Performance Evaluation of improved QADA-KF and JPDAF for Multitarget Tracking in Clutter

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***Abstract*—This paper presents a performance evaluation of two types of multi-target tracking algorithms: 1) classical Kalman Filter based algorithms for multi-target tracking improved with Quality Assessment of Data Association (QADA) method using optimal data association, and 2) the Joint Probabilistic Data Association Filter (JPDAF). QADA technique is improved by using new basic belief assignment (bba) modelling, and also modified by means of the new Belief interval distance applied for computing the quality indicator associated with the pairings in the optimal data association solution. The evaluation is based on Monte Carlo simulations for maneuvering multiple-target tracking (MTT) problem in clutter.**

Keywords: Data association, JPDAF, Belief Functions, PCR6 fusion rule, QADA, Multitarget Tracking.

I. INTRODUCTION

The main function of each radar surveillance system is to keep targets tracks maintenance. It becomes a crucial and challenging problem especially in complicated situations of closely spaced or crossing targets.
The main objective of multiple-target tracking (MTT) is to estimate jointly, at each observation time moment, the number of targets continuously moving in a given region and their trajectories from the noisy sensor data.

Data Association (DA) is a central problem in MTT systems' design [1, 2]. It relates to the process of associating uncertain measurements (observations) to the tracked tracks. In the presence of a dense MTT environment, with false alarms and sensor detection probabilities less than unity, the problem of DA becomes more complex, because it should contend with many possibilities of pairings, some of which are in practice very imprecise, unreliable, and could lead to critical association mistakes in the overall tracking process.

In order to deal with these complex associations the most recent method to evaluate the Quality Assessment of Data Association (QADA) encountered in multiple target tracking applications in a mono-criterion context was proposed by Dezert and Benameur [4], and extended in [5] for the multicriteria context. It is based on belief functions (BF) for achieving the quality of pairings belonging to the optimal data assignment solution based on its consistency with respect to all the second best solutions, provided by a chosen algorithm. Recently, in [6, 19] the authors discussed and proposed the way in which Kalman filter (KF) could be enhanced in order to reflect the knowledge obtained based on the QADA method, called QADA-KF method. QADA assumes that the reward matrix is known, regardless of the manner in which it is obtained by the user.

In this paper QADA method is improved by using new bba modelling, and also modified by means of the new Belief Interval distance (BId) [18] applied for computing the quality indicator associated with the pairing in the optimal DA solution. The results are compared with those obtained by using Pignistic Probabilities [16]. We propose and test the performance of two versions of QADA-KF. The first one utilizes the assignment matrix, provided by the Global Nearest Neighbor (GNN) method, called QADA-GNN KF approach. The second one utilizes the assignment matrix, provided by the Probabilistic Data Association (PDA) method, called QADA-PDA KF method. These two QADA-KF methods are compared with the well-known Joint Probabilistic Data Association Filter (JPDAF) [7]–[9] which is an extension of the Probabilistic Data Association Filter (PDAF) [1] to a fixed and known number of targets. JPDAF uses joint association events and joint association probabilities in order to avoid conflicting measurement-to-track assignments by making a soft (probabilistic) assignment of all validated measurements to multiple targets.

The main objective of this paper is to: (1) improve QADA method by using new bba modelling;
(2) modify the improved QADA method by means of the new Belief interval distance for computing the quality indicator; (3) compare the performances of: (a) classical MTT algorithms based on the GNN approach for data association, utilizing Kinematic only Data (KDA) based MTT; (b) QADA-GNN KF based MTT; (c) QADA-PDA KF based MTT; (d) JPDAF based MTT.

The evaluation is based on a Monte Carlo simulation for particular difficult maneuvering MTT problem in clutter.

This paper is organized as follows. Section II is devoted to the improved QADA method. Section III discusses the Kalman Filter improved by QADA. The two variants of the assignment matrix, utilized by QADA are discussed in Section IV. In Section V the JPDAF is described and discussed. A particular simulation MTT scenario and results are presented for the KDA, QADA-GNN KF, QADA-PDA KF, and JPDAF in Section VI. Conclusions are made in Section VII.

II. THE IMPROVED QUALITY ASSESSMENT OF OPTIMAL DATA ASSOCIATION

II.*A Improvement of QADA bba modelling.*

DA is a decisive step in MTT systems [1], [2]. It consists in finding the global optimal assignments of targets $T_i, i=1,...,m$ to some measurements $z_j, j=1,...,n$ at a given time k by maximizing the overall gain in such a way, that no more than one target is assigned to a measurement, and reciprocally. The m×n reward matrix $\Omega=[\omega(i,j)]$ is defined by its elements $\omega(i,j)>0$, representing the gain of the association of target T_i with the measurement z_j.

The first and the second best assignments matrices A_1 and A_2 are used [4], in order to establish the quality of the specific associations (pairings) satisfying the condition $a_1(i,j)=1$. The main idea behind QADA method is to compare the values $a_1(i,j)$ in A_1 with the corresponding ones $a_2(i,j)$ in A_2 and to identify the change (if any) of the optimal pairing (i,j). In our MTT context, (i,j) means that measurement z_j is associated with target T_i. A quality indicator is established, depending on both the stability of the pairing and its relative impact on the global reward. The proposed method works also when the 1-st and 2-nd best optimal assignment A_1 and A_2 are not unique, i.e. there are multiplicities available. The construction of the quality indicators is based on Belief Functions (BF) theory and Proportional Conflict Redistribution fusion rule no.6 (PCR6), defined within DSm theory [16].
It depends on the type of pairing matching in the way, described below:

- In case, when $a_1(i,j)=a_2(i,j)=0$, one has a full agreement on “non-association” of the given pairing (i,j) in A_1 and A_2. This “non-association” has no impact on the global reward values $R_1(\Omega,A_1)$ and $R_2(\Omega,A_2)$, so it will be useless to utilize it in DA. Hence, the quality indicator value is set to $q(i,j)=1$.

