

Change Detection from Remote Sensing Images Based on Evidential Reasoning

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Abstract—Theories of evidence have already been applied more or less successfully in the fusion of remote sensing images. In the classical evidential reasoning, all the sources of evidence and their fusion results are related with the same invariable (static) frame of discernment. Nevertheless, there are possible change occurrences through multi-temporal remote sensing images, and these changes need to be detected efficiently in some applications. The invariable frame of classical evidential reasoning can't efficiently represent nor detect the changes occurrences from heterogenous remote sensing images. To overcome this limitation, Dynamical Evidential Reasoning (DER) is proposed for the sequential fusion of multi-temporal images. A new state transition frame is defined in DER and the change occurrences can be precisely represented by introducing a state transition operator. The belief functions used in DER are defined similarly to those defined in the Dempster-Shafer Theory (DST). Two kinds of dynamical combination rules working in free model and constrained model are proposed in this new framework for dealing with the different cases. In the final, an experiment using three pieces of real satellite images acquired before and after an earthquake are provided to show the interest of the new approach.

Keywords: Evidence Theory, Change Detection, Dynamical Evidential Reasoning, DST, DSMT, Remote Sensing.

I. INTRODUCTION

Information fusion resulting from multi-temporal and multi-sources remote sensing images remains an open and important problem [1]. The remote sensing images can be quite different in their modality [2]: orbits may be ascending and descending, parameters of acquisitions may differ from one image to another even when the two acquisitions are issued from the same sensor. That is why, for change detection purpose, the use of the difference image is not an appropriate point of view due to the number of false alarms it induces. The situation is even worse on high resolution images over urban areas since many building appears differently in the two images to be compared due to the geometry of sensors, the perspective, the light condition and shadows... Hence the comparison of the classified images seems to be more appropriated. But, this yields to deal with uncertain, imprecise and even conflicting information. Evidence theories including Dempster-Shafer Theory (DST) [3] and Dezert-Smarandache Theory (DSMT) [4] are good for dealing with such information, and they have been applied for remote sensing applications [1, 5, 6]. In past works, a particular attention was paid to obtain very specific results for decision-making support through efficient

fusion of sources of evidence. Thus many works focused mainly on the redistribution of the conflicting beliefs [7, 8]. These combination approaches can be called static approaches, since they work under the assumption that the frame, on which is based the decision-making support, is temporally invariable in the fusion process. However, in the fusion of the multi-temporal remote sensing images, unexpected change occurrences can arise in some parts of the images. So both the image classification and change detection will be involved together. The classical combination rules in evidence theories provide specific classification results in the invariable parts of images, but it cannot precisely detect the change occurrences in the variable parts.

Therefore, a dynamical evidential reasoning (DER) working under the condition that the frame does not necessarily remain invariable in the fusion is proposed. In this paper, a new frame called "State-transition power-set" is defined, and the change occurrences among different hypotheses can be precisely represented by the state transition operator in this frame. The dynamical belief functions $Bel(\cdot)$, plausibility functions $Pl(\cdot)$ and pignistic probability $BetP(\cdot)$ [9, 10] are defined similarly as in DST. Dynamical combination rules are then proposed to work either in the free model, or in the constrained model. The free model is well adapted when no prior knowledge is known on elements of the frame. The constrained model can be used if some integrity constraints about the change occurrences are known. The dynamical approach improves the performance of the classification of areas and estimation of the changes through the fusion of multi-temporal sources of evidence. Our proposed approach is finally applied for the fusion of three sequential pieces of satellite images acquired before and after an earthquake.

II. DYNAMICAL EVIDENTIAL REASONING APPROACH

A. A brief review of DSMT

We need to introduce briefly DSMT framework because the Dynamical Evidential Reasoning approach shares some common ideas with DSMT, in particular the necessity to deal with hybrid models of the frames in some applications for changes detection. The purpose of DSMT is to overcome the limitations of DST [3] mainly by proposing new underlying models for the frames of discernment in order to fit better with the nature of real problems, and proposing new efficient combination and conditioning rules. In DSMT framework, the

elements θ_i , $i = 1, 2, \dots, n$ of a given frame Θ are not necessarily exclusive, and there is no restriction on θ_i but their exhaustivity. The hyper-power set D^Θ in DSMT is defined as the set of all composite propositions built from elements of Θ with operators \cup and \cap . For instance, if $\Theta = \{\theta_1, \theta_2\}$, then $D^\Theta = \{\emptyset, \theta_1, \theta_2, \theta_1 \cap \theta_2, \theta_1 \cup \theta_2\}$. A (generalized) basic belief assignment (bba for short) is defined as the mapping $m : D^\Theta \rightarrow [0, 1]$. The generalized belief and plausibility functions are defined in almost the same manner as in DST.

Two models¹ (the free model and hybrid model) in DSMT can be used to define the bba's to combine. In the free DSMT model, the sources of evidence are combined without taking into account integrity constraints. When the free DSMT model does not hold because the true nature of the fusion problem under consideration, we can take into account some known integrity constraints and define bba's to combine using the proper hybrid DSMT model. All details of DSMT with many examples can be easily found in [4].

