mathbf{x}(k) = \mathbf{F}[M(k)]\mathbf{x}(k-1) + \mathbf{v}[k-1, M(k)]$$

$$\mathbf{z}(k) = \mathbf{H}[M(k)]\mathbf{x}(k) + \mathbf{w}[k, M(k)]$$

where $M(k)$ is the mode in effect during the sampling period ending at time k belonging in $\mathcal{M}_r$. $\mathbf{x}(k)$ and

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$\mathbf{z}(k)$ are the target state and observation vectors. The set of all available measurements up to k is denoted $\mathbf{Z}^k$. $\mathbf{F}[M(k)]$ and $\mathbf{H}[M(k)]$ are known matrices depending on the dynamic model $M(k)$. The statistics of the process and observation noises $\mathbf{v}[k-1, M(k)]$ and $\mathbf{w}[k, M(k)]$ can differ from mode to mode. Usually one considers $\mathbf{v}[k-1, M(k) = M_j] \sim \mathcal{N}(\bar{\mathbf{v}}_j, \mathbf{Q}_j)$ and $\mathbf{w}[k, M(k) = M_j] \sim \mathcal{N}(\bar{\mathbf{w}}_j, \mathbf{R}_j)$ with known covariance matrices $\mathbf{Q}_j$ and $\mathbf{R}_j$ respectively. The Mode jump process is modeled as a Makov chain with known a priori probabilities $P\{M(0) = M_j\} = \mu_j(k = 0)$ and known transition probabilities $P\{M(k) = M_j | M(k-1) = M_i\} = \pi_{ij}$. A cycle of the classical IMM algorithm $(k-1) \mapsto k$ consists in the following steps:

• **Step 0** (Initialization at $k = 0$): Definition of dynamic and observation matrices, choice of process and observation noise levels, sampling period, initialization of the filters adapted to each mode, choice of the prior mode probabilities P_j and the transition probability matrix $\mathbf{P}_t \triangleq [\pi_{ij} = P\{M_j(k)|M_i(k-1)\}]'$ assumed known and time-invariant.

• **Step 1** (Interaction-mixing ($j = 1, \ldots, r$)): Mixing of the previous cycle mode-conditioned state estimates $\hat{\mathbf{x}}_i(k-1|k-1)$ and covariance, using the mixing probabilities $\mu_{i|j}(k-1|k-1)$, to initialize the current cycle of each mode-conditioned filter $\hat{\mathbf{x}}_j^0(k-1|k-1)$. This is done by

$$\hat{\mathbf{x}}_j^0(k-1|k-1) = \sum_{i=1}^{r} \mu_{i|j}(k-1|k-1)\hat{\mathbf{x}}_i(k-1|k-1) \quad (1)$$

$$\begin{aligned} \mathbf{P}_j^0(k-1|k-1) = \sum_{i=1}^{r} \mu_{i|j}(k-1|k-1)\{\mathbf{P}_i(k-1|k-1) \\ - [\hat{\mathbf{x}}_i(k-1|k-1) - \hat{\mathbf{x}}_j^0(k-1|k-1)] \cdot \\ [\hat{\mathbf{x}}_i(k-1|k-1) - \hat{\mathbf{x}}_j^0(k-1|k-1)]'\} \end{aligned} \quad (2)$$

where the elements $\mu_{i|j}(k-1|k-1)$ of the mixing probability (vertical) vector $\boldsymbol{\mu}_{k-1|k-1}(.|M_j(k)) = [\mu_{i|j}(k-1|k-1), i = 1, \ldots r]'$ are calculated by

$$\begin{aligned} \mu_{i|j}(k-1|k-1) &\triangleq P\{M_i(k-1)|M_j(k), \mathbf{Z}^{k-1}\} \\ &= \frac{\pi_{ij}\mu_i(k-1)}{\mu_j^-(k)} \end{aligned} \quad (3)$$

with

$$\mu_j^-(k) \triangleq P\{M_j(k)|\mathbf{Z}^{k-1}\} = \sum_{i=1}^{r} \pi_{ij}\mu_i(k-1) \quad (4)$$

The equation (4) can be written more concisely as:

$$\boldsymbol{\mu}_k^-(.) = \mathbf{P}_t \cdot \boldsymbol{\mu}_{k-1}(.) \quad (5)$$

where $\mathbf{P}_t' = [\pi_{ij}]$ and $\boldsymbol{\mu}_{k-1}(.)$ represents the (vertical) vector of prior probability of modes, i.e.

$$\boldsymbol{\mu}_{k-1}(.) = [P(M_i(k-1)|\mathbf{Z}^{k-1})]' = [\mu_1(k-1) \ldots \mu_r(k-1)]'$$

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[1]For simplicity, we assume here linear systems.

