Determination of Basic Belief Assignment Using Fuzzy Numbers

Zhe Zhang, Deqiang Han
Institute of Integrated Automation
School of Electronic and Information Engineering
Xi'an Jiaotong University
Xi'an, Shaanxi, China 710049
Email: zhezhang@stu.xjtu.edu.cn; deqhan@gmail.com

Jean Dezert
ONERA
The French Aerospace Lab
Chemin de la Hunière,
F-91761 Palaiseau, France
Email: jean.dezert@onera.fr

Yi Yang
SKLSVMS
School of Aerospace
Xi'an Jiaotong University
Xi'an, Shaanxi, China 710049
Email: jiafeiyy@mail.xjtu.edu.cn

***Abstract*—Dempster-Shafer evidence theory (DST) is a theoretical framework for uncertainty modeling and reasoning. The determination of basic belief assignment (BBA) is crucial in DST, however, there is no general theoretical method for BBA determination. In this paper, a method of generating BBA using fuzzy numbers is proposed. First, the training data are modeled as fuzzy numbers. Then, the dissimilarities between each test sample and the training data are measured by the distance between fuzzy numbers. In the final, the BBAs are generated from the normalized dissimilarities. The effectiveness of this method is demonstrated by an application of classification problem. Experimental results show that the proposed method is robust to outliers.**

***Keywords*—Evidence theory, basic belief assignment (BBA), fuzzy numbers, outliers.**

I. Introduction

The theory of belief functions also called Dempster-Shafer evidence theory (DST) [1], [2], is a theoretical framework for uncertainty modeling and reasoning. The expression of uncertainty, i.e., the determination of basic belief assignment (BBA) is one of the most crucial problems to deal with. BBA is a kind of random set in nature and its determination is actually the problem of modeling the distribution of random set, which is still unsolved in mathematics [3]. Therefore, the determination of BBA is a challenging problem in DST and has aroused widespread concerns.

One category for generating BBA is the application-based empirical approach. Shafer [1] generates BBA based on statistic evidence. Selzer [12] generates BBA according to the class number and the neighborhood of the target for automatic target classification. Bi [13] proposed focal element triplet for text categorization. Valente [4] proposed several BBA determination methods for speech recognition based on the membership. Zhang [5] generates BBA based on evidential Markov random field for image segmentation. Salzenstein [14] proposed an iterative estimation method to generate BBA based on the Gaussian model for multisensor image segmentation. Dezert [6] generates BBA to describe the uncertainty of threshold choosing in edge detection. Han [7] generates BBA based on the intervals of the expected payoffs for different alternatives to deal with multi-criteria decision making problems.

The another category for generating BBA is the application-free approach. Boudraa [8] proposed a method based on fuzzy membership functions. Deng [9] generates BBA based on the similarity measure described by the radius of gyration. Han [10] proposed a method based on uncertain optimization. Kang [11] proposed a method based on interval numbers.

In Kang's method [11], the training data are modeled as interval numbers determined by their lower and upper bound values. Since the interval number is a special case of the fuzzy number and only keeps minimum and maximum values, other important information, such as mean value and median, are lost when modeling the data. To deal with this, other types of fuzzy numbers are used to model the training data in this paper, i.e., the mean value and median are also kept to describe the training data. Then the BBAs are generated from the dissimilarities between the test sample and the training data using the distance between fuzzy numbers. Compared with the distance between interval numbers in Kang's method, the distance between fuzzy numbers is more robust when there exist outliers in training data. To verify the effectiveness of the proposed BBA determination method, we consider its application on the classification problem. The experiment results show that the proposed method can achieve high classification accuracies.

