

# Evaluation of Efficiency of Torrential Protective Structures With New BF-TOPSIS Methods

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**Abstract**—Decision-Aid Methods (DAMs) such as the Cost-Benefit Analysis (CBA) and the Analytical Hierarchy Process (AHP) help decision-makers to rank alternatives or to choose the best one among several potential ones. The new Belief Function based Technique for Order Preference by Similarity to Ideal Solution (BF-TOPSIS) methods have been recently developed for Multi-Criteria Decision-Making problems. In this paper, we compare CBA, AHP and BF-TOPSIS methods through an actual application case to natural risks in mountains. The CBA is the most used approach but is limited. Classical Multi-Criteria Decision-Aid methods (MCDAs) such as the AHP help to go further. The BF-TOPSIS methods notably show a robustness to rank reversal problems in the problem under concern, with a tractable complexity. Moreover, some steps of these new methods can be included in other MCDAs developed under the belief function theory framework.

**Keywords:** multi-criteria, decision-making, BF-TOPSIS, belief functions, efficiency, torrential protection works.

## I. INTRODUCTION

Real life Decision-Making Problems (DMPs) depend on several criteria for which precise or imperfect evaluation can be provided. Economic aspects are also of main importance to compare and choose strategies or measures. For instance, in mountainous areas, risk managers must rank several protective actions against torrential floods to choose the best one.

Several Decision-Aid Methods (DAMs) are useful to solve such DMPs. In CBA<sup>1</sup> approach, all relevant elements, such as environmental issues and damage caused by natural hazards, are transformed into monetary terms revealing people's preferences [1]. Since damage knowledge is uncertain, their expected value is generally used to introduce the risk concept [2]. In practice, CBA thus compares all mitigation alternatives taking into account three economic criteria: 1) the initial investment cost, 2) the global maintenance cost, and 3) the global risk reduction [3]. CBA remains the most used method but monetary valuation of non material assets is still an ethical issue in practice [4]. In this context, other methods have been developed and used.

Multi-Criteria Decision-Aid methods (MCDAs) use scoring against weighted criteria to express preferences [5]. To rank all

alternatives and choose the best among them, single synthesizing criterion approaches such as AHP<sup>2</sup> [6] and MAUT<sup>3</sup> [7] use a step of data normalization or utility transformation. Out-ranking methods such as ELECTRE<sup>4</sup> [8] reject the hypothesis of a total pre-order of the decision-maker (DM) preferences and cope with their non-transitivity. Alternatives are compared pairwise, sometimes without being able to specify any strict preference. Other methods such as ERV<sup>5</sup> [9] and TOPSIS<sup>6</sup> [10] evaluate more precisely how each alternative is, to some extent, better or worse than the others.

To support MCDM problems based on imperfect evaluations, belief function theory makes it possible to represent all kinds of imperfect information given by a source through a basic belief assignment (BBA) [11]. The new BF-TOPSIS<sup>7</sup> methods presented in a companion paper [12] improves classical MCDM support. This paper aims to demonstrate it through an actual example of DMP about choosing the most efficient protection alternative against torrential floods. Classical CBA is first applied to highlight difficulties of practical monetary valuation of all criteria. AHP is then used to aggregate all benefits and cost criteria, as proposed in [13]. In this paper, we finally show that the new BF-TOPSIS methods not only improve the CBA process taking into account all decision criteria, but are also more robust to rank reversal problems than the AHP.

In section II, we describe the classical MCDM problem and briefly introduce CBA and AHP. We then recall some basics of belief functions in Section III. The BF-TOPSIS methods are only briefly introduced in Section IV because they are detailed in [12] with examples. The CBA, AHP and BF-TOPSIS are implemented to a real case of a classical MCDM problem in Section V. In Section VI, we finally discuss the improvements obtained by the BF-TOPSIS and imagine some potential combinations with other MCDAs to support unclassical MCDM problems under epistemic uncertainty.

<sup>2</sup>Analytic Hierarchy Process

<sup>3</sup>Multi-Attribute Utility Theory

<sup>4</sup>elimination and choice translating reality

<sup>5</sup>Estimated Ranking Vector

<sup>6</sup>Technique for Order Preference by Similarity to Ideal Solution

<sup>7</sup>Belief Function based TOPSIS

<sup>1</sup>Cost-Benefit Analysis

## II. CLASSICAL MCDM PROBLEM AND METHODS

### A. Classical MCDM problem

A classical MCDM problem can be formalized by a given  $M \times N$  score matrix  $\mathbf{S} = [S_{ij}]$  as expressed in Eq. (1), where  $M \geq 2$  is the number of alternatives (or measures, choices, solutions)  $A_i$  ( $i = 1, \dots, M$ ), and  $N \geq 1$  is the number of criteria  $C_j$  ( $j = 1, \dots, N$ ). Each  $S_{ij}$  is the score of  $A_i$  for a criterion  $C_j$ . Each  $C_j$  has an importance (or preference) weight  $w_j \in [0, 1]$ , and we assume that the weighting factors are normalized,  $\sum_{j=1}^N w_j = 1$ .

$$\mathbf{S} \triangleq \begin{matrix} & \begin{matrix} C_1, w_1 & \dots & C_j, w_j & \dots & C_N, w_N \end{matrix} \\ \begin{matrix} A_1 \\ \vdots \\ A_i \\ \vdots \\ A_M \end{matrix} & \begin{bmatrix} S_{11} & \dots & S_{1j} & \dots & S_{1N} \\ \vdots & & \vdots & & \vdots \\ S_{i1} & \dots & S_{ij} & \dots & S_{iN} \\ \vdots & & \vdots & & \vdots \\ S_{M1} & \dots & S_{Mj} & \dots & S_{MN} \end{bmatrix} \end{matrix} \quad (1)$$

The MCDM problem is about choosing the best alternative  $A_{i^*} \in A \triangleq \{A_1, A_2, \dots, A_M\}$  given the score matrix  $\mathbf{S}$  and the importance weighting factors  $w_j$ ,  $j = 1, \dots, N$ .

### B. The CBA actually used to support natural hazard DMP

Torrential floods can damage several elements such as housing, roads, and factories. Nearby a torrent, we denote  $\Omega$  the total number of individual elements  $z_e$  ( $e = 1, \dots, \Omega$ ) at torrential risk.  $R_{z_e}$  is the annual risk, or potential damage (losses), for each  $z_e$ . Their sum for  $e = 1, \dots, \Omega$  gives in Eq. (2) the annual total risk denoted  $R_\Omega$ . To calculate each  $R_{z_e}$ , Eq. (2) considers  $p(y)$  the continuous annual probability distribution of an intensity criterion of interest  $y$  (e.g. volume for debris flows),  $V(z_e, y)$  the potential damaging rate (or vulnerability) of  $z_e$  given  $y$ , and  $q(z_e)$  its exposure rate [14]. Each  $z_e$  is monetary valued in €. From an operational standpoint, several discrete scenarii represent  $p(y)$  [3].

$$R_\Omega \triangleq \sum_{e=1}^{\Omega} R_{z_e} \triangleq \sum_{e=1}^{\Omega} q(z_e) \cdot z_e \cdot \int p(y) V(z_e, y) dy \quad (2)$$

$R_{\Omega,0}$  is the baseline annual risk, calculated through Eq. (2) with baseline  $p(y)$ ,  $V(z_e, y)$  and  $q(z_e)$ ,  $e = 1, \dots, \Omega$ . One considers  $M$  potential protective solutions  $A_i$ ,  $i = 1, \dots, M$ . Each one reduces  $R_{\Omega,0}$  to a residual annual risk  $R_{\Omega,i}$  ( $i = 1, \dots, M$ ). For each  $A_i$ ,  $R_{\Omega,i}$  is calculated through Eq. (2) with corresponding  $p(y)$ ,  $V(z_e, y)$ , and  $q(z_e)$ , different from the baseline ones. One finally computes the annual risk reduction  $R_{\Omega,0} - R_{\Omega,i}$  ( $i = 1, \dots, M$ ).

