Full-dimension Attitude Determination Based on Two-antenna GPS/SINS Integrated Navigation System

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***Abstract*—A practical method for full-dimension attitude determination based on the combination of two-antenna global positioning system (GPS) and strapdown inertial navigation system (SINS) is presented. In view of the fact that two-antenna GPS can only provide two attitude angles using carrier phase difference measurements, not all of the SINS attitude errors can be directly corrected by the integrated navigation system. The proposed method makes use of the coordinate transformation about the attitude and the related measurements provided by two-antenna GPS and SINS to solve the full-dimension attitude determination problem through information fusion. The simulation results indicate that the proposed method can effectively determine the required attitude in real time.**

Keyword: Attitude determination, two-antenna GPS, integrated navigation system, sensor fusion.

I. Introduction

Navigation plays an important role in both military and civilian applications. It can provide required information for guidance and control of systems [1]. Researchers have made significant progress in navigation technology over the past few decades, proposing various methods of position and attitude determination. As probably the most common method, strapdown inertial navigation system (SINS) can output full-dimension attitude, velocity and position in real time at a high data rate. On the other hand, SINS cannot maintain high accuracy during long-term running and especially maneuvers, due to the error accumulation [2]. Small errors in the measurements from the accelerometers and gyroscopes accumulate to gradually larger errors in the attitude, velocity and position through the integral operation. Therefore, it is necessary to aid SINS with other navigation systems.

Integrated navigation system is a well-studied problem. The most widely used solution is to combine global positioning system (GPS) and SINS. GPS has characteristics such as the stable accuracy and low data rate [3]. The integrated navigation system exploits the complementarity between GPS and SINS to improve the navigation performance based on information fusion [4]. The researches of GPS/SINS integrated navigation system are mainly focused on the use of single-antenna GPS, where the velocity and position errors of SINS can be well corrected while the divergence of the attitude errors still exists [5]. However, since used for the determination of other states, the attitude has much importance in practical applications, especially in the areas of aviation and aerospace, where the attitude variations are more complex. Attitude determination methods based on multi-antenna carrier phase differential GPS aim to determine the attitude of the concerned moving vehicle. Ref. [6][7] show that the full-dimension attitude can be accurately determined by three-antenna GPS. Thus, it makes sense to correct the SINS attitude errors with three-antenna GPS. Compared with single-antenna GPS/SINS integrated navigation system, the integration of three-antenna GPS and SINS greatly improves the attitude accuracy, but it consequently incurs complex structure, much space occupation, difficult installation and high cost, which limit its applications on small vehicles [8][9]. It is worth noting that two-antenna GPS can still determine two attitude angles despite giving up the other one, and single baseline composed of two antennas is easier to deploy. Therefore, two-antenna GPS/SINS integrated navigation system is a compromise between cost and performance. It has been studied and applied in many cases, where the problem is how to determine all attitude angles accurately. The method in Ref. [10] estimates all attitude errors using the equivalent baseline derived from the SINS as the measurement. The method in Ref. [11] uses the difference between the accelerations derived from GPS and SINS as the measurement to estimate the attitude directly, but can only be applied to land vehicles. The method in Ref. [12] uses two-antenna real-time kinematic GPS, so the application is limited by the requirement of a GPS base station. These methods all introduce nonlinearity into the system models. It consequently causes linearization errors and increases the computational complexity.

This paper proposes a full-dimension attitude determination method based on two-antenna GPS/SINS integrated navigation system. The errors of two SINS attitude angles are directly corrected with the corresponding attitudes provided by two-antenna GPS through filtering. Inspired by the coordinate transformation between the body frame and the local frame, the proposed method makes full use of the coordinate relation between the accelerations to determine the other one attitude angle. The simulation results indicate that the proposed method is rational and effective.

II. Attitude Determination in Navigation Systems

A brief description of the concepts, definitions and principles used in the attitude determination is presented in this section.

