# Vérification et validation en Calcul de Structure : Etat de l'Art et Challenges


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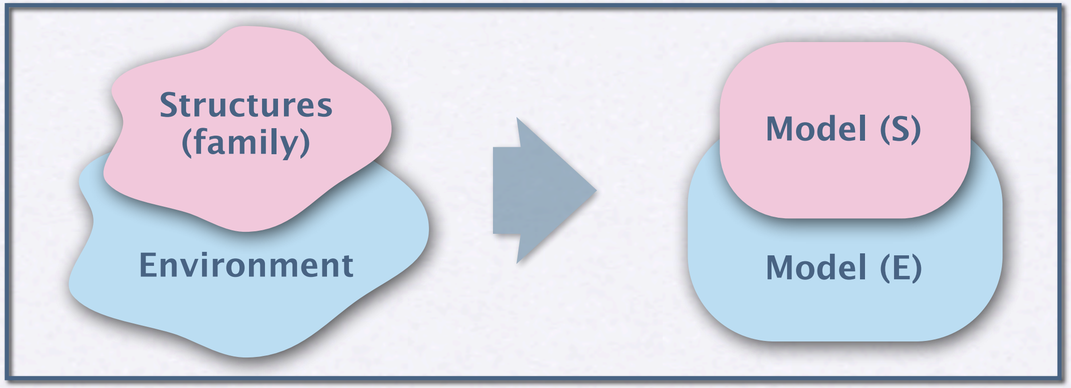
# Transfer to virtual world




- New
  - to **identify** and to **model** the (S) and the (E) **variability** (uncertainties)
  - to use more **physics-based** material models (nano or micro or meso or hybrid)
- Research issues
  - model **verification** (error estimators, adaptive calculations)
  - model **validation**
  - **multiscale computation** (local-global)
  - **model order reduction** (real time simulation-design)
  - **isogeometric FEA (**Integration CAD and FEA) Hughes et all 2003,.....

# The control of models

## A constant concern, what has changed?

Reference model

Quantitative tools to measure quality of the approximated model (begin: 1975)

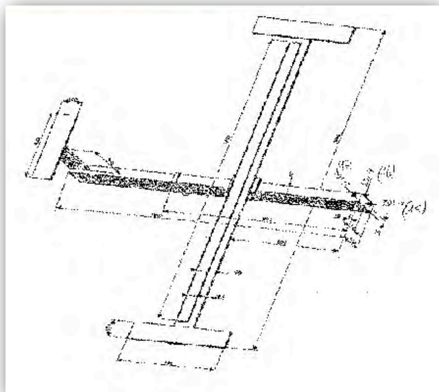
Approximated model

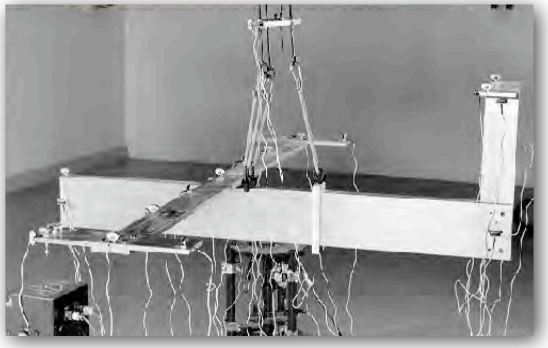
## The standard situation: 3 models

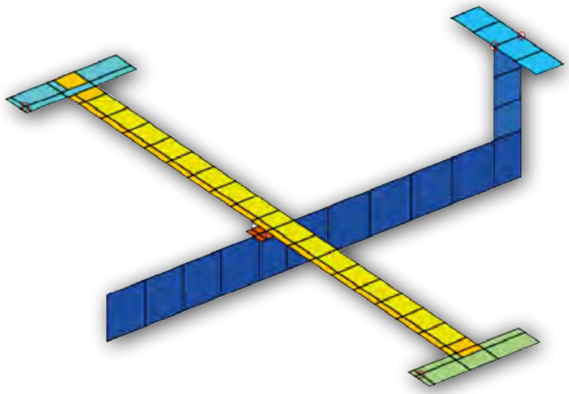
Conceptual model

Reality

Computerized model







# The control of models

| Situation 1: verification | | Situation 2: validation | |
|---|---|---|---|
| **Conceptual model** | reference model | **Reality** | reference model |
| **Computerized model** | approximated model | **Conceptual model** | approximated model |

**Situation 1: verification**

**Question 1**
to quantify the quality of the approximated model

**Question 2**
to optimize the computational parameters for a prescribed quality

**Situation 2: validation**

**Question 1**
to quantify the quality of the approximated model

**Question 2**
to correct (to update) the approximated model for a better fit with the reality

# Part 1: Verification

# Verification: history in brief

Approximated model: computerized model

## The state of the art: 3 families of *global* error estimators

**Smoothing of the finite element solution**

- [Zienckiewicz-Zhu 87], [Ainsworth], [Beckers], [Huerta], [Diez], [Craig], [Haque], [Fourment], [Le Dain], [Strouboulis], [Wiberg], [Rodenas], [Zong] ...

**Verification of the equilibrium equations**

- [Babuska-Rheinboldt 78], [Ainsworth], [Aubry], [Bank], [Gago], [Kelly], [Oden], [Prudhomme], [Stein], [Strouboulis], [Tie], [Weiser], [Zienckiewicz] ...

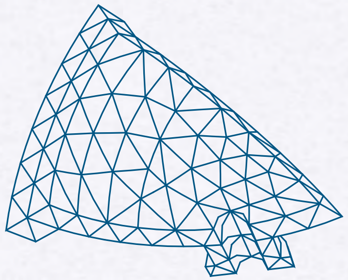
**Constitutive relation errors**

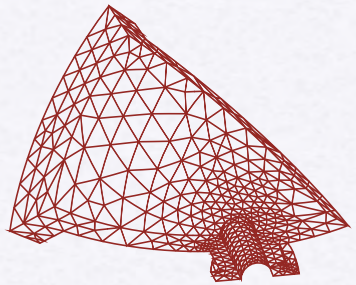
- [Ladevèze 75], [Pelle], [Boisse], [Coffignal], [Coorevits], [Combe], [Destuynder], [Florentin], [Gallimard], [Leguillon], [Marin], [Moës], [Ramananjanahary], [Rougeot], [Ryckelynck], [Cartensen], [Diez] [Babuska], [Bouillard], [Beckers], [Maunder], [Oden], [Perego et al.], [Stein], [Strouboulis],[Rodenas], [Orlando], [Patera], [Warzee] ...

# Verification: history in brief

Approximated model: computerized model

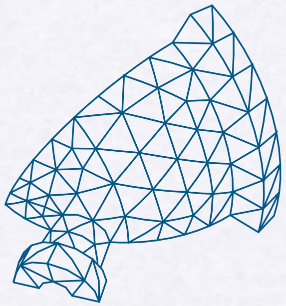
- Illustration of the state of the art

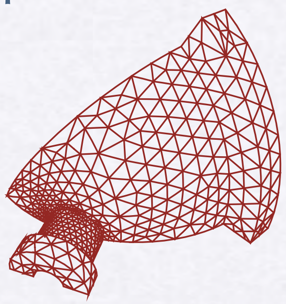




Initial error **30 %**

Target error **2 %**
Computed error **2,2 %**





| Control of the computation |
| --- |
| **Error estimator** (*a posteriori*) |
| **Modification (adaptation)** of computational parameters |

- Extension to nonlinear problems ant to transient dynamics

  book : Ladevèze-Pelle (Springer) 2004 (CRE-method)

# Verification: history in brief

Approximated model: computerized model

## Research topic in progress:local quantities(outputs of interest)

- Per[aire-Patera 95], [Rannacher], [Babuska-Strouboulis],[Cirak-Ramm], [Stein], [Diez-Huerta], [Oden-Prudhomme], [Ladevèze-Rougeot]...

- **idea**

$$\boxed{\alpha = \int_\Omega \overbrace{\Sigma}^{\text{Extractor}} \cdot \overbrace{\sigma}^{\text{Stress}} \, d\Omega \qquad \left( \alpha_h = \int_\Omega \Sigma \cdot \sigma_h \, d\Omega \right)}$$

**Coming back to global error estimators**

- **remark**: accurate error estimation
  (solving of the direct problem and adjoint one)

# Verification: history in brief

Approximated model: computerized model

- Extension to nonlinear case
  [Jonhson 92, Rannacher et al, Oden et al 06]

→ Linearization at each time step

- The state-of-the-art: Books
  Babuska-Strouboulis (Oxford) 2001
  Ladevèze-Pelle (Springer) 2004

**A scientific challenge: to derive *guaranteed* and *accurate* error bounds**

# Central question discussed here

$$\alpha_{ex} = \int_\Omega \mathrm{Tr}[\sigma_{ex} \tilde{\Sigma}] \mathrm{d}\Omega$$

sol. reference problem

From a f.e. approach, find guaranteed bounds such that:

$$\alpha_h^- \leq \alpha_{ex} \leq \alpha_h^+$$

or

$$|\alpha_{ex} - \frac{1}{2}(\alpha_h^+ + \alpha_h^-)| \leq \frac{1}{2}(\alpha_h^+ - \alpha_h^-)$$

all sources of errors:

- modeling error
- space and time discretisation
- iteration technique

# The state of the art

- Rather few works
  - [Rannacher, Peraire et al, Ladevèze et al, Strouboulis-Babuska, Diez-Huerta et al, Oden-Prudhomme, de Almeida et al, Stein, Cirak-Ramm,...]
  - Outside F.E. context: [Greenberg 1948, Washizu 1953,...]
    - ➡*analytical Green functions*
- Methods: only **one** approach with variants
  - Cachan's presentation based on Constitutive Relation Error (general!)