B. The space of state transitions in DER

DSMT has been already applied for the fusion of multi-temporal satellite images in [1]. However, the classical frame used is not well adapted for measuring the changes among its elements. The conjunctive elements (intersections) in hyper-power set D^Θ represent either the overlap between hypotheses in free DSMT model, or the conflict produced by the conjunctive combination in Hybrid DSMT model when this is an integrity constraint. Actually, the conjunction $A \cap B$ is unable to characterize the transition A changing to B (denoted $A \rightarrow B$), or B changing to A (denoted $B \rightarrow A$). Therefore, if we need to distinguish two possible state transitions for changes detection, we need to define new operator and cannot use the classical conjunctive/intersection operator as in classical approaches. We propose the state transition operator "changing to", denoted \rightarrow , satisfying the following reasonable conditions:

(C1) Impossible (forward) state-transition

$$A \rightarrow \emptyset \triangleq \emptyset$$

(C2) Impossible (backward) state-transition

$$\emptyset \rightarrow A \triangleq \emptyset$$

(C3) Distributivity of \cup w.r.t. \rightarrow

$$(A \cup B) \rightarrow C = (A \rightarrow C) \cup (B \rightarrow C)$$

(C4) Distributivity of \rightarrow w.r.t. \cup

$$A \rightarrow (B \cup C) = (A \rightarrow B) \cup (A \rightarrow C)$$

(C5) Associativity of state-transition

$$(A \rightarrow B) \rightarrow C = A \rightarrow (B \rightarrow C) = A \rightarrow B \rightarrow C.$$

For notation convenience, a (state) transition $A \rightarrow B$ will be denoted $t_{A,B}$. It is important to note that the order of indexes does matter because $t_{A,B} \neq t_{B,A}$ in general, but if $A = B$

¹Actually, Shafer's model, considering all elements of the frame as truly exclusive, can be viewed as a special case of hybrid model.

obviously. $t_{A,A} = A \rightarrow A$ represents a particular invariable transition. A chain of transitions $\theta_1 \rightarrow \theta_2 \dots \rightarrow \theta_n$ will be denoted $t_{1,2,\dots,n}$. A transition $\theta_i \rightarrow (\theta_j \cup \theta_k)$ will be denoted $t_{i,j \cup k}$, etc.

In the theories of belief functions (DST, DSMT or Transferable Belief Model [9]), the result of the fusion of sources of evidence defined on a same frame of discernment Θ , is obtained by a given rule of combination relatively to a fusion space G^Θ , where G^Θ can be either the classical power set 2^Θ , a hyperpower-set D^Θ , or a superpower set (the power set of the refined frame) depending on the theory used. In this paper, we propose to use another fusion space (the space of transitions), denoted T^Θ , in order to deal explicitly with all possible state transitions we want to detect.

Firstly, the transition frame is given by:

$$\begin{aligned} \Theta_{1 \rightarrow n} &= \Theta_1 \times \Theta_2 \times \dots \times \Theta_n \\ &= \{t_{X_1, X_2, \dots, X_n} | X_i \in \Theta_i, i = 1, 2, \dots, n\} \end{aligned}$$

where Θ_i is the frame associated with the i -th source and \times is the Cartesian product operator.

In this paper, we assume to work in a more simple case where all frames Θ_i , $i = 1, 2, \dots, n$ are the same and equal to Θ , and where $G^\Theta = 2^\Theta$. In other words, we will work with the simpler space denoted T_n^Θ and defined by

$$T_n^\Theta = 2^{\Theta_{1 \rightarrow n}} = 2^{\overbrace{\Theta \times \Theta \times \dots \times \Theta}^{n \text{ times}}}$$

T_n^Θ can be called state transition power-set, which is composed by all the elements in $\Theta_{1 \rightarrow n}$ with the union operator \cup . We define \cup as componentwise operator in the following way:

$$\forall t_X, t_Y \in T_n^\Theta, \quad t_X \cup t_Y = t_{X \cup Y}. \quad (1)$$

Following conditions C1-C5, we note that generally

$$\begin{aligned} t_{(X_1, X_2, \dots, X_n)} \cup t_{(Y_1, Y_2, \dots, Y_n)} &= t_{(X_1, X_2, \dots, X_n) \cup (Y_1, Y_2, \dots, Y_n)} \\ &\neq t_{X_1 \cup Y_1, X_2 \cup Y_2, \dots, X_n \cup Y_n} \end{aligned}$$

If $\forall X_i \neq Y_i$; X_i, Y_i are singletons, $t_{(X_1, X_2, \dots, X_n) \cup (Y_1, Y_2, \dots, Y_n)}$ indicates only two kinds of possible transitions: $t_{X=(X_1, X_2, \dots, X_n)}$ or $t_{Y=(Y_1, Y_2, \dots, Y_n)}$, whereas the element $t_{X_1 \cup Y_1, X_2 \cup Y_2, \dots, X_n \cup Y_n}$ represents $2 \times 2 \times \dots \times 2 = 2^n$ kinds of possible transitions. It is obvious that they are quite different, and $t_{X_1 \cup Y_1, X_2 \cup Y_2, \dots, X_n \cup Y_n}$ is much more imprecise than $t_{(X_1, X_2, \dots, X_n) \cup (Y_1, Y_2, \dots, Y_n)}$.