- In case, when $a_1(i,j)=a_2(i,j)=1$, one has a full agreement on “association” of the given pairing (i,j) in A_1 and A_2. This “association” has different impacts on the global reward values $R_1(\Omega,A_1)$ and $R_2(\Omega,A_2)$. In order to estimate the quality of this matching association, one establish two basic belief assignments (bba), $m_S(.)$ ($s=1,2$) according to the both sources of information (A_1 and A_2). The Frame of Discernment (FoD), one reasons on, consists of a single hypothesis $X=(T_i,z_j)$: measurement z_j belongs to track T_i. The ignorance is modeled by the proposition $X\cup\overline{X}$, where $\overline{X}$ is a negation of hypothesis X.

$$\begin{cases} m_s(X)=a_s(i,j).\,\omega(i,j)/R_s(\Omega,A_s) \\ m_s(X\cup\overline{X})=1-m_s(X) \end{cases}$$

Applying the conjunctive rule of combination $m_1(.)\otimes m_2(.)$ one gets:

$$\begin{cases} m_{12}(X)=m_1(X)m_2(X)+m_1(X)m_2(X\cup\overline{X})+m_1(X\cup\overline{X})m_2(X) \\ m_{12}(X\cup\overline{X})=m_1(X\cup\overline{X})m_2(X\cup\overline{X}) \end{cases} \tag{1}$$

In our previous works [6, 17, 19], we did propose to use the pignistic transform BetP to establish the quality indicator.

- In case, when $a_1(i,j)=1$ and $a_2(i,j)=0$, then a disagreement (conflict) on the association (T_i,z_j) in A_1 and A_2 is detected. One could find the association (T_i,z_{j_2}) in A_2, where j_2 is the measurement index, such that $a_2(i,j_2)=1$. In order to define the quality of such conflicting association (T_i,z_j), one establish two basic belief assignments (bba), $m_S(.)\,(s=1,2)$ according to the both sources of information (A_1 and A_2). The FoD, one reasons on, consists of the following two propositions: $X=(T_i,z_j)$, and $Y=(T_i,z_{j_2})$. The ignorance is modeled by the proposition $X \cup Y$.

In our previous works [4], we did define the bbas by:

$$\begin{cases} m_1(X) = a_1(i,j) \cdot \dfrac{\omega(i,j)}{R_1(\Omega, A_1)} \\ m_1(X \cup Y) = 1 - m_1(X) \end{cases} \tag{2}$$

$$\begin{cases} m_2(Y) = a_2(i,j_2) \cdot \dfrac{\omega(i,j_2)}{R_2(\Omega, A_2)} \\ m_2(X \cup Y) = 1 - m_2(Y) \end{cases} \tag{3}$$

This modeling in fact does not work efficiently in some cases and that is why we need to revise it to make the QADA approach working more efficiently. For example, let's consider only one target T and two validated measurements z_1 and z_2 with the following payoff matrix $\Omega=[100 \quad 1]$. The two possible associations are represented by $A_1=[1 \quad 0]$ providing a rewards $R_1=100$, and $A_2=[0 \quad 1]$ providing a rewards $R_2=1$. In this simple case, one has $\Theta=\{X=(T,z_1), Y=(T,z_2)\}$. In applying formulas (2)–(3), one gets

$$\begin{cases} m_1(X) = a_1(i,j) \cdot \dfrac{\omega(i,j)}{R_1(\Omega, A_1)} = 1 \times \dfrac{100}{100} = 1 \\ m_1(X \cup Y) = 1 - m_1(X) = 0 \end{cases} \tag{4}$$

$$\begin{cases} m_2(Y) = a_2(i,j_2) \cdot \dfrac{\omega(i,j_2)}{R_2(\Omega, A_2)} = 1 \times \dfrac{1}{1} = 1 \\ m_2(X \cup Y) = 1 - m_2(Y) = 0 \end{cases} \tag{5}$$

The conjunctive combination rule gives:

$$\begin{aligned} m_{12}(X) &= m_1(X) m_2(X \cup Y) = 1 \times 0 = 0 \\ m_{12}(Y) &= m_1(X \cup Y) m_2(Y) = 0 \times 1 = 0 \\ m_{12}(X \cup Y) &= m_1(X \cup Y) m_2(X \cup Y) = 0 \times 0 = 0 \\ m_{12}(\theta) &= m_1(X) m_2(Y) = 1 \times 1 = 1 \end{aligned}$$