Let us consider the following example to establish the score matrix  $\mathbf{S}$ .  $t$  is the temporal horizon of the project and  $q_p \in [0, 1]$ ,  $p = 1, \dots, t$  is the annual discount rate. We consider  $N = 3$  with  $C_1$  the investment cost,  $C_2$  the global maintenance cost within  $n$  years, and  $C_3$  the global risk reduction within  $n$  years. All  $S_{ij}$  values are assessed in €. For each  $A_i$ ,  $C_1$  is directly valued to give  $S_{i1}$ .  $S_{i2}$  depends on the annual maintenance cost  $\kappa_i$ .  $S_{i3}$  is related to the annual

risk reduction  $R_{\Omega,0} - R_{\Omega,i}$ . The higher are  $S_{i1}$  and  $S_{i2}$ , the more costly is  $A_i$ : the DM wants to minimize them. The higher is  $S_{i3}$ , the better is  $A_i$ : the DM wants to maximize it.  $A_{i^*}$  is thus chosen with  $i^* \triangleq \arg \max_i \{NPV_i\}$ , or with  $i^* \triangleq \arg \max_i \{r_i\}$ , where the net present value  $NPV_i$  and the benefit cost ratio  $r_i$  are defined by:

$$NPV_i \triangleq \sum_{p=1}^t \frac{1}{(1+q_p)^p} (R_{\Omega,0} - R_{\Omega,i}) - \sum_{p=1}^t \frac{1}{(1+q_p)^p} \kappa_i - S_{i1} \quad (3)$$

$$r_i \triangleq \frac{\sum_{p=1}^t \frac{1}{(1+q_p)^p} (R_{\Omega,0} - R_{\Omega,i})}{S_{i1} + \sum_{p=1}^t \frac{1}{(1+q_p)^p} \kappa_i} \quad (4)$$

### C. The single synthesizing criterion approaches

One of the criticisms of CBA is the monetary valuation of  $S_{i3}$ . Other scales, and thus risk  $R_\Omega$  defined in Eq. (2), should be considered to assess human life, environmental, and psychological damage or losses [4]. MCDAs can overcome this lack [5]. We focus on the widely used AHP method [6]. We briefly recall its principles (see Fig. 1) to draw decision according to the score matrix  $\mathbf{S}$ , because we will compare our new BF-TOPSIS methods to AHP in the sequel.

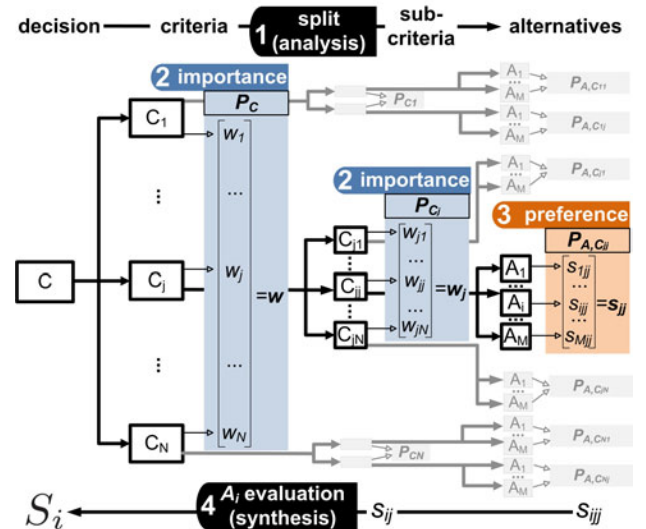


Fig. 1. Main steps of the AHP method.

• **AHP Step 1:** Analyze and split the DMP arranging criteria in a hierarchical structure descending from a decision goal to criteria, sub-criteria and finally alternatives in successive levels.

• **AHP Step 2:** For each level of the hierarchical structure, preference matrices are built on pairwise comparisons. For any  $X$  and  $Y$  assessed through the same unit, each hypothesis  $X \succ Y$  ( $X$  is preferred to  $Y$ ) is quantified by a Preference Level (PL) value  $PL_{X \succ Y} \in [1; 9]$  through the assessment of Table I. For instance, for the set  $C$ , one can compute an  $N \times N$  matrix  $\mathbf{P}_C$  as in (5). In the classical AHP, the column  $N$ -vector  $\mathbf{w} = [w_1, \dots, w_j, \dots, w_N]^T$  corresponds to the eigenvector of

$\mathbf{P}_C$  with well-known issues such as the rank reversal [15]. For each  $j^{\text{th}}$ -column, one normalizes each element by the sum of all its elements, giving a normalized  $N \times N$  matrix. For each  $j^{\text{th}}$ -line, one calculates the sum of all normalized elements, giving an  $N$  column vector. One normalizes each of its element by the sum of all elements, giving  $\mathbf{w}$ . As pairwise comparisons may not be totally consistent, a last step consists in checking the consistency of these results.

TABLE I  
PREFERENCE LEVEL  $PL_{X \succ Y}$  ASSESSMENT TABLE.

| Qualitative preference $X \succ Y$ | $PL_{X \succ Y}$ | $PL_{Y \succ X}$ |
|------------------------------------|------------------|------------------|
| (X) is as important as (Y)         | 1                | 1                |
| is somewhat more important than    | 3                | 1/3              |
| is more important than             | 5                | 1/5              |
| is far more important than         | 7                | 1/7              |
| is absolutely more important than  | 9                | 1/9              |

$$\mathbf{P}_C \triangleq \begin{matrix} & C_1 & \dots & C_j & \dots & C_N \\ \begin{matrix} C_1 \\ \vdots \\ C_j \\ \vdots \\ C_N \end{matrix} & \begin{bmatrix} 1 & \dots & PL_{C_1 \succ C_j} & \dots & PL_{C_1 \succ C_N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ PL_{C_j \succ C_1} & \dots & 1 & \dots & PL_{C_j \succ C_N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ PL_{C_N \succ C_1} & \dots & PL_{C_N \succ C_j} & \dots & 1 \end{bmatrix} \end{matrix} \quad (5)$$

• **AHP Step 3:** For the lowest sub-criteria level and for each sub-criteria, for instance  $C_j$ , the classical AHP method, called *criteria-solution*, uses the same approach as in the Step 2 to get an  $M \times M$  preference matrix  $\mathbf{P}_{A, C_j}$  not for criteria but for alternatives. The  $M$ -eigenvector of normalized score  $\mathbf{s}_j \triangleq [s_{1j}, \dots, s_{ij}, \dots, s_{Mj}]^T$  is then extracted.  $N$  vectors  $\mathbf{s}_j$  are established. For DMPs with numerous alternatives, another *criteria-evaluation-solution* exists [16] since the pairwise comparison cannot be reasonably done on large matrices [17].

• **AHP Step 4:** Finally, the synthesizing score  $S_i$  is deduced for each  $A_i$  by an additive aggregation operation according to (6).  $A_{i^*}$  is chosen with  $i^* \triangleq \arg \max_i \{S_i\}$ .

$$S_i = \sum_{j=1}^M w_j \cdot s_{ij} \quad (6)$$

For quantitative MCDM problems, other methods make it possible to extract the normalized  $M$ -vector  $\mathbf{s}_j$  replacing the AHP step 3. For instance, a second way is to normalize each  $S_{ij}$  by the maximum or the sum of  $\mathbf{s}_j$ . To some extent, the approach is similar to the Multi-Attribute Utility Theory (MAUT) which establishes the utility function  $u_j$ , specific to each  $C_j$ , giving  $s_{ij} = u_j(S_{ij})$ . Once  $\mathbf{s}_j$  is computed, the step 4 can be replaced by other aggregation rules such as the multiplicative one [15].