As we see, the important and major difference between the classical approaches (DST, TBM, DSMT) and DER approach is the choice of the fusion space we are working with. With DST, TBM or DSMT, the fusion space we work with is always the same (independent of the number of sources) as soon as the sources are defined with respect to same frame Θ , whereas with DER approach the fusion space is always increasing with the number of sources, even if the sources are all referring to the same frame Θ . This of course increases the complexity of DER approach, but this is the "price to pay" to identify and estimate the possible change occurrences

in remote sensing images sequence as it will be shown in last section of this paper. Clearly, DST and DSmT are not well adapted for detecting change occurrences. For example, the transition $t_{A,B,A}$ from state A in source 1, to state B in source 2, and back to state A in source 3 cannot be represented in the fusion space proposed with TBM, DST, nor in DSmT.

Example 1: Let's consider $\Theta = \{\theta_1, \theta_2\}$ with Shafer's model, then $2^\Theta = \{\emptyset, \theta_1, \theta_2, \theta_1 \cup \theta_2\}$. We first consider at time 1 the initial set of *invariable transitions* defined as follows:

$$\begin{aligned} \Theta_{1 \rightarrow 1} &= \Theta; \\ T_1^\Theta &\equiv 2^\Theta = \{t_\emptyset \equiv \emptyset, t_1 \equiv \theta_1, t_2 \equiv \theta_2, t_{1 \cup 2} \equiv \theta_1 \cup \theta_2\} \end{aligned}$$

If we want to consider all the possible transitions from time stamp 1 to stamp 2, one starts with the cross product frame

$$\Theta_{1 \rightarrow 2} = \Theta \times \Theta = \{t_{1,1}, t_{1,2}, t_{2,1}, t_{2,2}\}$$

which has $|\Theta| \times |\Theta| = 2 \times 2 = 4$ distinct elements, and we build its power set $T_2^\Theta = 2^{\Theta_{1 \rightarrow 2}}$ including its 16 elements as in the classical way. The \emptyset element can be interpreted as the following set of impossible state transitions corresponding to $t_{\emptyset, \emptyset}, t_{\emptyset, 1}, t_{\emptyset, 2}, t_{\emptyset, 1 \cup 2}, t_{1, \emptyset}, t_{2, \emptyset}$ and $t_{1 \cup 2, \emptyset}$. We recall that the imprecise elements are derived from application of conditions C3 and C4 and not from the componentwise union of n -uples involved in transition indexes. The cardinality of T_n^Θ increases with the value of n as $|T_n^\Theta| = 2^{|\Theta|^n}$.

The power set of transitions we want to work with for such very simple example will be given by:

$$\begin{aligned} T_2^\Theta &= 2^{\Theta_{1 \rightarrow 2}} \\ &= \{\emptyset, t_{1,1}, t_{1,2}, t_{2,1}, t_{2,2}, \\ &\quad t_{1,1} \cup t_{1,2} = t_{1,1 \cup 2}, t_{1,1} \cup t_{2,1} = t_{1 \cup 2, 1}, \\ &\quad t_{1,1} \cup t_{2,2} = t_{(1,1) \cup (2,2)}, t_{1,2} \cup t_{2,1} = t_{(1,2) \cup (2,1)}, \\ &\quad t_{1,2} \cup t_{2,2} = t_{1 \cup 2, 2}, t_{2,1} \cup t_{2,2} = t_{2, 1 \cup 2}, \\ &\quad t_{1,1} \cup t_{1,2} \cup t_{2,1} = t_{(1,1) \cup (1,2) \cup (2,1)}, \\ &\quad t_{1,1} \cup t_{1,2} \cup t_{2,2} = t_{(1,1) \cup (1,2) \cup (2,2)}, \\ &\quad t_{1,1} \cup t_{2,1} \cup t_{2,2} = t_{(1,1) \cup (2,1) \cup (2,2)}, \\ &\quad t_{1,2} \cup t_{2,1} \cup t_{2,2} = t_{(1,2) \cup (2,1) \cup (2,2)}, \\ &\quad t_{1,1} \cup t_{1,2} \cup t_{2,1} \cup t_{2,2} = t_{1 \cup 2, 1 \cup 2}\}. \end{aligned}$$

C. Basic definitions in DER

Belief function $Bel(\cdot)$, plausibility function $Pl(\cdot)$ and pignistic probability $BetP(\cdot)$ [9, 10]² are basic and important functions for the decision making. They can be also used in DER approach as well. Indeed, all the elements in $\Theta^{1 \rightarrow n}$ composed by the state transitions through the singleton elements have specific and unique meaning, and they are considered the singleton elements. All the focal elements in T_n^Θ can be decomposed in the disjunctive canonical form using these singleton elements with the operator \cup , and we call that canonical

² $DSmP(\cdot)$ transformation proposed in [4] which provides of better probabilistic informational content than $BetP(\cdot)$ can also be chosen instead. But $DSmP(\cdot)$ is more complicated to implement than $BetP(\cdot)$ and it has not been tested in our application for now.

