International Workshop on Wavelets & CFD
École Normale Supérieure, Paris, 11 June 2019

Multiwavelet-Based Grid Adaptation with Discontinuous-Galerkin Schemes for Conservation Laws

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Abstract

Solutions of conservation laws typically are very heterogeneous because they may exhibit discontinuities. Thus, high resolution is needed to adequately resolve these local features whereas a moderate resolution is sufficient to accurately resolve smooth parts of the solution. Therefore these problems are well suited for local grid adaptation.

In general, there are no rigorous a-priori or a-posteriori error estimates available to control local grid refinement. The purpose of the talk is to present a strategy that is based on data analysis rather than error estimates. In particular, the adaptive strategy is designed for solving unsteady compressible flows by a discontinuous Galerkin method.

The underlying idea of the adaptive strategy is to perform a multiresolution analysis using multiwavelets on a hierarchy of nested grids. This provides information on the difference between successive refinement levels that may become negligibly small in regions where the solution is smooth. Applying thresholding, the data is compressed thereby triggering local grid adaptation. Furthermore, this information is also used as an indicator for limiting. A heuristic strategy is given how to choose the threshold value such that the accuracy of a reference solution on a uniform discretization is maintained asymptotically. Although there is no rigorous proof available, except for the one-dimensional scalar case [1], the reliability of the strategy has been verified by numerous computations and parameter studies for different conservations laws, e.g., Burgers equation [2], Euler equations [3], shallow water equations [4], in one and two space dimensions.

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