Belief function theory based decision support methods: application to torrent protection work effectiveness and reliability assessment

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ABSTRACT: Civil engineering protection works mitigate natural risks in mountains, such as torrents. Analysing their effectiveness at several scales is an essential issue in the risk management. Based on expert knowledge, used methods have been developed under risky environment. However, decision is made under uncertainty because of 1) the lack of information and knowledge on natural phenomena and 2) the heterogeneity of available information and 3) the reliability of sources.
In this paper, we propose to help decision-makers with advanced multicriteria decision making methods (MCDMs). Combining classical MCDM approaches, belief function, fuzzy sets and possibility theories, they make it possible decisions based on heterogeneous, imprecise and uncertain evaluation of criteria provided by more or less reliable sources in an uncertain context. COWA-ER (Cautious Ordered Weighted Averaging with Evidential Reasoning), Fuzzy-Cautious OWA or ER-MCDA (Evidential Reasoning for Multi Criteria Decision Analysis) are thus applied to several scales of effectiveness assessment.

1 INTRODUCTION

Mountain natural phenomena such as torrential floods put people and buildings at risk. Protection works influence both causes and effects of phenomena to limit induced risks. For instance, check-dams control material volume and flow of torrential floods. Their design allow them to reduce sediment production (Figure 1). Defining the strategy for investment and maintenance is an essential issue in the risk management process. It is based on their effectiveness assessment. Decision support tools help assessing their economic efficiency depending on their structural state and functional effects on phenomena (stopping, braking, guiding, etc.) (Carladous et al. 2014).

Cost Benefit Analysis (CBA) is the most used decision-aid method in the natural hazard context. It helps assessing efficiency of potential actions comparing investment and maintenance costs with direct and indirect losses (Bründl et al. 2009). Actually, natural risk analysis is limited to a set of scenarii which can be discussed (Eckert et al. 2012). Each scenario is defined by a probability value exceedance for a criterion of interest. However, probability knowledge (distribution or scenarii) is affected by the lack of information on phenomena, but also by heterogeneity and reliability of available sources (Tacnet 2009).

Concepts of failure mode and effects analysis (FMEA), already used for hydraulic dams (Peyras et al. 2006), are extended to assess the effectiveness of check-dams (Ghariani et al. 2014). Those methodologies elicit the expert reasoning process and consider structural, functional and economic features (Carladous et al. 2014): indicators formalise information processing to make it repeatable and reproducible (Curt et al. 2010). Nevertheless, assessment is based on heterogeneous and imprecise information provided by more or less reliable sources (Tacnet 2009).

Methods to represent information imperfection are needed to aid decisions including check-dam effectiveness assessment. Advanced MCDMs combining classical MCDM approaches (Saaty 1980, Roy 1985), belief function (Shafer 1976, Smarandache and Dezert 2015), fuzzy sets (Zadeh 1965) and possibility theories (Zadeh 1978) have been developed to help decisions under risk or uncertainty such as COWA (Tacnet & Dezert 2011), Fuzzy-Cautious OWA (Han et al. 2012) and ER-MCDA (Tacnet et al. 2010).

This paper first recalls the context of information imperfection related to check-dams. We secondly introduce the principles of new belief function theory based evolutions of MCDMs. We then apply them to cases related to effectiveness of check-dams. We finally discuss remaining issues for new decision-making methods in risky and uncertain contexts.

2 EFFECTIVENESS OF PROTECTION WORKS IN AN UNCERTAIN ENVIRONMENT

Assessing effectiveness of existing check-dams is based on their structural state, functional capacity and relative risk reduction. We describe below the decision context and information imperfection all over the decision process.

2.1 *Formalization of decision context*

2.1.1 *Several system scales as alternatives*

Protecting exposed elements with check-dams is based on interdependent systems. A check-dam E^l belongs to a device D^o. Several devices protect exposed elements at the watershed scale (F). Each E^l is considered as an alternative belonging to a set of m check dams $D^o = \{E^1, \ldots, E^l, \ldots, E^m\}$. D^o is a device alternative in the set $F = \{D^1, \ldots, D^o, \ldots, D^t\}$. F represents all t devices which protect exposed elements in the watershed (Figure 1).

2.1.2 *Possible actions on systems as alternatives*

For each system scale E^l, D^o and F, several actions a_i can be proposed: for example, no action (a_1), maintenance of check-dams (a_2) or building new works (a_3). E^l_i, D^o_i and F_i represent all possible actions on each system scale (resp. a single check-dam, a set of check-dams, all sets in the watershed).

2.1.3 *Decision objects and linked problems*

A decision-making problem consists in choosing, ranking or sorting alternatives on the basis of quantitative or qualitative criteria g_j (Roy 1985). Effectiveness is the level of objective achievement (AFNOR 2005). Sorting alternatives E^l, D^o and F in effectiveness classes (e.g., optimal, correct, partial, deficient) is a recurrent issue. Choosing between several alternatives a_i, or ranking them, are other practical issues.