### III. BASICS OF THE BELIEF FUNCTION THEORY

Originally proposed by Shafer in [11], the belief function theory is based on representing imperfect knowledge through

basic belief assignments (BBAs). We briefly introduce it in the context of MCDM problems. One starts specifying the frame of discernment (FoD) of the DMP under consideration denoted by the finite discrete set  $\Theta$ . Each element of the FoD is a potential alternative (or solution) of the DMP, and the elements of  $\Theta$  are assumed exhaustive and mutually exclusive. The powerset of  $\Theta$  denoted  $2^\Theta$  is the set of all subsets of  $\Theta$ , empty set included. A body of evidence is a source of information that will help the decision-maker to identify the best element of the FoD as the solution of the problem. The interest of belief functions is their ability to model epistemic uncertainties. Each body of evidence is characterized by a BBA, or a mass of belief, which is a mapping  $m(\cdot) : 2^\Theta \rightarrow [0, 1]$  that satisfies  $m(\emptyset) = 0$ , and for all  $A \neq \emptyset \in 2^\Theta$  the condition  $\sum_{A \subseteq \Theta} m(A) = 1$ . The Belief function  $\text{Bel}(\cdot)$  and the Plausibility function  $\text{Pl}(\cdot)$  are defined from  $m(\cdot)$  by:

$$\text{Bel}(A) \triangleq \sum_{B \subseteq A | B \in 2^\Theta} m(B) \quad (7)$$

$$\text{Pl}(A) \triangleq \sum_{B \cap A \neq \emptyset | B \in 2^\Theta} m(B) \quad (8)$$

The credibility  $\text{Bel}(A)$  and the plausibility  $\text{Pl}(A)$  are often interpreted as lower and upper bounds of the unknown probability of  $A$ . The vacuous BBA defined as  $m_v(\Theta) = 1$  models the full ignorant source of evidence. In 1976, Shafer did propose Dempster's rule to combine distinct sources of evidence [11] which has been subject to strong debates in fusion community starting from Zadeh's first criticism in 1979 [18]. Since 1990's many alternatives have been proposed to combine more or less efficiently belief functions, as well as an extension of belief function in the framework of Dezert-Smarandache Theory (DSmT) as shown and discussed in [19].

### IV. THE BF-TOPSIS METHODS

Four BF-TOPSIS methods have been recently proposed in [12] with an increasing complexity and robustness to rank reversal phenomenon. All these methods start with constructing BBAs from the score values of the matrix  $\mathbf{S}$  as explained in the next subsection. Only the way those BBAs are processed differs from one BF-TOPSIS method to another one.

#### A. Construction of BBAs from the score matrix $\mathbf{S}$

In [12], one has proved that BBAs can be consistently built from  $\mathbf{S}$  by computing  $\text{Bel}_{ij}(A_i)$  and  $\text{Bel}_{ij}(\bar{A}_i)$  defined by:

$$\text{Bel}_{ij}(A_i) \triangleq \begin{cases} \frac{\text{Sup}_j(A_i)}{A_{\max}^j} & \text{if } A_{\max}^j \neq 0 \\ 0 & \text{if } A_{\max}^j = 0 \end{cases} \quad (9)$$

$$\text{Bel}_{ij}(\bar{A}_i) \triangleq \begin{cases} \frac{\text{Inf}_j(A_i)}{A_{\min}^j} & \text{if } A_{\min}^j \neq 0 \\ 0 & \text{if } A_{\min}^j = 0 \end{cases} \quad (10)$$

where  $\bar{A}_i$  is the complement of  $A_i$  in  $\Theta$ , and

$$\text{Sup}_j(A_i) \triangleq \sum_{k \in \{1, \dots, M\} | S_{kj} \leq S_{ij}} |S_{ij} - S_{kj}| \quad (11)$$

$$\text{Inf}_j(A_i) \triangleq - \sum_{k \in \{1, \dots, M\} | S_{kj} \geq S_{ij}} |S_{ij} - S_{kj}| \quad (12)$$

The denominators involved in Eqs. (9)-(10), are defined by  $A_{\max}^j \triangleq \max_i \text{Sup}_j(A_i)$  and  $A_{\min}^j \triangleq \min_i \text{Inf}_j(A_i)$ , and they are supposed different from zero<sup>8</sup>. With this principle of construction,  $\text{Bel}_{ij}(A_i)$  and  $\text{Bel}_{ij}(\bar{A}_i)$  belong to  $[0; 1]$  and are consistent because the equality  $\text{Pl}_{ij}(A_i) = 1 - \text{Bel}_{ij}(\bar{A}_i)$  holds (see the mathematical proof in [12]). Therefore, the belief interval of choosing  $A_i$  considering  $C_j$  is given by:

$$[\text{Bel}_{ij}(A_i); \text{Pl}_{ij}(A_i)] \triangleq \left[ \frac{\text{Sup}_j(A_i)}{A_{\max}^j}, 1 - \frac{\text{Inf}_j(A_i)}{A_{\min}^j} \right] \quad (13)$$

From this belief interval, we deduce the BBA  $m_{ij}(\cdot)$  which is the triplet  $(m_{ij}(A_i), m_{ij}(\bar{A}_i), m_{ij}(A_i \cup \bar{A}_i))$  defined by:

$$m_{ij}(A_i) \triangleq \text{Bel}_{ij}(A_i) \quad (14)$$

$$m_{ij}(\bar{A}_i) \triangleq \text{Bel}_{ij}(\bar{A}_i) = 1 - \text{Pl}_{ij}(A_i) \quad (15)$$

$$m_{ij}(A_i \cup \bar{A}_i) \triangleq m_{ij}(\Theta) = \text{Pl}_{ij}(A_i) - \text{Bel}_{ij}(A_i) \quad (16)$$

If a numerical value  $S_{ij}$  is missing in  $\mathbf{S}$ , one uses  $m_{ij}(\cdot) \triangleq (0, 0, 1)$ , i.e. one takes the vacuous belief assignment.

Using the formulae (9)-(16), we obtain from any  $M \times N$  score matrix  $\mathbf{S}$  the general  $M \times N$  matrix of BBAs for  $\Theta = A$  defined by:

$$\mathbf{M} \triangleq \begin{matrix} & C_1, w_1 & \dots & C_j, w_j & \dots & C_N, w_N \\ \begin{matrix} A_1 \\ \vdots \\ A_i \\ \vdots \\ A_M \end{matrix} & \begin{bmatrix} m_{11}(\cdot) & \dots & m_{1j}(\cdot) & \dots & m_{1N}(\cdot) \\ \vdots & & \vdots & & \vdots \\ m_{i1}(\cdot) & \dots & m_{ij}(\cdot) & \dots & m_{iN}(\cdot) \\ \vdots & & \vdots & & \vdots \\ m_{M1}(\cdot) & \dots & m_{Mj}(\cdot) & \dots & m_{MN}(\cdot) \end{bmatrix} \end{matrix} \quad (17)$$

### B. The BF-TOPSIS methods

Four BF-TOPSIS methods have been presented in details in [12]. We only briefly recall them. All these methods use the BBA construction presented in the subsection IV-A first to establish BBA matrix  $\mathbf{M}$  and then, the Belief Interval-based Euclidean distance between BBAs proposed in [20].

**Principle of BF-TOPSIS1:** From the BBA matrix  $\mathbf{M}$  and for each alternative  $A_i$ , one computes the Belief Interval-based Euclidean distances  $d_{BI}^E(m_{ij}, m_{ij}^{\text{best}})$  (defined in [20]) between  $m_{ij}(\cdot)$  and the ideal best BBA defined by  $m_{ij}^{\text{best}}(A_i) \triangleq 1$ , and the distances  $d_{BI}^E(m_{ij}, m_{ij}^{\text{worst}})$  between  $m_{ij}(\cdot)$  and the ideal worst BBA defined by  $m_{ij}^{\text{worst}}(\bar{A}_i) \triangleq 1$ . Then, one computes the weighted average of  $d_{BI}^E(m_{ij}, m_{ij}^{\text{best}})$  values with relative importance weighting factor  $w_j$  of criteria  $C_j$ . Similarly, one computes the weighted average of  $d_{BI}^E(m_{ij}, m_{ij}^{\text{worst}})$  values. More specifically, one computes

$$d^{\text{best}}(A_i) \triangleq \sum_{j=1}^N w_j \cdot d_{BI}^E(m_{ij}, m_{ij}^{\text{best}}) \quad (18)$$

$$d^{\text{worst}}(A_i) \triangleq \sum_{j=1}^N w_j \cdot d_{BI}^E(m_{ij}, m_{ij}^{\text{worst}}) \quad (19)$$

<sup>8</sup>If  $A_{\max}^j = 0$  then  $\text{Bel}_{ij}(A_i) = 0$ , and if  $A_{\min}^j = 0$  then  $\text{Pl}_{ij}(A_i) = 1$ , so that  $\text{Bel}_{ij}(\bar{A}_i) = 0$ .

