QADA-PDA versus JPDA for Multi-Target Tracking in Clutter

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**QADA = Quality Assessment of Data Association**

This presentation is an *extension/improvement* of the Fusion 2017 paper


This QADA-PDA method for MTT has been published very recently (June 2017) in

Multi-Target Tracking (MTT) ⇒ Data Association + Tracking Filter

1. **Data Association (DA)** - important task of MTT [Bar-Shalom 1990]
   DA purpose is to find the assignment matrix with most likely observation-to-track associations to keep and improve target tracks maintenance performance
   
   - Classical DA approach:
     - Use all observations-to-tracks pairings selected in the 1st optimal global DA solution to update tracks, even if some pairings solutions are doubtful (have poor quality)
   
   - Sophisticate approaches:
     - Use all possible joint DA solutions and their a posteriori probas ⇒ **Joint Probabilistic Data Association Filter (JPDAF)** [Bar-Shalom Fortmann Scheffe 1980]
     - Use the 1st best global DA solution, evaluate its quality and modify the tracking filter accordingly ⇒ **Quality Assessment of Data Association (QADA)** approach introduced in [Dezert Benameur 2014]

2. **Tracking filters**  The CMKF (Converted Measurement Kalman Filter) [Lerro Bar-Shalom 1993] and JPDAF are used in this work

3. **In this presentation**  We compare performances of
   - QADA-PDA KF based MTT (QADA using PDA matrix and min dist. decision strategy)
   - JPDAF based MTT.
Joint Probabilistic Data Association Filter

→ proposed in [Bar-Shalom Fortmann Scheffe 1980] as an extension of PDAF for MTT

- **Main idea:**
  - The meas.-to-target association probas \( \beta^t_i(k) = \sum_{\Theta(k)} P(\Theta(k)|Z^k) \hat{\omega}_{it}(\Theta(k)) \) and \( \beta^0_t(k) = 1 - \sum_{i=1}^{m_k} \beta^t_i(k) \) are computed jointly across the targets from the joint posteriori probas \( P(\Theta(k)|Z^k) \) and only for the latest set of measurements. The target track updates are done for \( t = 1, \ldots, N_T \) by
    \[
    \hat{x}^t(k|k) = \hat{x}^t(k|k-1) + K^t(k) \sum_{i=1}^{m_k} \beta^t_i(k) \tilde{z}^t_i(k) \\
    P^t(k|k) = \beta^0_t(k) P^t(k|k-1) + (1 - \beta^0_t(k)) P^t_c(k) + \tilde{P}^t(k)
    \]

- **Assumptions of JPDAF**
  - the number \( N_T \) of targets is known and tracks have been initialized;
  - \( p(x^t(k)|Z^k) \sim \mathcal{N}(x^t(k); \hat{x}^t(k|k), P^t(k|k)) \), for \( t = 1, \ldots, N_T \)
  - each target generates at most one meas. at each scan and there are no merged meas.;
  - each target is detected with some known detection probability \( P^t_d \leq 1 \);
  - false alarms (FA) are uniformly distributed with known FA density \( \lambda_{FA} \) (Poisson pmf).

- **Advantages**
  - very good theoretical framework, and 0-scan-back filter (memoryless filter)
  - work well with moderate FA densities and non persisting interferences

- **Drawbacks**
  - often intractable for complex dense MTT scenarios
  - track coalescence effects in difficult scenarios
Data Association in Multi Target Tracking

Data Association (DA) Problem

Find the global optimal assignments of measurements $z_j, j = 1, \ldots, N$ available at time $k$ to targets $T_i, i = 1, \ldots, N_T$ by maximizing the overall gain (rewards):

$$R(\Omega, A) \triangleq \sum_{i=1}^{N_T} \sum_{j=1}^{N} \omega(i, j) a(i, j).$$