II. Basis of Evidence Theory

Dempster-Shafer evidence theory (DST) [1], [2] is a theoretical framework for uncertainty modeling and reasoning. In DST, the frame of discernment (FOD) Θ contains l mutually exclusive and exhaustive elements: $\Theta = \{\theta_1, \theta_2, \ldots, \theta_l\}$. The power set of Θ (the set of all subsets of Θ) is denoted by 2^Θ. The basic belief assignment (BBA, also called a mass function) m is defined from 2^Θ to $[0, 1]$ satisfying

$$\sum\nolimits_{A \subseteq \Theta} m(A) = 1\ ,\ m(\emptyset) = 0 \tag{1}$$

$m(A)$ represents the evidence support to the proposition A. If $m(A) > 0$, A is called a focal element.

The plausibility function (Pl) and belief function (Bel) are defined respectively as:

$$Pl(A) = \sum\nolimits_{A \cap B \neq \emptyset} m(B) \tag{2}$$

$$Bel(A) = \sum\nolimits_{B \subseteq A} m(B) \tag{3}$$

Dempster's rule of combination [1], used for combining two distinct sources of evidence in the DST framework, is defined as

$$m_1 \oplus m_2(A) = \begin{cases} 0, \ A = \emptyset \\ \frac{1}{1-K} \sum\limits_{B \cap C = A} m_1(B) m_2(C), \ A \neq \emptyset \end{cases} \tag{4}$$

where $K = \sum_{B \cap C = \emptyset} m_1(B) m_2(C)$ represents the total conflict or contradictory mass assignments.

For a probabilistic decision-making based on the BBA, Smets defined the pignistic probability transformation [15] to transform a BBA into a probability measure $BetP$:

$$BetP(\theta_i) \triangleq \sum\nolimits_{\theta_i \in A} \frac{m(A)}{|A|} \quad \forall \theta_i \in \Theta \tag{5}$$

where $|A|$ denotes the cardinality of A. The final decision is often made by choosing the element in FOD which has the highest $BetP$ value.

III. The determination of BBA based on interval numbers

In DST, the expression of uncertainty is the process of generating BBA. Therefore, the determination of BBA is the first step and crucial in the applications of DST. However, BBA is a kind of random set and its determination is actually the problem of modeling the distribution of random set, which is still unsolved in mathematics [3]. Kang [11] proposed a BBA determination method based on interval numbers (IN). The basis of interval numbers is briefly introduced first.

A. Basis of interval numbers

An interval number $\tilde{a}$ in $\mathbb{R}$ is a set of real numbers that lie between two real numbers, i.e., $\tilde{a} = [a_1, a_2] = \{x | a_1 \leq x \leq a_2\}$, a_1, $a_2 \in \mathbb{R}$ and $a_1 \leq a_2$.

The dissimilarity between two interval numbers $\tilde{a} = [a_1, a_2]$ and $\tilde{b} = [b_1, b_2]$ can be measured by the distance between them [16]:

$$\begin{aligned} D^2(\tilde{a}, \tilde{b}) = & \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} \left\{ \left[\left(\frac{a_1 + a_2}{2} \right) + x(a_2 - a_1) \right] \right. \\ & \left. - \left[\left(\frac{b_1 + b_2}{2} \right) + y(b_2 - b_1) \right] \right\} \mathrm{d}x \mathrm{d}y \\ = & \left[\left(\frac{a_1 + a_2}{2} \right) - \left(\frac{b_1 + b_2}{2} \right) \right]^2 \\ & + \frac{1}{3} \left[\left(\frac{a_2 - a_1}{2} \right)^2 + \left(\frac{b_2 - b_1}{2} \right)^2 \right] \end{aligned} \tag{6}$$

The larger $D(\tilde{a}, \tilde{b})$ is, the larger dissimilarity between $\tilde{a}$ and $\tilde{b}$ is.

B. IN-based method

In IN-based method, the training data belonging to the same focal element $A \subseteq \Theta$ are modeled as an interval number $\tilde{a} = [a_1, a_2]$, where a_1 and a_2 are the minimum and maximum values of the training data respectively. For a single test sample, it is also modeled as a degenerate interval number $\tilde{t} = [t, t]$, where t is its value. If the test sample $\tilde{t}$ is similar to the training data $\tilde{a}$, the corresponding proposition (the test sample belongs to A) should be assigned a large belief.