# An answer : the Constitutive Relation Error Method

[Ladevèze, Pelle, Springer NY 05]
[Ladevèze, Comp. Mech. 08, Pled et al IJNME13,Ladeveze-Chamoin IJNME 10,CMAME 11]

- all other answers for linear quasi-static problems included

- $$|\alpha_{ex} - \alpha_h - \alpha_{hh}| \leq E^h_{CRE} \tilde{E}^h_{CRE}$$

"small" → accurate bounds

Accurate solving of
the adjoint problem

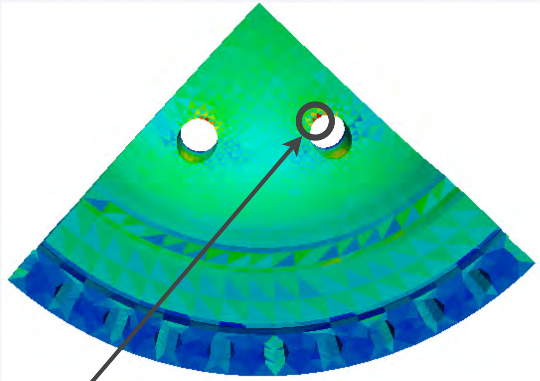
# Technical point : equilibration techniques

- **"Element equilibration technique"EET**
  - General explicit technique
  - [Ladevèze 1975, Pelle, Rougeot, Coffignal, Maunder, Strouboulis-Babuska,...
- **Star patch equilibration technique SPET**
  - General technique
  - [Cartensen-Funken 2000, Morin et al 2002, Choi-Paraschivoiu 2004, Diez et al 2006-2008]
- **Hybrid equilibration technique EESPT**
  - [Ladeveze-Chamoin-Pled 2010]
- **Other equilibration techniques**
  - [Kelly 1984, Ainsworth-Oden 1993, Bank-Weiser 1985...]