Applying PCR6 fusion rule [16] (Vol.3):

$$\begin{aligned} m(X) &= m_1(X) m_2(X \cup Y) + m_1(X) \cdot \frac{m_1(X) m_2(Y)}{m_1(X) + m_2(Y)} \\ m(Y) &= m_1(X \cup Y) m_2(Y) + m_2(Y) \frac{m_1(X) m_2(Y)}{m_1(X) + m_2(Y)} \\ m(X \cup Y) &= m_1(X \cup Y) m_2(X \cup Y) \end{aligned} \tag{6}$$

one gets finally

$$m_{PCR6}(X) = 1/2$$
$$m_{PCR6}(Y) = 1/2$$

which yields (using Pignistic transformation) $BetP(X) = BetP(Y) = 1/2$. This result is counter-intuitive (not realistic) because in this very simple case one knows that $X=(T,z_1)$ is obviously the best data association solution. To circumvent this serious problem, we propose to modify the bba modeling by taking a new model of bba construction as follows:

$$\begin{cases} m_1(X) = a_1(i,j)\cdot \dfrac{\omega(i,j)}{R_1(\Omega, A_1) + R2(\Omega, A_2)} \\ m_1(X \cup Y) = 1 - m_1(X) \end{cases} \tag{7}$$

$$\begin{cases} m_2(Y) = a_2(i,j_2)\cdot \dfrac{\omega(i,j_2)}{R_1(\Omega, A_1) + R_2(\Omega, A_2)} \\ m_2(X \cup Y) = 1 - m_2(Y) \end{cases} \tag{8}$$

If we apply this modeling on the previous example, we obtain

$$\begin{cases} m_1(X) = \dfrac{a_1(i,j)\cdot \omega(i,j)}{R_1(\Omega, A_1) + R_2(\Omega, A_2)} = 1 \times \dfrac{100}{101} \approx 0.99 \\ \\ m_1(X \cup Y) = 1 - m_1(X) = 0.01 \end{cases}$$

$$\begin{cases} m_2(Y) = \dfrac{a_2(i,j_2)\cdot \omega(i,j_2)}{R_1(\Omega, A_1) + R_2(\Omega, A_2)} = 1 \times \dfrac{1}{101} \approx 0.01 \\ m_2(X \cup Y) = 1 - m_2(Y) = 0.99 \end{cases}$$

Hence, one gets now

$$\begin{aligned} m_{12}(X) &= m_1(X)m_2(X \cup Y) = 0.9801 \\ m_{12}(Y) &= m_1(X \cup Y)m_2(Y) = 0.0001 \\ m_{12}(X \cup Y) &= m_1(X \cup Y)m_2(X \cup Y) = 0.0099 \\ m_{12}(\theta) &= m_1(X)m_2(Y) = 0.0099 \end{aligned}$$

Applying PCR6 redistribution principle, one gets finally

$$\begin{aligned} m(X) &= 0.9801 + (0.99 \times 0.0099)/1 = 0.989901 \\ m(Y) &= 0.0001 + (0.01 \times 0.0099)/1 = 0.010099 \end{aligned}$$

which yields $BetP(X) = 0.989901$ and $BetP(Y) = 0.010099$. This result fits now perfectly with what we expect, that is $X = (T, z_1)$ is obviously the best data association solution.

II.*B. Improvement of quality indicator calculating by using Belief Interval (BI) distance*

In [11], [20], the Euclidean belief interval distance between two bbas $m_1(.)$ and $m_2(.)$ is defined on the powerset of a given $\Theta = \{\theta_1, \ldots, \theta_n\}$ as follows

$$d_{BI}(m_1, m_2) \triangleq \sqrt{N_c \cdot \sum_{X \in 2^\Theta} d_W^2(BI_1(X), BI_2(X))} \tag{9}$$

where $N_c = 1/2^{n-1}$ is a normalization factor to have $d_{BI}(m_1, m_2) \in [0,1]$, and $d_W(BI_1(X), BI_2(X))$ is the Wasserstein's distance [22] between belief intervals $BI_1(X) \triangleq [Bel_1(X), Pl_1(X)] = [a_1, b_1]$ and $BI_2(X) \triangleq [Bel_2(X), Pl_2(X)] = [a_2, b_2]$. More specificly,

$$d_W = ([a_1, b_1], [a_2, b_2]) \triangleq \sqrt{\left[\frac{[a_1 + b_1]}{2} - \frac{[a_2 + b_2]}{2}\right]^2 + \frac{1}{3}\left[\frac{[b_1 - a_1]}{2} - \frac{[b_2 + a_2]}{2}\right]^2} \tag{10}$$

In [20], we have proved that $d_{BI}(x, y)$ is a true distance metric because it satisfies the properties of non-negativity $(d(x,y) \geq 0)$, non-degeneracy $(d(x,y) = 0 \Leftrightarrow x = y)$, symmetry $(d(x,y) = d(y,x))$, and the triangle inequality $(d(x,y) + d(y,z) \geq d(x,z))$, for any bba x, y and z defined on 2^Θ. The choice of Wasserstein's distance in d_{BI} definition is justified by the fact that Wasserstein's distance is a true distance metric and it fits well with our needs because we have to compute a distance between $[Bel_1(X), Pl_1(X)]$ and $[Bel_2(X), Pl_2(X)]$.

For notation convenience, we denote m_X the categorical bba having only X as focal element, where $X \neq \emptyset$ is an element of the powerset Θ. More precisely, m_X is the particular (categorical) bba defined by $m_X(X)=1$ and $m_X(Y)=0$ for any $Y \neq X$. Such basic bba plays an important role in our new decision scheme because its corresponding belief interval reduces to the degenerate interval [1 1] which represents the certainty on X. The basic principle of the new decision scheme we propose is very simple and intuitively makes sense. It consists in selecting as the final decision (denoted by $\hat{X}$) the element of the powerset for which the belief interval distance between the bba $m(\cdot)$ and m_X, $X \in 2^\Theta \setminus \{\emptyset\}$ is the smallest one. Therefore, take as the final decision $\hat{X}$ given by

$$\hat{X} = \arg\min_{X \in 2^\Theta \setminus \{\emptyset\}} d_{BI}(m, m_X) \tag{11}$$

where $d_{BI}(m, m_X)$ is computed according to (9). $m(\cdot)$ is the bba under test and $m_X(\cdot)$ the categorical bba focused on X defined above.