and $\boldsymbol{\mu}_k^-(.)$ represents the (vertical) vector of predicted prior probability of modes

$$\boldsymbol{\mu}_k^-(.) = [P(M_j(k)|\mathbf{Z}^{k-1})]' = [\mu_1^-(k) \ \ldots \mu_r^-(k)]'$$

• **Step 2** (Mode conditioned filter): From prior mixed statistics $\hat{\mathbf{x}}_j^0(k-1|k-1)$ and $\mathbf{P}_j^0(k-1|k-1)$ and the target measurement $\mathbf{z}(k)$, one calculates $\hat{\mathbf{x}}_j(k|k)$ and $\hat{\mathbf{P}}_j(k|k)$ for each possible mode in effect (r filters running in parallel) by a specific filter matched to mode M_j, typically a Kalman filter if the dynamic and observation system are linear, or Extended Kalman Filter (EKF) to deal with linear or non linear equations, or any other sophisticated filters if necessary for dealing for example with miss-detections and false alarms [3]. The likelihood $\Lambda_j(k)$ of the filter j is assumed to be Gaussian with

$$\Lambda_j(k) = \frac{1}{(2\pi)^{n_z/2}\sqrt{|\mathbf{S}_j(k)|}} \exp^{-\frac{1}{2}\tilde{\mathbf{z}}_j'(k)\mathbf{S}_j^{-1}(k)\tilde{\mathbf{z}}_j(k)} \quad (6)$$

where $\tilde{\mathbf{z}}_j(k) \triangleq \mathbf{z}(k) - \hat{\mathbf{z}}_j(k|k-1)$ is the innovation and $\mathbf{S}_j(k)$ is the covariance of the innovation provided by the filter j.

• **Step 3** (Mode probability update): The probability $\mu_j(k)$ of each mode j for $j = 1, \ldots, r$ is calculated by

$$\mu_j(k) = P\{M_j(k)|\mathbf{Z}^k\} = \Lambda_j(k)\mu_j^-(k)/\sum_{i=1}^{r}\Lambda_i(k)\mu_i^-(k) \tag{7}$$

• **Step 4** (Global estimation for output purpose): The global estimate $\hat{\mathbf{x}}(k|k)$ and the covariance of estimation error $\mathbf{P}(k|k)$ are given by:

$$\hat{\mathbf{x}}(k|k) = \sum_{j=1}^{r}\mu_j(k)\hat{\mathbf{x}}_j(k|k) \tag{8}$$

$$\mathbf{P}(k|k) = \sum_{j=1}^{r}\mu_j(k)\{\mathbf{P}_j(k|k) \\ - [\hat{\mathbf{x}}_j(k|k) - \hat{\mathbf{x}}(k|k)] \cdot [\hat{\mathbf{x}}_j(k|k) - \hat{\mathbf{x}}(k|k)]'\} \tag{9}$$

3 Belief-based IMM algorithm

In 2009, Nassreddine et al. have proposed in [13] an extension of classical IMM filter in the framework of Dempster-Shafer Theory (DST) [14] for dealing with an unknown and variant motion models. The idea was to select a set of candidate models[2], and then estimate a current basic belief assignment (bba) defined on the power-set of this set of models based on the fusion of bba's built from measurement likelihoods with the predicted bba of the models using Smets' rule[3] denoted $\odot$. From the result of Smets' fusion, the mixed

state of classical IMM filter is replaced with the *pignistic averaging* of the mode-conditioned state estimates. This new extension of IMM filter was called BIMM (Belief-based IMM) since it uses belief function theory to represent the uncertainty in the switches between the modes. This section presents succinctly the principle of the BIMM filter. We justify also our motivation for developing a new Belief-based IMM algorithm. The steps of BIMM are actually very close to the steps of classical IMM, except that predicted and updated mode probabilities are estimated from pignistic probabilities derived from a basic belief assignment updated with the conjunctive rule of combination. The main changes of BIMM concern the Step 1 and the Step 3 of IMM algorithm. The frame of discernment chosen in BIMM coincides with the set of possible models, i.e. $\Theta(k) \equiv \mathcal{M}_r(k) = \{M_i(k), i = 1, \ldots, r\}$. Instead of computing recursively the mixed $\mu_{i|j}(.)$ and updated $\mu_j(.)$ probabilities with eqs. (3) and (4) as done with the classical IMM, one deals with bba's defined on the power-set 2^Θ of the frame of discernment. Mathematically, a normal bba $\mathbf{m}(.)$ is defined[4] as a mapping from $2^\Theta \mapsto [0,1]$ such that $m(\emptyset) = 0$ and $\sum_{A \in 2^\Theta} m(A) = 1$. A is a focal element of $\mathbf{m}(.)$ if $m(A) > 0$. Any discrete probability measure can be interpreted as a special belief function, called Bayesian belief [14] whose focal elements are singletons of 2^Θ. Any belief function with a bba $\mathbf{m}(.)$ can be approximated into subjective probability measure thanks to the pignistic transformation [17] defined for all $M_i \in \Theta(k)$ by

$$BetP\{M_i\} = \sum_{A \in 2^\Theta | A \cap M_i = M_i} \frac{1}{|A|} \cdot \frac{m(A)}{1 - m(\emptyset)} \tag{10}$$

where $|A|$ is the cardinality of A.