The relative closeness of the alternative  $A_i$  with respect to the ideal best solution  $A^{\text{best}}$  defined by:

$$C(A_i, A^{\text{best}}) \triangleq \frac{d^{\text{worst}}(A_i)}{d^{\text{worst}}(A_i) + d^{\text{best}}(A_i)} \quad (20)$$

is used to make the preference ordering according to the descending order of  $C(A_i, A^{\text{best}}) \in [0, 1]$ , where a larger  $C(A_i, A^{\text{best}})$  value means a better alternative.

**Principle of BF-TOPSIS2:** In this method, one computes at first the relative closeness of each alternative  $A_i$  w.r.t. its ideal best solution  $A^{\text{best}}$  for each criteria  $C_j$  by:

$$C_j(A_i, A^{\text{best}}) \triangleq \frac{d_{BI}^E(m_{ij}, m_{ij}^{\text{worst}})}{d_{BI}^E(m_{ij}, m_{ij}^{\text{worst}}) + d_{BI}^E(m_{ij}, m_{ij}^{\text{best}})} \quad (21)$$

The global relative closeness of each alternative  $A_i$  with respect to its ideal best solution  $A^{\text{best}}$  is then obtained by the weighted average of  $C_j(A_i, A^{\text{best}})$ . More specifically by

$$C(A_i, A^{\text{best}}) \triangleq \sum_{j=1}^N w_j \cdot C_j(A_i, A^{\text{best}}) \quad (22)$$

which is used to make the final preference ordering.

**Principle of BF-TOPSIS3:** This method is more complicated to implement because it uses PCR6 fusion rule [19] to combine BBAs also taking into account the importance factors  $w_j$ . For each alternative  $A_i$ , one fuses the  $N$  BBAs  $m_{ij}(\cdot)$  discounted with importance factor  $w_j$  (see [21] for details) from the  $i$ -th row of the BBA matrix  $\mathbf{M}$  with PCR6 rule of combination. One gets the fused BBA  $m_i^{\text{PCR6}}$  for each  $A_i$ , from which one computes the Belief Interval-base Euclidean distance  $d^{\text{best}}(A_i) = d_{BI}^E(m_i^{\text{PCR6}}, m_i^{\text{best}})$  between  $m_i^{\text{PCR6}}(\cdot)$  and its ideal best BBA  $m_i^{\text{best}}(A_i) \triangleq 1$ . Similarly, one computes the distance  $d^{\text{worst}}(A_i) = d_{BI}^E(m_i^{\text{PCR6}}, m_i^{\text{worst}})$  between  $m_i^{\text{PCR6}}(\cdot)$  and  $m_i^{\text{worst}}(\bar{A}_i) \triangleq 1$ . The relative closeness of each  $A_i$  with respect to ideal best solution  $C(A_i, A^{\text{best}})$  is computed by Eq. (20), and is used to make the preference ordering according to the descending order of  $C(A_i, A^{\text{best}})$ .

**Principle of BF-TOPSIS4:** This method is similar to BF-TOPSIS3 except that we use the more complicate ZPCR6 fusion rule which is a modified version of PCR6 rule taking into account Zhang's degree of intersection of focal elements in the conjunctive consensus operator. ZPCR6 rule is explained in details with examples in [22].

## V. APPLICATION OF BF-TOPSIS TO PROTECTIVE ACTION TO TAKE WITHIN A TORRENTIAL WATERSHED

### A. Formalization of the actual MCDM problem

To mitigate the risk of an area exposed to torrential floods, one considers, as an example, four solutions:

- $A_1$ : doing nothing;
- $A_2$ : building several check dam series in the headwaters;
- $A_3$ : building a sediment trap upstream the torrential fan;
- $A_4$ : mixing implementation of  $A_2$  and  $A_3$ .

Moreover, public authorities often wish to compare those solutions with another one:

- $A_5$ : individually protecting each element at risk.

Therefore, to help the DM, we assume that the expert must compare the first four alternatives (let us call it **case 1** with  $M=4$ ) and then integrate the  $A_5$  in a global comparison (let us call it **case 2** with  $M=5$ ) (see Fig. 2).

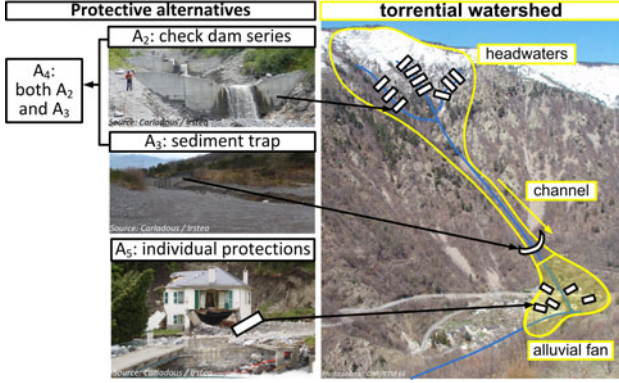


Fig. 2. Potential alternatives of the real DMP.

The DM considers five criteria ( $N = 5$ ) and wants to minimize  $C_1$ ,  $C_2$  and to maximize  $C_3$ ,  $C_4$  and  $C_5$  with:

- $C_1$ : the investment cost in €;
- $C_2$ : the annual maintenance cost in € ( $S_{i2} = \kappa_i$ );
- $C_3$ : the annual risk reduction in  $m^2$  of houses damaged;
- $C_4$ : the annual risk reduction in *human casualties*;
- $C_5$ : the annual risk reduction in *number of sites dangerous to environment in area at risk*.

To compare criteria in pairs, the DM gives the preference matrix  $\mathbf{P}_C$  according to the AHP method [6]. Implementing the AHP step 2 gives the importance weighting vector  $\mathbf{w} = [0.08, 0.04, 0.10, 0.46, 0.32]^T$  recalled in the Table II. It corresponds to the importance ranking:  $C_4 \succ C_5 \succ C_3 \succ C_1 \succ C_2$ .

$$\mathbf{P}_C = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} & \begin{bmatrix} 1 & 4 & 0.5 & 1/7 & 0.25 \\ 0.25 & 1 & 0.25 & 1/9 & 1/6 \\ 2 & 4 & 1 & 0.2 & 1/7 \\ 7 & 9 & 5 & 1 & 2 \\ 4 & 6 & 7 & 0.5 & 1 \end{bmatrix} \end{matrix} \quad (23)$$

For each  $A_i$  and  $C_j$ , the DM assesses the  $S_{ij}$  value to establish the score matrices  $\mathbf{S}_{\text{case1}}$  and  $\mathbf{S}_{\text{case2}}$  shown in Table II.

In the next subsections, we provide intermediary results for each method to help the reader to check by himself the validity of our final ranking vectors gathered in comparative Tables XIII (for case 1) and XIV (for case 2) in the subsection V-E.