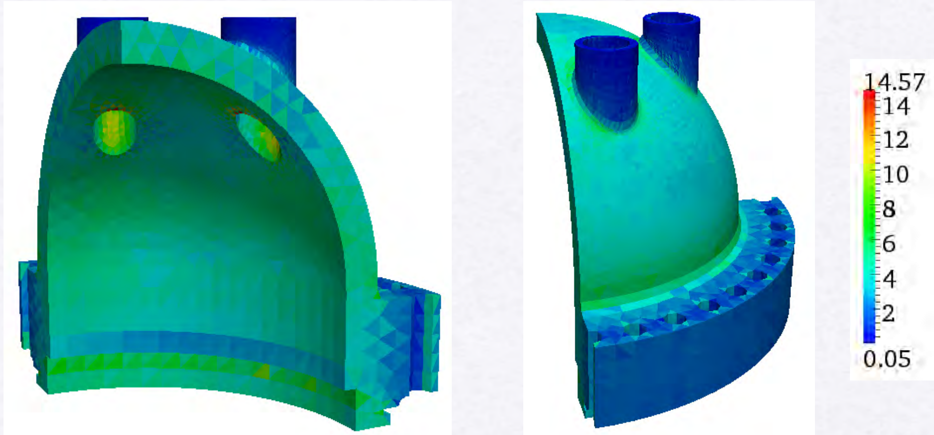
# Results

- Industrial cases

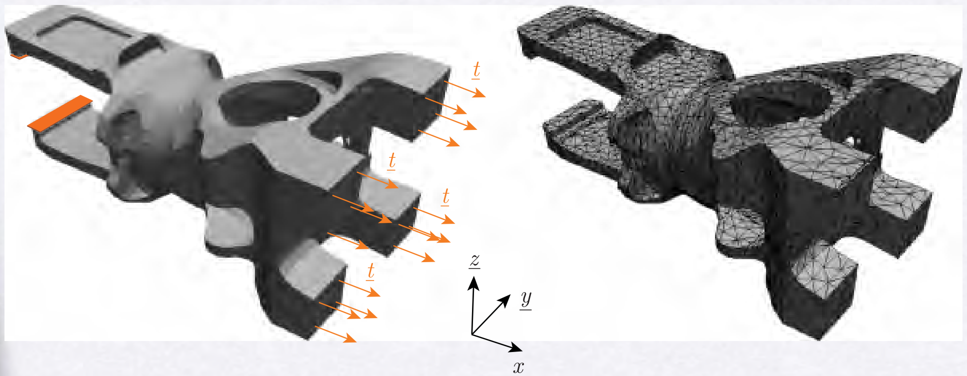
Pled-Chamoin-Ladeveze IJMNE 2011

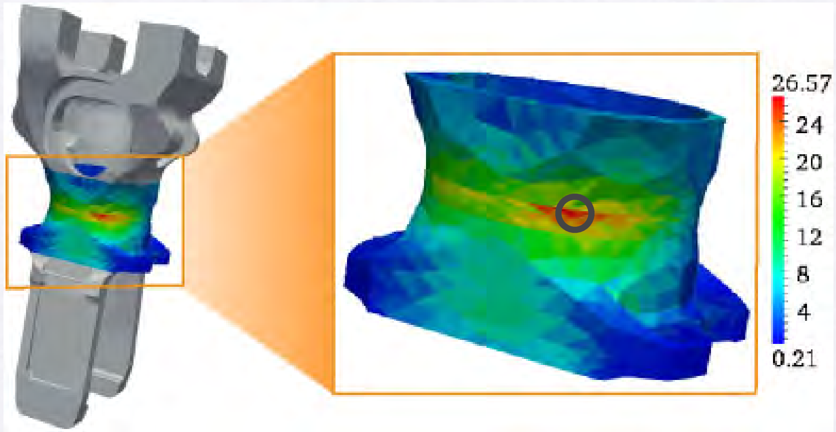


$$I = \frac{1}{|\omega|}\int_\omega \sigma_{VM}\, d\omega$$




$$\boxed{14.08 \leq I_{ex} \leq 14.93}$$



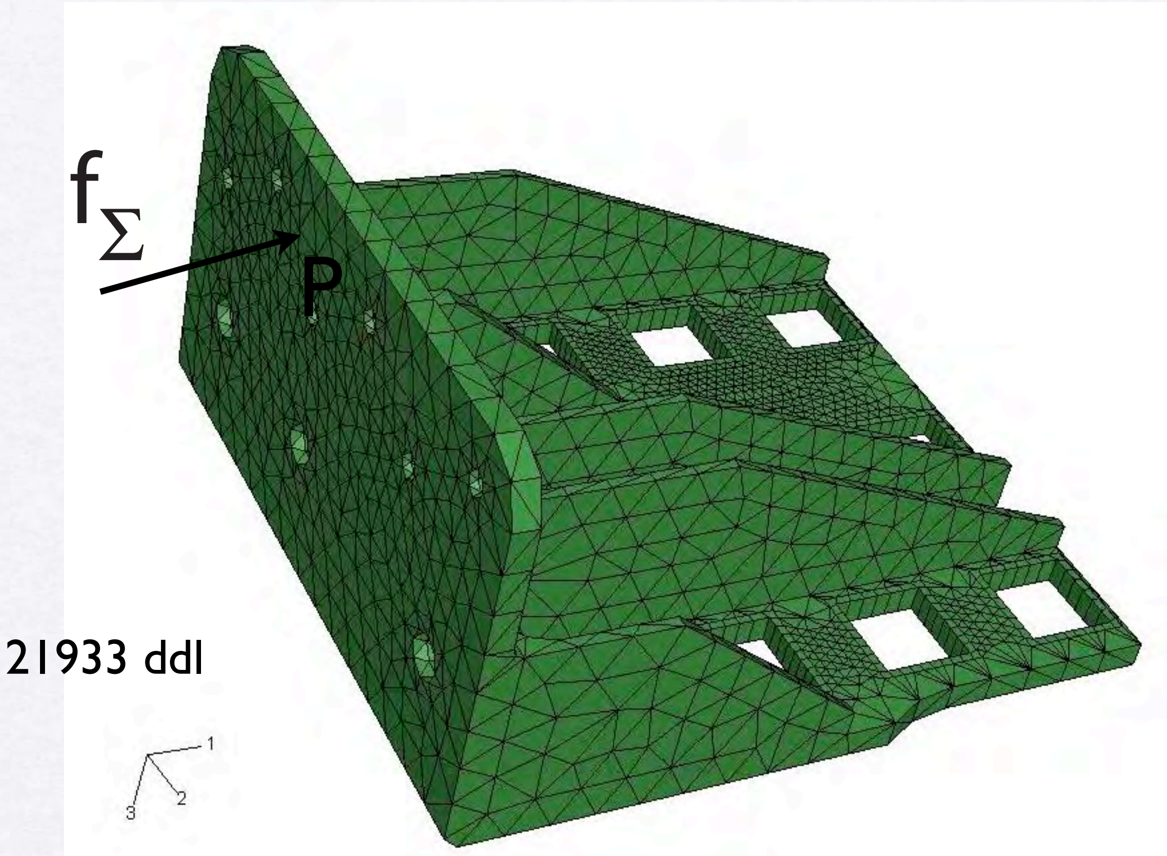



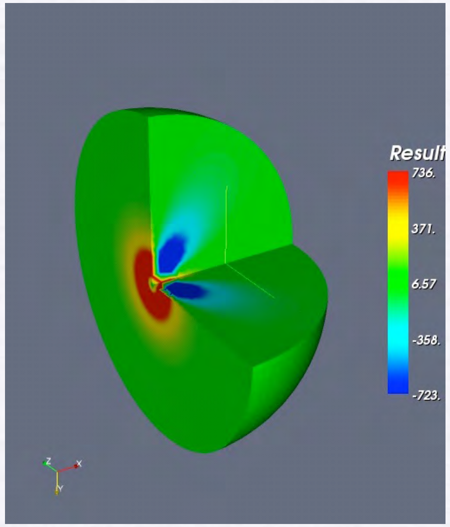

$$\boxed{25.32 \leq I_{ex} \leq 28.24}$$

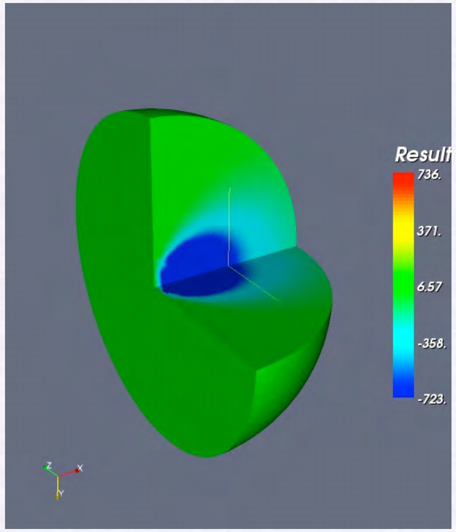
# Results

- Linear elasticity

$$I = u_x(P)$$




$$\frac{\chi_{inf}}{I_{ex}} = 0.93 \qquad \frac{\chi_{inf}}{I_{ex}} = 1.06$$

# To sum up

**MATURE**

- **Goal-oriented verification thank to guaranteed accurate error bounds**
  - linear problems (cost↘,implementation↘ :MOR techniques)
  - stochastic linear problems
  - vibrations
  - X-FEM,Isogeometric FEA
- **Global error estimators and adaptive approaches**
  - multiscale computation
  - nonlinear problems
  - transient dynamics

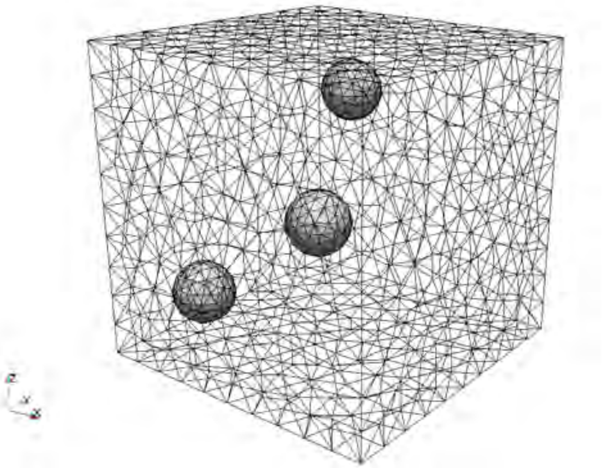
# To sum up

RESEARCH

- **Goal-oriented verification thank to guaranteed accurate error bounds**
  - reduced order model
  - multiscale computation
  - transient dynamics
  - nonlinear problems, instabilities

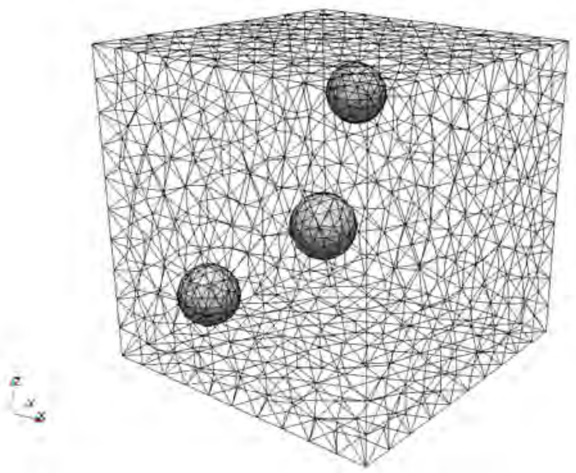
# Ilustration 3 : PGD -virtual chart

[Allier *et al., in preparation*]



- elasticity problem (small perturbations)
- 3 extra-parameters: $E_i$ (in each inclusion;+- 70%)
- quantity of interest: $Q = \max_{K \in T_h} \frac{1}{|K|} \int_K \sigma_{11} dx$