- $\Omega = [\omega(i, j)]$ is the DA matrix representing the gain of the associations of target $T_i$ with the measurement $z_j$ (usually homogeneous to the likelihood).
- Assignment solution: $N_T \times N$ binary matrix $A = [a(i, j)]$ with $a(i, j) \in \{0, 1\}$

$$a(i, j) = \begin{cases} 1, & \text{if } z_j \text{ is associated to track } T_i \\ 0, & \text{otherwise.} \end{cases}$$

How to get the optimal solution(s)

- by Kuhn-Munkres/Hungarian algorithm (1955/1957)
- by Bourgeois and Lassalle (1971) for rectangular DA matrix.
- by Murty’s method (1968) which gives the $m$-best assignments in order of increasing cost ⇒ used in QADA method
Main Idea behind QADA method [Dezert Bennameur 2014]

- compare \((z_j, T_i)\) in the 1st-best DA solution with \((z_j, T_i)\) in the 2nd-best DA solution
- establish a quality indicator, associated with pairing in 1st-best DA solution, based on belief functions, PCR6 fusion rule [DSmT Books], and some decision strategy.

**QADA assumes the DA (reward) matrix is known, regardless of the manner in which it is obtained.**  ⇒  Several QADA-based MTT are possible depending of the choice of DA matrix construction

The QADA method

1. based on a (modified) Basic Belief Assignment (BBA) modeling
2. the computation of quality of DA (i.e. confidence) \(q(i, j) \in [0, 1]\) of pairings \((T_i, z_j), i = 1, ..., N_T; j = 1, ..., N\) chosen in the 1st-best DA solution is based on its stability in the 2nd best DA solution and a belief Interval distance decision strategy

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1 In our Fusion 2017 paper, it is based on Pignistic Probability transformation decision strategy, which is a lossy transformation.
Choice of DA matrix for QADA

Two choices of DA matrix $\Omega$ for QADA-KF have been tested

1. **QADA-GNN** ⇒ DA matrix $\Omega$ is based on distances (as used in Global Nearest Neighbours (GNN) approach)
   - Elements $\omega_{ij}$ are the normalized distances $d(i, j)$ s.t. $d^2(i, j) \leq \gamma$ given by
     \[
     \omega(i, j) \equiv d(i, j) \triangleq [(z_j(k) - \hat{z}_i(k|k - 1))^T S^{-1}(k)(z_j(k) - \hat{z}_i(k|k - 1))]^{1/2}
     \]

2. **QADA-PDA** ⇒ DA matrix $\Omega$ is based on Posterior Data Association (PDA) probas as given in PDAF
   - Elements $\omega_{ij}$ of $\Omega$ are the posterior DA probas $p_{ij}$ given by PDAF
     \[
     p_{ij} = \begin{cases} 
     \frac{b}{b + \sum_{l=1}^{N} \alpha_{il}} & \text{for } j = 0 \text{ (no valid observ.)} \\
     \frac{\alpha_{ij}}{b + \sum_{l=1}^{N} \alpha_{il}} & \text{for } 1 \leq j \leq N
     \end{cases}
     \]
     where $b \triangleq (1 - P_g P_d) \lambda_{FA} (2\pi)^{M/2} \sqrt{|S_{ij}|}$ and $\alpha_{ij} \triangleq P_d \cdot e^{-\frac{d^2_{ij}}{2}}$
   - The $(N + 1)$th column of $\Omega$ will include the values $p_{i0}$ associated with $H_0(k)$ DA hypothesis (i.e. no one of the validated measurements originated from the target $T_i$ at time $k$).
Derivation of DA quality in QADA method

1. Build DA matrix $\Omega$ and find 1st-best and 2nd-best global DA solutions $A_1$ and $A_2$ using Murty’s algo.
2. Compare $a_1(i, j)$ in $A_1$ (1st best solution) with $a_2(i, j)$ value in $A_2$ (2nd best sol.).
3. Establish a quality indicator $q(i, j) \in [0, 1]$ for each optimal pairing $(T_i, z_j)$.