The steps of BIMM proposed in [13] are[5]:

• **Step 0** (Initialization at $k = 0$): Definition of dynamic and observation matrices, choice of process and observation noise levels, sampling period, initialization of the filters adapted to each mode. The prior probabilities of modes $\{P_j = P\{M(0) = M_j\}, j = 1, \ldots, r\}$ used in IMM, are replaced[6] by the vacuous belief assignment $m(\Theta(k = 0) = M_1 \cup M_2 \cup \ldots \cup M_r) = 1$. The probability transition matrix $\mathbf{P}_t' = [\pi_{ij}]$ is replaced by a bba transition matrix[7] $\mathbf{M}_t \triangleq [m_{ij}]$ having a very simple structure defined by the r implication rules: "R_i: if $M(k) = M_i(k)$ then $M(k+1) = M_i(k+1)$" with known belief coefficients $\beta_i \in [0,1]$ for $i = 1, 2, \ldots, r$ with $\beta_i = m(M_i(k+1)|M_i(k))$ and $1 - \beta_i = m(\Theta(k+1) = M_1 \cup \ldots \cup M_r(k+1)|M_i(k))$.

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[2]Corresponding to the so-called frame of discernment and usually denoted Θ in DST.

[3]Smet's rule is nothing but the non normalized Dempster's rule of combination, i.e. the conjunctive rule.

[4]We use boldface letters to denote vectors or matrices.

[5]We use a more classical notation generally adopted in the tracking community.

[6]Note that this initialization can also be done by taking $m(M_j(k = 0) = P_j$ as well if one considers that prior probabilities of modes is accurate enough.

[7]Called switching mass function in [13].

• **Step 1** (Interaction-mixing): The mixing probability $\mu_{i|j}(k-1|k-1)$ are calculated as follows:

1. The derivation of probabilities vector $\boldsymbol{\mu}_k^-(.) = [\mu_1^-(k) \ \dots \ \mu_r^-(k)]'$ in classical IMM is replaced by the derivation of the predicted bba $\mathbf{m}_k^-(.)$ given by

$$\mathbf{m}_k^-(.) \triangleq \mathbf{M}_t \cdot \mathbf{m}_{k-1}(.) \tag{11}$$

2. The derivation of probabilities $\mu_{i|j}(k-1|k-1) \triangleq P\{M_i(k-1)|M_j(k), \mathbf{Z}^{k-1}\}$ is replaced by the derivation of bba $m_{k-1|k-1}(.)$ thanks to the Generalized Bayesian Theorem (GBT) [18]. More precisely,

$$\mathbf{m}_{k-1|k-1}(.|M_j(k)) =$$
$$[\odot\mathbf{m}_k^{\Uparrow\Theta(k-1)\times\Theta(k)}(.|M_i(k-1))](.|M_j(k)]^{\downarrow\Theta(k-1)} \tag{12}$$

where $\Uparrow \Theta(k-1) \times \Theta(k)$ is the ballooning extension [18] of the bba on the Cartesian product frame $\Theta(k-1) \times \Theta(k)$, and where $\downarrow \Theta(k-1)$ represents the marginalization operation of the bba on the frame $\Theta(k-1)$. See [18], for details and examples.

3. The derivation of the mixing probability $\mu_{i|j}(k-1|k-1) = P\{M_i(k-1)|M_j(k), \mathbf{Z}^{k-1}\}$ of classical IMM is replaced by the pignistic probability drawn from $\mathbf{m}_{k-1|k-1}(.|M_j(k))$, that is:

$$\mu_{i|j}(k-1|k-1) = BetP\{M_i(k-1)|M_j(k), \mathbf{Z}^{k-1}\}$$

where $BetP\{.\}$ is calculated with the transformation (10) using $\mathbf{m}_{k-1|k-1}(.|M_j(k))$ given by (12).

$\hat{\mathbf{x}}_j^0(k-1|k-1)$ and $\mathbf{P}_j^0(k-1|k-1)$ are calculated as in IMM Step 1.

• **Step 2**: Same as IMM Step 2.