focal element. For example, $m(t_{(\theta_1 \cup \theta_2), \theta_3}) = m(t_{\theta_1, \theta_3} \cup t_{\theta_2, \theta_3})$ because of the condition (C3). The belief, plausibility functions and the pignistic transformation are defined in DER similarly as in DST; that is:

$$Bel(A) = \sum_{A, B \in T_n^\Theta; B \subset A} m(B) \quad (2)$$

$$Pl(A) = \sum_{A, B \in T_n^\Theta; A \cap B \neq \emptyset} m(B) \quad (3)$$

The interval $[Bel(A), Pl(A)]$ is then interpreted as the lower and upper bounds of imprecise probability for decision-making support [3] and the pignistic probability $BetP(A)$ commonly used to approximate the unknown probability $P(A)$ in $[Bel(A), Pl(A)]$ is calculated by:

$$BetP(A) = \sum_{A, B \in T_n^\Theta; A \subset B} \frac{|A \cap B|}{|B|} m(B) \quad (4)$$

where $|X|$ is the cardinal of the element X . In DER, the cardinal of $A \in T_n^\Theta$ is the number of the singleton elements it contains in its canonical form. For example, $|t_{(\theta_1 \cup \theta_2), \theta_3}| = |t_{\theta_1, \theta_3} \cup t_{\theta_2, \theta_3}| = 2$.

D. Combination rules in DER

The classical combination rules usually work under the assumption that all the sources of information refer to the same common frame. The results of existing combination rules do not deal with unexpected changes in the frame and they manage the conflicting beliefs without taking into account these possible changes in the frame. Nevertheless, the conflicting information is more important than the information from the agreement for changes detections.

The fundamental difference between the classical approach and this new Dynamical Evidence Reasoning (DER) approach is that the frame of the fusion process is considered possibly variable over time, and the fusion process is adapted for the changes detection and identification. When the elements of the frame are invariable, they can be considered as a special case of changes corresponding to invariable transition. The dynamical combination rules will be defined by using the state transition operator to take benefit of the useful information included in the conflict between sources. The combination rules work in free model and constrained model as well.

1) *Combination rule in the free model of transitions:* In the free model, there is no prior knowledge on state transitions, and all kinds of changes among the elements are considered possible to happen. We start with the combination of the two temporal sources of evidences at first. Let \mathbf{m}_1 and \mathbf{m}_2 be two bba's provided by two temporal sources of evidence over the frame of discernment Θ satisfying Shafer's model. Its corresponding power-set is $2^\Theta = \{\emptyset, \theta_1, \theta_2, \theta_1 \cup \theta_2, \dots, \Theta\}$. $m_1(A), A \in 2^\Theta$ is the mass of belief committed to the hypothesis A by the source no. 1, and $m_2(B), B \in 2^\Theta$ is the mass committed to B by the source no. 2. The sources no. 1 and no. 2 are considered as independent.

In this work, we propose to compute the mass of belief of a forward transition $t_{A_k, B_l} = A_k \rightarrow B_l$, $l \geq k$ as $m(t_{A_k, B_l}) = m_k(A)m_l(B)$ where k and l are temporal stamps/indexes. Note that because $m_k(A)m_l(B) = m_l(B)m_k(A)$, this mass of belief also can be associated to the backward transition $t_{B_l \rightarrow A_k} = B_l \rightarrow A_k$. We assume always working with ordered products corresponding to forward temporal transitions. By convention, the bba \mathbf{m}_i provided by the source no. i is assumed to be available at time i and before the bba \mathbf{m}_j provided by the source no. j . Indexes of sources correspond actually to temporal stamps. Following this very simple principle, the conjunctive combination rule in the free model of transitions, denoted DERf, is defined by:

$$\forall t_{X_1, X_2} \in T_2^\Theta, \quad m_{1 \rightarrow 2}(t_{X_1, X_2}) = m_1(X_1)m_2(X_2) \quad (5)$$

where $X_1 \in 2^{\Theta_1}$ and $X_2 \in 2^{\Theta_2}$.

For simplicity and in our application, we consider that the frames are all the same; that is $\Theta_1 = \Theta_2 = \dots = \Theta_n = \Theta$. This simple conjunctive rule can be extended easily for combining n sequential sources of evidence as follows:

- **Direct joint fusion of n sources:** $\forall t_{X_1, X_2, \dots, X_n} \in T_n^\Theta$

$$m_{1 \rightarrow n}(t_{X_1, X_2, \dots, X_n}) = m_1(X_1)m_2(X_2) \cdots m_n(X_n) \quad (6)$$

where $X_i \in 2^{\Theta_i}$, $i = 1, \dots, n$.

In this free model, the result of the combination is very specific since all kinds of change occurrences are distinguished in the results, but the computation burden is very large because of the large increase of the cardinality of T_n^Θ with n .

The following example will show the difference between DER and DSMT in free model.