$$Q : \sum_{i=1}^{m} \alpha_i(E_1)\beta_i(E_2)\gamma_i(E_3)$$

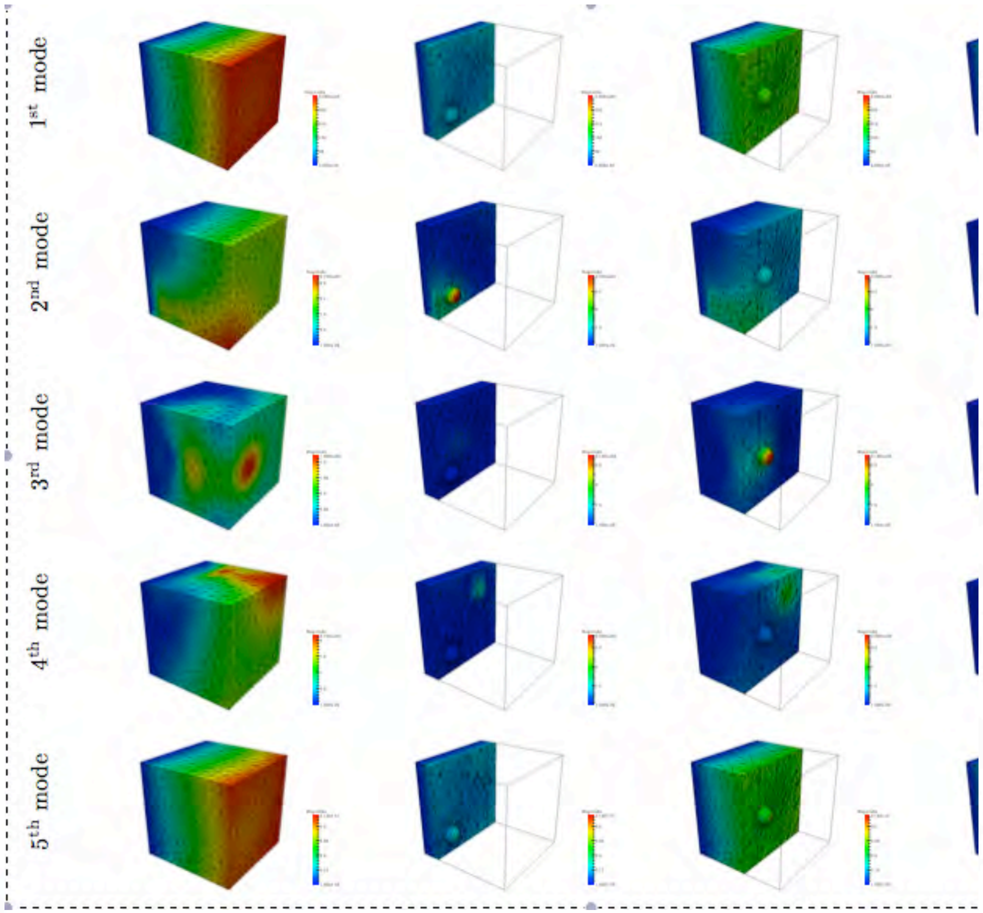


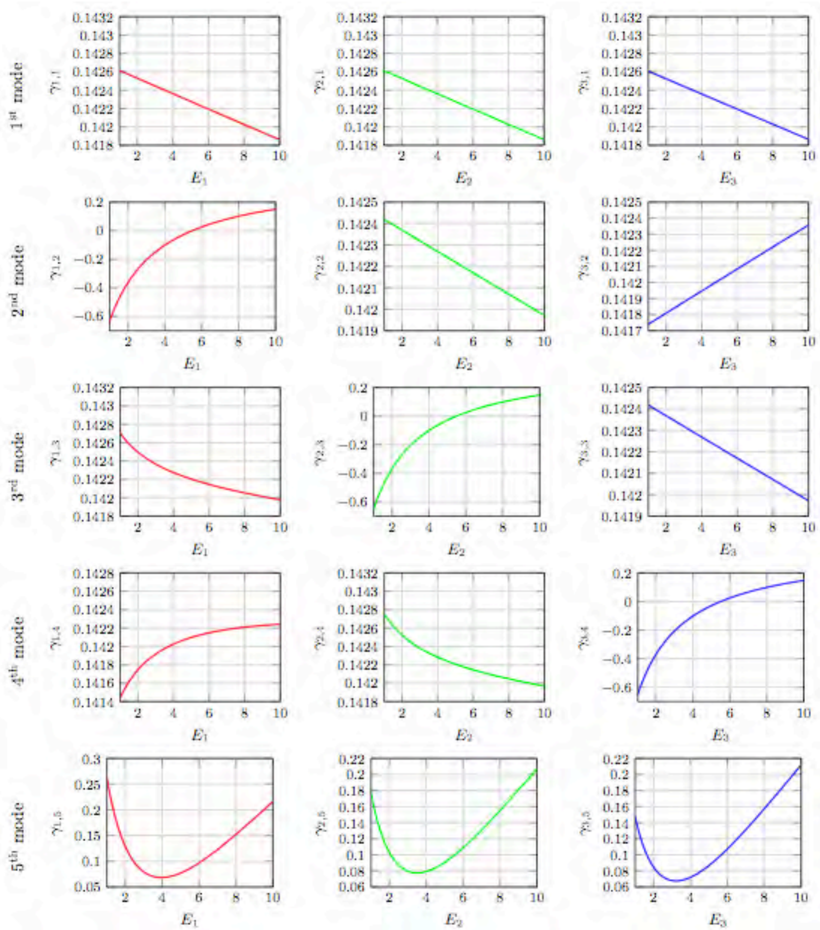
# 3D Example

[Allier *et al., in preparation*]

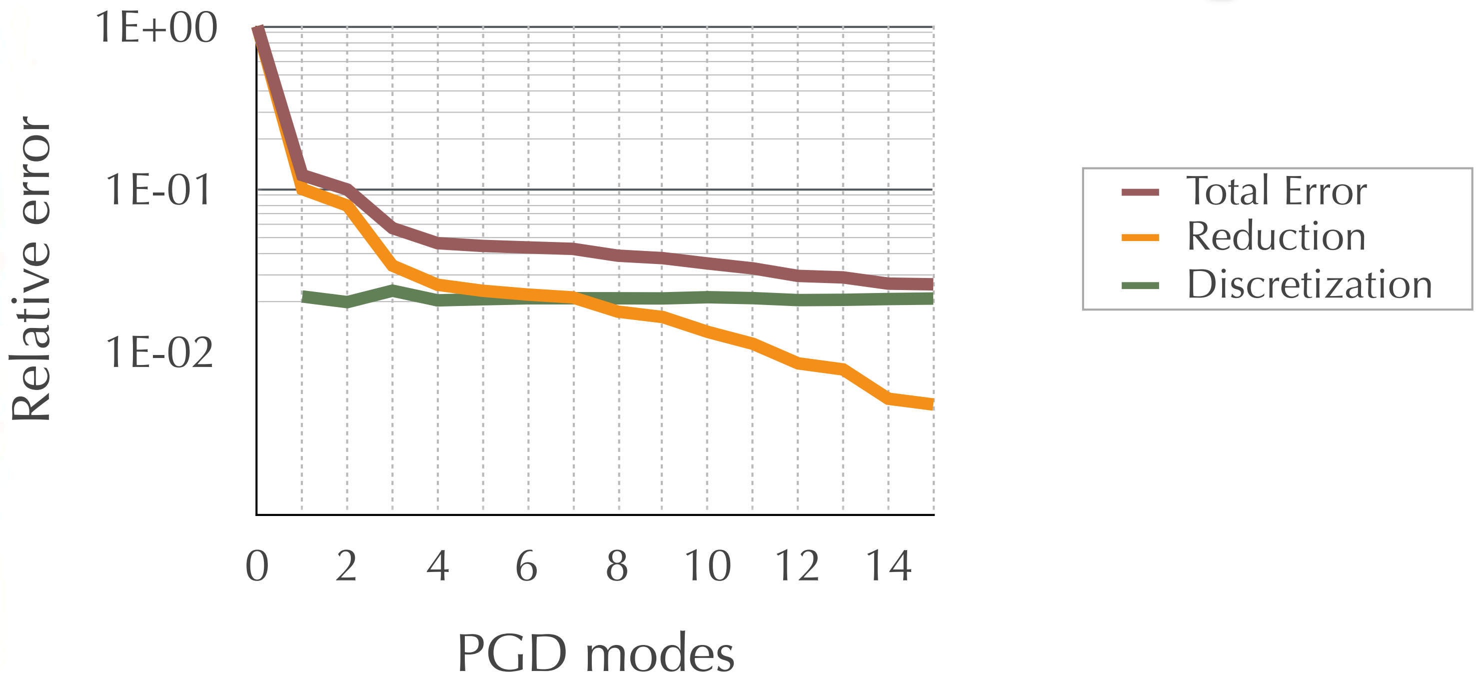
- elasticity problem (small perturbations)
- 3 extra-parameters: $E_i$ (in each inclusion)
- quantity of interest: $Q = \max_{K \in T_h} \frac{1}{|K|} \int_K \sigma_{11} dx$

## First PGD Modes



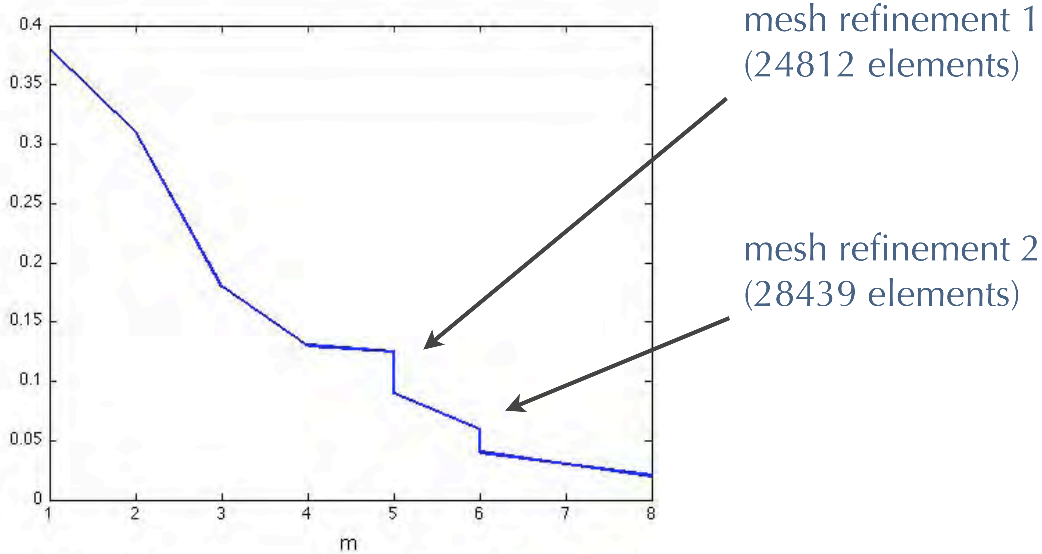



# Error Estimation & Adaptive Strategy




## Adaptive strategy for the quantity of interest

$$\frac{E_{CRE} \cdot \tilde{E}_{CRE}}{|Q(u_m) + Q_{corr}|}$$



# Part 2 : Validation

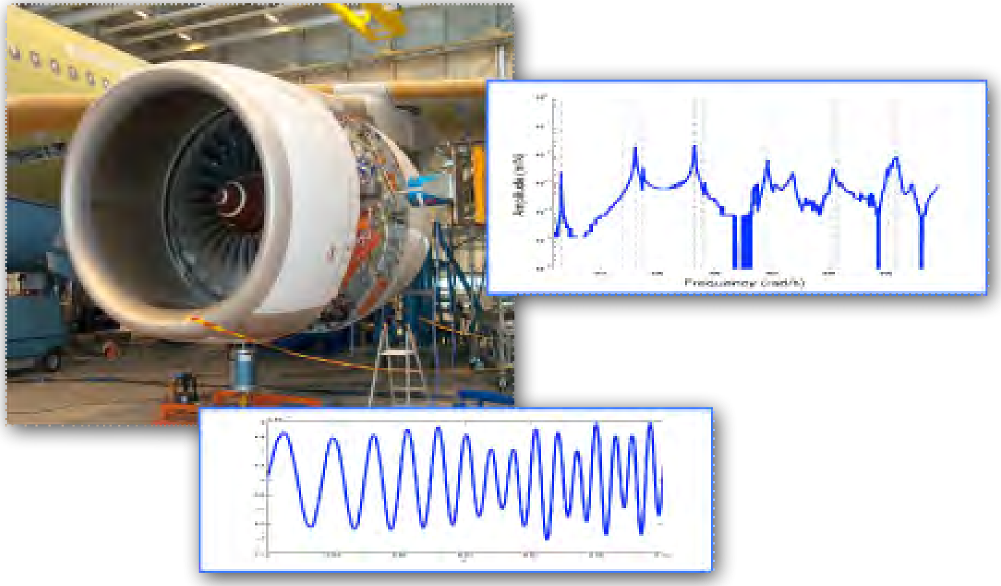
# Model validation related to a set of well-chosen tests

- **example**: validation and updating of dynamic structural models (LF)

Model error

Error: X%

Adaptive improvement (updating)



# The control of models - validation

Reference model: a set of tests

**Approach** **Strong a priori knowledge of the model**

- **Residual on test results**
  [Ohayon et al. 80], [Baruch 82], [Link et al. 91], [Natke 91], [Ewins 88], [Lallement et al. 83], [Chen et al. 89], [Mottershead and Friswell 93], [Berman 95], [Zimmerman 95], [Hemez and Farhat 95], [Hemez et al 02] ...