• **Step 3** (Mode bba update): The updated bba $\mathbf{m}_k(.)$ of modes is computed from the conjunctive combination of the predicted bba $\mathbf{m}_{k-1}^-(.)$ given in (11) with *observed bba's*[8] $\mathbf{m}_{k,j}(.)$, $j = 1, 2, \dots r$ by

$$\mathbf{m}_k(.) = [\mathbf{m}_{k,1}\odot\dots\odot\mathbf{m}_{k,r}\odot\mathbf{m}_{k-1}^-](.) \tag{13}$$

where the *observed bba's* $\mathbf{m}_{k,j}(.)$ for $j = 1, \dots, r$ are given[9] by [13]:

$$\begin{cases} m_{k,j}(M_j(k)) &= 0 \\ m_{k,j}(\bar{M}_j(k)) &= \alpha_j(1 - R\Lambda_j(k)) \\ m_{k,j}(\Theta(k)) &= 1 - \alpha_j(1 - R\Lambda_j(k)) \end{cases} \tag{14}$$

α_j is a discounting coefficient associated with the likelihood of the mode $M_j(k)$ and R is a normalization constant.

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[8]We mean that the bba $\mathbf{m}_{k,j}(.)$ is built from the likelihood $\Lambda_j(k)$ which depends on the mode $M_j(k)$ and on the observation available $\mathbf{z}(k)$.

[9]This is Appriou's model no. 1 in [1].

• **Step 4** (Global estimation for output purpose): The global estimate $\hat{\mathbf{x}}(k|k)$ and the covariance of estimation error $\mathbf{P}(k|k)$ are given as in step 4 of classical IMM by taking $\mu_j(k) = BetP\{M_j(k)|\mathbf{Z}^k\}$ where $BetP\{M_j(k)|\mathbf{Z}^k\}$ is the pignistic probability that the mode M_j is effective at time k. $BetP\{M_j(k)|\mathbf{Z}^k\}$ is computed from the updated bba $\mathbf{m}_k(.)$ given by (13).

A mistake in Step 1 of BIMM filter: The aforementioned Step 1 of BIMM algorithm described with an example in [13] is clearly incorrect because the derivation of the predicted bba $\mathbf{m}_k^-(.)$ by (5) is wrong because the sum of masses of focal elements is not equal to one. It is easy to verify from example in [13] when considering only two models, when taking $\beta_1 = m_($M_1(k)|M_1(k-1)) = 0.9$, $1-\beta_1 = 0.1 = m_($M_1(k) \cup \bar{M}_1(k)|M_1(k-1))$ and $\beta_2 = m_($M_2(k)|M_2(k-1)) = 0.89$, $1-\beta_2 = 0.11 = m_($M_2(k) \cup \bar{M}_2(k)|M_2(k-1))$ and taking the prior bba $\mathbf{m}_{k-1}(.) = [m(\emptyset) = 0 \ m(M_1(k-1)) = 0.45 \ m(M_2(k-1)) = 0.20 \ m(M_1(k-1) \cup M_2(k-1)) = 0.35]'$. Applying the wrong formula (11), one gets precisely:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0.1 \\ 0 & 0 & 0.89 & 0.11 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{M}_t} \underbrace{\begin{bmatrix} 0 \\ 0.45 \\ 0.20 \\ 0.35 \end{bmatrix}}_{\mathbf{m}_{k-1}(.)} = \underbrace{\begin{bmatrix} 0 \\ 0.4400 \\ 0.2165 \\ 0.3500 \end{bmatrix}}_{\mathbf{m}_k^-(.)} \neq \underbrace{\begin{bmatrix} 0 \\ 0.44 \\ 0.21 \\ 0.35 \end{bmatrix}}_{\text{Result in [13]}}$$

One can see that the sum of components of $\mathbf{m}_k^-(.)$ equals 1.0065 !!! This mistake is not due to rounding approximation of the result, but to a more serious mistake in the choice of the transition matrix $\mathbf{M}_t$. This mistake actually comes from the confusion in indices of the classical IMM transition matrix. It is easy to verify that the correct transition matrix must be actually taken as the transpose of $\mathbf{M}_t$. Therefore, the correct derivation of $\mathbf{m}_k^-(.)$ must be done by

$$\mathbf{m}_k^-(.) \triangleq \mathbf{M}_t' \cdot \mathbf{m}_{k-1}(.) \tag{15}$$

For the example 1 of [13], one will get correctly

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0.89 & 0 \\ 0 & 0.1 & 0.11 & 1 \end{bmatrix}}_{\mathbf{M}_t'} \underbrace{\begin{bmatrix} 0 \\ 0.45 \\ 0.20 \\ 0.35 \end{bmatrix}}_{\mathbf{m}_{k-1}(.)} = \underbrace{\begin{bmatrix} 0 \\ 0.4050 \\ 0.1780 \\ 0.4170 \end{bmatrix}}_{\mathbf{m}_k^-(.)}$$

Remarks on BIMM filter: The BIMM is based on two[10] pillars: 1) the conjunctive rule of combination, and 2) the pignistic transformation to approximate a bba into a subjective probability measure because. These two pillars are disputable because:

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[10]Actually, Smets' Generalized Bayesian Theorem (GBT) could be also considered as the third pillar of BIMM.