2.2 *Various information is needed but is imperfect*

2.2.1 *The states of the nature S or S^o*

Debris flows and torrential floods with bed-load transport are the two main torrential processes (Meunier 1991). Choosing a specific criterion of interest for each process is needed (e.g., flow volume or deposit depth).

The states of the nature analysis depends on its location in the watershed. They can be represented by a finite or a continuous set according to available information. For torrential floods, field experts define a finite set $S = \{S_1, \ldots, S_k, \ldots, S_n\}$ for F scale and another set $S^o = \{S^o_1, \ldots, S^o_k, \ldots, S^o_n\}$ for D^o and E^l scales (Carladous et al. 2014).

2.2.2 *The decision-maker (DM) preferences on g_j*

Assessing each alternative in a MCDM context requires three elements from the DM about g_j: 1) the list of g_j, 2) weights w_j: preferences between g_j, 3) g_j assessment scale: preferences between alternative evaluations through a total or a partial pre-order (von Neumann & Morgenstern 1953, Roy 1985).

2.2.3 *Decision-making and imperfect information*

To compare several alternatives, decision support tools need evaluations of their consequences (payoffs/gains) under S (or S^o). For example, each F_i is evaluated given the knowledge on S and the payoff matrix defined by $C = [C_{ik}]$ where $i = 1, \ldots, q$ and $k = 1, \ldots, n$ (Eq. (1)). The decision problem consists in choosing the alternative $F_{i^*} \in F_i$ which maximizes the payoff to the DM. We assume that C_{ik} assessment can be based on several g_j.

$$\begin{array}{c} \\ F_1 \\ \vdots \\ F_i \\ \vdots \\ F_q \end{array} \begin{array}{c} \begin{array}{ccccc} S_1 & \ldots & S_k & \ldots & S_n \end{array} \\ \begin{pmatrix} C_{11} & \ldots & C_{1k} & \ldots & C_{1n} \\ & & \vdots & & \\ C_{i1} & \ldots & C_{ik} & \ldots & C_{in} \\ & & \vdots & & \\ C_{q1} & \ldots & C_{qk} & \ldots & C_{qn} \end{pmatrix} \end{array} = C, \tag{1}$$

Whatever the decision context, all decisions relate to imperfection of used information to assess S, C_{ik} and g_j (Tacnet 2009): **inconsistency** (conflict between sources); **imprecision** (e.g., interval of numerical values); **incompleteness** (lack of information while data exist); **aleatory uncertainty** (aleatory events); **epistemic uncertainty** (lack of knowledge).

Depending on his knowledge about S the DM is face on different decision-making problems (Tacnet & Dezert 2011): **under certainty** (only one S_k is known); **under risk** (the true S is unknown but one knows all the probabilities $p_k = P(S_k)$); **under ignorance** (one assumes no knowledge about the true S but that it belongs to S); **under uncertainty** (the knowledge on S is characterized by a belief structure).

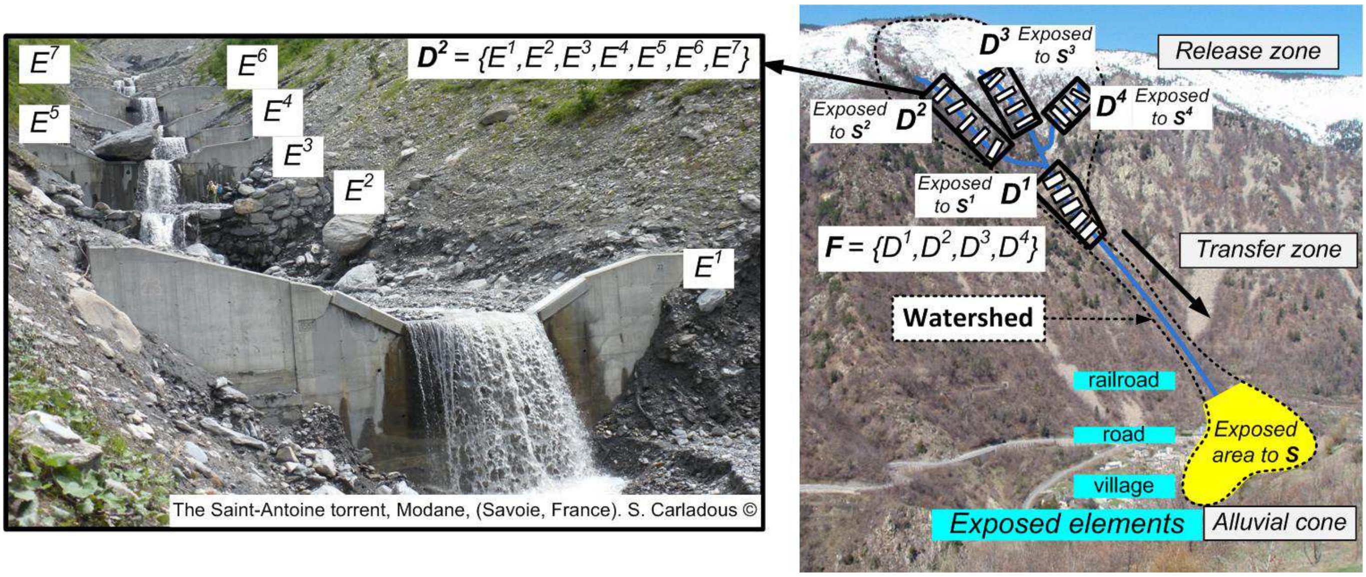

Figure 1: Multi-system formalization for check-dams in a torrential watershed.