Several cases are possible

- **Case 1**: $a_1(i, j) = a_2(i, j) = 0 \Rightarrow$ Agreement on non-association of $T_i$ with $z_j$
  A useless stable case. We set $q(i, j) = 0$.

- **Case 2**: $a_1(i, j) = a_2(i, j) = 1 \Rightarrow$ Agreement on association $(T_i, z_j)$
  Stable case with different impacts on $R_1(\Omega, A_1)$ and $R_2(\Omega, A_2)$.

  - BBAs construction on frame $\Theta = \{X = (T_i, z_j), \tilde{X}\}$ done as follows for $s = 1, 2$
    \[
    m_s(X) = a_s(i, j) \cdot \omega(i, j)/R_s(\Omega, A_s) \quad \text{and} \quad m_s(X \cup \tilde{X}) = 1 - m_s(X)
    \]
  
  - Conjunctive rule of combination (here no conflict occurs)
    \[
    \begin{cases}
    m_{12}(X) = m_1(X)m_2(X) + m_1(X)m_2(X \cup \tilde{X}) + m_1(X \cup \tilde{X})m_2(X) \\
    m_{12}(X \cup \tilde{X}) = m_1(X \cup \tilde{X})m_2(X \cup \tilde{X})
    \end{cases}
    \]
  
  - In [1], QADA quality/confidence indicator is based on lossy pignistic proba transf.
    \[q(i, j) \triangleq \text{BetP}(X) = m_{12}(X) + \frac{1}{2} m_{12}(X \cup \tilde{X})\]
  
  - In [2] and here, QADA quality/confidence indicator is based on min distance strategy
**Case 3:** \(a_1(i, j) = 1\) and \(a_2(i, j) = 0\) \(\Rightarrow\) **conflict in solutions** between \(A_1\) and \(A_2\)

- find index \(j_2\), such that \(a_2(i, j_2) = 1\)
- BBAs construction on frame \(\Theta = \{X \triangleq (T_i, z_j), Y \triangleq (T_i, z_{j_2})\}\)

**Basic Belief Assignment (BBA) modeling**

\[
\begin{aligned}
    m_1(X) &= a_1(i, j) \cdot \frac{\omega(i,j)}{R_1(\Omega,A_1)+R_2(\Omega,A_2)} \\
    m_1(X \cup Y) &= 1 - m_1(X) \\
    m_2(Y) &= a_2(i, j_2) \cdot \frac{\omega(i,j_2)}{R_1(\Omega,A_1)+R_2(\Omega,A_2)} \\
    m_2(X \cup Y) &= 1 - m_2(Y)
\end{aligned}
\]

- BBAs fusion with PCR6 fusion rule

\[
\begin{aligned}
    m(X) &= m_1(X) m_2(X \cup Y) + m_1(X) \cdot \frac{m_1(X) m_2(Y)}{m_1(X) + m_2(Y)} \\
    m(Y) &= m_1(X \cup Y) m_2(Y) + m_2(Y) \cdot \frac{m_1(X) m_2(Y)}{m_1(X) + m_2(Y)} \\
    m(X \cup Y) &= m_1(X \cup Y) m_2(X \cup Y)
\end{aligned}
\]

- In [1], QADA quality/confidence indicator is based on lossy pignistic proba transf.

\[
q(i, j) \triangleq \text{BetP}(X) = m_{12}(X) + \frac{1}{2} m_{12}(X \cup Y)
\]

- In [2] and here, QADA quality/confidence indicator is based on min distance strategy
Belief interval of $A$

$$BI(A) \triangleq [\text{Bel}(A), \text{Pl}(A)] = \left[ \sum_{B \in 2^\Theta | B \subseteq A} m(B), \sum_{B \in 2^\Theta | B \cap A \neq \emptyset} m(B) \right]$$