1. The efficiency of Smets' rule for combining bba's is very questionable in this belief-based extension of IMM because it has been already proved in [15], Vol. 3, and specially in sequential Target Type Tracking problem [7] that such rule doesn't perform well in general for mode change detection. Smets' rule doesn't respond to new information since very quickly all the mass of belief concentrates on the empty set. See example in [15], Vol. 3, Chap. 1, freely downloadable from the web and not reported here due to space limitation.

2. The real interest and efficiency of the pignistic transformation is also disputable because there exists other probabilistic transformations which perform better than $BetP$ in term of probabilistic informational content, in particular the DSmP transformation developed in [15], Vol. 3, Chap 1 & 3 and also in [8].

3. The justification for the use of Appriou's model no. 1 in step 3 of BIMM is missing and probably other (and maybe better) models could be developed to derive the updated bba $\mathbf{m}_k(.)$. This question has not been investigated in this paper and will be a source for future research.

Interest of BIMM w.r.t. IMM: The potential advantage of the belief-based IMM approach is to offer some robustness of the filter when replacing the strong constraint on the knowledge of probability of transitions π_{ij} (usually based on ad-hoc assumptions on the mean sojourn time of the target in each mode) by a more flexible constraint on the transitions based on (very simple and less specific) uncertain implication rules. With BIMM, one can also relax the knowledge of the prior probabilities of the modes by starting the tracking directly with a vacuous belief prior of the modes. Of course, if one has good reasons to use a given prior of modes, this can be done easily in belief-based IMM approach which is also a nice features of such filter.

4 PCR-BIMM algorithm

To preserve the potential advantages of BIMM and to overcome its aforementionned problems, we propose to keep its general structure as a belief-based extension of classical IMM but we replace Smets' rule by the more effective Proportional Conflict Redistribution rule no. 5 (PCR5), or eventually the more simple PCR rule no. 6 (PCR6), and to replace the pignistic transformation by the more effective DSmP transformation to estimate modes probabilities required in the IMM filter. We call this new algorithm, the PCR-BIMM filter. Before giving the sketch of our PCR-BIMM filter, we just recall what are the PCR5 fusion rule and the DSmP transformation. All details, justifications with examples on PCR5 and DSmP can be found freely from the web in [15], Vols. 2 & 3 and will not be reported here.

4.1 PCR5 and PCR6 fusion rules

In DSmT (Dezert-Smarandache Theory) framework, the Proportional Conflict Redistribution Rule no. 5 (PCR5) is used generally to combine bba's. PCR5 transfers the conflicting mass only to the elements involved in the conflict and proportionally to their individual masses, so that the specificity of the information is entirely preserved in this fusion process. Let $m_1(.)$ and $m_2(.)$ be two independent[11] bba's, then the PCR5 rule is defined as follows (see [15], Vol. 2 for full justification and examples): $m_{PCR5}(\emptyset) = 0$ and $\forall X \in 2^{\Theta} \setminus \{\emptyset\}$

$$m_{PCR5}(X) = \sum_{\substack{X_1, X_2 \in 2^{\Theta} \\ X_1 \cap X_2 = X}} m_1(X_1) m_2(X_2) +$$

$$\sum_{\substack{X_2 \in 2^{\Theta} \\ X_2 \cap X = \emptyset}} \left[\frac{m_1(X)^2 m_2(X_2)}{m_1(X) + m_2(X_2)} + \frac{m_2(X)^2 m_1(X_2)}{m_2(X) + m_1(X_2)} \right] \quad (16)$$

where all denominators in (16) are different from zero. If a denominator is zero, that fraction is discarded. Additional properties of PCR5 can be found in [9]. Extension of PCR5 for combining qualitative bba's can be found in [15], Vol. 2 & 3. All propositions/sets are in a canonical form. A variant of PCR5, called PCR6 has been proposed by Martin and Osswald in [15], Vol. 2, for combining $s > 2$ sources. The general formulas for PCR5 and PCR6 rules are given in [15], Vol. 2 also. PCR6 coincides with PCR5 when one combines two sources. The difference between PCR5 and PCR6 lies in the way the proportional conflict redistribution is done as soon as three or more sources are involved in the fusion. For example, let's consider three sources with bba's $m_1(.)$, $m_2(.)$ and $m_3(.)$, $A \cap B = \emptyset$ for the model of the frame Θ, and $m_1(A) = 0.6$, $m_2(B) = 0.3$, $m_3(B) = 0.1$. With PCR5 the partial conflicting mass $m_1(A)m_2(B)m_3(B) = 0.6 \cdot 0.3 \cdot 0.1 = 0.018$ is redistributed back to A and B only with respect to the following proportions respectively: $x_A^{PCR5} = 0.01714$ and $x_B^{PCR5} = 0.00086$ because the proportionalization requires