The similarity between $\tilde{a}$ and $\tilde{t}$ is defined as:

$$S(\tilde{a}, \tilde{t}) = \frac{1}{1 + \alpha D(\tilde{a}, \tilde{t})} \tag{7}$$

where $\alpha > 0$ is a parameter to control the degree of dispersion of the normalized similarities and $D(\tilde{a}, \tilde{t})$ is the distance between the interval numbers $\tilde{a}$ and $\tilde{t}$. Finally, the BBA can be generated from the normalized similarities.

In IN-based method, when modeling the training data, only the minimum and maximum values are kept and used to calculate similarities. However, when the distribution of the data is not uniform, the extreme values are insufficient to well describe the data. Actually, any interval number is a special case of a fuzzy number. Other types of fuzzy numbers, such as triangular fuzzy number (TFN) and trapezoidal fuzzy number (TrFN), can keep more useful information of the data, such as the mean value and median. Thus, TFN and TrFN are used to model the data in this paper.

IV. BBA construction from fuzzy numbers

A. Basis of fuzzy numbers

The generalized left right fuzzy number (GLRFN) $\tilde{b} = [b_1, b_2, b_3, b_4]$ is a special case of a convex, normalized fuzzy set of the real line when its membership function is defined by [17]:

$$\mu(x) = \begin{cases} L\left(\frac{b_2 - x}{b_2 - b_1}\right) & \text{for } b_1 \leq x \leq b_2 \\ 1 & \text{for } b_2 \leq x \leq b_3 \\ R\left(\frac{x - b_3}{b_4 - b_3}\right) & \text{for } b_3 \leq x \leq b_4 \\ 0 & \text{else} \end{cases} \tag{8}$$

where L and R are strictly decreasing functions defined on $[0, 1]$ and satisfy the conditions:

$$\begin{aligned} L(x) = R(x) = 1 \quad \text{if } x \leq 0, \\ L(x) = R(x) = 0 \quad \text{if } x \geq 1. \end{aligned} \tag{9}$$

The interval number is a special case of GLRFN with $b_1 = b_2$ and $b_3 = b_4$. The triangular fuzzy number (TFN) and trapezoidal fuzzy number (TrFN) [16] are two of the most common fuzzy numbers encountered in applications involving fuzzy numbers.

For TrFN, $L(x) = R(x) = 1-x$. The distance between two TrFNs $\tilde{a} = [a_1, a_2, a_3, a_4]$ and $\tilde{b} = [b_1, b_2, b_3, b_4]$ is defined as:

$$\begin{aligned}
&D^2(\tilde{a}, \tilde{b}) \\
&= \tfrac{1}{4}[(a_2 + a_3) - (b_2 + b_3)]^2 \\
&+ \tfrac{1}{4}[(a_2 + a_3) - (b_2 + b_3)] \\
&\times (a_4 - a_3 - a_2 + a_1 - b_4 + b_3 + b_2 - b_1) \\
&+ \tfrac{1}{12}(a_3 - a_2)^2 + \tfrac{1}{12}(b_3 - b_2)^2 \\
&+ \tfrac{1}{12}(a_3 - a_2)[a_4 - a_3 + a_2 - a_1] \\
&+ \tfrac{1}{12}(b_3 - b_2)[b_4 - b_3 + b_2 - b_1] \\
&+ \tfrac{1}{9}\left[(a_4 - a_3)^2 + (a_2 - a_1)^2\right] \\
&+ \tfrac{1}{9}\left[(b_4 - b_3)^2 + (b_2 - b_1)^2\right] \\
&- \tfrac{1}{9}\left[(a_2 - a_1)(a_4 - a_3) + (b_2 - b_1)(b_4 - b_3)\right] \\
&+ \tfrac{1}{6}\left[(a_4 - a_3)(b_2 - b_1) + (a_2 - a_1)(b_4 - b_3)\right] \\
&- \tfrac{1}{6}\left[(a_4 - a_3)(b_4 - b_3) + (a_2 - a_1)(b_2 - b_1)\right]
\end{aligned} \tag{10}$$

The larger $D(\tilde{a}, \tilde{b})$ is, the larger dissimilarity between $\tilde{a}$ and $\tilde{b}$ is.