A. Attitude Representation

For the attitude determination, two coordinate frames should be defined first.

Body coordinate frame (Ox_b, y_b, z_b): The origin is located at the center of the vehicle, the x_b axis points to the right of the body, the y_b axis points to the nose and the z_b axis is orthogonal to these two axes to form a right-handed coordinate frame.

Local coordinate frame (Ox_n, y_n, z_n): A right-handed coordinate system with the origin located at the same position as body frame, the x_n axis pointing to the East, the y_n axis pointing to the North and the z_n axis pointing upward.

The attitude of the vehicle describes the spatial orientation of the body frame with respect to the local frame. A general attitude representation is Euler angles, which consist of the yaw, pitch and roll angles. The three components are defined as the rotation angles around the body frame z_b axis, x_b axis and y_b axis, respectively. Different orders in which the rotations are performed can lead to different coordinate transformations with the same set of Euler angles due to the non-commutativity of finite rotations.

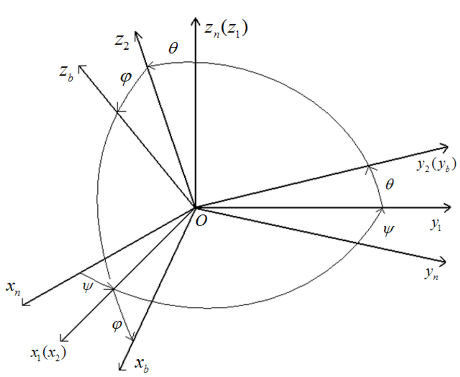

Fig. 1. Coordinate transformation process of Euler angles.

Here, the rotations are performed in the order of yaw-pitch-roll. As shown in Fig. 1, the coordinate transformation process can be expressed as

$$Ox_n, y_n, z_n \xrightarrow{\psi} Ox_1, y_1, z_1 \xrightarrow{\theta} Ox_2, y_2, z_2 \xrightarrow{\varphi} Ox_b, y_b, z_b$$

where Ox_1, y_1, z_1 and Ox_2, y_2, z_2 are intermediate frames; ψ, θ and φ denote the yaw, pitch and roll angles, respectively. The matrix form of the coordinate transformation from Ox_n, y_n, z_n to Ox_b, y_b, z_b is

$$\alpha^b = C_\varphi C_\theta C_\psi \alpha^n \tag{1}$$

where α^b is the coordinate in Ox_b, y_b, z_b and α^n is the counterpart in Ox_n, y_n, z_n; C_ψ, C_θ and C_φ are the rotation matrices with the yaw, pitch and roll angles, respectively. They are given [2] by

$$C_\psi = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$C_\theta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \tag{3}$$

$$C_\varphi = \begin{bmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ \sin\varphi & 0 & \cos\varphi \end{bmatrix} \tag{4}$$

In view of Eq. (2), (3) and (4), the attitude matrix determined by Euler angles in this order is

$$\begin{aligned} C_n^b &= C_\varphi C_\theta C_\psi \\ &= \begin{bmatrix} c_\varphi c_\psi - s_\varphi s_\theta s_\psi & c_\varphi s_\psi + s_\varphi s_\theta c_\psi & -s_\varphi c_\theta \\ -c_\theta s_\psi & c_\theta c_\psi & s_\theta \\ s_\varphi c_\psi + c_\varphi s_\theta s_\psi & s_\varphi s_\psi - c_\varphi s_\theta c_\psi & c_\varphi c_\theta \end{bmatrix} \end{aligned} \tag{5}$$

with the property $C_b^n = (C_n^b)^T$, where s_ψ, s_θ, s_φ, c_ψ, c_θ and c_φ denote $\sin(\psi)$, $\sin(\theta)$, $\sin(\varphi)$, $\cos(\psi)$, $\cos(\theta)$ and $\cos(\varphi)$, respectively.