3 NEW BELIEF FUNCTION THEORY BASED EVOLUTION OF MCDMS

Comparing alternatives requires assessment of 1) DM preferences on w_j and g_j 2) information imperfection to evaluate S and g_j 3) MCDM choice to aggregate several g_j to define C. In this part, we introduce the principles of methods based on new MCDMs evolutions based on belief function theory.

3.1 *Basics of belief functions*

Shafer (1976) originally proposed the basics of belief functions. One starts with a finite set Θ (called the frame of discernment of the decision problem). Each element of Θ is a potential answer of the decision problem and they are assumed exhaustive and exclusive. The powerset of Θ denoted 2^Θ is the set of all subsets of Θ, empty set included. A body of evidence is a source of information that will help the DM to identify the best element of Θ. The interest of belief functions is their ability to model epistemic uncertainties. Each body of evidence is characterized by basic belief assignment (bba), or a mass of belief, which is a mapping $m(.) : 2^\Theta \rightarrow [0,1]$ that satisfies $m(\emptyset) = 0$, and for all $A \neq \emptyset \in 2^\Theta$ the condition $\sum_{A\subseteq\Theta} m(A) = 1$. The Belief function Bel$(.)$ and the plausibility function Pl$(.)$ are defined from $m(.)$ by :

$$\text{Bel}(A) = \sum_{B\subseteq A | B\in 2^\Theta} m(B) \qquad (2)$$

$$\text{Pl}(A) = \sum_{B\cap A\neq\emptyset | B\in 2^\Theta} m(B) \qquad (3)$$

Bel(A) and the plausibility function Pl(A) are often interpreted as lower and upper bounds of the unknown probability of A. The vaccuous bba defined as $m_v(\Theta) = 1$ models the full ignorant source of evidence. Shafer (1976) proposed Dempster's rule to combine distinct sources of evidence which has been subject to strong debates in fusion community starting from Zadeh's first criticism in 1979. Since the 90's many alternatives have been proposed to combine more or less efficiently belief functions, as well as an extension of belief function in the framework of Dezert-Smarandache Theory (DSmT) as shown and discussed in Smarandache and Dezert (2015).

According to the DM attitude, credibilities, plausibilities, Smets' Pignistic probability $BetP$ (Smets 2005) or Dezert-Smarandache probability $DSmP_{\epsilon=0}$ (Smarandache & Dezert 2015),Vol. 3 can be computed to compare alternatives.

3.2 *ER-MCDA*

Tacnet (2009) proposed the ER-MCDA methodology. Its originality consists in the association of different theories. It dissociates imperfect evaluations from their combination in the fusion process considering both evaluation imperfection and heterogeneity, reliability of sources. It uses developments for MCDM based on the combination of Analytic Hierarchic Process (AHP) approach developed by Saaty (1980) and DSmT (Smarandache & Dezert 2015). AHP allows to build bbas from DM preferences on solutions which are established with respect to several g_j. DSmT allows to aggregate efficiently the (possibly highly conflicting) bbas based on each criterion. DSmT-AHP method also allows to take into account the different importances of g_j and/or of the different members of the DM group.

ER-MCDA exploits the following general principles into independent steps:

• The AHP methodology helps to analyze the decision problem through a hierarchical structure and to define the evaluation classes for decision through a common frame of discernment Θ.

• The imprecise evaluation and mapping of g_j: qualitative or quantitative criteria are evaluated through possibility distributions representing both imprecision and uncertainty (Zadeh 1978). Possibility distribution can be derived into bbas (Baudrit et al. 2005). We use a mapping process that projects the bbas expressed on fuzzy sets expressed on Θ (Zadeh 1965).

• The fusion of mapped evaluations and g_j: a first fusion process is done for all evaluations of the different sources for a same g_j. bbas can be discounted according to the reliability level of each source. We finally get bbas for each g_j whose weights ω_j have been defined according to the classical AHP method. Those ω_j are derived into importance discounting factors. bbas corresponding to each g_j are then fused a second time to get the final result which is called a decision profile. This profile shows not only the decision to take but provides also an evaluation of the distribution of knowledge on the other levels and uncertainty. It is possible to check if all sources agree about the decision and also to have an idea about the uncertainty of their evaluation. The quality of information leading to decision is linked to the decision itself. The results can be bbas or belief, plausibility values that correspond to pessimistic or optimistic choice of a decision level. With ER-MCDA, one uses PCR6 (Proportional Conflict Redistribution Rule no 6) developed in DSmT (Smarandache & Dezert 2015) (Vol. 3) to palliate disadvantages of the classical Dempster fusion rule discussed in (Dezert & Tchamova 2014). The importance of criteria is a different concept than the classical reliability concept developed and used in the belief theory context. In order to make a difference between importance of criteria, uncertainty related to the evaluations of criteria and reliability of the different sources, specific methods such as DSmT-AHP (Dezert et al. 2010, Dezert and Tacnet 2011) have extended Saaty's AHP method.