For TFN, $L(x) = R(x) = 1 - x$ and $b_2 = b_3$. The distance between two TFNs $\tilde{a} = [a_1, a_2, a_3]$ and $\tilde{b} = [b_1, b_2, b_3]$ is defined as:

$$\begin{aligned}
&D^2(\tilde{a}, \tilde{b}) \\
&= (a_2 - b_2)^2 + \tfrac{1}{2}(a_2 - b_2)\left[(a_3 + a_1) - (b_3 + b_1)\right] \\
&+ \tfrac{1}{9}\left[(a_3 - a_2)^2 + (a_2 - a_1)^2\right] \\
&+ \tfrac{1}{9}\left[(b_3 - b_2)^2 + (b_2 - b_1)^2\right] \\
&- \tfrac{1}{9}\left[(a_2 - a_1)(a_3 - a_2) + (b_2 - b_1)(b_3 - b_2)\right] \\
&+ \tfrac{1}{6}(2a_2 - a_1 - a_3)(2b_2 - b_1 - b_3)
\end{aligned} \tag{11}$$

The larger $D(\tilde{a}, \tilde{b})$ is, the larger dissimilarity between $\tilde{a}$ and $\tilde{b}$ is.

B. *Fuzzy-number-based methods*

1) Data modeling: To generate BBAs, the fuzzy numbers are used to model the training data and test samples in this paper. For the training data belonging to $A \subseteq \Theta$ and the test sample t, we can use three different kinds of fuzzy numbers to model them:

(1) TFNmean: the training data are modeled as a triangular fuzzy number $\tilde{a} = [a_1, a_2, a_3]$, where a_1 and a_3 are the minimum and maximum values of the training data respectively and a_2 is the mean value. The test sample is modeled as $\tilde{t} = [t, t, t]$.

(2) TFNmed: the training data are modeled as a triangular fuzzy number $\tilde{b} = [b_1, b_2, b_3]$, where b_1 and b_3 are the minimum and maximum values of the training data respectively and b_2 is the median. The test sample is modeled as $\tilde{t} = [t, t, t]$.

(3) TrFN: the training data are modeled as a trapezoidal fuzzy number $\tilde{c} = [c_1, c_2, c_3, c_4]$, where c_1 and c_4 are the minimum and maximum values of the training data respectively, c_2 is either the mean value or median, whichever is smaller and c_3 is either the mean value or median, whichever is larger. The test sample is modeled as $\tilde{t} = [t, t, t, t]$.

In these ways, besides the maximum and minimum values, the mean value and (or) median can be also kept to describe the training data.

2) Calculate the similarities: Similar to the IN-based method, the similarity between the training data and test sample are measured from the distance between them (Eq. (11) for TFN or Eq. (10) for TrFN) using Eq. (7). Actually, other normalization functions can be used here.

3) Generate the BBAs: The BBAs are generated from the normalized similarities. If the test sample $\tilde{t}$ is similar to the training data $\tilde{a}$, the corresponding proposition ($\tilde{t}$ belongs to the same focal element with $\tilde{a}$) should be assigned with a large belief.

In the next section, we consider the classification problem to verify the effectiveness of our proposed BBA determination method.

V. Classification example based illustration of the proposed BBA determination method

We give a classification example on a set of artificial data to illustrate the process of our BBA determination method and verify its effectiveness.