This decision scheme is very general in the sense that the decision making can be done on any type of element of the power-set 2^Θ, and not necessarily only on the elements (singletons) of the FoD. This method not only provides the final decision $\hat{X}$ to make, but also it evaluates how good this decision is with respect to its alternatives if we define the quality indicator $q(\hat{X})$ as follows

$$q(\hat{X}) \triangleq 1 - \frac{d_{BI}(m, m_{\hat{X}})}{\sum_{X \in 2^\Theta \setminus \{\emptyset\}} d_{BI}(m, m_X)} \tag{12}$$

One sees that the quality indicator $q(\hat{X})$ of the decision $\hat{X}$ made will become maximum (equal to one) when the distance between the bba $m(\cdot)$ and $m_{\hat{X}}$ is zero, which means that the bba $m(\cdot)$ is focused in fact only on the element $\hat{X}$. The higher $q(\hat{X})$ is, the more confident in the decision $\hat{X}$ we should be.

Of course, if a decision must be made with some extra constraint defined by a (or several) condition(s), denoted $c(X)$, then we must take into account $c(X)$ in Eq.(11), that is

$$\hat{X} = \arg\min_{X \in 2^\Theta \setminus \{\emptyset\} s.t\ c(X)} d_{BI}(m, m_X)$$

and also in the derivation of quality indicator by taking $\sum_{X \in 2^\Theta \setminus \{\emptyset\} s.t\ c(X)} d_{BI}(m, m_X)$ as denominator in (12). Theoretically any other strict distance metric, for instance Jousselme's distance [23]–[24], could be used instead of $d_{BI}(.,.)$. We have chosen d_{BI} distance because of its ability to provide good and reasonable behavior [20] as will be shown. When there exists a tie between multiple decisions $\{\hat{X}_j, j > 1\}$, then the prudent decision corresponding to their disjunction $\hat{X} = \cup \hat{X}_j$ should be preferred (if allowed), otherwise the final decision $\hat{X}$ is made by a random selection of elements $\hat{X}_j$.

III. QADA BASED KALMAN FILTER

The aim of this paper is to compare the performance of the JPDAF based MTT algorithm with the classical MTT algorithm, using the CMKF based on kinematics measurements, but improved by the QADA method.

In [6], the authors discuss and propose the way in which Kalman filter could be improved in order to reflect the knowledge obtained based on the QADA method.

Let's briefly recall what kind of information is obtained, having in hand the quality matrix, derived by QADA, in the MTT context. It gives knowledge about the confidence $q(i,j)$ in all pairings (T_i, z_j), $i = 1,\dots,m$; $j = 1,\dots,n$, chosen in the first best assignment solution. The smaller quality (confidence) of hypothesis " z_j belongs to T_i " means, that the particular measurement error covariance R was increased and the filter should not trust fully in the actual (true) measurement $z(k+1)$.

Having this conclusion in mind, the authors propose, such a behaviour of the measurement error covariance to be modeled by $R = R / q(T_i, z_j)$, for every pairing, chosen in the first best assignment and based on the corresponding quality value obtained. Then, when the Kalman filter gain decreases the true measurement $z_j(k+1)$ is trusted less in the updated state estimate $\hat{x}(k+1|k+1)$.

IV. BUILDING ASSIGNMENT MATRIX FOR QADA

QADA assumes the reward matrix is known, regardless of the manner in which it is obtained by the user. In this paper we propose two versions of QADA-KF. The first one utilizes the assignment matrix built from the single normalized distances, provided by the Global Nearest Neighbor method, called QADA-GNN KF method. The second one utilizes the assignment matrix, built from the posterior association probabilities, provided by the Probabilistic Data Association (PDA) method, called QADA-PDA KF method.

A. Assignment matrix based on GNN method

The GNN method finds and propagates the single most likely hypothesis during each scan to update KF. It is a hard (i.e., binary) decision approach, as compared to the JPDAF which is a soft (i.e., probabilistic) decision approach using all validated measurements with their probabilities of association. GNN method was applied in [6] and [17] to obtain the assignment matrix, utilized in QADA. In this case the elements of assignment matrix $\omega(i,j), i=1,\dots,m;\ j=1,\dots,n$ represent the normalized distances, $d(i,j) \triangleq \left[(z_j(k)-\hat{z}_i(k|k-1))' S^{-1}(k)(z_j(k)-\hat{z}_i(k|k-1))\right]^{1/2}$ between the validated measurement z_j and target T_i satisfying the condition $d^2(i,j)\le\gamma$. The distance $d(i,j)$ is computed from the measurement $z_j(k)$ and its prediction $\hat{z}_i(k|k-1)$ (see [1] for details), and the inverse of the covariance matrix $S(k)$ of the innovation, computed by the tracking filter. The threshold γ, for which the probability of given observation to fall in the gate is 0.99, could be defined from the table of the Chi-square distribution with M degrees of freedom and allowable probability of a valid observation falling outside the gate. In this case the DA problem consists in finding the best assignment that minimizes the overall cost.

B. Assignment matrix based on PDA method

The Probabilistic Data Association (PDA) method [1] calculates the association probabilities for validated measurements at a current time moment to the target of interest. PDA assumes the following hypotheses according to each validated measurement:

- $H_i(k): z_j(k)$ is a measurement, originated from the target of interest, $i=1,\dots,m$
- $H_0(k)$: no one of the validated measurement originated from the target of interest

If N observations fall within the gate of track i, $N+1$ hypotheses will be formed.