3.3 *COWA-ER and Fuzzy Cautious OWA*

Tacnet and Dezert (2011) proposed the COWA-ER method for decision-making under uncertainty taking into account imperfect evaluations and unknown beliefs about groups of the possible states of the world. COWA-ER mixes cautiously the principle of Ordered Weighted Averaging (OWA) approach (Yager 2008) with the fusion of belief functions proposed in DSmT (Smarandache and Dezert 2015). Fuzzy Cautious OWA (Han et al. 2012) is an improvement of COWA-ER using fuzzy sets.

3.3.1 *The OWA approach*

To recall it, we take into account the **decision-making** problems introduced in 2.2.3 and Eq. (1).

1 – under certainty: one chooses F_{i^*} with $i^* \triangleq \arg\max_i\{C_{ik}\}$.

2 – under risk: as for the CBA (cf 1), for each F_i, we compute expected payoff $E[C_i] = \sum_k p_k \cdot C_{ik}$, then we choose F_{i^*} with $i^* \triangleq \arg\max_i\{E[C_i]\}$.

3 – under ignorance: Yager (2008) uses the OWA operator as a weighted average of ordered values of a variable. For each F_i, one chooses a weighting vector $W_i = [w_{i1}, w_{i2}, \dots w_{in}]$ and computes its OWA value $V_i \triangleq \text{OWA}(C_{i1}, C_{i2}, \dots, C_{in}) = \sum_k w_{ik} \cdot b_{ik}$ where b_{ik} is the kth largest element in the collection of payoffs C_{i1}, C_{i2},..., C_{in}. Then one chooses F_{i^*} with $i^* \triangleq \arg\max_i\{V_i\}$. W_i depends on the decision attitude of the DM (pessimistic, optimistic, normative/neutral, etc.).

4 – under uncertainty: one assumes that a priori knowledge on the frame S is given by a bba $m(.) : 2^S \to [0, 1]$. This case includes all previous cases depending on the choice of $m(.)$. Yager's OWA under uncertainty is based on the derivation of a *generalized expected value* C_i of payoff for each F_i as follows:

$$C_i = \sum_{l=1}^{r} m(X_l) V_{il} \tag{4}$$

where r is the number of focal elements of the belief structure $(S, m(.))$. $m(X_l)$ is the mass of belief of $X_l \in 2^S$, and V_{il} is the payoff we get when we select F_i and the state of the nature lies in X_l.

For F_i and a focal element X_l, instead of using all payoffs C_{ik}, we consider only the payoffs in the set $M_{il} = \{C_{ik} | S_k \in X_l\}$ and $V_l = \text{OWA}(M_{il})$ for some decision-making attitude chosen a priori. Once generalized expected values C_i, $i = 1, 2, \dots, q$ are computed, we compare alternatives through these results.

The principle of this method is simple, but its implementation can be quite greedy in computational resources specially if one wants to adopt a particular attitude for a given level of optimism, specially if the dimension of the frame S is large.

3.3.2 *The COWA-ER approach*

Yager's OWA approach is based on the choice of a given attitude measured by an optimistic index in $[0, 1]$ to get the weighting vector W_i. What should be done in practice if we don't know which attitude to adopt? An answer to this question has been proposed in Cautious OWA with Evidential Reasoning (COWA-ER) which exploits the results of the two extreme attitudes jointly (pessimistic and optimistic ones) to take a decision under uncertainty based on the imprecise valuation of alternatives. In COWA-ER, the pessimistic and optimistic OWA are used respectively to construct the intervals of expected payoffs for different alternatives. For example, for q alternatives, the expected payoffs are:

$$E[C] = \begin{bmatrix} E[C_1] \\ E[C_2] \\ \vdots \\ E[C_q] \end{bmatrix} \subset \begin{bmatrix} [C_1^{\min}, C_1^{\max}] \\ [C_2^{\min}, C_2^{\max}] \\ \vdots \\ [C_q^{\min}, C_q^{\max}] \end{bmatrix}$$

Therefore, one has q sources of information before using the belief functions framework. Basically, the COWA-ER methodology requires four steps:

• Step 1: normalization of imprecise values in $[0, 1]$;
• Step 2: conversion of each normalized imprecise value into elementary bba $m^o(.)$;
• Step 3: fusion of bba $m^i(.)$ with some combination rule (typically the PCR6 rule);
• Step 4: choice of the final decision based on the resulting combined bba.

With COWA-ER, we consider as Θ, the finite set of alternatives $\Theta = \{Z_1, Z_2, \ldots, Z_q\}$ and the sources of belief associated with them obtained from the normalized imprecise expected payoff vector $E^{Imp}[C_i]$. The modeling for computing a bba associated to hypothesis F_i from any imprecise value $[a; b] \subseteq [0; 1]$ is done by:

$$\begin{cases} m_i(F_i) = a, \\ m_i(\overline{F_i}) = 1 - b \\ m_i(F_i \cup \overline{F_i}) = m_i(\Theta) = b - a \end{cases} \tag{5}$$

where $\overline{F_i}$ is the F_i's complement in Θ.