Example 2: Let's consider three bba's on $\Theta = \{\theta_1, \theta_2\}$ as

	θ_1	θ_2	$\theta_1 \cup \theta_2$
\mathbf{m}_1	0.6	0	0.4
\mathbf{m}_2	0	1	0
\mathbf{m}_3	0	0.5	0.5

- **With DSMT free model:**

- 1) DSMT: $m(\theta_1 \cap \theta_2) = 0.6, m(\theta_2) = 0.4$
- 2) DERf: $m(t_{1,2,2}) = 0.3, m(t_{1,2,1 \cup 2}) = 0.3$
 $m(t_{1 \cup 2, 2, 1 \cup 2}) = 0.2, m(t_{1 \cup 2, 2, 2}) = 0.2$

For the singleton elements, based on DERf, one gets:

	$Bel(\cdot)$	$BetP(\cdot)$	$Pl(\cdot)$
$t_{1,2,2}$	0.3	0.60	1
$t_{1,2,1}$	0	0.20	0.5
$t_{2,2,1}$	0	0.05	0.2
$t_{2,2,2}$	0	0.15	0.4

The results of DERf precisely represents the belief of change occurrences, whereas the elements in DSMT cannot reflect the process of state transition because of its invariable frame. In the decision making, it indicates that the hypothesis $t_{1,2,2}$ is most possible to happen according to $Bel(\cdot)$, $Pl(\cdot)$ or $BetP(\cdot)$, which means θ_1 in source 1 changes to θ_2 in source 2 and to θ_2 in source 3. It implies DSMT is not adapted for the change detection because of its invariable frame.

2) **Combination rule in the constrained model of transitions:** In the constrained model of transitions, one knows that some kinds of changes among the different elements can't occur according to our prior knowledge. The set $\emptyset \triangleq \{\emptyset_{\mathcal{M}}, \emptyset\}$ can be defined in introducing some integrity constraints as done in the hybrid model of DSMT. $\emptyset_{\mathcal{M}}$ includes all the transitions in T_i^Θ , $i = 1, 2, \dots, n$, which have been forced to be empty because of the chosen integrity constraints in the model \mathcal{M} , and \emptyset is classical empty set. If the sources of evidence share the same reliability in the combination, the conflict among the evidences will be regarded as possible changes or as empty sets depending on the constraints we have. The mass of empty sets arising from integrity constraints can be distributed to the other focal elements. The notation $t_{A,B} \stackrel{\mathcal{M}}{=} t$, means that the transition $t_{A,B}$ is equivalent to the transition t in the underlying model \mathcal{M} given the integrity constraints.

- **DER_{DS} rule of combination:** The mass of empty sets is proportionally distributed to the other focal elements similarly to Dempster-Shafer's rule here and we denote this rule by DER_{DS} for short. Of course, the conflicting mass can also be redistributed by some other ways. The combination rule DER_{DS} is mathematically defined as follows:

$$\forall t_{n-1} \in T_{n-1}^\Theta, \forall X_n \in 2^{\Theta_n} \equiv 2^\Theta, \forall t_n \in T_n^\Theta, n \geq 2$$

$$m_{1 \rightarrow n}(t_n) = \frac{\sum_{t_{n-1}, X_n \stackrel{\mathcal{M}}{=} t_n} m_{1 \rightarrow n-1}(t_{n-1})m_n(X_n)}{1 - K} \quad (7)$$

where K represents the mass of belief committed to the empty sets (i.e. the degree of conflict) which is given by

$$K = \sum_{t_{n-1}, X_n \in \emptyset} m_{1 \rightarrow n-1}(t_{n-1})m_n(X_n). \quad (8)$$

When considering the direct combination of n sequential sources altogether, one has $\forall t_n \in T_n^\Theta$, and $X_i \in 2^{\Theta_i}$, $i = 1, 2, \dots, n$

$$m_{1 \rightarrow n}(t_n) = \frac{\sum_{t_{X_1, X_2, \dots, X_n} \stackrel{\mathcal{M}}{=} t_n} m_1(X_1) \cdots m_n(X_n)}{1 - K} \quad (9)$$

where

$$K = \sum_{t_{X_1, X_2, \dots, X_n} \in \emptyset} m_1(X_1) \cdots m_n(X_n). \quad (10)$$

Remark: The summation introduced in (7) and (9) allows to take into account the integrity constraints of the model of the space of transitions as shown in the next example. If all kinds of transitions among different elements are constrained to be empty sets, the frame will become Shafer's model, and DER_{DS} will reduce to Dempster's rule. \square

Example 3: Let's consider the frame $\Theta = \{\theta_1, \theta_2\}$ with following two integrity constraints representing the impossible state transitions $\emptyset_{\mathcal{M}} \triangleq \{t_{1,2}, t_{2,2}\}$. Therefore, due to these

integrity constraints, the fusion space T_2^Θ , as given in details in the Example 1, reduces to the simple following set

$$T_2^\Theta = \{\emptyset, t_{1,1}, t_{2,1}, t_{1 \cup 2,1}\}$$

We also consider the following bba's inputs:

	θ_1	θ_2	Θ
\mathbf{m}_1	0.4	0.6	0
\mathbf{m}_2	0.5	0.2	0.3

According to the underlying hybrid model \mathcal{M} , only the products $m_1(\theta_1)m_2(\theta_2)$ and $m_1(\theta_2)m_2(\theta_2)$ take part in the conflict as: $K = m_1(\theta_1)m_2(\theta_2) + m_1(\theta_2)m_2(\theta_2) = 0.2$.