$$\frac{x_A^{PCR5}}{m_1(A)} = \frac{x_B^{PCR5}}{m_2(B)m_3(B)} = \frac{m_1(A)m_2(B)m_3(B)}{m_1(A) + m_2(B)m_3(B)}$$

that is $\dfrac{x_A^{PCR5}}{0.6} = \dfrac{x_B^{PCR5}}{0.03} = \dfrac{0.018}{0.6 + 0.03} \approx 0.02857$

thus $\begin{cases} x_A^{PCR5} = 0.60 \cdot 0.02857 \approx 0.01714 \\ x_B^{PCR5} = 0.03 \cdot 0.02857 \approx 0.00086 \end{cases}$

With the PCR6 fusion rule, the partial conflicting mass $m_1(A)m_2(B)m_3(B) = 0.6 \cdot 0.3 \cdot 0.1 = 0.018$ is redistributed back to A and B only with respect to the following proportions respectively: $x_A^{PCR6} = 0.0108$ and

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[11]I.e. each source provides its bba independently of the other sources.

$x_B^{PCR6} = 0.0072$ because the PCR6 proportionalization is done as follows:

$$\frac{x_A^{PCR6}}{m_1(A)} = \frac{x_{B,2}^{PCR6}}{m_2(B)} = \frac{x_{B,3}^{PCR6}}{m_3(B)} = \frac{m_1(A)m_2(B)m_3(B)}{m_1(A) + m_2(B) + m_3(B)}$$

that is

$$\frac{x_A^{PCR6}}{0.6} = \frac{x_{B,2}^{PCR6}}{0.3} = \frac{x_{B,3}^{PCR6}}{0.1} = \frac{0.018}{0.6 + 0.3 + 0.1} = 0.018$$

thus

$$\begin{cases} x_A^{PCR6} = 0.6 \cdot 0.018 = 0.0108 \\ x_{B,2}^{PCR6} = 0.3 \cdot 0.018 = 0.0054 \\ x_{B,3}^{PCR6} = 0.1 \cdot 0.018 = 0.0018 \end{cases}$$

and therefore with PCR6, one gets finally the following redistributions to A and B:

$$\begin{cases} x_A^{PCR6} = 0.0108 \\ x_B^{PCR6} = x_{B,2}^{PCR6} + x_{B,3}^{PCR6} = 0.0054 + 0.0018 = 0.0072 \end{cases}$$

From the implementation point of view, PCR6 is much more simple to implement than PCR5. For convenience, Matlab codes of PCR5 and PCR6 fusion rules can be found in [15, 16].

4.2 The DSmP transformation

The DSmP probabilistic transformation is a serious alternative to the classical pignistic transformation which allows to increase the probabilistic information content (PIC), i.e. to minimize the Shannon entropy, of the approximated subjective probability measure drawn from any bba. Justification and comparisons of $DSmP(.)$ w.r.t. $BetP(.)$ and to other transformations can be found in details in [8, 15], Vol. 3, Chap. 3. $DSmP$ transformation is defined[12] by $DSmP_\epsilon(\emptyset) = 0$ and $\forall X \in 2^\Theta \setminus \{\emptyset\}$ by

$$DSmP_\epsilon(X) = \sum_{Y \in 2^\Theta} \frac{\displaystyle\sum_{\substack{Z \subseteq X \cap Y \\ \mathcal{C}(Z)=1}} m(Z) + \epsilon \cdot \mathcal{C}(X \cap Y)}{\displaystyle\sum_{\substack{Z \subseteq Y \\ \mathcal{C}(Z)=1}} m(Z) + \epsilon \cdot \mathcal{C}(Y)} m(Y)$$

(17)

where $\mathcal{C}(X \cap Y)$ and $\mathcal{C}(Y)$ denote the cardinals of the sets $X \cap Y$ and Y respectively; $\epsilon \geq 0$ is a small number which allows to reach a highest PIC value of the approximation of $m(.)$ into a subjective probability measure. Usually $\epsilon = 0$, but in some particular degenerate cases, when the $DSmP_{\epsilon=0}(.)$ values cannot be derived, the $DSmP_{\epsilon>0}$ values can however always be derived by choosing ϵ as a very small positive number, say $\epsilon = 1/1000$ for example in order to be as close as we want to the highest value of the PIC. The smaller ϵ, the better/bigger PIC value one gets. When $\epsilon = 1$ and when the masses of all elements Z having $\mathcal{C}(Z) = 1$ are zero, $DSmP_{\epsilon=1}(.) = BetP(.)$.

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[12]Here we work on classical power-set, but DSmP can be defined also for working with other fusion spaces, hyper-power sets or super-power sets if necessary.