B. SINS Attitude Determination

SINS uses gyroscopes to determine the attitude. Three gyroscopes installed as a translation of the body frame can provide the angular rates or increments of body rotations, and the attitude can be estimated through different algorithms based on the output forms. The quaternion algorithm is generally used for angular rate gyroscopes [13]. The attitude update process is described by

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{6}$$

where $Q = [\ q_0 \quad q_1 \quad q_2 \quad q_3\ ]^T$ is the quaternion representing the rotation of the body frame with respect to the local frame; $\omega = [\ \omega_x \quad \omega_y \quad \omega_z\ ]^T$ is the angular rate of the same rotation as Q, which can be derived from the gyroscope outputs. Given the one-to-one correspondence between Q and C_n^b, the attitude can be determined once the quaternion is updated by Eq. (6).

C. Two-antenna GPS Attitude Determination

The attitude determination system based on GPS requires the use of multiple antennas. Attitude angles can be calculated from the relative position of the antennas. Therefore, GPS carrier phase measurement must be used in this case, since the key is to determine the relative position accurately.

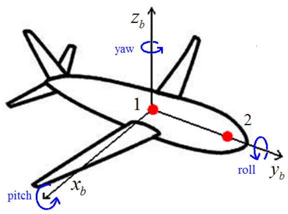

Fig. 2. Configuration of two antennas represented by the two red dots.

Suppose that two antennas are installed on the vehicle, and the configuration of the antennas is shown in Fig. 2. Antenna 1 and 2 are fixed along the y_b axis with known baseline length. The carrier phase differences that eliminate the common measurement errors of the two antennas can be described [14] by

$$\Delta p_{1,2}^{j} = \frac{1}{\lambda}\left(S^{j} \cdot b_{1,2}^{e}\right) - N_{1,2}^{j} \tag{7}$$

where the antennas receive from the same satellite j; $\Delta p_{1,2}^{j}$ is the carrier phase difference; λ is the carrier wavelength; S^j is the unit vector from the antennas to the satellite; $b_{1,2}^{e}$ is the baseline vector from antenna 1 to 2; $N_{1,2}^{j}$ is the difference between the integer ambiguities. Once the integer ambiguities are determined [15][16], the baseline vector in earth-centered earth-fixed (ECEF) coordinate frame can be obtained. Then the baseline vector in the local frame is calculated by

$$b_{1,2}^{n} = C_e^n b_{1,2}^{e} \tag{8}$$

with

$$C_e^n = \begin{bmatrix} -\sin\phi & \cos\phi & 0 \\ -\sin L\cos\phi & -\sin L\sin\phi & \cos L \\ \cos L\cos\phi & \cos L\sin\phi & \sin L \end{bmatrix} \tag{9}$$

where L and ϕ denote the latitude and longitude of the vehicle. Let $b_{1,2}^{n} = (x, y, z)$. According to the geometrical relationship shown in Fig. 1, the yaw and pitch angles can be directly calculated by

$$\psi = -\arctan\frac{x}{y} \tag{10}$$

$$\theta = \arctan\frac{z}{\sqrt{x^2 + x^2}} \tag{11}$$

Applying the error propagation rule, the standard deviations of the yaw and pitch angles can be derived [17] as

$$\sigma_\psi = \sqrt{\sigma_x^2\cos^2\psi + \sigma_y^2\sin^2\psi}/l\cos\theta \tag{12}$$

$$\sigma_\theta = \sqrt{\sigma_z^2\cos^2\theta + \sigma_x^2\sin^2\theta\cos^2\psi + \sigma_y^2\sin^2\theta sin^2\psi}/l \tag{13}$$

where σ_x, σ_y, σ_z are the standard deviations of the baseline vector components and l is the baseline length. Assume that the errors of baseline vector components are independent identically distributed. Eq. (12) and (13) become

$$\sigma_\psi = \sigma/l\cos\theta \tag{14}$$

$$\sigma_\theta = \sigma/l \tag{15}$$

with $\sigma = \sigma_x = \sigma_y = \sigma_z$. Eq. (14) and (15) indicate that the attitude accuracy is proportional to the baseline length while inversely proportional to the position error. Moreover, the pitch angle has influence on the yaw angle accuracy.