COWA-ER can help to take a decision if one wants on a group/subset of alternatives satisfying a min of credibility (or plausibility level) selected by the DM. It can also be extended directly for the fusion of several sources of informations when each source can provide a payoffs matrix. We can also discount each source easily if needed.

3.3.3 *The Fuzzy-COWA-ER approach*

Unfortunately, COWA-ER has a serious limitation because the computational time depends on the number of alternatives. In COWA-ER, each expected interval is used as an information source, however, these expected intervals are jointly obtained and thus these information sources are relatively correlated. For these reasons, a modified version of COWA-ER, called Fuzzy-COWA-ER (or FCOWA-ER for short) has been developed in Han et al. (2012). With FCOWA-ER, we consider the 2 columns of the expected payoff $E[C_i]$ as 2 information sources, representing pessimistic and optimistic attitudes. The column-wise normalized expected payoff is:

$$E^{Fuzzy}[C] = \begin{bmatrix} N_1^{\min}, N_1^{\max} \\ N_2^{\min}, N_2^{\max} \\ \vdots \\ N_q^{\min}, N_q^{\max} \end{bmatrix}$$

where $N_i^{\min} \in [0, 1]$ ($i = 1, ..., q$) represents the normalized value in the column of pessimistic attitude and $N_i^{\max} \in [0, 1]$ represents the normalized value in the column of optimistic attitude. The vectors $[N_1^{\min}, ..., N_q^{\min}]$ and $[N_1^{\max}, ..., N_q^{\max}]$ can be seen as two fuzzy membership functions (FMFs) representing the possibilities of all the alternatives $F_1, ..., F_q$.

The FCOWA-ER method requires also four steps:
• Step 1: normalize each column in $E[C]$, respectively, to obtain $E^{Fuzzy}[C]$;
• Step 2: conversion of two normalized columns, i.e., two FMFs (Fuzzy Membership Functions) into two bbas $m_{Pess}(.)$ and $m_{Opti}(.)$ using the α-cut approach introduced in Florea et al. (2008);
• Step 3: fusion of bbas $m_{Pess}(.)$ and $m_{Opti}(.)$ with some combination rule (typically the PCR6 rule);
• Step 4: choice of the final decision based on the resulting combined bba.

In FCOWA-ER, only one combination step is needed. Furthermore, the bba's obtained by using α-cuts are consonant support (nested in order).

4 APPLICATION TO PROBLEMS OF PROTECTION WORKS EFFECTIVENESS

4.1 *Assessment of structural effectiveness of a single check-dam through ER-MCDA*

4.1.1 *AHP methodology*

The problem consists in choosing the observed structural effectiveness level of a given E^l. It is assessed through 6 criteria g_j (Tacnet & Curt 2013) (Figure 2). ω_j in Table 1 are defined by experts.

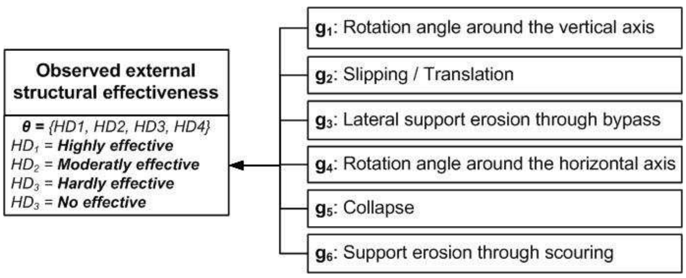

Figure 2: Hierarchical structure to assess observed structural effectiveness of E^l.

According to *DST* (Dempster-Shafer Theory) framework (Shafer 1976), Θ is composed of 4 exclusive elements of effectiveness levels: HD_1 = 'High', HD_2 = 'Medium', HD_3 = 'Low' and HD_4 = 'None'.

4.1.2 *Imprecise evaluation and mapping of g_j*

For E^l, we assume evaluations of g_j by two experts (sources) s_1 and s_2 through possibility distributions (Table 1). Through expert elicitation, a set of fuzzy intervals $L - R$ links each g_j evaluation scale and Θ. Quotations used are extracted from Bouchon-Meunier (1995) (Table 2). Using this mapping process, bbas are established in Table 3.

To take into account the reliability of each source, we discount the input masses of Table 3 by applying the classical Shafer's discounting method (Shafer 1976). We use here discounting factors $\alpha_{s1} = 0.7$ and $\alpha_{s2} = 0.5$. We obtain 12 discounted bba's (noted m'_1 and m'_2) in Table 4.

4.1.3 *Two steps of fusion*

The step 1 consists in combining the bbas $m'_1(.)$ and $m'_2(.)$ for each g_j with PCR6 fusion rule (Table 5).

Table 1: Criteria evaluations of E^l.