A. *Artificial training data*

Suppose there are three classes in a set of artificial data: $\Theta = \{\theta_1, \theta_2, \theta_3\}$. Each sample has three features, f_1, f_2 and f_3, and each feature is correspondent to a normal distribution. The deviation parameters for each class are 0.25, 1 and 0.25 respectively and the mean parameters for each feature of each class are given in Table I.

TABLE I
THE MEAN PARAMETERS FOR EACH FEATURE OF EACH CLASS

Class	f_1	f_2	f_3
θ_1	9	5	10
θ_2	10	9	5
θ_3	5	10	9

We generate 60 training data for each class. Among the 60 samples belonging to class θ_1, there is an outlier whose value of feature f_1 is much larger than others belonging to class θ_1. The generated training data are shown in Fig. 1.

In this case, each class can be distinguished easily from other classes using one feature (when its mean parameter is 5), but are difficult distinguished from other classes using other features.

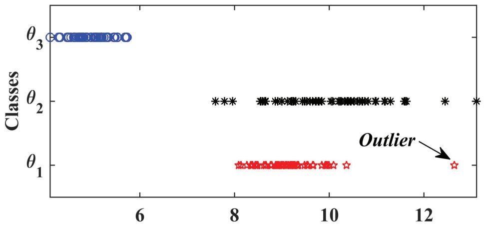
B. *The process of classification*

For a given test sample, the process of labeling its class can be outlined below:

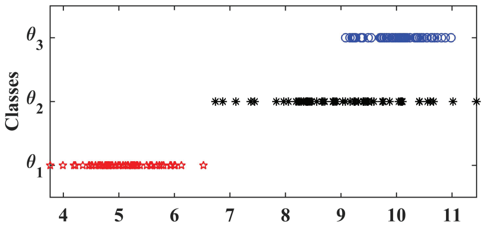
Step 1 Generate three mass functions m_1, m_2 and m_3 according to the corresponding features of the training data respectively.

TABLE II
MODELING THE TRAINING DATA ON FEATURE f_1

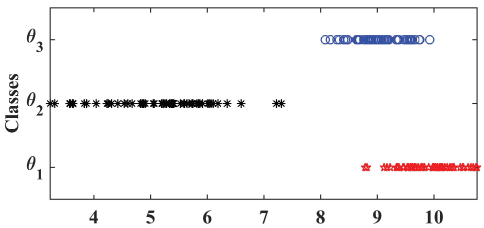
Focal element	IN	TFNmean	TFNmed	TrFN
$\{\theta_1\}$	$[8.1, 12.6]$	$[8.1, 9.1, 12.6]$	$[8.1, 9.0, 12.6]$	$[8.1, 9.0, 9.1, 12.6]$
$\{\theta_2\}$	$[7.6, 13.1]$	$[7.6, 10.0, 13.1]$	$[7.6, 10.0, 13.1]$	$[7.6, 10.0, 10.0, 13.1]$
$\{\theta_3\}$	$[4.1, 5.7]$	$[4.1, 5.0, 5.7]$	$[4.1, 4.8, 5.7]$	$[4.1, 4.8, 5.0, 5.7]$
$\{\theta_2, \theta_3\}$	$[8.1, 12.6]$	$[8.1, 9.6, 12.6]$	$[8.1, 9.3, 12.6]$	$[8.1, 9.3, 9.6, 12.6]$



(a) Values of the training data for feature f_1.



(b) Values of the training data for feature f_2.



(c) Values of the training data for feature f_3.

Fig.1. Values of the training data.

Step 2 Combine m_1, m_2 and m_3 using Eq. (4) to obtain the combined mass function m.

Step 3 Transform m into the probability measure $BetP$ using Eq. (5).

Step 4 The class of the test sample is labeled as class $\theta_i \in \Theta$ which has the highest $BetP$ value.

We take a test sample $t = (t_1, t_2, t_3) = (11.8, 9.8, 3.9)$ (whose class is θ_2) as an example to explain how to generate m_1 based on fuzzy numbers in Step 1 in detail. The result of interval-number-based method is also given for comparison.