D. Two-antenna GPS/SINS Integrated Navigation System

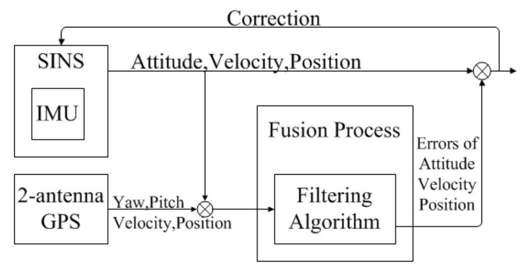

Fig. 3. Two-antennas GPS/SINS integrated navigation system.

The system structure is shown in Fig. 3. A state space model is established for the filtering algorithm. The SINS errors are chosen as the system states, which can be expressed as

$$\begin{aligned} X(t) = [&\beta_E \quad \beta_N \quad \beta_U \quad \delta v_E \quad \delta v_N \quad \delta v_U \quad \delta L \quad \delta\phi \\ &\delta h \quad \varepsilon_x \quad \varepsilon_y \quad \varepsilon_z \quad \nabla_x \quad \nabla_y \quad \nabla_z]^T \end{aligned} \tag{16}$$

where β_E, β_N and β_U are angle errors of the local frame; δv_E, δv_N and δv_U are velocity errors; δL, $\delta\phi$ and δh are latitude, longitude and height errors; ε_x, ε_y, ε_z and ∇_x, ∇_y, ∇_z are gyroscope and accelerometer errors, respectively. Given the SINS error model [2], the state equation is expressed as

$$\dot{X}(t) = F(t)X(t) + G(t)W(t) \tag{17}$$

where $F(t)$ and $G(t)$ can be derived from the SINS error model, and

$$W(t) = [\omega_{gx} \quad \omega_{gy} \quad \omega_{gz} \quad \omega_{ax} \quad \omega_{ay} \quad \omega_{az}]^T \tag{18}$$

where ω_{gx}, ω_{gy}, ω_{gz} and ω_{gx}, ω_{gy}, ω_{gz} are gyroscope and accelerometer measurement noises, respectively.

Suppose that SINS and GPS are loosely coupled, the differences between the outputs of SINS and GPS are chosen as the measurements. Let subscripts I and G denote SINS and GPS, respectively. The attitude measurement is expressed as

$$Z_a(t) = \begin{bmatrix} \psi_I - \psi_G \\ \theta_I - \theta_G \end{bmatrix} = H_a(t)X(t) + V_a(t) \tag{19}$$

with

$$H_a(t) = \left[\begin{array}{cc} C_a & 0_{2\times 12} \end{array} \right] \tag{20}$$

$$V_a(t) = \left[\begin{array}{cc} \delta\psi_G & \delta\theta_G \end{array} \right]^T \tag{21}$$

where $\delta\psi_G$ and $\delta\theta_G$ are GPS attitude errors, and C_a is given [18] by

$$C_a = \left[\begin{array}{ccc} \sin\psi\tan\theta & -\cos\psi\tan\theta & -1 \\ -\cos\psi & -\sin\psi & 0 \end{array} \right] \tag{22}$$