- **Constitutive Relation Error estimation**
  [Ladevèze 83, 94, 05], [Balmès 96], [Bonnet et al 94], [Bricout et al 97], [Aubry et al 98], [Cogan et al. 01], [Golinval 96], [Reynier 90], [Chouaki 97], [Deraemaeker 01],[Constantinescu 05] [Bouillard 05], [Allix et al 06,08,14],[Ladeveze ,Puel et al 06] [Chrysokkos et al 08 05][Bonnet-Aquino 14]…

**+ Inverse Problems litterature**

# The control of models - validation

Reference model: a set of tests

**Error estimation based on the concept of Constitutive Relation Error**

- **Idea : to take as reference**
  - the reliable experimental data (ex: sensor location, eigenfrequencies)
  - a part of the model : the reliable equations (ex: equilibrium equations)
- **Computation of an error estimation and the local contributions**
  - if updating is unable to decrease the error :the used model is too coarse

# The control of models - validation

- **Validation/updating : State-of-the-Art**
  - linear models with damping/ vibration tests (stochastic models, noisy measures)
  - quasi-static linear model

# The control of models - validation

## Illustration 1: modeling↗↗↗

**Validation of a structure model with damping**

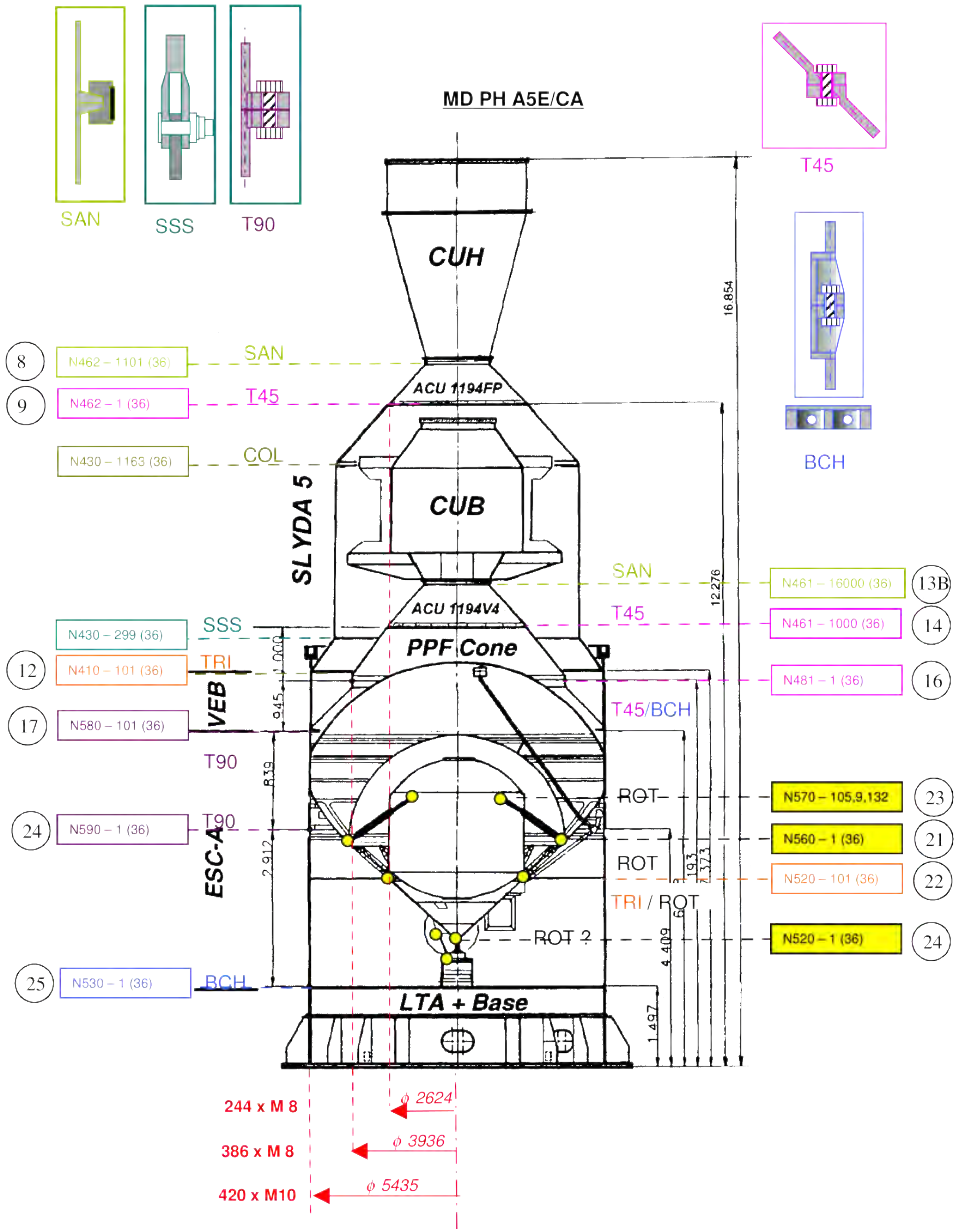
**Damping data**

—damping coefficient of the composite layer

— connections : friction coefficient, prestress

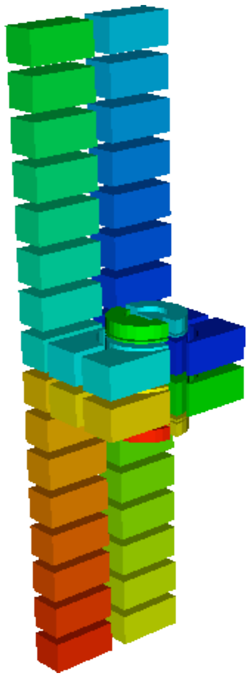
LMT-Cachan/CNES/ AIRBUS D&E

# Global modes of MS-ESCA

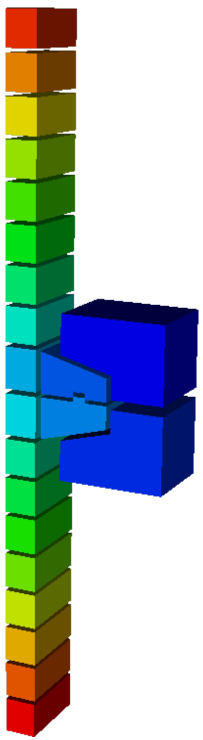


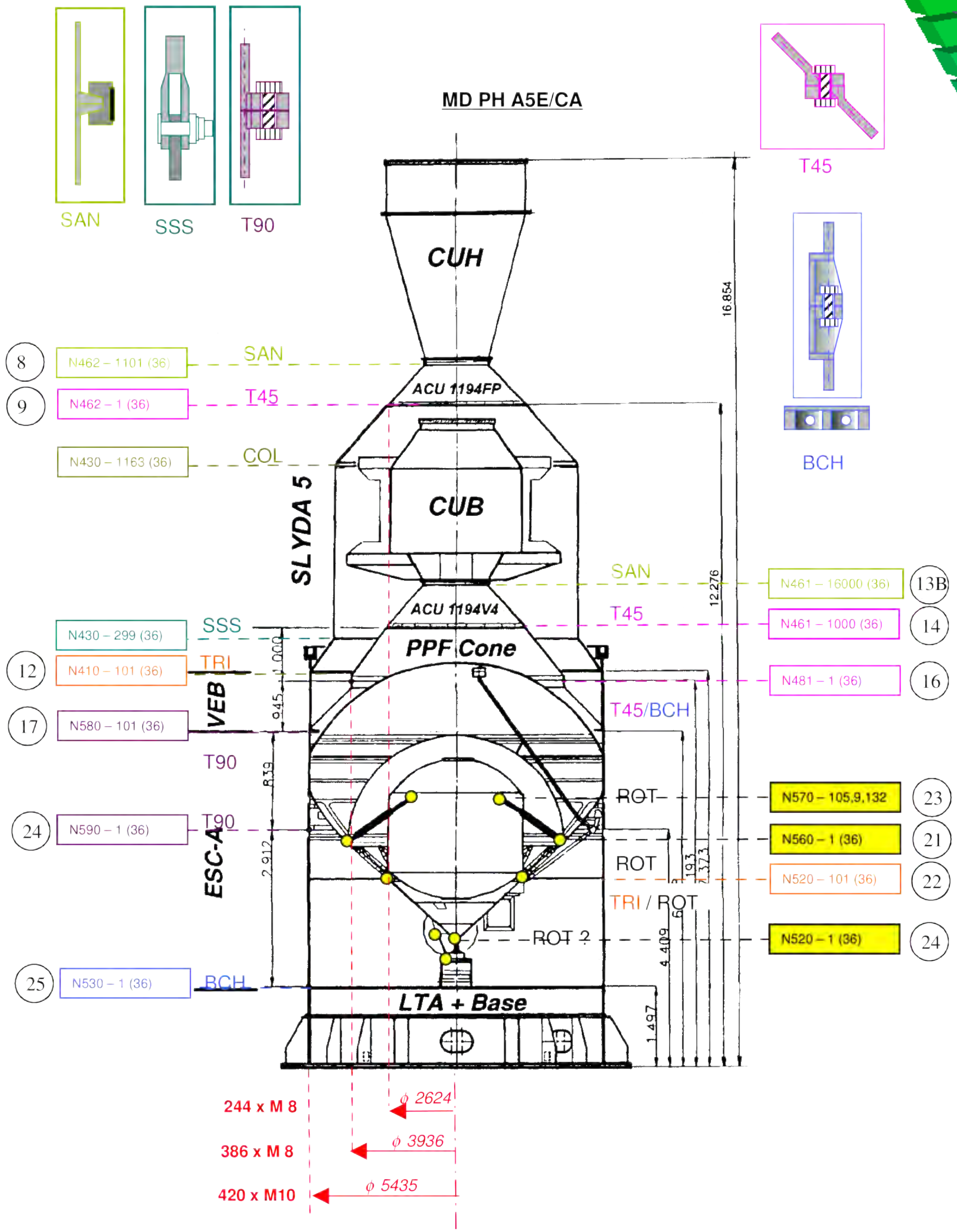
# Global modes of MS-ESCA

BT90

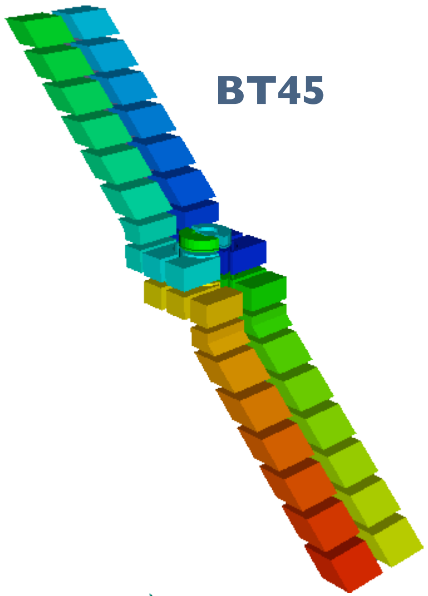


STRAP

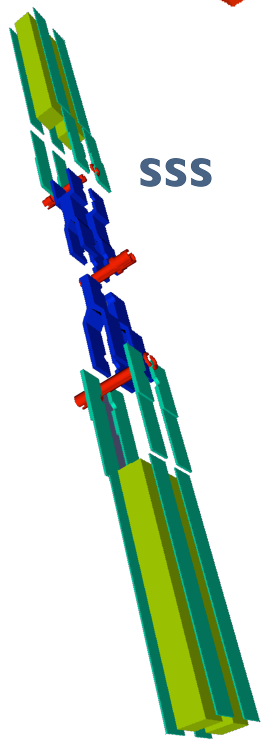




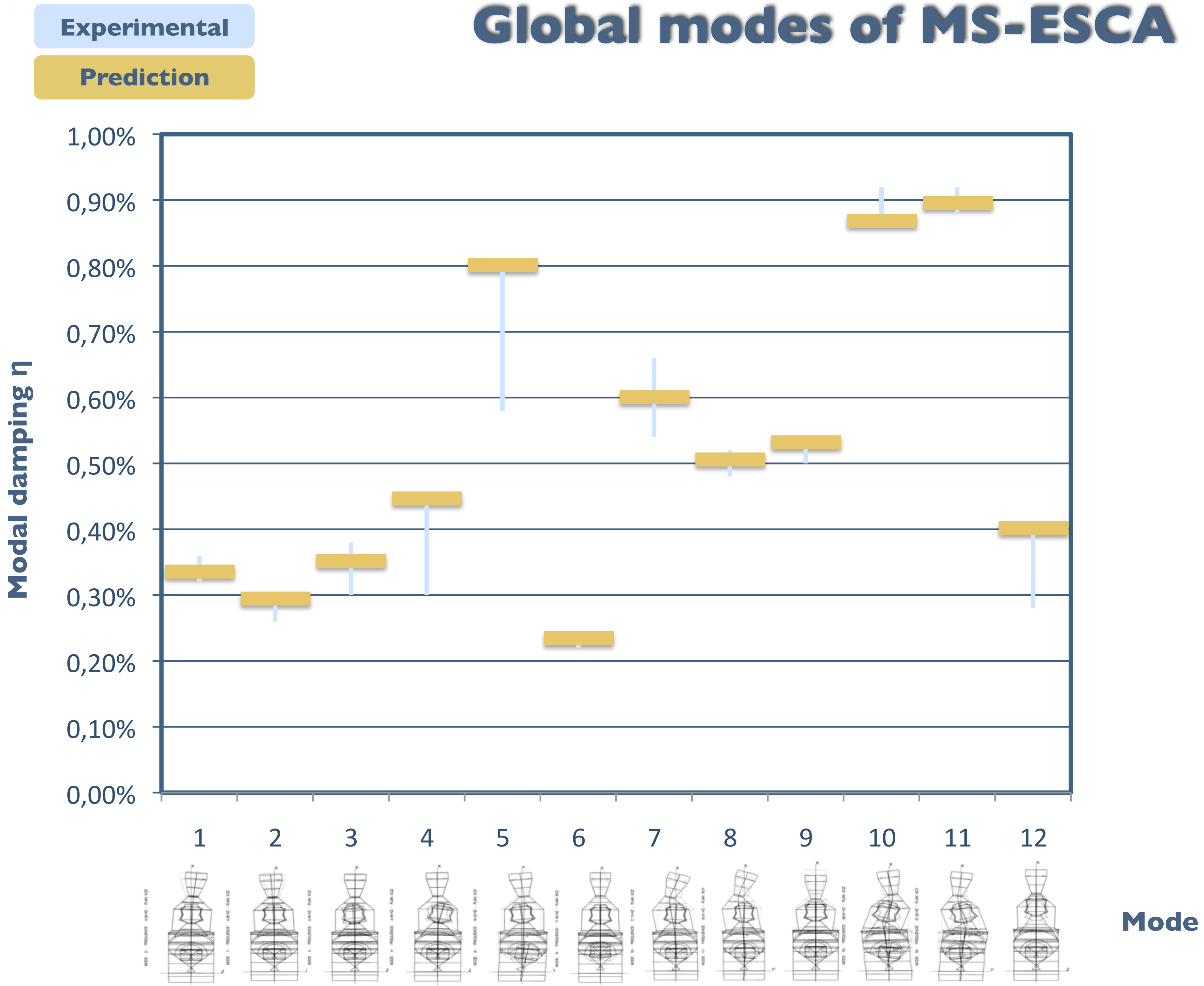

BT45



SSS



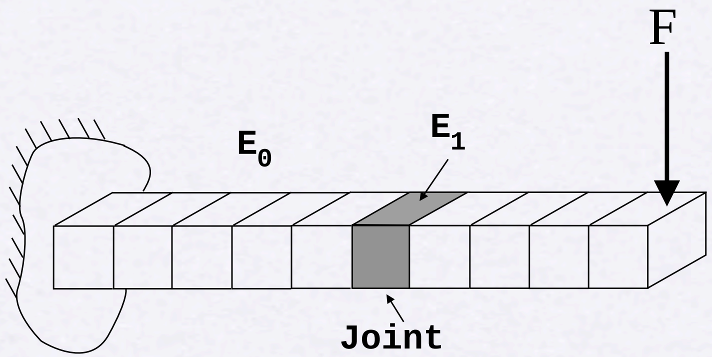
# Global modes of MS-ESCA



# The control of models - validation

**Illustration 2:** Validation/updating of a stochastic model

- The model




| | |
|---|---|
| Length | 10 m |
| Section | $1.10^{-2}$ m$^2$ |
| Moment of Inertia | $8.33.10^{-6}$ m$^4$ |
| Density | 2700 kg/m$^3$ |