The probability of H_0 is proportional to $p_{i0}=\lambda_{FA}^N(1-P_d)$, and the probability of $H_j(j=1,2,\dots,N)$ is proportional to

$$p_{ij} \triangleq \frac{\lambda_{FA}^{N-1} P_g P_d \cdot e^{-\frac{d_{ij}^2}{2}}}{(2\pi)^{M/2}\cdot\sqrt{|S_{ij}|}} \tag{13}$$

where P_g is the a priory probability that the correct measurement is in the validation gate [1]; P_d is the target detection probability; λ_{FA} is the spatial density of FA. The probabilities p_{ij} can be rewritten as [1]

$$p_{ij} = \begin{cases} \dfrac{b}{b+\sum_{l=1}^{N}\alpha_{il}} & for \quad j=0 \quad (no\ valid\ observ.) \\ \dfrac{\alpha_{ij}}{b+\sum_{l=1}^{N}\alpha_{il}} & for \quad 1\le j\le N \end{cases} \tag{14}$$

where

$$b \triangleq (1-P_g P_d)\lambda_{FA}(2\pi)^{M/2}\cdot\sqrt{|S_{ij}|} \tag{15}$$

and

$$\alpha_{ij} \triangleq P_d \cdot e^{-\frac{d_{ij}^2}{2}} \tag{16}$$

The assignment matrix used in QADA method is established from all p_{ij} given by (14) related with all association hypotheses. This matrix will have m rows (where m is the number of all targets of interest), and

$N+1$ columns for the hypotheses generated. The $(N+1)$th column will include the values p_{i0} associated with $H_0(k)$.

V. JOINT PROBABILISTIC DATA ASSOCIATION FILTER

The Joint Probabilistic Data Association Filter (JPDAF) is an extension of the Probabilistic Data Association Filter (PDAF) for tracking multiple targets in clutter [1], [2], [10] – [12]. This Bayesian tracking filter uses the probabilistic assignment of all validated measurements belonging to the target gate to update its estimate. The preliminary version of JPDAF was proposed by Bar-Shalom in 1974 [13], then updated and finalized in [7]–[9]. The assumptions of JPDAF are the following:

• the number N_T of established targets in clutter is known;

• all the information available from the measurements Z^k up to time k is summarized by the sufficient statistic $\hat{x}^t(k)$ (the approximate conditional mean), and covariance $P^t(k \mid k)$ for each target T;

• the real state $x^t(k)$ of a target t at time k is modeled by a Gaussian pdf $N\left(x^t(k); \hat{x}^t(k), P^t(k \mid k)\right)$;

• each target T follows its own dynamic model;

• each target generates at most one measurement at each observation time and there are no merged measurements;

• each target is detected with some known detection probability P_d^t

• the false alarms (FA) are uniformly distributed in surveillance area and their number follows a Poisson pmf with FA density λ_{FA}.

In JPDAF, the measurement to target association probabilities are computed jointly across the targets and only for the latest set of measurements. This appealing theoretical approach however can give rise to very high combinatorics complexity if there are several persistent interferences, typically when several targets are crossing or if they move closely during several consecutive scans. Moreover some track coalescence effects may also appear which degrades substantially the JPDAF performances as it will shown in section VI. These limitations of JPDAF have already been reported in [14].

Let's consider a cluster (a cluster is a group of targets which have some measurements in common in their validation gates, i.e. non-empty intersections) of $T \geq 2$ targets $t = 1,\ldots,T$. The set of m_k measurements available at scan k is denoted $Z(k) = \{z_i(k), i = 1,\ldots,m_k\}$. Each measurement $z_i(k)$ of $Z(k)$ either originates from a target or from a FA. Denote $\hat{z}^t(k \mid k-1)$ as the predicted measurement for target T, and all the possible innovations that could be used in the Kalman Filter to update the target state estimate are denoted $\tilde{z}_i^t(k) \triangleq z_i(k) - \hat{z}^t(k \mid k-1), i = 1,\ldots,m_k$. In JPDAF, instead of using a particular innovation $\tilde{z}_i^t(k)$, it uses the weighted innovation $\tilde{z}^t(k) = \sum_{i=1}^{m_k} \beta_i^t(k) \tilde{z}_i^t(k)$ where $\beta_i^t(k)$ is the probability that the measurement $z_i(k)$ originates from target t. $\beta_0^t(k)$ is the probability that none measurements originate from the target *t*. The core of JPDAF is the computation of the a posteriori association probabilities $\beta_i^t(k), i = 0,1,\ldots,m_k$ based on all possible joint association events $\Theta(k) = \bigcap_{i=1}^{m_k} \Theta_i^{t_i}(k)$, where $\Theta_i^{t_i}(k)$ is the event that measurement $z_i(k)$ originates from target t_i (by convention and notation convenience, $t_i = 0$ means that the origin of measurement z_i is a FA.), $0 \leq t_i \leq N_T$. More precisely, one has to compute for $i = 1,\ldots,m_k$, $\beta_i^t(k) = \sum_{\Theta(k)} P\{\Theta(k) \mid Z^k\} \hat{\omega}_{it}(\Theta(k))$ and $\beta_0^t(k) = 1 - \sum_{i=1}^{m_k} \beta_i^t(k)$, where Z^k is the set of all measurements available up to time k, and $\hat{\omega}_{it}(\Theta(k))$ are the corresponding components of the association matrix characterizing the possible joint association $\Theta(k)$.

