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Exploiting underlying approximation properties in the discontinuous Galerkin method for improved trouble cell indication

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Abstract

Nonlinear hyperbolic equations are often used to describe the behaviour of a quantity of interest in many areas such as climate modeling, shallow water equations, and computational fluid dynamics. Solutions of these equations may develop discontinuities or shocks. The discontinuous Galerkin (DG) method allows for great flexibility in creating accurate and efficient simulations. However, near a discontinuity or shock, solutions may develop non-physical spurious oscillations. Areas where these occur are called troubled cells. To counteract these oscillations, typically some stabilization is used such as limiting, filtering, or artificial dissipation. To ensure these techniques are only applied in necessary regions, an accurate troubled-cell indicator is needed. In this presentation we discuss how to exploit the underlying approximation space of the DG method. Specifically, the use of multiwavelets [1] for troubled-cell indication. We explain how the DG coefficients easily relate to the multiwavelet decomposition [2]. This decomposition is written as a sum of a global average and finer details on different levels. Using such a decomposition allows the multiwavelet expansion to act as a troubled-cell indicator, where the averages of this contribution are computed on each element, [3, 4]. Additionally, we will discuss the performance of this indicator which typically relies on a problem-dependent parameter. Optimal performance of these indicators should produce an approximation that is free of spurious oscillations. In general, many tests are required to obtain this optimal parameter for each problem. However, to avoid choosing parameters, it is possible to combine the underlying approximation properties with techniques from statistics such as Tukey's boxplot approach [5]. This allows for investigating sudden increases or decreases of the indicator value with respect to the neighboring values and hence, indication basically reduces to detecting outliers. This allows for an automated algorithm that can easily be applied to various troubled-cell indication variables.

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