Euclidean belief interval based distance [Han Dezert Yang 2014]

$$d_{BI}^E(m_1, m_2) \triangleq \sqrt{\frac{1}{2|\Theta|-1} \cdot \sum_{A \in 2^\Theta} d^I(BI_1(A), BI_2(A))^2}$$

$$d^I([a_1, b_1], [a_2, b_2]) = \sqrt{\left[ \frac{a_1 + b_1}{2} - \frac{a_2 + b_2}{2} \right]^2 + \frac{1}{3} \left[ \frac{b_1 - a_1}{2} - \frac{b_2 - a_2}{2} \right]^2}$$

Decision-making from a BBA

$$\delta = \hat{X} = \arg \min_{\chi \in \Theta} d(m, m_\chi)$$

Quality of the decision

$$q(\hat{X}) = 1 - \frac{d_{BI}(m, m_\hat{X})}{\sum_{\chi \in \Theta} d_{BI}(m, m_\chi)} \in [0, 1]$$

Higher is $q(\hat{X})$ more trustable is the decision $\delta = \hat{X}$
QADA-PDA Kalman Filter for MTT

**Classical Kalman Filter (KF) state estimate**

\[
\hat{x}(k|k) = \hat{x}(k|k-1) + K(k)(z(k) - \hat{z}(k|k-1))
\]

with Kalman filter gain matrix

\[
K(k) = P(k|k-1)H^T(k)[H(k)P(k|k-1)H^T(k) + R]^{-1}
\]

In KF, \(z(k)\) is assumed to be correct with the measurement noise characterized by the given covariance matrix \(R\)

- If \(R\) decreases \(\Rightarrow \) \(z(k)\) is more precise \(\Rightarrow \) Gain \(K(k)\) increases
- If \(R\) increases \(\Rightarrow \) \(z(k)\) is less precise \(\Rightarrow \) Gain \(K(k)\) decreases

**Improved KF with QADA quality factor**

The quality factor \(q(i, j) \equiv q(T_i, z_j)\) expresses the confidence in the association \((T_i, z_j)\)

- If \(q(T_i, z_j) \rightarrow 0\) \(\Rightarrow\) we don’t trust \((T_i, z_j)\) \(\Rightarrow\) \(z_j\) is incorrect and so we increase \(R\)
- If \(q(T_i, z_j) \rightarrow 1\) \(\Rightarrow\) we trust \((T_i, z_j)\) \(\Rightarrow\) \(z_j\) is correct and we keep \(R\) as it is

**KF gain adjustment with QADA**

\[
R_{\text{QADA}} = \frac{1}{q(T_i, z_j)} \cdot R \quad \Rightarrow \quad K(k) = P(k|k-1)H^T(k)[H(k)P(k|k-1)H^T(k) + R_{\text{QADA}}]^{-1}
\]
Measures of performances

We compare the performances several MTT algorithms (using Monte Carlo simul.)

- **KDA-GNN KF** based MTT (GNN matrix based on Kinematic meas.)
- **QADA-GNN KF** based MTT (QADA using GNN matrix)
- **QADA-PDA KF** based MTT (QADA using PDA matrix)
- **JPDAF**

**Criteria for MTT performance evaluation**

1. **TL = Track Life** → average number of track updates before track deletion
   - A track is removed after 3 successive incorrect associations, or missed detections
   - With JPDAF, a track is removed if the true measurement is out of the gate during 3 successive scans

2. **pMC = Percentage of miscorrelation**
   → Percentage of incorrect measurement-to-track association during the scans
   → for JPDAF, pMC = % of time the true measurement is outside its target gate
   pMC for JPDAF is not equivalent to pMC for other algos
   It does reflect only partially the performances of JPDAF

3. **TP = Track purity**
   \[ TP = \frac{\text{Number of correct associations}}{\text{Total number of associations}} \]

4. **PPI = Probabilistic Purity Index** → used only with JPDAF
   \[ \text{PPI} = \% \text{ of correctness of measurement having the highest proba} \]
Scenario 1 - Targets merging in a close formation