### B. Implementation of the CBA method

The CBA steps are applied to the two  $4 \times 5$  and  $5 \times 5$  score matrices  $\mathbf{S}_{\text{case1}}$  and  $\mathbf{S}_{\text{case2}}$  given in the Table II. For each alternative  $A_i$  and each criterion  $C_j$ , the score  $S_{ij}$  must be

TABLE II  
SCORE MATRICES  $\mathbf{S}_{\text{case1}}$  (WITHOUT  $A_5$ ) AND  $\mathbf{S}_{\text{case2}}$  (WITH  $A_5$ ).

|                             |                             | $C_j$     | $C_1$   | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|-----------------------------|-----------------------------|-----------|---------|-------|-------|-------|-------|
|                             |                             | $w_j$     | 0.08    | 0.04  | 0.10  | 0.46  | 0.32  |
| $\mathbf{S}_{\text{case2}}$ | $\mathbf{S}_{\text{case1}}$ | $A_1$     | 0       | 0     | 0     | 0     | 0     |
|                             |                             | $A_2$     | 300 000 | 6 000 | 5     | 0.007 | 0.02  |
|                             |                             | $A_3$     | 300 000 | 1 500 | 5     | 0.008 | 0.04  |
|                             |                             | $A_4$     | 600 000 | 7 500 | 7     | 0.008 | 0.05  |
|                             | $A_5$                       | 1 000 000 | 0       | 7     | 0.008 | 0.1   |       |

transformed into monetary value  $S_{ij}^\epsilon$ . For  $C_3$ , we assume an average price of 2 300 €/m<sup>2</sup> for houses built in any of the eleven French mountainous departments<sup>9</sup>:  $S_{i3}^\epsilon = 2\,300 \cdot S_{i3}$  ( $S_{i3}$  given in the Table II). As transforming  $C_5$  in monetary value is a difficult process, it is not generally done in practice: we assume  $S_{i5}^\epsilon = 0$ . Practical monetary valuation of human life is discussed and we extract two methods:

- **Method 1**: with no human life monetary valuation for  $C_4$ , one assumes  $S_{i4}^\epsilon = 0$ ;
- **Method 2**: considering 2.5 million € as the average price of one human life [23], one assumes  $S_{i4}^\epsilon = 2\,500\,000 \cdot S_{i4}$  ( $S_{i4}$  given in the Table II).

To apply the CBA, the two  $4 \times 5$  and  $5 \times 5$  score matrices  $\mathbf{S}_{\text{case1}}$  and  $\mathbf{S}_{\text{case2}}$  are transformed into two  $4 \times 3$  and  $5 \times 3$  score matrices  $\mathbf{S}_{\text{case1}}^\epsilon$  and  $\mathbf{S}_{\text{case2}}^\epsilon$  in Table III. For each  $A_i$ , the  $C_3$  score of global risk reduction in € is  $S_{i3}^\epsilon + S_{i4}^\epsilon + S_{i5}^\epsilon$ .

TABLE III  
TRANSFORMED IN € SCORE MATRICES  $\mathbf{S}_{\text{case1}}^\epsilon$  AND  $\mathbf{S}_{\text{case2}}^\epsilon$ .

|                                      |                                      | $C_1$     | $C_2$   | $C_3$    |          |        |
|--------------------------------------|--------------------------------------|-----------|---------|----------|----------|--------|
|                                      |                                      |           |         | Method 1 | Method 2 |        |
| $\mathbf{S}_{\text{case2}}^\epsilon$ | $\mathbf{S}_{\text{case1}}^\epsilon$ | $A_1$     | 0       | 0        | 0        | 0      |
|                                      |                                      | $A_2$     | 300 000 | 6 000    | 11 500   | 29 000 |
|                                      |                                      | $A_3$     | 300 000 | 1 500    | 11 500   | 31 500 |
|                                      |                                      | $A_4$     | 600 000 | 7 500    | 16 100   | 36 100 |
|                                      | $A_5$                                | 1 000 000 | 0       | 16 100   | 36 100   |        |

As recommended in [24], we assume the temporal horizon  $n = 50$  years and a fixed rate  $q_p = 3\%$ ,  $\forall p \in [1, n]$ . For each  $A_i$ , applying the formulae (3) and (4) gives the CBA results in the Table IV.

TABLE IV  
NPV AND  $r$  RESULTS BY CBA.

|       | Method 1                              |       | Method 2                           |       |
|-------|---------------------------------------|-------|------------------------------------|-------|
|       | Without monetary human life valuation |       | With monetary human life valuation |       |
|       | $NPV_i$                               | $r_i$ | $NPV_i$                            | $r_i$ |
| $A_1$ | 0                                     | –     | 0                                  | –     |
| $A_2$ | -158 487                              | 0.65  | 291 784                            | 1.64  |
| $A_3$ | -42 702                               | 0.87  | 471 894                            | 2.39  |
| $A_4$ | -378 724                              | 0.52  | 135 871                            | 1.17  |
| $A_5$ | -585 751                              | 0.41  | -71 156                            | 0.93  |

$NPV$  and  $r$  give the same preference ranking. We use  $NPV$  results to establish the decision ranking vector (see Tables XIII and XIV in the subsection V-E).

<sup>9</sup>www.meilleursagents.com/prix-immobilier

### C. Implementation of the AHP method

After AHP Steps 1 and 2 (see the subsection V-A), the AHP step 3 is applied on the two score matrices  $\mathbf{S}_{\text{case1}}$  and  $\mathbf{S}_{\text{case2}}$  given in the Table II. Two methods of normalization have been tested to normalize each  $j$ -th column of  $\mathbf{S}_{\text{case1}}$  and  $\mathbf{S}_{\text{case2}}$ .

**Method 1:** For each  $C_j$ , the DM gives its preference matrix on the set of alternatives to get the normalized score matrices for case 1 and case 2 given respectively by Eqs. (24)-(25).

$$\mathbf{S}_{\text{case1}}^1 = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ A_1 & \begin{bmatrix} 0.643 & 0.576 & 0.038 & 0.037 & 0.042 \\ 0.158 & 0.071 & 0.197 & 0.219 & 0.128 \\ 0.158 & 0.308 & 0.197 & 0.372 & 0.320 \\ 0.041 & 0.045 & 0.568 & 0.372 & 0.510 \end{bmatrix} \\ A_2 \\ A_3 \\ A_4 \end{matrix} \quad (24)$$

$$\mathbf{S}_{\text{case2}}^1 = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ A_1 & \begin{bmatrix} 0.515 & 0.368 & 0.028 & 0.028 & 0.032 \\ 0.190 & 0.046 & 0.124 & 0.156 & 0.071 \\ 0.190 & 0.186 & 0.124 & 0.272 & 0.137 \\ 0.073 & 0.032 & 0.363 & 0.272 & 0.228 \\ 0.032 & 0.368 & 0.363 & 0.272 & 0.532 \end{bmatrix} \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{matrix} \quad (25)$$

**Method 2:** One uses the classical normalization procedure<sup>10</sup> and gets the normalized score matrices for case 1 and case 2 given respectively by Eqs. (26)-(27).

$$\mathbf{S}_{\text{case1}}^2 = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ A_1 & \begin{bmatrix} 1.000 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.750 & 0.600 & 0.294 & 0.304 & 0.182 \\ 0.750 & 0.900 & 0.294 & 0.348 & 0.364 \\ 0.500 & 0.500 & 0.412 & 0.348 & 0.455 \end{bmatrix} \\ A_2 \\ A_3 \\ A_4 \end{matrix} \quad (26)$$

$$\mathbf{S}_{\text{case2}}^2 = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ A_1 & \begin{bmatrix} 1.000 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.864 & 0.600 & 0.208 & 0.226 & 0.095 \\ 0.864 & 0.900 & 0.208 & 0.258 & 0.190 \\ 0.727 & 0.500 & 0.292 & 0.258 & 0.238 \\ 0.545 & 1.000 & 0.292 & 0.258 & 0.476 \end{bmatrix} \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{matrix} \quad (27)$$

For those four normalized score matrices, the ranking results  $S_i$  given by the formula (6) of AHP step 4 are gathered in the Tables XIII (case 1) and XIV (case 2), see the subsection V-E.

### D. Implementation of the BF-TOPSIS methods

The decreasing preference according to  $C_1$  and  $C_2$  must be first taken into account. The corresponding initial scores  $S_{i1}$  and  $S_{i2}$ ,  $i = 1, \dots, 5$  given in the Table II are thus multiplied by -1 to get the corresponding  $4 \times 5$  (case 1) and  $5 \times 5$  (case 2) new score matrices  $\mathbf{S}_{\text{case1}}^{\text{pref}}$  and  $\mathbf{S}_{\text{case2}}^{\text{pref}}$  in the Table V.