The velocity measurement is expressed as

$$Z_v(t) = \left[\begin{array}{c} v_{IE} - v_{GE} \\ v_{IN} - v_{GN} \\ v_{IU} - v_{GU} \end{array} \right] = H_v(t)X(t) + V_v(t) \tag{23}$$

with

$$H_v(t) = \left[\begin{array}{ccc} 0_{3\times 3} & I_{3\times 3} & 0_{3\times 9} \end{array} \right] \tag{24}$$

$$V_v(t) = \left[\begin{array}{ccc} \delta v_{GE} & \delta v_{GN} & \delta v_{GU} \end{array} \right]^T \tag{25}$$

where δv_{GE}, δv_{GN} and δv_{GU} are GPS velocity errors. The position measurement is expressed as

$$Z_p(t) = \left[\begin{array}{c} (L_I - L_G)R_M \\ (\phi_I - \phi_G)R_N\cos L \\ h_I - h_G \end{array} \right] = H_p(t)X(t) + V_p(t) \tag{26}$$

with

$$H_p(t) = \left[\begin{array}{ccc} 0_{3\times 6} & diag(R_M \quad R_N\cos\phi \quad 1) & 0_{3\times 6} \end{array} \right]^T \tag{27}$$

$$V_p(t) = \left[\begin{array}{ccc} \delta p_{GE} & \delta p_{GN} & \delta p_{GU} \end{array} \right]^T \tag{28}$$

where R_M and R_N denote the radius of curvature in the meridian and the prime vertical, repectively; δp_{GE}, δp_{GN} and δp_{GU} are GPS position errors.

The total measurement equation composed of Eq. (19), (23) and (26) is

$$Z(t) = \left[\begin{array}{c} H_a(t) \\ H_v(t) \\ H_p(t) \end{array} \right] X(t) + \left[\begin{array}{c} V_a(t) \\ V_v(t) \\ V_p(t) \end{array} \right] \tag{29}$$

Eq. (17) and (29) are both linear. Assuming that all measurement noises are zero mean Gaussian white noise, Kalman filter can be used for the estimation of attitude angles after the system model being discretized. The output frequency of SINS is much higher than GPS, so the estimation is carried out once GPS measurements are available. Moreover, the data provided by GPS and SINS should be synchronized before making the estimation in order to reduce the time synchronization errors [19][20]. Since this paper is mainly focused on the attitude determination, all the data are assumed to be synchronized.

The attitude, velocity and position errors of SINS are estimated by Kalman filter and then the corrections are fed back to SINS. The integrated navigation system can provide the yaw, pitch, velocity and position with high accuracy. However, the roll accuracy is lower than the other two angles, because the roll error cannot be well estimated without the corresponding measurement from two-antenna GPS. Therefore, the application of two-antenna GPS/SINS is limited in some cases.

III. Full-dimension attitude determination

In view of the coordinate transformation about the attitude, a method is proposed to determine the roll angle. With the accurate velocity provided by GPS, the acceleration can be approximately deduced by

$$\dot{v}^n \approx \frac{v_{t_1}^n - v_{t_0}^n}{t_1 - t_0} \tag{30}$$

where superscript n denotes the local frame and t_0, t_1 are sampling times. According to the principle of inertial navigation [2], the acceleration equation is

$$\dot{v}^n = a^n - (2\omega_{ie}^n + \omega_{en}^n) \times v^n + g^n \tag{31}$$

where a^n is the non-gravitational acceleration, also called specific force, ω_{ie}^n is the rotation angular rate of Earth, ω_{en}^n is the rotation angular rate of the local frame with respect to ECEF frame, and g^n is gravity acceleration. ω_{ie}^n and ω_{en}^n can be calculated by

$$\omega_{ie}^n = \left[\begin{array}{c} 0 \\ \omega_{ie}\cos L \\ \omega_{ie}\sin L \end{array} \right] \tag{32}$$

$$\omega_{en}^n = \left[\begin{array}{c} -v_N/\left(R_M + h\right) \\ v_E/\left(R_N + h\right) \\ v_E\tan L/\left(R_N + h\right) \end{array} \right] \tag{33}$$

where ω_{ie} is the scalar for the rotation angular rate of Earth.