1) Data modeling: For feature f_1, the training data belonging to each focal element $A \in \Theta$ can be modeled as an interval number (IN) or a fuzzy number (TFNmean, TFNmed or TrFN), as shown in Table II. The test sample can be modeled as $\tilde{t}_1 = [11.8, 11.8]$ (IN), $\tilde{t}_1 = [11.8, 11.8, 11.8]$ (TFNmean or TFNmed) or $\tilde{t}_1 = [11.8, 11.8, 11.8, 11.8]$ (TrFN).

In this case, the training data from class θ_1 has an overlapped region with the data from θ_2. For a test sample belonging to this region, it is difficult to distinguish whether class θ_1 or θ_2 it should be labeled as and its belief assigned to focal element $\{\theta_1, \theta_2\}$ ($m_1\{\theta_1, \theta_2\}$) should also be considered. Thus, the training data belonging to the overlapped region are also modeled.

2) Calculate the distance between fuzzy numbers: The distances between the test sample $\tilde{t}_1$ and the training data from different focal elements are calculated using Eq. (6) (for IN), Eq. (11) (for TFNmean and TFNmed) or Eq. (10) (for TrFN), as given in Table III.

TABLE III
THE DISTANCE BETWEEN THE TEST SAMPLE AND TRAINING DATA

Focal element	IN	TFNmean	TFNmed	TrFN
$\{\theta_1\}$	1.928	3.439	3.541	2.217
$\{\theta_2\}$	2.134	2.552	2.529	1.838
$\{\theta_3\}$	6.875	9.660	9.738	6.862
$\{\theta_1, \theta_2\}$	1.928	2.977	3.265	2.040

In Table III, according to the IN-based method, the test sample is closer to $\{\theta_1\}$ than $\{\theta_2\}$. However, without the outlier, the actual range of the training data from $\{\theta_1\}$ is $[8.1, 10.4]$ and the test sample 11.8 should be assigned a smaller distance to $\{\theta_2\}$, whose range is $[7.6, 13.1]$. By only considering the minimum and maximum values, the IN-based method can easily get counterintuitive distances, especially when there are outliers. However, the mean value and median are relatively insensitive to outliers, so that the fuzzy-number-based methods can obtain more reasonable distances. In this case, the fuzzy-number-based methods assign the test sample a smaller distance to $\{\theta_2\}$ than $\{\theta_1\}$.

3) Calculate the similarities: The similarities between the test sample $\tilde{t}_1$ and the training data from different focal elements are calculated from the above distances using Eq. (7), where α is taken as 5, as shown in Table IV.

TABLE IV
THE SIMILARITIES BETWEEN THE TEST SAMPLE AND TRAINING DATA

Focal element	IN	TFNmean	TFNmed	TrFN
$\{\theta_1\}$	0.094	0.055	0.054	0.083
$\{\theta_2\}$	0.086	0.073	0.073	0.098
$\{\theta_3\}$	0.028	0.020	0.020	0.028
$\{\theta_1, \theta_2\}$	0.094	0.063	0.058	0.089

4) Generate m_1: m_1 is generated from the normalized similarities, as shown in Table V. Our fuzzy-number-based methods assign the largest mass of belief to $\{\theta_2\}$ rather than $\{\theta_1\}$ or $\{\theta_1, \theta_2\}$, which is more reasonable compared with the IN-based method.

TABLE V
THE GENERATED m_1

Focal element	IN	TFNmean	TFNmed	TrFN
$\{\theta_1\}$	0.311	0.261	0.261	0.277
$\{\theta_2\}$	0.284	0.345	0.358	0.329
$\{\theta_3\}$	0.094	0.096	0.099	0.095
$\{\theta_1, \theta_2\}$	0.311	0.298	0.282	0.299

In the same way, m_2 and m_3 can be generated from feature f_2 and f_3 repectively, as shown in Table VI and Table VII.