			Expert 1 (s1)			Expert 2 (s2)		
criterion g_j	ω_j	unity	E	Π(E)	N(E)	E	Π(E)	N(E)
g_1	0.1	Degree (d)	d=0	1	1	d=0	1	1
g_2	0.1	Meter (m)	m=0	1	1	m=0	1	1
g_3	0.3	Meter (m)	$2 \leqslant$ m $\leqslant 4$	1	1	$1 \leqslant$ m $\leqslant 6$	1	1
			$2.5 \leqslant$ m $\leqslant 3.5$	1	0.3	$2 \leqslant$ m $\leqslant 4$	1	0.7
						m=3	1	0.2
g_4	0.1	Degree (d)	d=0	1	1	$15 \leqslant$ m $\leqslant 20$	1	1
						$10 \leqslant$ m $\leqslant 20$	1	0.55
g_5	0.1	Meter (m)	$0.1 \leqslant$ m $\leqslant 0.5$	1	1	m=0	1	1
			$0.2 \leqslant$ m $\leqslant 0.4$	1	0.5			
g_6	0.3	Meter (m)	$0.2 \leqslant$ m $\leqslant 1.2$	1	1	$0.2 \leqslant$ m $\leqslant 0.8$	1	1
			$0.4 \leqslant$ m $\leqslant 0.8$	1	0.7			
			$0.5 \leqslant$ m $\leqslant 0.7$	1	0.3			

Table 2: Mapping models for each criterion.

	HD1		HD2		HD3		HD4	
criterion g_j	supp	noy	supp	noy	supp	noy	supp	noy
g_1	$0 \leqslant$ d $\leqslant 5$	d=0	$0 \leqslant$ d $\leqslant 15$	$5 \leqslant$ d $\leqslant 10$	$10 \leqslant$ d $\leqslant 30$	$15 \leqslant$ d $\leqslant 25$	$25 \leqslant$ d	$30 \leqslant$ d
g_2	$0 \leqslant$ m $\leqslant 0.3$	m=0	$0 \leqslant$ m $\leqslant 1$	$0.3 \leqslant$ m $\leqslant 0.7$	$0.7 \leqslant$ m $\leqslant 2$	$1 \leqslant$ m $\leqslant 1.7$	$1.7 \leqslant$ m	$2 \leqslant$ m
g_3	$0 \leqslant$ m $\leqslant 0.5$	m=0	$0 \leqslant$ m $\leqslant 2.5$	$0.5 \leqslant$ m $\leqslant 2$	$2 \leqslant$ m $\leqslant 4$	$2.5 \leqslant$ m $\leqslant 3.5$	$3.5 \leqslant$ m	$4 \leqslant$ m
g_4	$0 \leqslant$ d $\leqslant 5$	d=0	$0 \leqslant$ d $\leqslant 15$	$5 \leqslant$ d $\leqslant 10$	$10 \leqslant$ d $\leqslant 30$	$15 \leqslant$ d $\leqslant 25$	$25 \leqslant$ d	$30 \leqslant$ d
g_5	$0 \leqslant$ m $\leqslant 0.2$	m=0	$0 \leqslant$ m $\leqslant 0.5$	$0.2 \leqslant$ m $\leqslant 0.3$	$0.3 \leqslant$ m $\leqslant 1$	$0.5 \leqslant$ m $\leqslant 0.8$	$0.8 \leqslant$ m	$1 \leqslant$ m
g_6	$0 \leqslant$ m $\leqslant 0.3$	m=0	$0 \leqslant$ m $\leqslant 1.5$	$0.3 \leqslant$ m $\leqslant 1.1$	$1.1 \leqslant$ m $\leqslant 2.5$	$1.5 \leqslant$ m $\leqslant 2.2$	$2.2 \leqslant$ m	$2.5 \leqslant$ m

Table 3: E^l bba's after mapping process in a *DST* framework.

criterion g_j	g_1	g_2	g_3	g_4	g_5	g_6
				$m_1(.)$		
HD1	1	1	0	1	0.03125	0.005
HD2	0	0	0.0875	0	0.78125	0.9912
HD3	0	0	0.825	0	0.1875	0.00037
HD4	0	0	0.0875	0	0	0
				$m_2(.)$		
HD1	1	1	0	0	1	0.1875
HD2	0	0	0.1375	0.1125	0	0.8125
HD3	0	0	0.665	0.8875	0	0
HD4	0	0	0.1975	0	0	0

Table 4: Shafer's discounting of input masses with reliability factors $\alpha_{s1} = 0.7$ and $\alpha_{s2} = 0.5$.

criterion g_j	g_1	g_2	g_3	g_4	g_5	g_6
				$m'_1(.)$		
HD1	0.7	0.7	0	0.7	0.021875	0.0035
HD2	0	0	0.06125	0	0.546875	0.69384
HD3	0	0	0.57750	0	0.13125	0.00259
HD4	0	0	0.06125	0	0	0
Θ	0.3	0.3	0.3	0.3	0.3	0.3
				$m'_2(.)$		
HD1	0.5	0.5	0	0	0.5	0.09375
HD2	0	0	0.06875	0.05625	0	0.40625
HD3	0	0	0.33250	0.44375	0	0
HD4	0	0	0.09875	0	0	0
Θ	0.5	0.5	0.5	0.5	0.5	0.5

In step 2, we apply to each bba of Table 5 the importance discounting method presented in Smarandache et al. (2010). We use ω_j (Table 1) to get the Table 6. After combining its 6 bba with a variant of $PCR6$ to take into account positive masses on $\emptyset$, noted $PCR6_\emptyset$ and a normalization procedure (Smarandache et al. 2010), we finally get the Table 7.