The conjunctive mass of belief of possible transitions are given by

$$\begin{aligned} m_\cap(t_{1,1}) &= m_1(\theta_1)m_2(\theta_1) = 0.20 \\ m_\cap(t_{1,1 \cup 2}) &= m_1(\theta_1)m_2(\theta_1 \cup \theta_2) = 0.12 \\ m_\cap(t_{2,1}) &= m_1(\theta_2)m_2(\theta_1) = 0.30 \\ m_\cap(t_{2,1 \cup 2}) &= m_1(\theta_2)m_2(\theta_1 \cup \theta_2) = 0.18 \end{aligned}$$

Due to the integrity constraints $t_{1,2} \stackrel{\mathcal{M}}{=} \emptyset$, $t_{2,2} \stackrel{\mathcal{M}}{=} \emptyset$, one has

$$\begin{cases} t_{1,1 \cup 2} = t_{1,1} \cup t_{1,2} \stackrel{\mathcal{M}}{=} t_{1,1} \cup \emptyset = t_{1,1} \\ t_{2,1 \cup 2} = t_{2,1} \cup t_{2,2} \stackrel{\mathcal{M}}{=} t_{2,1} \cup \emptyset = t_{2,1} \end{cases}$$

Therefore, the mass $m_\cap(t_{1,1 \cup 2})$ must be transferred to $t_{1,1}$, whereas $m_\cap(t_{2,1 \cup 2})$ must be transferred to $t_{2,1}$ only. Finally, the result given by DER_{DS} rule is

$$\begin{aligned} m(t_{1,1}) &= \frac{m_\cap(t_{1,1}) + m_\cap(t_{1,1 \cup 2} \equiv t_{1,1})}{1 - K} = 0.40 \\ m(t_{2,1}) &= \frac{m_\cap(t_{2,1}) + m_\cap(t_{2,1 \cup 2} \equiv t_{2,1})}{1 - K} = 0.60 \end{aligned}$$

It shows that the combination rule in constrained model can provide more specific results than in the free model when the associated constrained information is known.

III. APPLICATION ON REAL REMOTE SENSING IMAGES

Three pieces of multi-temporal satellite images are analyzed by using DER in this experiment. Three QuickBird images have been acquired before and after the may 21st 2003 earthquake in the region of Boumerdes, Algeria. They have been corrected geometrically by using SRTM elevation data and resample by a P+Xs pan-sharpening technique to yield a 60cm resolution color images (see Fig. 1) [11].

It has been analyzed in theory and shown by the numerical example that DSmt does not work well for the change detection, especially when the number of sources is larger than 2. So only DER will be applied in this experiment. The unsupervised clustering method ECM (Evidential C-Means), detailed in [12], is applied for the image classification only by using the radiometric point of view. ECM is adapted to the classification of uncertain data, and the imprecise classes can be acquired by ECM. The membership about the classification of each pixel acquired by ECM are directly used as bba's here.

The three sequential images are clustered in $C = 5$ groups, and the classifications are defined as (colors refer to Fig. 1):

$$\begin{aligned} \theta_1 &= \{\text{Dark area: green plant}(w_1) \text{ or shadow}(w_2)\} \\ \theta_2 &= \{\text{Maroon area: bared soil}(w_3)\} \\ \theta_3 &= \{\text{Dark gray area: gray building}(w_4)\} \\ \theta_4 &= \{\text{Gray area: road}(w_5) \text{ or incomplete building}(w_6)\} \\ \theta_5 &= \{\text{White area: white building}(w_7) \text{ or ruin}(w_8)\} \\ \Theta &= \{\text{Ignorance}\} \end{aligned}$$

The other tuning parameters are defined by: Maximum number of iterations $T = 20$, weighting exponent for cardinality $\alpha = 2$, weighting exponent $\beta = 2$, termination threshold $\varepsilon = 1$. This classification technique has been used here because it is unsupervised and the results can be directly used as the mass functions (bba's). Nevertheless, any kind of classifiers (including supervised ones) may be used at this level. For the decision making, we take the criteria that true hypothesis gets the maximum of pignistic probability.

The time analysis of the three images is mainly taken for detecting the important change occurrences to evaluate the damage, and we just pay attention to some particular transitions rather than all the possible transitions. So the constrained model of DER is applied here. The state transitions considered from the image Fig. 1-(a) to Fig. 1-(b) includes the change occurrences from the gray building to ruin as $t_{3,5}(t_{i,j} = t_{\theta_i, \theta_j})$ and from incomplete building to white building as $t_{4,5}$, and all invariable transitions as $t_{1,1}, t_{2,2}, t_{3,3}, t_{4,4}, t_{5,5}$. The change occurrences from gray building to ruin as $t_{3,5}$ and from gray building to bared soil as $t_{3,2}$ which means the ruin has been cleaned, and all the invariable transitions as $t_{1,1}, t_{2,2}, t_{3,3}, t_{4,4}, t_{5,5}$ are involved from the image Fig. 1-(b) to Fig. 1-(c). Therefore, the constrained available transitions in the three images are given by $t_{1,1,1}, t_{2,2,2}, t_{3,3,3}, t_{4,4,4}, t_{5,5,5}, t_{3,5,5}, t_{4,5,5}, t_{3,3,5}, t_{3,3,2}$. All the other possible transitions considered useless here are defined as empty sets through constrained model in DER.