- Targets $T_1, T_2, T_3, T_4$ move from West to East with constant velocity $100 \text{ m/sec}$ during 30 scans.
- The stationary sensor is located at the origin with range $10000 \text{ m}$. The sampling period is $T_{\text{scan}} = 5 \text{ sec}$.
- Measurement precision: Azimuth $\rightarrow \sigma_{\text{Az}} = 0.2 \text{ deg}$ and range $\rightarrow \sigma_{\text{D}} = 40 \text{ m}$.
- From scans 15 to 30, targets move in parallel with inter distance of $150 \text{ m}$.
- FA are uniformly distributed in the surveillance region with know density $P_d = 0.999$ is associated with the sensor.
Monte Carlo results based on 200 runs

<table>
<thead>
<tr>
<th>(in %)</th>
<th>QADA-PDA</th>
<th>QADA-GNN</th>
<th>JPDAF</th>
<th>KDA-GNN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average TL</td>
<td><strong>89.39</strong> (88.12)</td>
<td><strong>89.13</strong> (84.31)</td>
<td>78.42</td>
<td>70.02</td>
</tr>
<tr>
<td>Average pMC</td>
<td><strong>2.45</strong> (2.67)</td>
<td><strong>2.39</strong> (3.28)</td>
<td>5.92</td>
<td>5.71</td>
</tr>
<tr>
<td>Average TP</td>
<td><strong>86.14</strong> (84.54)</td>
<td><strong>85.92</strong> (79.86)</td>
<td>PPI=32.96</td>
<td>61.95</td>
</tr>
</tbody>
</table>

Performance results for 0.15 FA per gate on average

Note: JPDAF requires almost **3 times more computational time** than other methods because an exponential growing of number of joint association hypotheses.
Simulation results for scenario 1

Results with **0.15 FA per gate** on average

Averaged RMSE on X for track 1 with the tracking methods.

Averaged RMSE on Y for track 1 with the tracking methods.
Simulation results for scenario 1 (cont’d)

Results with **0.15 FA per gate** on average

Averaged RMSE on X for track 3 with the tracking methods.

Averaged RMSE on Y for track 3 with the tracking methods.
Scenario 2 - Targets merging in a close formation and then splitting

Simulation of groups of target for scenario 2

- Five air targets ($T_1$, $T_2$, $T_3$, $T_4$, $T_5$) moving from North-West to South-East with constant velocity 100 m/sec during 65 scans.
- The stationary sensor is located at the origin with range 20000 m. The sampling period is $T_{scan} = 5$ sec.
- Measurement precision: **Azimuth** $\rightarrow \sigma_{Az} = 0.35$ deg and **range** $\rightarrow \sigma_D = 25$ m.
- Targets move in three groups: **Group 1** = $T_1$, **Group 2** = ($T_2$, $T_3$, $T_4$), **Group 3** = $T_5$
- The number of false alarms (FA) follows a Poisson distribution. FA are uniformly distributed in the surveillance region.
- $P_d = 0.999$ is associated with the sensor.
Monte Carlo results based on 300 runs

<table>
<thead>
<tr>
<th>(In %)</th>
<th>GROUPS of Targets Scenario SigmaD=35, SigmaAz=0.2 FAingate=0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>KDA-GNN</td>
</tr>
<tr>
<td>Average TL</td>
<td>50.27</td>
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<tr>
<td>Average pMC</td>
<td>3.35</td>
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<tr>
<td>Average TP</td>
<td>45.61</td>
</tr>
</tbody>
</table>

Performances of QADA KF methods with 0.2 FA per gate
Simulation results for scenario 2 (cont’d)

Results with **0.2 FA per gate** on average

Averaged RMSE on X for track 1 with the four tracking methods.

Averaged RMSE on Y for track 1 with the four tracking methods.
Simulation results for scenario 2 (cont’d)

Results with **0.2 FA per gate** on average

Averaged RMSE on X for track 3 with the four tracking methods.