TABLE VI
THE GENERATED m_2

Focal element	IN	TFNmean	TFNmed	TrFN
$\{\theta_1\}$	0.062	0.038	0.039	0.047
$\{\theta_2\}$	0.177	0.173	0.176	0.184
$\{\theta_3\}$	0.380	0.378	0.375	0.378
$\{\theta_2, \theta_3\}$	0.381	0.411	0.410	0.391

TABLE VII
THE GENERATED m_3

Focal element	IN	TFNmean	TFNmed	TrFN
$\{\theta_1\}$	0.160	0.138	0.141	0.147
$\{\theta_2\}$	0.484	0.547	0.537	0.522
$\{\theta_3\}$	0.183	0.162	0.167	0.171
$\{\theta_1, \theta_3\}$	0.173	0.153	0.155	0.160

After generating m_1, m_2 and m_3, the combined mass function m can be obtained by using the Dempster's rule of combination (Eq. (4)) and then the probability measure $BetP$ can be obtained using Eq. (5), as given in Table VIII.

TABLE VIII
THE GENERATED $BetP$

Class	IN	TFNmean	TFNmed	TrFN
θ_1	0.065	0.026	0.027	0.038
θ_2	0.808	0.873	0.866	0.853
θ_3	0.127	0.101	0.107	0.109

Finally, the test sample $t = (11.8, 9.8, 3.9)$ is labeled as class θ_2 since it has the highest $BetP$ value.

VI. EXPERIMENTS

To further compare the effectiveness of the proposed BBA determination methods with the IN-based method, we did the classification experiments on three UCI data sets (Iris, Wine and Wdbc).

In each experiment, the amounts of the samples from different classes are equal. Among the samples from the same class, 60% samples are used as the training data and the rest 40% samples are used as the test samples. We generate BBAs from all the features (one BBA generated from one feature) and the final classification result is obtained from the combined mass function. The value of α in Eq. (7) is set as 5. The accuracy of each classification is calculated from 100 runs of the Monte-Carlo experiments. The classification accuracies[1] are given in Table IX.

TABLE IX
THE ACCURACIES OF THE CLASSIFICATIONS (%)

Data set	IN	TFNmean	TFNmed	TrFN
Iris	92.67	93.83	93.85	93.92
Wine	91.48	93.29	94.23	92.79
Wdbc	67.71	86.91	88.32	81.27

From Table IX we can see, the proposed fuzzy-number-based methods can achieve higher accuracies than IN-based method.

Furthermore, we compared the robustness of our proposed method with IN-based method. We add one outlier to the training data for each class, whose values on each feature are set as:

$$O(f_i) = \max(f_i) + 0.2 \times (\max(f_i) - \min(f_i)) \qquad (12)$$

where $\max(f_i)$ and $\min(f_i)$ are the maximum and minimum values of the training data respectively on feature f_i. The accuracies are given in Table X.

TABLE X
THE ACCURACIES OF THE CLASSIFICATIONS WITH OUTLIERS (%)

Data set	IN	TFNmean	TFNmed	TrFN
Iris	88.72	93.08	93.07	92.13
Wine	80.89	91.75	92.53	90.06
Wdbc	61.80	82.87	84.28	73.69

From Table IX and Table X we can see, the accuracies of IN-based method drop significantly when the outliers are added while the accuracies of our fuzzy-number-based methods drop slightly. Therefore, the proposed fuzzy-number-based methods are more robust for outliers than IN-based method.

VII. CONCLUSION

In this paper we have proposed new methods for generating BBA based on fuzzy numbers. The experiments on its application of classification show that our proposed method is effective and robust for outliers and can achieve higher accuracies than the IN-based method.

In future work, we will focus on the distance between fuzzy numbers. More types of distance will be used and compared to describe the dissimilarity between the test sample and training data. Other normalization functions to establish similarities will be evaluated, as well as other possible decision-making strategies. Also, other evidence combination rules will be tested to make comparisons.

[1]The accuracy is defined as the percentage of correct classifications.

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