4.3 Sketch of PCR-BIMM

We briefly summarize the five steps of our PCR-BIMM filter.

• **Step 0** (Initialization at $k = 0$): Same as Step 0 of BIMM.

• **Step 1** (Interaction-mixing): Same as Step 1 of BIMM except that the predicted bba $\mathbf{m}_k^-(.)$ is computed by (15) instead of (11), that is

$$\mathbf{m}_k^-(.) \triangleq \mathbf{M}_t' \cdot \mathbf{m}_{k-1}(.) \quad (18)$$

and the derivation of the mixing probability $\mu_{i|j}(k-1|k-1) = P\{M_i(k-1)|M_j(k), \mathbf{Z}^{k-1}\}$ of classical IMM is replaced by the DSmP probability drawn from $\mathbf{m}_{k-1|k-1}(.|M_j(k))$, that is:

$$\mu_{i|j}(k-1|k-1) = DSmP_\epsilon(M_i(k-1)|M_j(k), \mathbf{Z}^{k-1})$$

where $DSmP_\epsilon(.)$ is calculated with the transformation (17) using $\mathbf{m}_{k-1|k-1}(.|M_j(k))$ given by (12).

• **Step 2**: Same as IMM Step 2.

• **Step 3** (Mode bba update): The updated bba $\mathbf{m}_k(.)$ of modes is computed from the PCR5 (or eventually PCR6) rule, denoted $\oplus$, of the predicted bba $\mathbf{m}_{k-1}^-(.)$ given in (15) with bba's $\mathbf{m}_{k,j}(.)$, $j = 1, 2, \ldots r$ by

$$\mathbf{m}_k(.) = [\mathbf{m}_{k,1} \oplus \ldots \oplus \mathbf{m}_{k,r} \oplus \mathbf{m}_{k-1}^-](.) \quad (19)$$

where the *observed bba's* $\mathbf{m}_{k,j}(.)$ for $j = 1, \ldots, r$ are given as in BIMM by (14).

• **Step 4** (Global estimation for output purpose): The global estimate $\hat{\mathbf{x}}(k|k)$ and the covariance of estimation error $\mathbf{P}(k|k)$ are given as in step 4 of classical IMM by taking $\mu_j(k) = DSmP_\epsilon\{M_j(k)|\mathbf{Z}^k\}$ computed from the updated bba $\mathbf{m}_k(.)$ by (17).

Remark: This preliminary version of PCR-BIMM is perfectible because it still shares several points with BIMM[13]. In particular, the Step 3 of PCR-BIMM calculates, as in BIMM, $\mathbf{m}_{k,j}(.)$ with a model based on likelihoods $\Lambda_j(k)$ whose strong justification is missing. Further investigations will be done to improve this step 3, as well as the Step 1 to get better performances of PCR-BIMM (if possible) in a future research.

5 Simulation results

In this section, we present the application of the PCR-BIMM to a ground target tracking problem. We consider a vehicle localized in $(1000m, 5000m)$ in the cartesian referential (X, Y). We simulate a ground sensor located in $(0,0)$ which is able to detect the moving target in range ρ and azimut θ. The gaussian measurement noise is supposed to be white and centered with the covariances $\sigma_\rho = 20\,m$ and $\sigma_\theta = 0.008\,rad$. The sampling time is fixed to 2 seconds. For tracking the ground target we only consider two motion models.

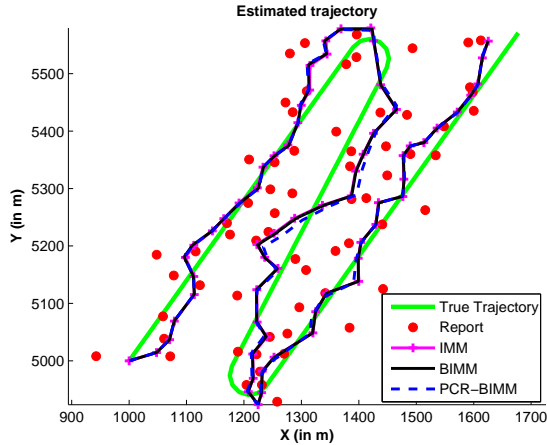[13]In particular, the GBT is still used in Step 2 of PCR-BIMM.

Figure 1: True target trajectory and estimated trajectories.

A constant velocity motion model called CV 1, with a small noise $\sigma_{CV_1} = 1\,m.s^{-2}$ and another constant velocity motion model called CV 2, with a bigger noise $\sigma_{CV_2} = 4\,m.s^{-2}$ to palliate the target maneuver. The initial state for each IMM, BIMM[14] and PCR-BIMM is the true initial target state $\mathbf{x}(0)$. The transition Matrix $\mathbf{P}_t$ is equal to :

$$\mathbf{P}_t = \begin{bmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{bmatrix} \quad (20)$$

and the mass transition matrix $\mathbf{M}_t$ for the BIMM and PCR-BIMM is same as in the paper [13]. The initial motion model mass is represented by the vacuous mass function.