According to it, E^l is mainly medium effective because the highest belief mass is $m(HD2)$. Because $m(HD3)$, we can say that E^l effectiveness is more between low and medium, but not high, nor none.

Table 5: E^l bbas after the step 1 of PCR6-MCDA.

criterion g_j	g_1	g_2	g_3	g_4	g_5	g_6
				$m_{Step1}(.)$		
HD1	0.85	0.85	0	0.57656	0.35445	0.03820
HD2	0	0	0.0674	0.0198	0.41628	0.81046
HD3	0	0	0.69909	0.25364	0.07927	0.00131
HD4	0	0	0.08351	0	0	0
Θ	0.15	0.15	0.15	0.15	0.15	0.15003

Table 6: E^l bbas after importance discounting.

criterion g_j	g_1	g_2	g_3	g_4	g_5	g_6
			$m_{Step1}(.)$ after importance discounting			
$\emptyset$	0.9	0.9	0.7	0.9	0.9	0.7
HD1	0.085	0.085	0	0.05765	0.03544	0.01146
HD2	0	0	0.02022	0.00198	0.04163	0.24314
HD3	0	0	0.20973	0.02537	0.00793	0.00039
HD4	0	0	0.02505	0	0	0
Θ	0.015	0.015	0.045	0.015	0.015	0.04501

Table 7: E^l bbas after the step 2 of PCR6-MCDA.

criterion g_j	$m_{PCR6_\emptyset}(.)$	$m^{normalized}_{PCR6_\emptyset}(.)$
$\emptyset$	0.96601	0
HD1	0.00547	0.16094
HD2	0.01560	0.45901
HD3	0.01141	0.33559
HD4	0.00017	0.00494
Θ	0.00134	0.03951

4.2 *Comparing actions on D^o using (F)COWA-ER*

4.2.1 *Decision problem elicitation*

g_j is the D^o effectiveness level. In a DST framework, one assumes 7 scenarii such as flood with bed-load transport (S^o_3) or debris flow (S^o_6). One considers 5 possible actions such as repair of all the degraded check-dams (a_3) or renewal of all check-dams (a_5).

4.2.2 *C_{ik} and S^o evaluations.*

One rates C_{ik} with an integer between 0 (no effective) and 10 (very high effective) (Curt et al. 2010). As Eq. (1), one assumes C where $q = 5$ and $n = 7$ (Eq. (6)).

$$C = \begin{bmatrix} 5 & 3 & 4 & 2 & 3 & 1 & 1 \\ 7 & 4 & 6 & 3 & 4 & 2 & 1 \\ 8 & 5 & 7 & 4 & 5 & 3 & 1 \\ 10 & 7 & 10 & 6 & 7 & 5 & 2 \\ 10 & 9 & 10 & 9 & 10 & 10 & 4 \end{bmatrix} \quad (6)$$

One considers 4 X_l: $X_1 = S_1^o \cup S_3^o \cup S_5^o$, $X_2 = S_2^o \cup S_4^o \cup S_5^o \cup S_6^o$, $X_3 = S_7^o$, $X_4 = \Theta$. One gives $m(X_1) = 0.6$, $m(X_2) = 0.2$, $m(X_3) = 0.01$ and $m(X_4) = 0.19$. Applying the OWA pessimistic and optimistic operators, one can assess the bounds of expected effectiveness levels for each actions given by Eq. (7).

$$E[C] \subset \begin{bmatrix} [2.20; 4.56] \\ [3.00; 6.34] \\ [3.80; 7.33] \\ [5.60; 9.32] \\ [8.60; 9.94] \end{bmatrix} (7) \quad E^{Imp}[C] \approx \begin{bmatrix} [0.22; 0.46] \\ [0.30; 0.64] \\ [0.38; 0.74] \\ [0.56; 0.94] \\ [0.87; 1.00] \end{bmatrix} (8)$$

4.2.3 *Results through COWA-ER*

Steps 1 and 2 make it possible to assess bbas of the 5 actions in the Table 8 passing by the normalized imprecise matrix $E^{Imp}[C]$ given in Eq. (8).

Table 8: Basic belief assignements of the 5 actions.

Alternatives D_i	$m_i(D_i)$	$mi(\overline{D_i})$	$m_i(D_i \cup \overline{D_i})$
D_1	0.22	0.54	0.24
D_2	0.30	0.36	0.34
D_3	0.38	0.26	0.36
D_4	0.56	0.06	0.38
D_5	0.86	0	0.14

Step 3 combines the 5 bba's altogether with choice of the PCR6 fusion rule (Table 9).