Therefore, a^n can be calculated by Eq. (31) with GPS outputs. On the other hand, the accelerometers directly measure the non-gravitational acceleration in the body frame. Given the coordinate transformation between the local frame and the body frame, the accelerations are related by

$$a^b = C_n^b a^n \tag{34}$$

where a^b is measured by inertial measurement unit (IMU). Once a^b and a^n are known, Eq. (34) turns into nonlinear equations including three unknown attitude angles.

$$\left\{ \begin{array}{c} a_x^b = a_x^n\left(c_\varphi c_\psi - s_\varphi s_\theta s_\psi\right) + a_y^n\left(c_\varphi s_\psi + s_\varphi s_\theta c_\psi\right) - a_z^n s_\varphi c_\theta \\ a_y^b = -a_x^n c_\theta s_\psi + a_y^n c_\theta c_\psi + a_z^n s_\theta \\ a_z^b = a_x^n\left(s_\varphi c_\psi + c_\varphi s_\theta s_\psi\right) + a_y^n\left(s_\varphi s_\psi - c_\varphi s_\theta c_\psi\right) + a_z^n c_\varphi c_\theta \end{array} \right. \tag{35}$$

where a_x^b, a_y^b, a_z^b and a_x^n, a_y^n, a_z^n are the components of a^b and a^n.

In fact, the yaw and pitch angles are already given by two-antenna GPS. The roll angle can be determined by solving the nonlinear equations. One can change the equations in Eq. (35) into the form of

$$f_i\left(\psi, \theta, \varphi\right) = 0 \tag{36}$$

with $i = 1, 2, 3$, and the objective function is expressed as

$$\Phi\left(\psi, \theta, \varphi\right) = \sum_{i=1}^{3} f_i^2\left(\psi, \theta, \varphi\right) \tag{37}$$

The gradient descent method is used to find a set of attitude angles that minimize the objective function, and the procedure is as follows.

1) Provide the initial value (ψ, θ, φ);

2) Calculate the objective funtion $\Phi(\psi, \theta, \varphi)$;

3) If the result satisfies

$$\Phi(\psi, \theta, \varphi) < \varepsilon \tag{38}$$

where ε is a small positive number, the solution with certain accuracy is (ψ, θ, φ) and stop the calculation. Otherwise, the calculation continues.

4) Modify the current value by

$$(\psi, \theta, \varphi) = (\psi, \theta, \varphi) - \rho \times \left(\frac{\partial \Phi}{\partial \psi}, \frac{\partial \Phi}{\partial \theta}, \frac{\partial \Phi}{\partial \varphi}\right)\Bigg|_{(\psi,\theta,\varphi)} \tag{39}$$

where

$$\rho = \Phi(\psi, \theta, \varphi) \Big/ \left[\left(\frac{\partial \Phi}{\partial \psi}\right)^2 + \left(\frac{\partial \Phi}{\partial \theta}\right)^2 + \left(\frac{\partial \Phi}{\partial \varphi}\right)^2\right]\Bigg|_{(\psi,\theta,\varphi)}$$

5) Repeat 2), 3), 4) until Eq. (38) is satisfied or the duration exceed the estimation period. If no solution is obtained within the period, the initial value can be used as the substitution.

The initial ψ and φ are given by GPS, and the initial θ is given by SINS. Moreover, the termination threshold ε has effects on the timeliness and accuracy, so it should be determined based on the calculation performance of the system. The duration of the gradient descent cannot exceed the estimation period under all circumstances.

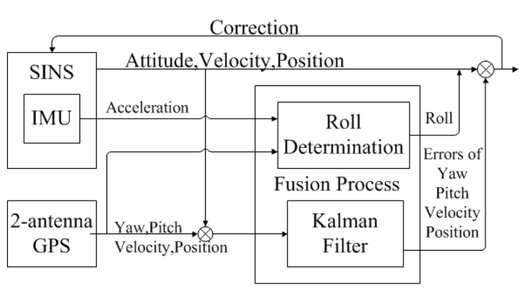

Fig. 4. Full-dimension attitude determination system.