The BBA construction step of all BF-TOPSIS methods described in the subsection IV-A from these two matrices gives us the following BBA matrices shown in Tables VI (for case 1) and VII (for case 2).

**BF-TOPSIS1 results:** for each  $A_i$ , the weighted averages of  $d_{BI}^E(m_{ij}, m_{ij}^{\text{best}})$  and  $d_{BI}^E(m_{ij}, m_{ij}^{\text{worst}})$ ,  $j = 1, \dots, 5$  are computed with Eqs. (18) and (19). The results are gathered for all  $A_i$ ,  $i = 1, \dots, 4$  (case 1) and  $i = 1, \dots, 5$  (case 2) in the Table VIII. For each  $A_i$ , applying Eq. (20) gives

<sup>10</sup>by dividing each  $S_{ij}$  value by  $\sum_j S_{ij}$ .

TABLE V  
SCORE MATRICES  $\mathbf{S}_{\text{CASE1}}^{\text{pref}}$  (WITHOUT  $A_5$ ) AND  $\mathbf{S}_{\text{CASE2}}^{\text{pref}}$  (WITH  $A_5$ ) TAKING INTO ACCOUNT THE PREFERENCE DIRECTION FOR EACH CRITERION  $C_j$ .

|   |   | $C_j$      | $C_1$    | $C_2$  | $C_3$ | $C_4$ | $C_5$ |
|---|---|------------|----------|--------|-------|-------|-------|
|   |   | $w_j$      | 0.08     | 0.04   | 0.10  | 0.46  | 0.32  |
| $\mathbf{S}_{\text{case2}}^{\text{pref}}$ | $\mathbf{S}_{\text{case1}}^{\text{pref}}$ | $A_1$      | 0        | 0      | 0     | 0     | 0     |
|   |   | $A_2$      | -300 000 | -6 000 | 5     | 0.007 | 0.02  |
|   |   | $A_3$      | -300 000 | -1 500 | 5     | 0.008 | 0.04  |
|   |   | $A_4$      | -600 000 | -7 500 | 7     | 0.008 | 0.05  |
|   | $A_5$                                     | -1,000,000 | 0        | 7      | 0.008 | 0.1   |       |

TABLE VI  
BBA MATRIX IN CASE 1 (WITHOUT  $A_5$ ).

|                            | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|----------------------------|-------|-------|-------|-------|-------|
| $m(A_1)$                   | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 |
| $m(A_2 \cup A_3 \cup A_4)$ | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| $m(\Theta)$                | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $m(A_2)$                   | 0.250 | 0.100 | 0.454 | 0.778 | 0.222 |
| $m(A_1 \cup A_3 \cup A_4)$ | 0.250 | 0.700 | 0.118 | 0.087 | 0.455 |
| $m(\Theta)$                | 0.500 | 0.200 | 0.428 | 0.135 | 0.323 |
| $m(A_3)$                   | 0.250 | 0.700 | 0.454 | 1.000 | 0.667 |
| $m(A_1 \cup A_2 \cup A_4)$ | 0.250 | 0.100 | 0.118 | 0.000 | 0.091 |
| $m(\Theta)$                | 0.500 | 0.200 | 0.428 | 0.000 | 0.242 |
| $m(A_4)$                   | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| $m(A_1 \cup A_2 \cup A_3)$ | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 |
| $m(\Theta)$                | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

TABLE VII  
BBA MATRIX IN CASE 2 (WITH  $A_5$ ).

|                                     | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|-------------------------------------|-------|-------|-------|-------|-------|
| $m(A_1)$                            | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 |
| $m(A_2 \cup A_3 \cup A_4 \cup A_5)$ | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| $m(\Theta)$                         | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $m(A_2)$                            | 0.455 | 0.100 | 0.454 | 0.778 | 0.069 |
| $m(A_1 \cup A_3 \cup A_4 \cup A_5)$ | 0.107 | 0.733 | 0.167 | 0.097 | 0.619 |
| $m(\Theta)$                         | 0.438 | 0.167 | 0.379 | 0.125 | 0.312 |
| $m(A_3)$                            | 0.455 | 0.700 | 0.454 | 1.000 | 0.207 |
| $m(A_1 \cup A_2 \cup A_4 \cup A_5)$ | 0.107 | 0.733 | 0.167 | 0.000 | 0.333 |
| $m(\Theta)$                         | 0.438 | 0.167 | 0.379 | 0.000 | 0.460 |
| $m(A_4)$                            | 0.182 | 0.000 | 1.000 | 1.000 | 0.310 |
| $m(A_1 \cup A_2 \cup A_3 \cup A_5)$ | 0.428 | 1.000 | 0.000 | 0.000 | 0.238 |
| $m(\Theta)$                         | 0.390 | 0.000 | 0.000 | 0.000 | 0.452 |
| $m(A_5)$                            | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $m(A_1 \cup A_2 \cup A_3 \cup A_5)$ | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $m(\Theta)$                         | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

the relative closeness  $C(A_i, A^{\text{best}})$ . The ranking vectors are deduced and shown in the comparative Tables XIII and XIV of the subsection V-E.

TABLE VIII  
BF-TOPSIS1: RESULTS OF WEIGHTED AVERAGE DISTANCE IN CASE 1 (WITHOUT  $A_5$ ) AND CASE 2 (WITH  $A_5$ ).

| $A_i$ | case 1                 |                         | case 2                 |                         |
|-------|------------------------|-------------------------|------------------------|-------------------------|
|       | $d^{\text{best}}(A_i)$ | $d^{\text{worst}}(A_i)$ | $d^{\text{best}}(A_i)$ | $d^{\text{worst}}(A_i)$ |
| $A_1$ | 0.762                  | 0.104                   | 0.741                  | 0.101                   |
| $A_2$ | 0.403                  | 0.477                   | 0.435                  | 0.417                   |
| $A_3$ | 0.183                  | 0.693                   | 0.293                  | 0.559                   |
| $A_4$ | 0.104                  | 0.762                   | 0.264                  | 0.585                   |
| $A_5$ | -                      | -                       | 0.067                  | 0.774                   |

**BF-TOPSIS2 results:** for each  $C_j$  and each  $A_i$ , the relative closeness  $C_j(A_i, A^{\text{best}})$  is computed applying the formula (21). Results are gathered for all  $C_j, j = 1, \dots, 5$  and all  $A_i, i = 1, \dots, 4$  (case 1, Table IX), and all  $A_i, i = 1, \dots, 5$  (case 2, Table X). For each  $A_i$ , the formula (22) gives the relative closeness  $C(A_i, A^{\text{best}})$ . The corresponding ranking vectors are given in the Tables XIII and XIV of the subsection V-E.

TABLE IX

BF-TOPSIS2: INTERMEDIARY  $C_j(A_i, A^{\text{BEST}})$  IN CASE 1 (WITHOUT  $A_5$ ).

|       | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|-------|-------|-------|-------|-------|-------|
| $A_1$ | 0.080 | 0.120 | 0.120 | 0.120 | 0.120 |
| $A_2$ | 0.028 | 0.034 | 0.086 | 0.543 | 0.544 |
| $A_3$ | 0.028 | 0.057 | 0.109 | 0.569 | 0.794 |
| $A_4$ | 0     | 0     | 0.100 | 0.560 | 0.880 |

TABLE X

BF-TOPSIS2: INTERMEDIARY  $C_j(A_i, A^{\text{BEST}})$  IN CASE 2 (WITH  $A_5$ ).

|       | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|-------|-------|-------|-------|-------|-------|
| $A_1$ | 0.080 | 0.120 | 0.120 | 0.120 | 0.120 |
| $A_2$ | 0.040 | 0.044 | 0.093 | 0.455 | 0.493 |
| $A_3$ | 0.040 | 0.068 | 0.117 | 0.577 | 0.660 |
| $A_4$ | 0.018 | 0.018 | 0.118 | 0.578 | 0.692 |
| $A_5$ | 0     | 0.040 | 0.140 | 0.600 | 0.920 |

**BF-TOPSIS3 results:** for each  $A_i$  and each case, the combination step through the PCR6 rule (with importance discounting) of the five BBAs  $m_{ij}(\cdot), j = 1, \dots, 5$  gives the results of the Table XI.  $d^{\text{best}}(A_i)$  and  $d^{\text{worst}}(A_i)$  are directly computed giving the Table XII. Applying Eq. (20) gives the relative closeness  $C(A_i, A^{\text{best}})$ . The corresponding ranking vectors are given in the comparative Tables XIII and XIV of the subsection V-E.