To compare the performances between the algorithms we used the root mean square error (RMSE) in location and velocity (figure 2) and the mean of the motion models probability obtained with 100 Monte-Carlo runs (figures 3, 4, 5). The first remark is, there is no significant improvement by using the belief function in the IMM. In fact, the RMSE of the IMM, BIMM and PCR-BIMM are globally the same. However, we can observe a short difference of the PCR-BIMM error after the target maneuvers between the time intervals $[20, 30]$ and $[40, 50]$. This observation carries along the second remark: the motion model transition duration is longer with the IMM (figure 3) and BIMM (figure 4) than the PCR-BIMM (figure 5). Then with the taken parameters for this simulation, the PCR-BIMM appears to be a good and fast detector of the motion models transition. However, its computed motion models probability is inferior to the probability obtained with the IMM and BIMM. More investigations need to be done to see if it is possible (and how) to improve PCR-BIMM in order to preserve both the good performance of the maneuver

[14]Our BIMM implementations uses algorithm described in section 3 with (15) and additional normalization step $m_k(.)$ in (7) since otherwise the BIMM algorithm doesn't work at all due to the problem mentioned in section 2.

detection and in the same time and get higher probability when the target is moving in the same mode.
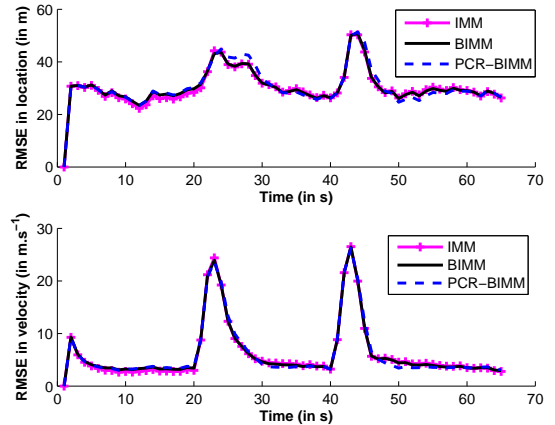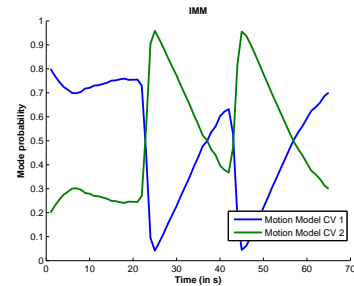

Figure 2: Root Mean Square Error.

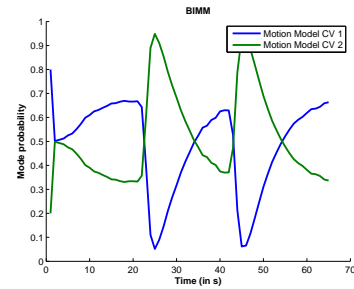Figure 3: Motion Model Probability of the IMM.

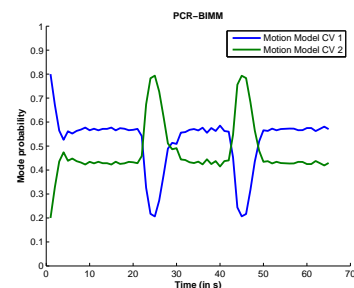Figure 4: Motion Model Probability of the BIMM.

Figure 5: Motion Model Probability of the PCR-BIMM

6 Conclusions

In this paper, we have examined in details the recent BIMM algorithm and have corrected a mistake in it, and also identified some of its limitations. To palliate the problems of BIMM algorithm, we have developed a more efficient belief-based algorithm, called PCR-BIMM, based on the Proportional Conflict Redistribution fusion rule and on the DSmP probabilistic transformation to replace the conjunctive rule and the pignistic transformation used in BIMM. The derivation of the predicted bba of modes done incorrectly in BIMM is also fixed in our PCR-BIMM filter. The perfomances of PCR-BIMM with respect to the (corrected) BIMM and to the classical IMM have been evaluated from a simple maneuvering target tracking scenario through Monte-Carlo simulations. The results obtained in this paper show the ability of the PCR-BIMM to track maneuvering targets and also to improve the maneuver detection. It is important to note that such PCR-BIMM filter can be considered as more robust than IMM since PCR-BIMM requires less specific prior information than IMM. Nevertheless, PCR-BIMM provides globally the same RMS estimation errors performances as those obtained with the classical IMM which requires more specific prior information. Application of PCR-BIMM for tracking multiple maneuvering ground targets in a battlefield surveillance context is under investigation and results will be published in forthcoming papers.

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