The system structure is modified to apply the proposed method as shown in Fig. 4. The yaw, pitch, velocity and position are still corrected through Kalman filter while the roll angle is determined by the new propose method. Thus, the whole system can provide the accurate full-dimension attitude.

IV. Simulation and Analysis

In order to verify the proposed method, the simulations on the designed two-antenna GPS/SINS integrated navigation system are carried out.

Fig. 5 shows the simulation scene, where the vehicle starts moving from the asterisk, and reaches the end point marked by the circle after 1200 seconds. The integrated navigation system is composed by a two-antenna GPS receiver and a low-cost IMU. Suppose that the constant errors of IMU have been compensated after the calibration and the random errors are modeled as the first-order Markov process. The parameter settings are shown in TABLE I.

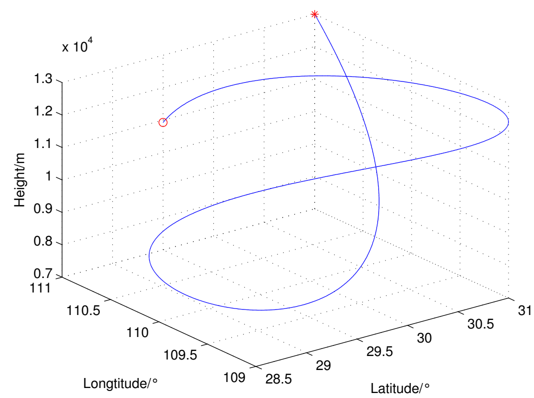

Fig. 5. Simulation scene.

TABLE I
SIMULATION PARAMETER SETTINGS.

Parameter	Value	Unit
IMU output frequency	100	Hz
gyro drift	1	$^\circ/h$
gyro drift correlation time	60	s
accelerometer error	10^{-3}	m/s^2
accelerometer error correlation time	15	s
GPS output frequency	1	Hz
GPS position error	10	m
GPS velocity error	0.2	m/s
baseline error	2	cm
two-antenna baseline length	2	m

The simulations compare the navigation performance between using and not using the proposed method under the same conditions. The results are shown in Figs. 6-8.

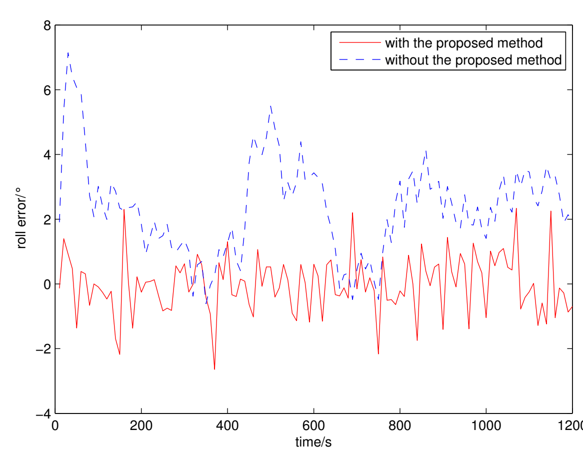

Fig. 6. The roll errors.

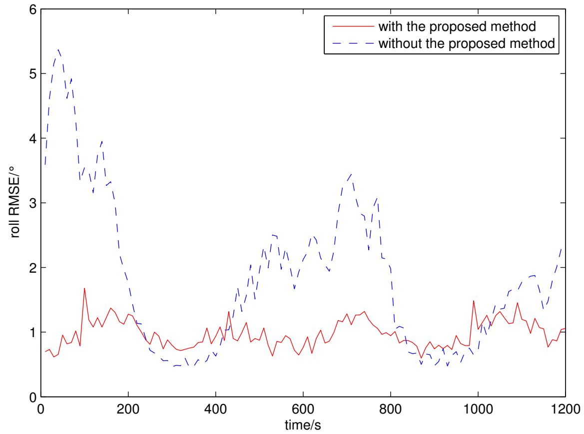

Fig. 7. RMSE of the roll angle.