Remark: The transitions $t_{3,5}$ and $t_{4,5}$ may involves many kinds of possible change occurrences, but some of the change occurrences are unavailable according to our prior knowledge. The constrained transitions can be defined by $t_{3,5} \stackrel{\mathcal{M}}{=} t_{w_4, w_8}, t_{4,5} \stackrel{\mathcal{M}}{=} t_{w_6, w_7}$. \square

The combination results are shown as Fig. 2. Some change occurrences are considered more important than the invariable transitions for the evaluation of the disaster, and they are extracted in Fig. 3.

The notations of transitions from 1st to 2nd image are:

$$\begin{aligned} t_{1,1} &: \{\text{Green area}\}, t_{2,2} : \{\text{Blue area}\}, t_{3,3} : \{\text{Gray area}\}, \\ t_{4,4} &: \{\text{White area}\}, t_{5,5} : \{\text{Dark red area}\}, \\ t_{3,5} &\stackrel{\mathcal{M}}{=} t_{w_4, w_8} : \{\text{Red area}\}, t_{4,5} \stackrel{\mathcal{M}}{=} t_{w_6, w_7} : \{\text{Dark yellow area}\}. \end{aligned}$$

$t_{3,5}$ and $t_{4,5}$ correspond to actual changes and they are linked to damage mapping (see Fig. 3). Fig. 3-(a) focus on those 2

classes. The notations of transitions from 2nd to 3rd image:

$$\begin{aligned} t_{1,1} &: \{\text{Green area}\}, t_{2,2} : \{\text{Blue area}\}, t_{3,3} : \{\text{Gray area}\}, \\ t_{4,4} &: \{\text{White area}\}, t_{5,5} : \{\text{Red area}\}, t_{3,2} : \{\text{Cyan area}\}, \\ t_{3,5} &: \stackrel{\mathcal{M}}{=} t_{w_4, w_8} : \{\text{Purple area}\}. \end{aligned}$$

$t_{3,5}, t_{3,2}$ corresponds to changes induced after the earthquake. The notations of transitions through the 1st, 2nd and 3rd images:

$$\begin{aligned} t_{1,1,1} &: \{\text{Green area}\}, t_{2,2,2} : \{\text{Blue area}\}, \\ t_{3,3,3} &: \{\text{Gray area}\}, t_{4,4,4} : \{\text{White area}\}, \\ t_{5,5,5} &: \{\text{Dark red area}\}, t_{3,3,2} : \{\text{Cyan area}\}, \\ t_{3,3,5} &: \stackrel{\mathcal{M}}{=} t_{w_4, w_4, w_8} : \{\text{Purple area}\}, \\ t_{3,5,5} &: \stackrel{\mathcal{M}}{=} t_{w_4, w_8, w_8} : \{\text{Red area}\}, \\ t_{4,5,5} &: \stackrel{\mathcal{M}}{=} t_{w_6, w_7, w_7} : \{\text{Dark yellow area}\}. \end{aligned}$$

We show more interest in the variable transitions than the invariable transitions, since the variable transitions reflect important change occurrences which is very valuable in the evaluation of disaster. As we can see, some buildings were entirely destroyed by the earthquake as represented by red color mainly on the left side of the image and some incomplete buildings has been completed represented by the yellow color in Fig. 3-(a), which reflects the disaster mainly happened on the left side. In Fig. 3-(b), the purple area indicates that another normal building also collapsed, and the cyan area means the run has been cleaned. Fig. 3-(c) shows the sequential transitions of the three images, and it represents all the particular change occurrences through the three images. These fusion results can be helpful for the disaster evaluation in the real application. There are some few false alarms of change detections which are mainly due to the difference in the geometry of the acquisitions. If some ancillary information are available, these noisy changes can be reduced.

IV. CONCLUSIONS

A Dynamic Evidential Reasoning (DER) approach has been proposed for the change detection in the multi-temporal remote sensing images. DER approach starts with the sequential construction of the power set of admissible state transitions taking into account, if necessary, some integrity constraints representing some known unacceptable (impossible) transitions. Based on a particular rule-based algebra, the mass of belief of the change occurrences can be computed using two different combination rules the DER_f or DER_{DS} rule. DER_f rule in the free model works under the condition that no prior knowledge about the change occurrences is known with a great computation burden. DER_{DS} rule in constrained model is preferred when some constraints on the impossible change occurrences is available to get better fusion results with less computational complexity. Several simple numerical examples were given to show how to use DER and to show its difference with classical fusion approaches. Finally, an experiment about the fusion of multi temporal satellite images

illustrates the interest and the efficiency of DER for changes detection and estimation. The DER fusion of images can well detect the change occurrences and classify the invariable areas. Our further research works will concern the extension of DER for working with other DSm fusion rules, etc.