TABLE XI

BF-TOPSIS3: INTERMEDIATE RESULTS AFTER PCR6 COMBINATION WITHOUT  $A_5$  (CASE 1) AND WITH  $A_5$  (CASE 2).

|                                     | case 1              |                                   | case 2              |                                   |
|-------------------------------------|---------------------|-----------------------------------|---------------------|-----------------------------------|
|                                     | $m_i^{\text{PCR6}}$ | $m_{i,\text{norm}}^{\text{PCR6}}$ | $m_i^{\text{PCR6}}$ | $m_{i,\text{norm}}^{\text{PCR6}}$ |
| $m(\emptyset)$                      | 0.904               | 0.000                             | 0.904               | 0.000                             |
| $m(A_1)$                            | 0.003               | 0.030                             | 0.003               | 0.030                             |
| $m(A_2 \cup A_3 \cup A_4 \cup A_5)$ | 0.093               | 0.970                             | 0.093               | 0.970                             |
| $m(\Theta)$                         | 0.000               | 0.000                             | 0.000               | 0.000                             |
| $m(\emptyset)$                      | 0.945               | 0.000                             | 0.942               | 0.000                             |
| $m(A_2)$                            | 0.041               | 0.743                             | 0.039               | 0.674                             |
| $m(A_1 \cup A_3 \cup A_4 \cup A_5)$ | 0.008               | 0.145                             | 0.014               | 0.234                             |
| $m(\Theta)$                         | 0.006               | 0.112                             | 0.005               | 0.092                             |
| $m(\emptyset)$                      | 0.920               | 0.000                             | 0.924               | 0.000                             |
| $m(A_3)$                            | 0.076               | 0.954                             | 0.064               | 0.847                             |
| $m(A_1 \cup A_2 \cup A_4 \cup A_5)$ | 0.001               | 0.006                             | 0.004               | 0.050                             |
| $m(\Theta)$                         | 0.003               | 0.040                             | 0.008               | 0.103                             |
| $m(\emptyset)$                      | 0.904               | 0.000                             | 0.922               | 0.000                             |
| $m(A_4)$                            | 0.093               | 0.970                             | 0.068               | 0.873                             |
| $m(A_1 \cup A_2 \cup A_3 \cup A_5)$ | 0.003               | 0.030                             | 0.003               | 0.038                             |
| $m(\Theta)$                         | 0.030               | 0.000                             | 0.007               | 0.089                             |
| $m(\emptyset)$                      | -                   | -                                 | 0.904               | 0.000                             |
| $m(A_5)$                            | -                   | -                                 | 0.094               | 0.977                             |
| $m(A_1 \cup A_2 \cup A_3 \cup A_5)$ | -                   | -                                 | 0.002               | 0.023                             |
| $m(\Theta)$                         | -                   | -                                 | 0.000               | 0.000                             |

TABLE XII

BF-TOPSIS3 & 4: RESULTS OF WEIGHTED AVERAGE DISTANCE IN CASE 1 (WITHOUT  $A_5$ ) AND CASE 2 (WITH  $A_5$ ).

| $A_i$ | case 1                 |                         | case 2                 |                         |
|-------|------------------------|-------------------------|------------------------|-------------------------|
|       | $d^{\text{best}}(A_i)$ | $d^{\text{worst}}(A_i)$ | $d^{\text{best}}(A_i)$ | $d^{\text{worst}}(A_i)$ |
| $A_1$ | 0.840                  | 0.026                   | 0.817                  | 0.025                   |
| $A_2$ | 0.208                  | 0.661                   | 0.268                  | 0.574                   |
| $A_3$ | 0.036                  | 0.832                   | 0.123                  | 0.720                   |
| $A_4$ | 0.026                  | 0.840                   | 0.102                  | 0.742                   |
| $A_5$ | -                      | -                       | 0.020                  | 0.822                   |

**BF-TOPSIS4 results:** As expected considering the comparative analysis made in [12], it provides the same results as with BF-TOPSIS3.

#### E. Comparing results of methods for the same real DMP

The comparative Tables XIII (case 1 - without  $A_5$ ) and XIV (case 2 - including  $A_5$ ) gather all the ranking results.

TABLE XIII

CBA, AHP & BF-TOPSIS RESULTS IN CASE 1 (WITHOUT  $A_5$ ).

| Methods   |   | Ranking vectors                    | Preference orders                   |
|-----------|---|------------------------------------|-------------------------------------|
| CBA       | 1 | $[0, -1.6, -0.4, -3.8] \cdot 10^5$ | $A_1 \succ A_3 \succ A_2 \succ A_4$ |
|           | 2 | $[0, 2.9, 4.7, 1.4] \cdot 10^5$    | $A_3 \succ A_2 \succ A_4 \succ A_1$ |
| AHP       | 1 | $[0.11, 0.18, 0.32, 0.40]$         | $A_4 \succ A_3 \succ A_2 \succ A_1$ |
|           | 2 | $[0.12, 0.31, 0.40, 0.41]$         | $A_4 \succ A_3 \succ A_2 \succ A_1$ |
| BF-TOPSIS | 1 | $[0.12, 0.54, 0.79, 0.88]$         | $A_4 \succ A_3 \succ A_2 \succ A_1$ |
|           | 2 | $[0.12, 0.54, 0.79, 0.88]$         | $A_4 \succ A_3 \succ A_2 \succ A_1$ |
|           | 3 | $[0.03, 0.76, 0.96, 0.97]$         | $A_4 \succ A_3 \succ A_2 \succ A_1$ |
|           | 4 | $[0.03, 0.76, 0.96, 0.97]$         | $A_4 \succ A_3 \succ A_2 \succ A_1$ |

TABLE XIV

CBA, AHP & BF-TOPSIS RESULTS IN CASE 2 (WITH  $A_5$ ).

| Methods   |   | Ranking vectors                          | Preference orders                             |
|-----------|---|--|---|
| CBA       | 1 | $[0, -1.6, -0.4, -3.8, -5.9] \cdot 10^5$ | $A_1 \succ A_3 \succ A_2 \succ A_4 \succ A_5$ |
|           | 2 | $[0, 2.9, 4.7, 1.4, -0.7] \cdot 10^5$    | $A_3 \succ A_2 \succ A_4 \succ A_1 \succ A_5$ |
| AHP       | 1 | $[0.07, 0.12, 0.20, 0.24, 0.35]$         | $A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$ |
|           | 2 | $[0.12, 0.25, 0.31, 0.30, 0.39]$         | $A_5 \succ A_3 \succ A_4 \succ A_2 \succ A_1$ |
| BF-TOPSIS | 1 | $[0.12, 0.49, 0.66, 0.69, 0.92]$         | $A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$ |
|           | 2 | $[0.12, 0.49, 0.66, 0.69, 0.92]$         | $A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$ |
|           | 3 | $[0.03, 0.68, 0.85, 0.88, 0.97]$         | $A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$ |
|           | 4 | $[0.03, 0.68, 0.85, 0.88, 0.97]$         | $A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$ |

Even if there is no rank reversal problem while comparing CBA results in cases 1 and 2, the CBA method is very sensitive to monetary valuation with complete changes of preference order if one monetary values the human life (Method 2) or not (Method 1).