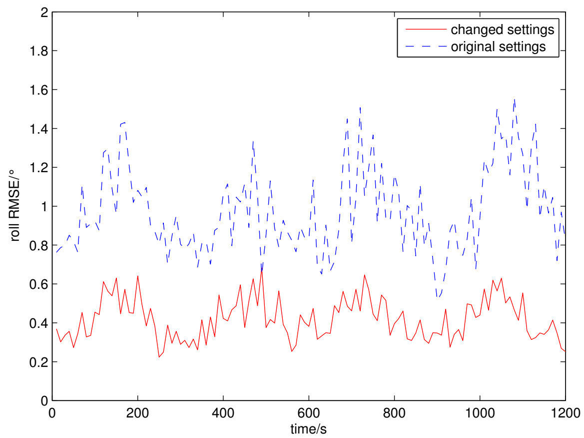

Fig. 8. RMSE of the roll angle with the different settings.

Fig. 6 shows that the proposed method can effectively estimate the roll angle and reduce the errors. In Fig. 7, the root-mean-square error (RMSE) of the roll angle are obtained by performing Monte Carlo simulation based on 100 independent runs, and the RMSE over $1200s$ is shown by the comparison 1 in TABLE II.

TABLE II
RMSE OF THE ROLL ANGLE.

	Condition	RMSE
Comparison 1	without the proposed method	2.1997°
	with the proposed method	0.9961°
Comparison 2	the original settings	0.9936°
	the changed settings	0.4319°

The results show that the proposed method can greatly improve the performance of the attitude determination based on two-antenna GPS/SINS. Given the fact that the roll angle is obtained by solving Eq. (35), the roll errors are mainly affected by a^b and a^n. a^b is directly measured by accelerometers and a^n is calculated by Eq. (31) with the GPS acceleration, velocity and position, so the roll errors are involved with the performance of GPS and accelerometers. Moreover, the gradient descent termination threshold and the initial values will influence the calculation efficiency. Fig. 8 shows the result of Monte Carlo simulation based on 100 independent runs with different settings shown in TABLE III, and the RMSE over $1200s$ is shown by the comparison 2 in TABLE II.

TABLE III
CHANGED PARAMETER SETTINGS.

Parameter	Value	Unit
accelerometer error	10^{-2}	m/s^2
GPS position error	5	m
GPS velocity error	0.1	m/s

The accuracy is significantly improved. Actually, the GPS acceleration calculated by Eq. (30) is the average acceleration over the estimation period, which is a kind of approximation. This will cause the approximation error even if GPS and accelerometers are absolutely accurate.

The results above indicate that the proposed method is rational and effective.

V. CONCLUSION

In this paper we have proposed a full-dimension attitude determination method based on the combination of two-antenna GPS and SINS. The method makes full use of the information provided by GPS and SINS to determine the full-dimension attitude. The attitude determination system has been implemented and tested through Monte Carlo simulations. We have shown in the analysis that the proposed method can effectively determine the roll angle, greatly improving the practicality of two-antenna GPS/SINS integrated navigation system. With this method, two-antenna GPS/SINS can provide accurate and real-time navigation information, achieving high performance at low cost. Future work will be carried out to reduce the approximation error in the GPS acceleration by using more accurate calculation methods. We will also see if the velocity can be used instead of the acceleration to avoid calculating the GPS acceleration. Moreover, field tests are required to further verify the proposed method on real data sets.

ACKNOWLEDGMENT

This work was supported by National Natural Science Foundation (Nos. 61573275, 61671370), the Grant for S-tate Key Program for Basic Research of China (973) (No. 2013CB329405), Science and Technology Project of Shaanxi Province (No. 2013KJXX-46), Postdoctoral Science Foundation of China (No. 2016M592790), and Fundamental Research Funds for the Central Universities (No. xjj2014122, xjj2016066).

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