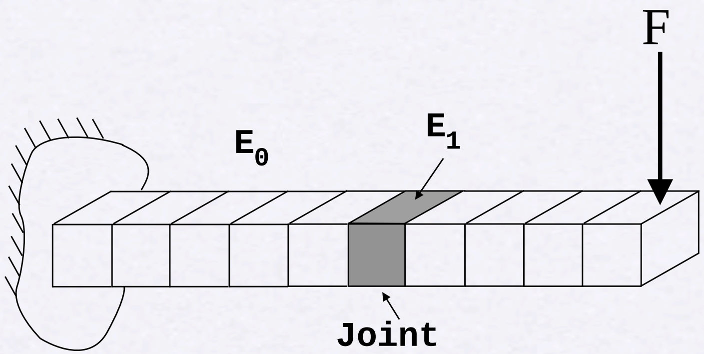
$E(1+\alpha\delta)$, $\delta$ is a Gaussian variable with zero mean and unitary standard deviation

| | Initial model | |
|---|---|---|
| | $E_0, \alpha_0$ | $E_1, \alpha_1$ |
| Case 1 | $71.10^9$ Pa, 0 | $71.10^9$ Pa , 0.1 |
| Case 2 | $71.10^9$ Pa, 0.02 | $71.10^9$ Pa , 0.02 |

# The control of models - validation

**Illustration 2:** Validation/updating of a stochastic model

■ Measurements




| | |
|---|---|
| Length | 10 m |
| Section | $1.10^{-2}$ m$^2$ |
| Moment of Inertia | $8.33.10^{-6}$ m$^4$ |
| Density | 2700 kg/m$^3$ |

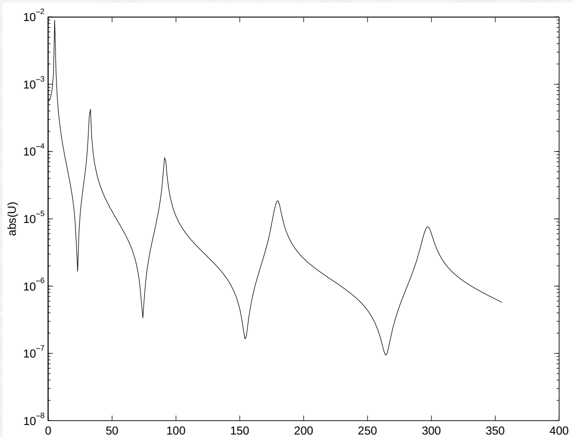
Frequency bandwidth contains **5 modes**

Hysteretic damping of 1%

Ten accelerometers equally spaced on beam

$E(1+\alpha\delta)$, $\delta$ is a Gaussian variable with zero mean and unitary standard deviation

| | Perturbed model | |
|---|---|---|
| | $E_0, \alpha_0$ | $E_1, \alpha_1$ |
| Case 1 | $71.10^9$ Pa, 0 | $142.10^9$ Pa, 0.1 |
| Case 2 | $71.10^9$ Pa, 0.05 | $71.10^9$ Pa, 0.05 |

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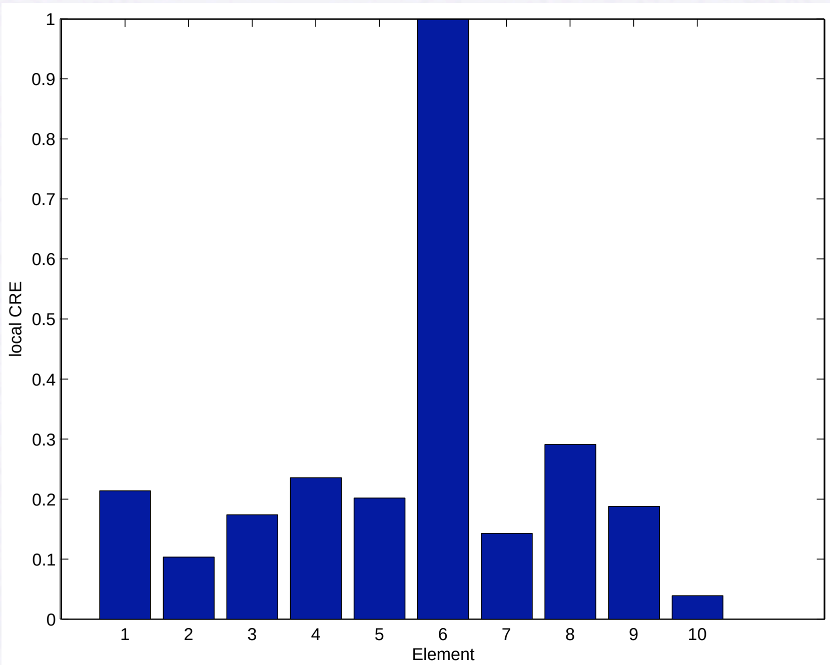
# The control of models - validation

**Illustration 2:** Validation/updating of a stochastic model

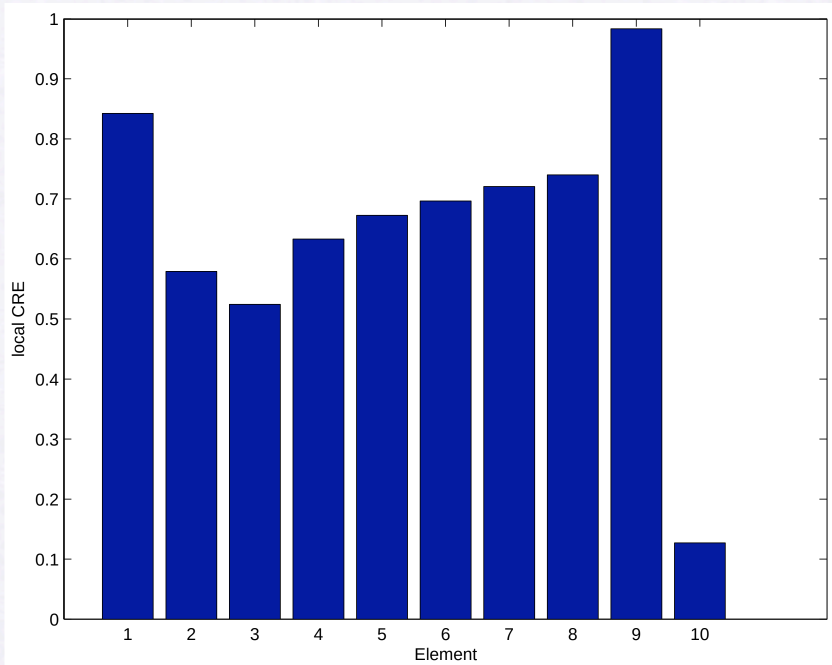
- CRE-estimation

Case 1




$\overline{E_{CRE}}(s) = 6.8\%, \bar{I} = 5.58\%$

Case 2




$\overline{E_{CRE}}(s) = 2\%, \bar{I} = 0.22\%.$

# The control of models - validation

RESEARCH

- Dynamic Data Driven Application Systems( DDDAS)

- Identification/validation of material model  (4D-images , ministructure)-Eikology

- The true validation model and the quantification of lacks of knowledge

# Dynamic Data Driven Application System

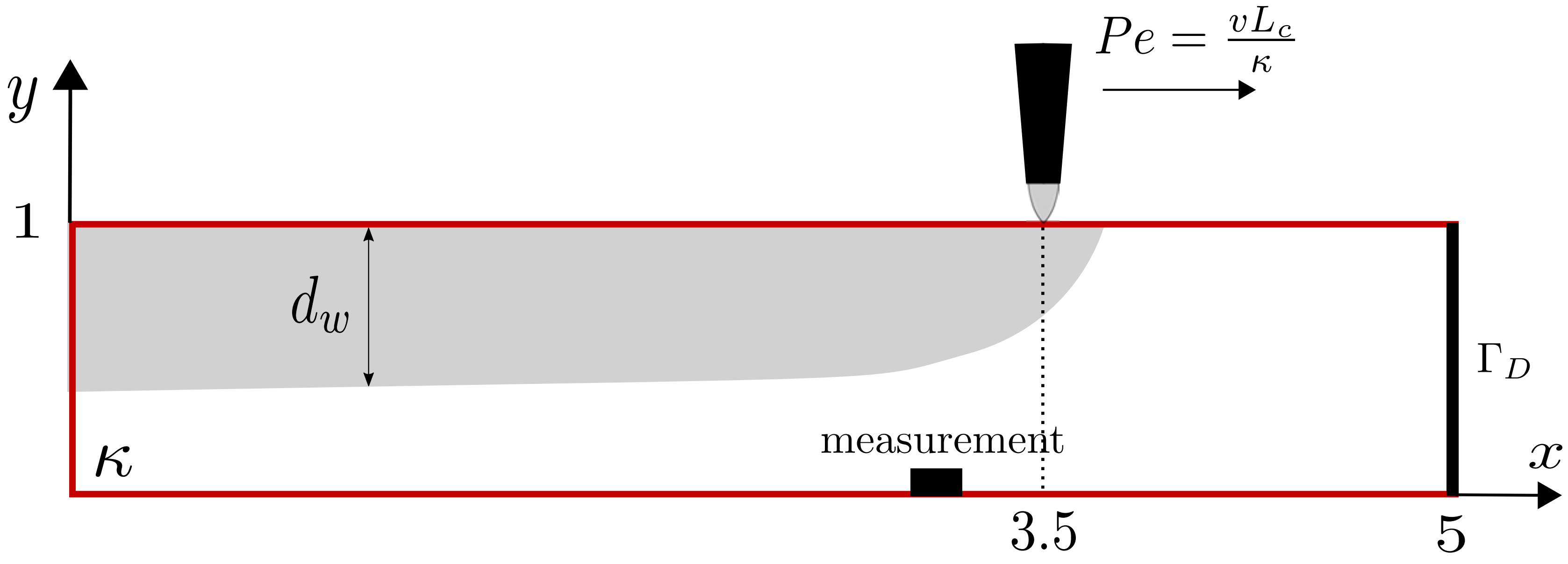
- realtime parameter-updating + reduced model)