Taking into account the environmental criterion  $C_5$  and the importance of each criterion  $C_j$ , the AHP changes the decision comparing to the CBA results. With the latter, the best choice is  $A_1$  (CBA method 1) or  $A_3$  (CBA method 2) whereas it is  $A_5$  or  $A_4$  including (case 2) or not (case 1)  $A_5$  into the AHP process. For this example, the AHP method 1 (normalization using AHP preference matrices) is more robust to the rank reversal problem than using the AHP normalization method

2. For the latter, the preference order is  $A_4 \succ A_3$  for case 1 (without  $A_5$ ) switching in  $A_3 \succ A_4$  for case 2 (including  $A_5$ ).

As the AHP, the BF-TOPSIS methods help to take into account all kind of quantitative criteria without monetary valuation step. For this example, they are robust to the rank reversal problem. Moreover, in comparison with the AHP method 1, there is no preference transformation which can be subjective and more complicated to implement.

## VI. CONCLUSION

To help choosing the best action to implement within a torrential watershed, the most used method in practice is the CBA. Assuming a precise classical MCDM problem, we have shown in this paper that the CBA only takes into account few decision criteria and therefore does not provide fully satisfactory results. Classical MCDA methods such as the AHP help to improve it. Nevertheless, for quantitative DMP where all scores correspond to numerical values, we show that a direct normalization step induces rank reversal problems. We also show that the new BF-TOPSIS methods are more robust to rank reversal for this DMP. Moreover, they are easier to implement than the classical AHP involving preference matrices step.

In this application context, precise score values are rarely available in actual DMP. Some methods, such as the ER-MCDA, have been developed to assess each alternative  $A_i$  through a  $m_i(\cdot)$  on a FoD which corresponds to an evaluation scale [16], [25]. They have yet been applied to similar actual cases, to assess effectiveness of torrential protection works [26]. To go further in application to actual cases, some developments are currently considered to integrate some steps of the BF-TOPSIS methods following the ER-MCDA methodology.

In the same way, some decision-making methods under uncertainty such as COWA-ER and Fuzzy COWA-ER exist and have been applied to similar actual cases [26]. Some challenging developments are needed at all to integrate those approaches taking into account several sources of imperfect information under high uncertainty on the states of the nature.

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## REFERENCES

- [1] N. Hanley and C. L. Spash, *Cost-Benefit Analysis and the Environment*. Edward Elgar, 1993.
- [2] R. A. Kramer, *Advantages and limitations of benefit-cost analysis for evaluating investments in natural disaster mitigation*. The World Bank, 1995, ch. 4, pp. 61–76.
- [3] M. Bründl, M. C. Mc Alpin, U. Gruber, and S. Fuchs, “Cost-benefit analysis of measures for avalanche risk reduction - a case study from davos,” in *RISK21 - Coping with Risks due to Natural Hazards in the 21st Century*, W. J. Ammann, S. Sannenmann, and L. Vulliet, Eds. Taylor & Francis Group, 2005, 255 p.
- [4] S. Fuchs and M. C. Mc Alpin, “The net benefit of public expenditures on avalanche defence structures in the municipality of Davos, Switzerland.” *Natural Hazards and Earth System Sciences*, vol. 5, pp. 319–330, 2005.
- [5] C. Gamper, M. Thöni, and H. Weck-Hannemann, “A conceptual approach to the use of Cost Benefit and Multi Criteria Analysis in natural hazard management,” *Natural Hazards and Earth System Science*, vol. 6, no. 2, pp. 293–302, 2006.
- [6] T. Saaty, *The analytic hierarchy process*. New York: McGraw-Hill, 1980.
- [7] R. L. Keeney and H. Raiffa, *Decisions with multiple objectives: preferences and value tradeoffs*. New York: John Wiley & Sons, 1976.
- [8] B. Roy, *Méthodologie multicritère d'aide à la décision*. Paris: Economica, 1985.
- [9] H. Yin, J. Lan, and X.-R. Li, “Measures for ranking estimation performance based on single or multiple performance metrics,” in *Proc. of 16th Int. Conf. on Information Fusion*, Istanbul, Turkey, July 9-12 2013, pp. 453–460.
- [10] Y. Lai, T. Liu, and C. Hwang, “TOPSIS for MODM,” *European Journal of Operational Research*, vol. 76, no. 3, pp. 486–500, 1994.
- [11] G. Shafer, *A Mathematical Theory of Evidence*. Princeton Univ. Press, 1976.
- [12] J. Dezert, D. Han, and H. Yin, “A new belief function based approach for multi-criteria decision-making support,” in *Proc. of 19th Int. Conf. on Information Fusion*, Heidelberg, Germany, July 5-8 2016.
- [13] E. Triantaphyllou and K. Baig, “The impact of aggregating benefit and cost criteria in four MCDA methods,” *Engineering Management, IEEE Transactions on*, vol. 52, no. 2, pp. 213–226, 2005.
- [14] N. Eckert, C. Keylock, D. Bertrand, E. Parent, T. Faug, P. Favier, and M. Naaim, “Quantitative risk and optimal design approaches in the snow avalanche field: review and extensions,” *Cold Regions Science and Technology*, vol. 79-80, pp. 1–19, 2012.
- [15] J. Barzilai and B. Golany, “AHP rank reversal, normalization and aggregation rules,” *INFOR*, vol. 32, no. 2, pp. 57–63, 1994.
- [16] J.-M. Tacnet, “Prise en compte de l'incertitude dans l'expertise des risques naturels en montagne par analyse multicritères et fusion d'information.” Ph.D. dissertation, Ecole Nationale Supérieure des Mines de Saint-Etienne, France, 2009.
- [17] G. Miller, “The magical number seven, plus or minus two: some limits on our capacity for processing information,” *Psychological Review*, vol. 101, no. 2, pp. 343–352, 1994.
- [18] L. A. Zadeh, *On the validity of Dempster's rule of combination*, Memo M79/24, Univ. of California, Berkeley, U.S.A., 1979.
- [19] F. Smarandache and J. Dezert, *Advances and applications of DSMT for information fusion - Collected works, vol. 1-4*. American Research Press, 2004, 2006, 2009, 2015.
- [20] D. Han, J. Dezert, and Y. Yang, “New distance measures of evidence based on belief intervals,” in *Proc. of 3rd Belief Int. Conf.*, Oxford, UK, Sept. 26-29 2014, pp. 432–441.
- [21] F. Smarandache, J. Dezert, and J.-M. Tacnet, “Fusion of sources of evidence with different importances and reliabilities,” in *Proc. of 13th Int. Conf. on Information Fusion*, Edinburgh, Scotland, UK, July 26-29 2010.
- [22] F. Smarandache and J. Dezert, “Modified PCR rules of combination with degrees of intersections,” in *Proc. of 18th Int. Conf. on Information Fusion*, Washington D.C, USA, July 6-9 2015, pp. 2100–2107.
- [23] V. Meunier and E. Mardsen, “L'analyse coût-bénéfices. Guide méthodologique.” Institut pour une culture de sécurité industrielle, Tech. Rep., 2009, (p.9).
- [24] K. Erdlenbruch, E. Gilbert, F. Grelot, and C. Lescoulier, “Une analyse coût-bénéfice spatialisée de la protection contre les inondations - Application de la méthode des dommages évités à la basse vallée de l'Orb.” *Ingénieries*, vol. 53, pp. 3–20, 2008.
- [25] J. Dezert and J.-M. Tacnet, “Evidential reasoning for multi-criteria analysis based on DSMT-AHP,” in *Proc. of 11th ISAHP Int. Conf.*, Sorrento, Italy, June 15-18 2011.
- [26] S. Carladou, J.-M. Tacnet, J. Dezert, and M. Batton-Hubert, “Belief function theory based decision support methods: Application to torrent protection work effectiveness and reliability assessment,” in *Proc. of 25th ESREL Int. Conf.*, Zürich, Switzerland, Sept. 7-10 2015, pp. 3643–3652.