Averaged RMSE on Y for track 3 with the four tracking methods.
Scenario 3 - Two crossing targets

Two maneuvering targets moving from West to East with constant velocity $38 \text{ m/sec}$ during 65 scans.

- The stationary sensor is located at the origin with range $1200 \text{ m}$.
- The sampling period is $T_{\text{scan}} = 1 \text{ sec}$.
- $\sigma_{\text{Az}} = 0.25 \text{ deg}$ and $\sigma_{\text{D}} = 25 \text{ m}$ for azimuth and range respectively.
- $P_d = 0.999$ is associated with the sensor.
Simulation results for scenario 3

Monte Carlo results based on 300 runs

<table>
<thead>
<tr>
<th>(In %)</th>
<th>KDA-GNN</th>
<th>QADA-GNN-BetP</th>
<th>JPDAF</th>
<th>QADA-PDA-BetP</th>
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<tr>
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<tr>
<td>Average TL</td>
<td>77.06</td>
<td>88.93</td>
<td>91.25</td>
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<td></td>
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<td>89.29</td>
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<td>93.54</td>
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<tr>
<td>Average pMC</td>
<td>2.40</td>
<td>2.24</td>
<td>2.08</td>
<td>2.11</td>
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<tr>
<td></td>
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<td>2.20</td>
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<td>2.15</td>
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<tr>
<td>Average TP</td>
<td>72.78</td>
<td>85.64</td>
<td>PPI=86.29</td>
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<td></td>
<td></td>
<td>86.11</td>
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<td>88.01</td>
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Performances of QADA-PDA versus JPDA for 0.2 FA per gate

<table>
<thead>
<tr>
<th>(In %)</th>
<th>KDA-GNN</th>
<th>QADA-GNN-BetP</th>
<th>JPDAF</th>
<th>QADA-PDA-BetP</th>
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<tbody>
<tr>
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</tr>
<tr>
<td>Average TL</td>
<td>58.80</td>
<td>77.20</td>
<td>82.87</td>
<td>83.18</td>
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<tr>
<td></td>
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<td>80.19</td>
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<td>83.21</td>
</tr>
<tr>
<td>Average pMC</td>
<td>3.61</td>
<td>3.63</td>
<td>2.94</td>
<td>3.40</td>
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<tr>
<td></td>
<td></td>
<td>3.54</td>
<td></td>
<td>3.41</td>
</tr>
<tr>
<td>Average TP</td>
<td>52.90</td>
<td>72.01</td>
<td>PPI=76.94</td>
<td>77.15</td>
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<tr>
<td></td>
<td></td>
<td>75.30</td>
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<td>77.23</td>
</tr>
</tbody>
</table>

Performances of QADA-PDA versus JPDA for 0.4 FA per gate
Simulation results for scenario 3 (cont’d)

Results with **0.2 FA per gate** on average

Averaged RMSE on X for track 1 with the four tracking methods.

Averaged RMSE on Y for track 1 with the four tracking methods.
Simulation results for scenario 3 (cont’d)

Results with \textbf{0.2 FA per gate} on average

Averaged RMSE on X for track 2 with the four tracking methods.

Averaged RMSE on Y for track 2 with the four tracking methods.
Conclusions

1. QADA-PDA is a zero-scan back method
2. QADA-PDA is quite simple to implement (mix of PDAF calculus and Optimal assignment search)
3. QADA-PDA is a good compromise between strict hard-assignment (GNN) and full soft-assignment (JPDA)
4. QADA-PDA avoids JPDA combinatorics/complexity
5. QADA-PDA works better than QADA-GNN, KDA-GNN and JPDA in difficult scenarios
6. QADA-PDA is a new and interesting practical method for MTT in clutter

Perspectives

1. making more precise evaluations of QADA-PDA method
2. development and test of better quality evaluation models (if any)
3. improvement of MTT performances using attribute information


D. Han, J. Dezert, Y. Yang, Belief interval Based Distances Measures in the Theory of Belief Functions, IEEE Trans. on SMC, 2017.