JPDAF is well theoretically founded and it does not require high memory. It provides pretty good results on simple MTT scenarios (with non-persisting interferences) with moderate FA densities. However the number of feasible joint association matrices increases exponentially with problem dimensions (m_k and N_T) which makes the JPDAF intractable for complex dense MTT scenarios. For more details about JPDAF, please refer to [1], [2], [10]–[12], [15].

VI. SIMULATION SCENARIO AND RESULTS

The Converted Measurement KF is used in our MTT algorithm. We assume constant velocity target model. The process noise covariance matrix is: $Q = \sigma_\upsilon^2 Q_T$, where T is the sampling period, σ_υ is the standard deviation of the process noise and Q_T is as given in [3]. Here are the results of KDA KF, QADA-GNN KF, QADA-PDA KF, and JPDAF for the MTT scenario with maneuvering targets.

The noise-free group of targets simulation scenario (Fig.1) consists of four air targets moving from left to right (or from West to East). For the clear explanation of the results, targets are numbered starting at the beginning with 1st target that has the greater y-coordinate and continuing to 4th target with the smallest y-coordinate. The stationary sensor is located at the origin with range $10000m$. The sampling period is $T_{scan} = 5\sec$ and the measurement standard deviations are 0.2 deg and 40 m for azimuth and range respectively. The targets move with constant velocity $V = 100\,m/s$. The first target for the first 8 scans moves without maneuvering keeping azimuth 120 deg from North. The group of two targets in the middle i.e. 2^{nd} and 3rd move without maneuvering keeping azimuth 90 deg from North that means, horizontally from West to East. It is the main direction of the group movement. The 4th target starts with azimuth 60 deg and moves towards the middle group of rectilinearly moving targets. When it approaches the group, it starts a turn to the right with 30 deg. Its initial azimuth of 120 deg is decreased by the angle of turn and becomes 90 deg, i.e. coincides with the main direction. From 15th scan, the four targets move rectilinearly in parallel. The distance between them is 150 m. The absolute value of the corresponding transversal acceleration for the two maneuvers is $1.495m/s^2$. The total number of scans for the simulations is 30. Fig.2 shows the noised scenario for yielding to 0.15 FA per gate on average.

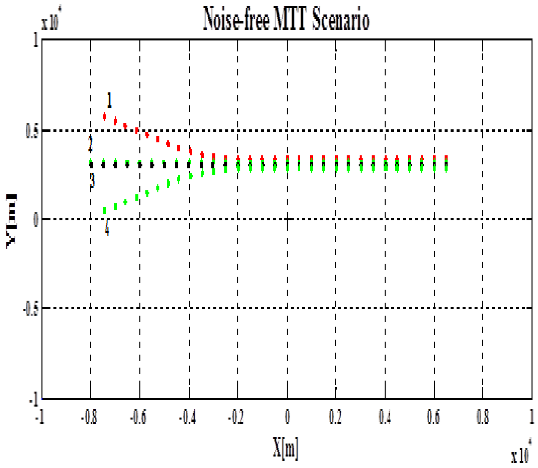

Figure 1. Noise-free group of targets Scenario.

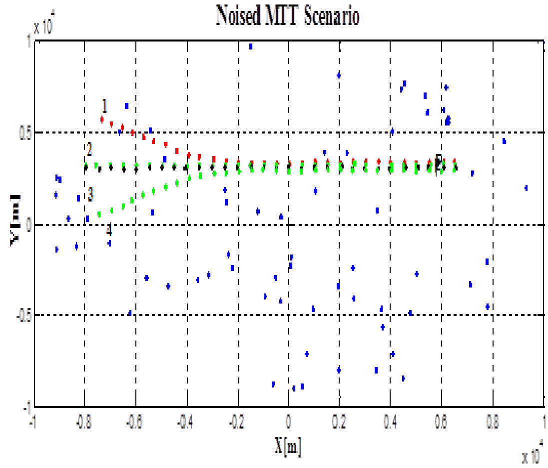

Figure 2. Noised group of targets Scenario.

Our results are based on Monte Carlo (MC) simulations with 200 independent runs in applying KDA based KF, QADAGNN KF, QADA-PDA KF, and JPDAF. We compare the performance of these methods with different criteria, and we use an idealized track initiation in order to prevent uncontrolled impact of this stage on the statistical parameters of the tracking process during MC simulations.

The true targets positions (known in our simulations) for the first two scans are used for track initiation. The evaluation of MTT performance is based on the criteria of Track Purity (TP), Track Life (TL), and percentage of miscorrelation (pMC):

1) TP criteria examines the ratio between the number of particular performed (jth observation - ith track) associations (in case of detected target) over the total number of all possible associations during the tracking scenario, but TP cannot be used with JPDAF because JPDAF is a soft assignment method.

Instead of TP, we define the Probabilistic Purity Index (PPI). It considers the measurement that has the highest association probability computed by the JPDAF and check, (and count) if this measurement originated from the target or not. PPI measures the ability of JPDAF to commit the highest probability to the correct target measurement in the soft assignment of all validated measurements.

2) TL is evaluated as an average number of scans before track's deletion. In our simulations, a track is canceled and deleted from the list of tracked tracks, when during 3 consecutive scans it cannot be updated with some measurement because there is no validated measurement in the validation gate. When using JPDAF, the track is canceled and deleted from the list of tracked tracks, when during 3 consecutive scans its own measurement does not fall in its gate. We call this, the "canceling/deletion condition". The status of the tracked tracks is denoted "alive".

3) pMC examines the relative number of incorrect observation-to-track associations during the scans.