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(a)

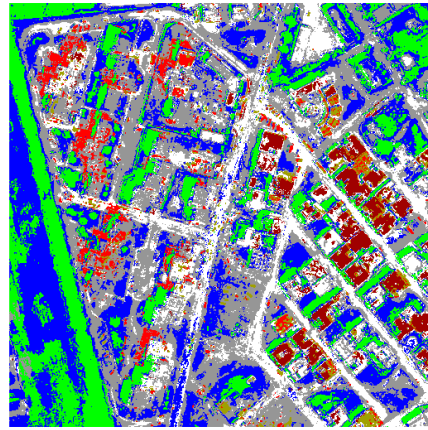


(b)

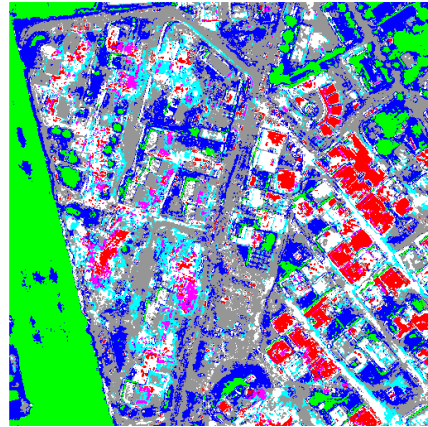


(c)

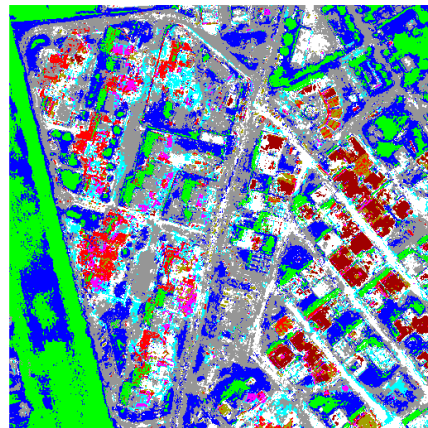
Figure 1. Multi-temporal QuickBird satellite images acquired before and after an earthquake. (a) before image: 04/22/2002, (b) after image: 05/13/2003, (c) latter acquisition: 06/13/2003. Dataset Boumerdes ©Copyright SERTIT, 2009, distribution CNES.



(a)

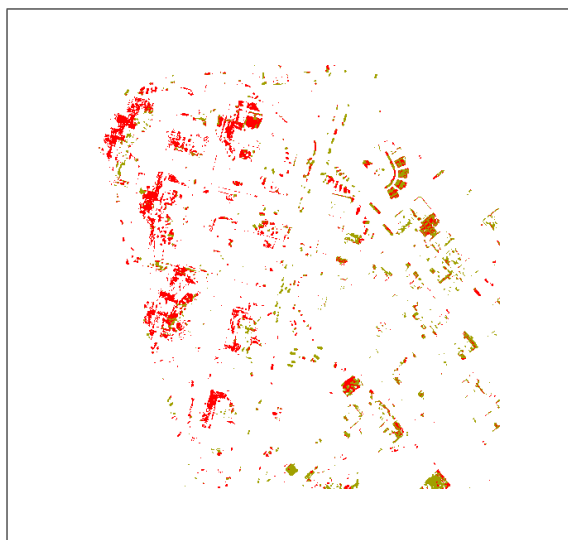


(b)

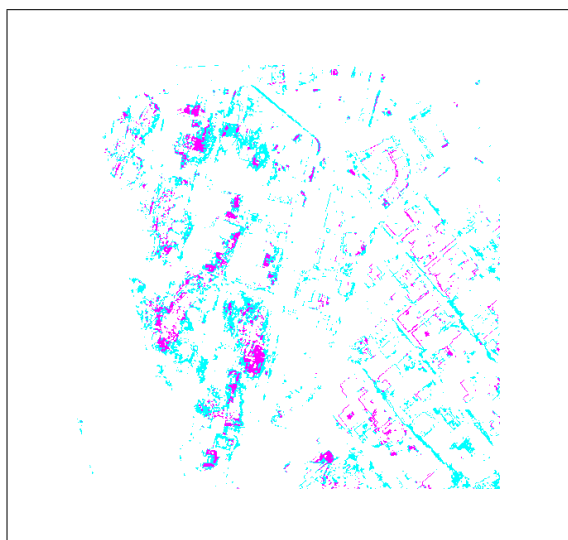


(c)

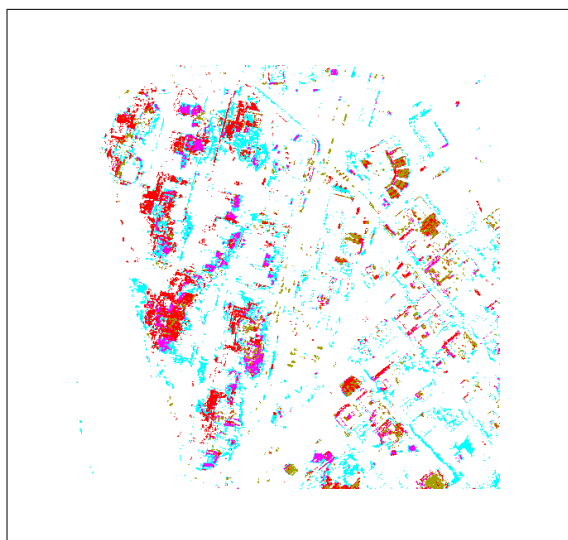
Figure 2. Fusion results of classified images by DER_{DS} . (a) Fusion of the 1st and 2nd image, (b) Fusion of the 2nd and 3rd image, (c) Fusion of the 1st, 2nd and 3rd image.



(a)



(b)



(c)

Figure 3. Significant damage map extracted from DER decision of Fig. 2). (a) Changes from the 1st to 2nd classified image, (b) Changes from the 2nd to 3rd classified image, (c) Changes through the 1st, 2nd and 3rd classified images.