Table 9: Fusion of the 5 elementary bbas with PCR6.

Focal Element	$m_{PCR6}(.)$
D_1	0.02835
D_2	0.04805
D_3	0.07318
D_4	0.15185
D_5	0.39179
$D_1 \cup D_5$	0.00019
$D_2 \cup D_5$	0.0004
$D_3 \cup D_5$	0.00059
$D_4 \cup D_5$	0.00269
$D_1 \cup D_4 \cup D_5$	0.0012
$D_2 \cup D_3 \cup D_5$	0.00056
$D_2 \cup D_4 \cup D_5$	0.00254
$D_3 \cup D_4 \cup D_5$	0.00372
$D_1 \cup D_2 \cup D_5$	0.00018
$D_1 \cup D_3 \cup D_5$	0.00026
$D_1^{\cup} D_2 \cup D_3 \cup D_5$	0.00138
$D_1^{\cup} D_2 \cup D_4 \cup D_5$	0.02194
$D_1 \cup D_3 \cup D_4 \cup D_5$	0.04123
$D_2 \cup D_3 \cup D_4 \cup D_5$	0.09063
$D_1 \cup D_2 \cup D_3 \cup D_4 \cup D_5$	0.13927

Choosing the decision-making rule is needed to implement the step 4. Results are compared in the Table 10. One sees that based on max of Bel, of BetP, of DSmP or of Pl, the best action is always D_5.

Table 10: Bel, BetP, DSmP and Pl of effectiveness levels of actions on D_i based on COWA-ER.

D_i	$Bel(D_i)$	$BetP(D_i^)$	$DSmP_{\epsilon=0}(D_i)$	$Pl(D_i)$
D_1	0.028	0.073	0.037	0.234
D_2	0.048	0.106	0.066	0.305
D_3	0.073	0.136	0.103	0.351
D_4	0.152	0.222	0.221	0.455
D_5	0.392	0.463	0.572	0.699

4.2.4 *Results through Fuzzy COWA-ER*

Step 1 makes it possible to get from Eq. (7) a normalized imprecise matrix $E^{Fuzzy}[C]$ in Eq. (9).

$$E^{Fuzzy}[C] \approx \begin{bmatrix} [0.26; 0.46] \\ [0.35; 0.64] \\ [0.44; 0.74] \\ [0.65; 0.94] \\ [1.00; 1.00] \end{bmatrix} \quad (9)$$

For the step 2, by using a 5 α-cut approach, we convert $E^{Fuzzy}[C]$ into 2 bbas $m_{Pess}(.)$ and $m_{Opti}(.)$. Step 3 combines them with choice of the PCR6 fusion rule. Results are given in the Table 11.

Table 11: The 2 bbas to combine and the result of PCR6 fusion

Focal Element	$m_{Pess}(.)$	$m_{Opti}(.)$	$m_{PCR6}(.)$
D_5	0.35	0.06	0.3895
$D_4 \cup D_5$	0.21	0.20	0.2847
$D_3 \cup D_4 \cup D_5$	0.09	0.10	0.1033
$D_2 \cup D_3 \cup D_4 \cup D_5$	0.09	0.18	0.1051
$D_1 \cup D_2 \cup D_3^{\cup} D_4 \cup D_5$	0.26	0.46	0.1174

The Table 12 shows the (approximate) values of $Bel(.)$, $BetP(.)$, $DSmP_{\epsilon=10^{-6}}(.)$ and $Pl(.)$ based on $m_{PCR6}(.)$ values of Table 11. One sees that based on max of Bel, of BetP, of DSmP or of Pl, the best action is always D_5 (similar decision as with COWA-ER).

Table 12: Credibility, BetP, DSmP and plausibility of effectiveness levels of D_i based on FCOWA-ER.

D_i	$Bel(D_i)$	$BetP(D_i)$	$DSmP(D_i)$	$Pl(D_i)$
D_1	0	0.023	0	0.117
D_2	0	0.050	0	0.222
D_3	0	0.084	0	0.326
D_4	0	0.227	0	0.611
A_5	0.389	0.616	1	1

5 CONCLUSIONS AND PERSPECTIVES

In this paper, we have both formalized the decision-problem and applied recent advanced MCDMs (ER-MCDA, COWA-ER, FCOWA-ER) to assess effectiveness of torrent protective check-dams in a context of imperfect information and more or less reliable sources. This application, based on expert knowledge, provides a class evaluation related to available knowledge. Others outranking methods such as the

Soft-Electre Tri (SET) methodology (Dezert & Tacnet 2012) can also be applied to sort protection systems in predefined effectiveness classes. Defining uncertain states of nature and corresponding belief mass m(.) remains challenging. Comparing belief functions theory with Bayesian probabilities or Choquet capacities in this actual context is a next step (Cohen & Tallon 2000). Effect on results of the fusion rules and order of combinations have also to be compared.

From an operational point of view, next steps will consist in DM and decision problem complete elicitation, criteria, importance, preferences on evaluation scale assessments. Afterwards, these methods will be combined in a global process taking into account all the system scales related to protection system devices.

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