The MTT performance results for KDA only KF, QADAGNN KF, QADA-PDA KF, and JPDAF for average false alarms in gate FA = 0.15 are given in Table 1. The MTT performance for QADA-PDA KF and QADA-

GNN KF are estimated for both: Pignistic probabilities, and minimum Belief distance d_{BI} principles to compute the quality indicator.

(in %)	*QADA-PDA*		*QADA-GNN*		*JPDAF*	*KDA*
	BetP	*BId*	*BetP*	*BId*		
Average TL	88.12	**89.39**	84.31	**89.13**	78.42	70.02
Average pMC	2.67	**2.45**	3.28	**2.39**	5.92	5.71
Average TP	84.54	**86.14**	79.86	**85.92**	PPI = 32.96	61.95

Table I: GROUP OF TARGETS SCENARIO: COMPARISON BETWEEN MTT PERFORMANCE RESULTS FOR 0.15 FA PER GATE.

According to all criteria, QADA-PDA KF method shows the best performance, followed by QADA-GNN KF, and JPDAF. The KDA based KF approach, as one could expect, shows the worst performance. It is obvious that minimum Belief distance interval principle for computing the quality indicator leads to improved MTT performance (compared to the results based on Pignistic probabilities - BetP) for both QADA-PDA KF and QADA-GNN KF. Still QADA-PDA KF outperforms QADA-GNN KF based MTT.

In order to make a fair comparison between QADA KF and JPDAF, we will discuss also the root mean square errors (RMSE), associated with the filtered X and Y values, presented in Figs. 3–6. The results for QADA-GNN KF and QADA-PDA KF are obtained on the base of the improved QADA method using minimal Belief Interval distance criteria and with the new bba modeling, proposed in the paper.

Figs. 3 and 4 show the mean square X and Y error filtered, associated with target 1, and compared for KDA KF, QADA-GNN KF, QADA-PDA KF, and JPDAF. Figs. 5 and 6 consider the same errors for the middle track 3. All the results are compared to the sensor’s errors along X and Y axis.

As a whole, one could see that rms errors, associated with QADA-PDA KF and QADA-GNN KF are a little bit less than the sensor’s measurement errors, except around the scan 15th, where all the targets move in parallel. We see that the RMSE on Y filtered error for track 1 associated with KDA-JPDAF grows extremely after scan 12. This behavior could be explained by the fact, that from this scan on target 1 starts moving in parallel with the rest of targets, causing that way spatial persisting interferences and track coalescence effects in JPDAF. These effects degrades significantly the quality of JPDAF performance as already reported in [14]. The same effect of track coalescence could be observed for track 3, moving in parallel during all the scans. The RMSE on Y filtered associated with JPDAF performance is high during the whole tracking region.

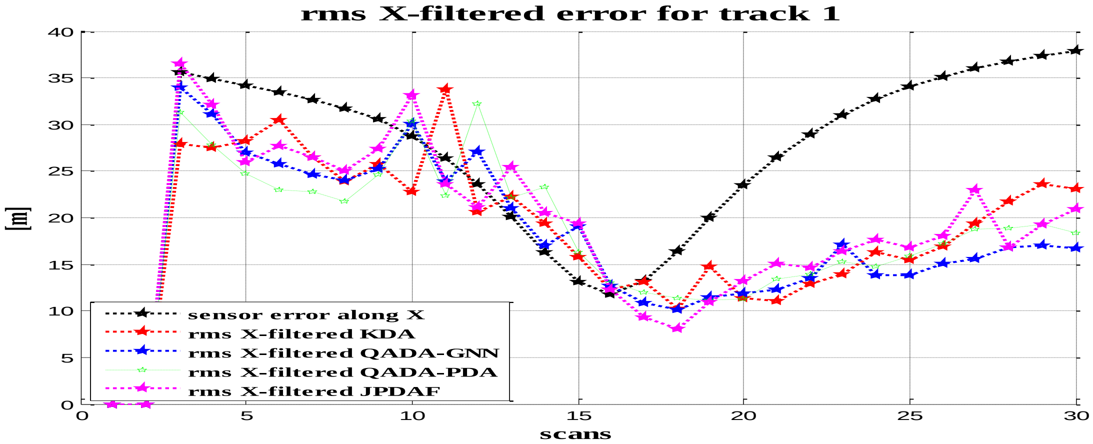

Figure 3. RMSE on X for track 1 with the four tracking methods.

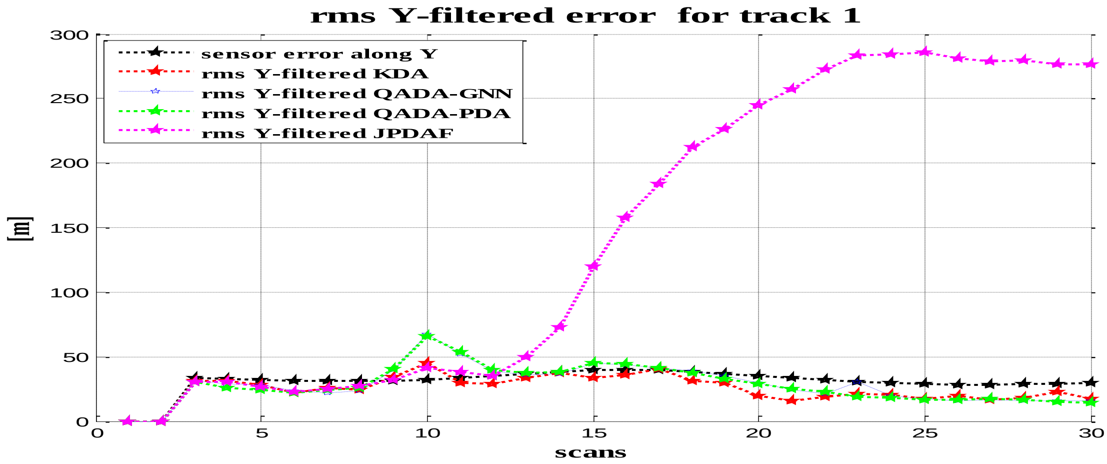

Figure 4. RMSE on Y for track 1 with the four tracking methods.

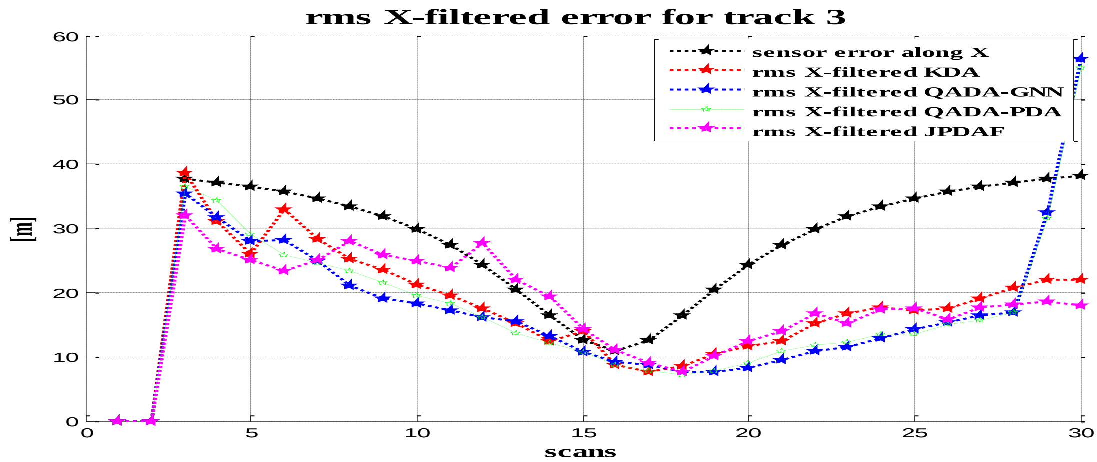

Figure 5. RMSE on X for track 3 with the four tracking methods.

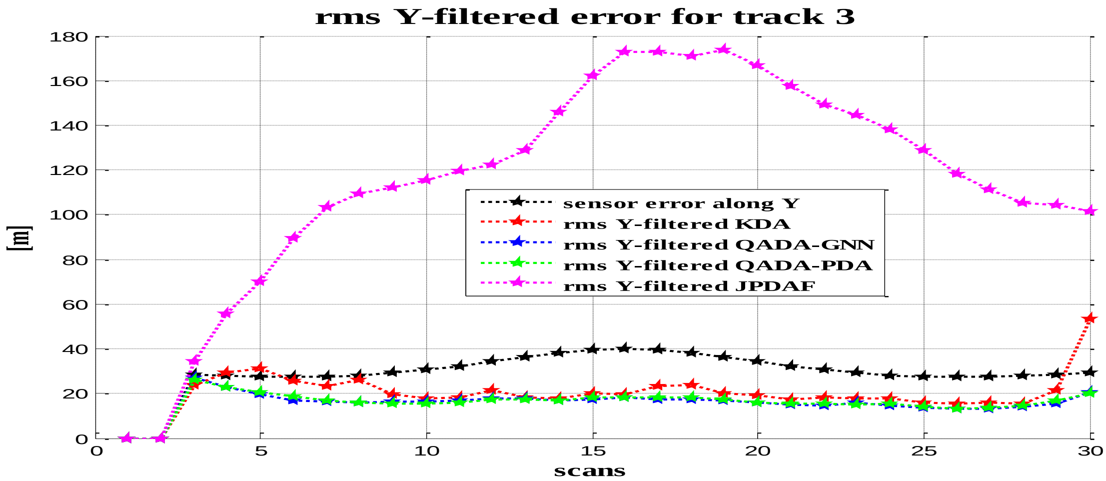

Figure 6. RMSE on Y for track 3 with the four tracking methods.

VII. CONCLUSIONS

This work evaluated with Monte Carlo simulations the efficiency of MTT performance in cluttered environment of four methods (a) classical MTT algorithm based on GNN approach for data association, utilizing Kinematic only Data; (b) QADA-GNN KF; (c) QADA-PDA KF; and (d) JPDAF. QADA technique was improved by using new bba modelling. It is also was modified by means of the new Belief interval distance applied for computing the quality indicator associated with the pairings in the optimal DA solution. The results were compared with those obtained by using Pignistic Probabilities. It was proved that this new approach leads to better MTT performance. The implemented groups of targets scenario shows the advantages of applying QADA-KF. According to all performance criteria, the QADA-PDA KF gives the best performance, followed by QADA-GNN KF, and JPDAF. The KDA KF approach shows the worst performance (as expected). This scenario is particularly difficult for JPDAF because of several closely spaced and rectilinearly moving targets in clutter during many consecutive scans, and it leads to track coalescence effects due to persisting interferences. As a result, the tracking performance of JPDAF is degraded. Because the complexity of the calculation for joint association probabilities grows exponentially with the number of targets, JPDAF requires almost 3 times more computational time in comparison to other methods in the first (complex) scenario.

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