# Illustration 4: Soudage en situation variable

- Problème adimensionné

$$T(x,y;t) \equiv \frac{\overline{T}(x,y;t) - \overline{T}_\infty}{\overline{T}_f - \overline{T}_\infty}$$

- Système de coordonnées se déplaçant avec la source



# The control of models - validation

RESEARCH

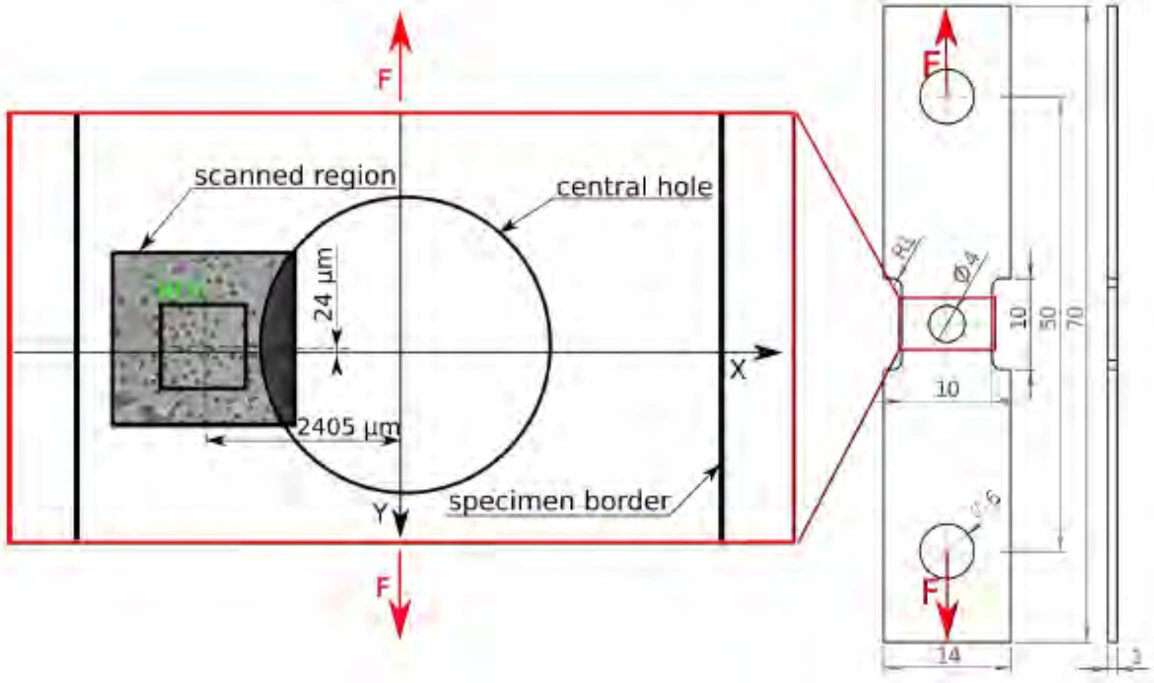
- Dynamic Data Driven Application Systems( DDDAS)
- Identification/validation of material model (4D-images , ministructure)-Eikology
- The true validation model and the quantification of lacks of knowledge

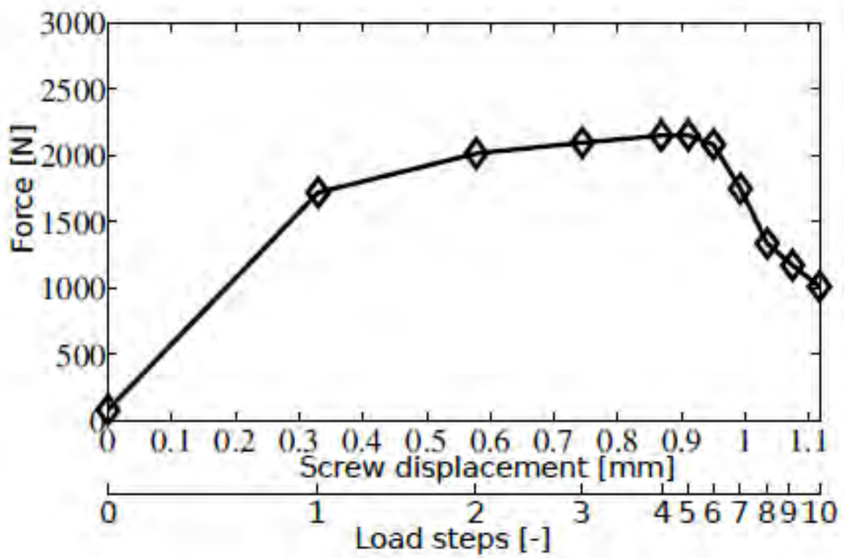
# Identification/validation of a material model-4D images (Eikology)

Hild , Roux et al 2016

# Central Hole Geometry

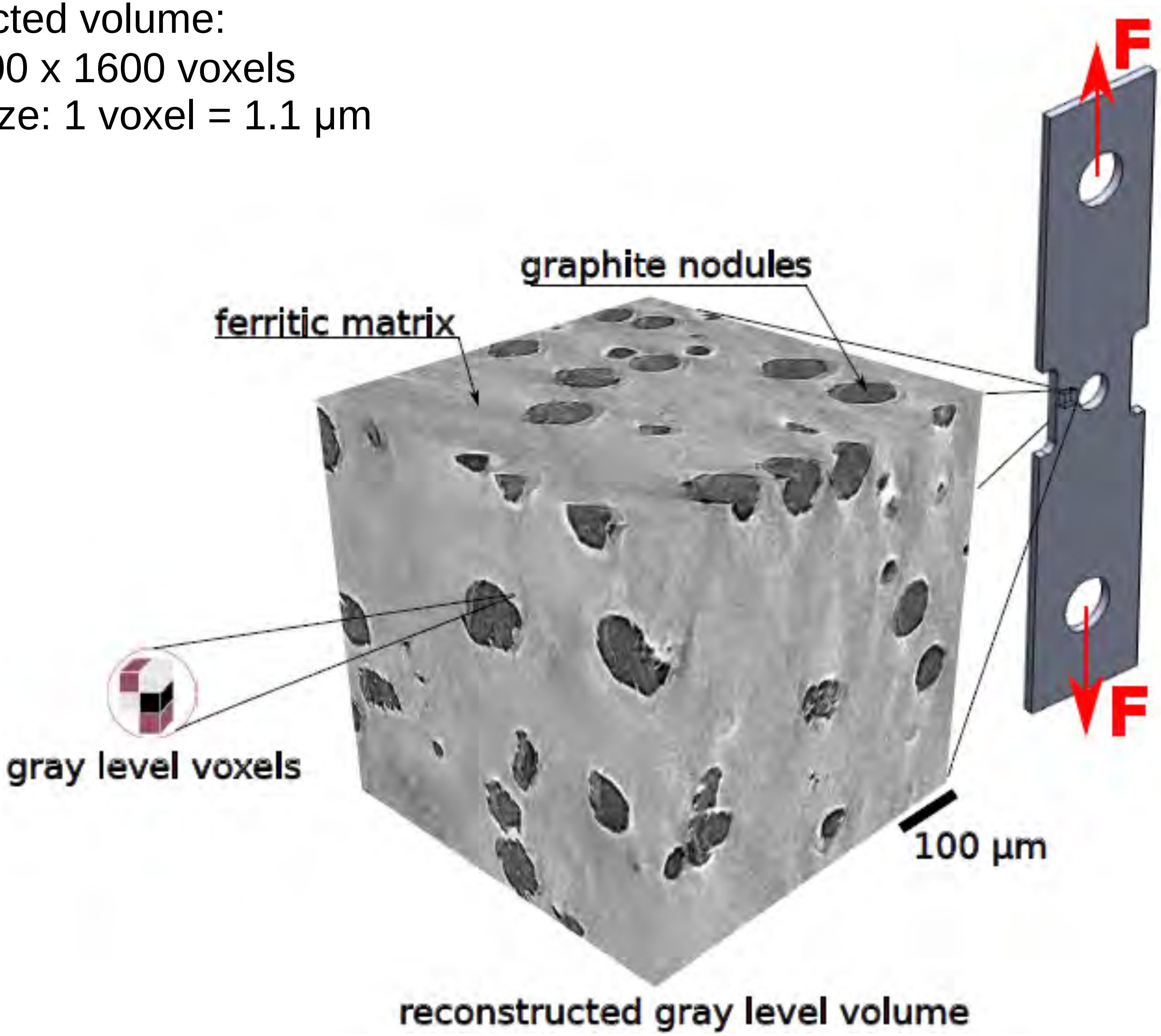
- Stepwise loading / scan procedure (laminography @ ESRF)



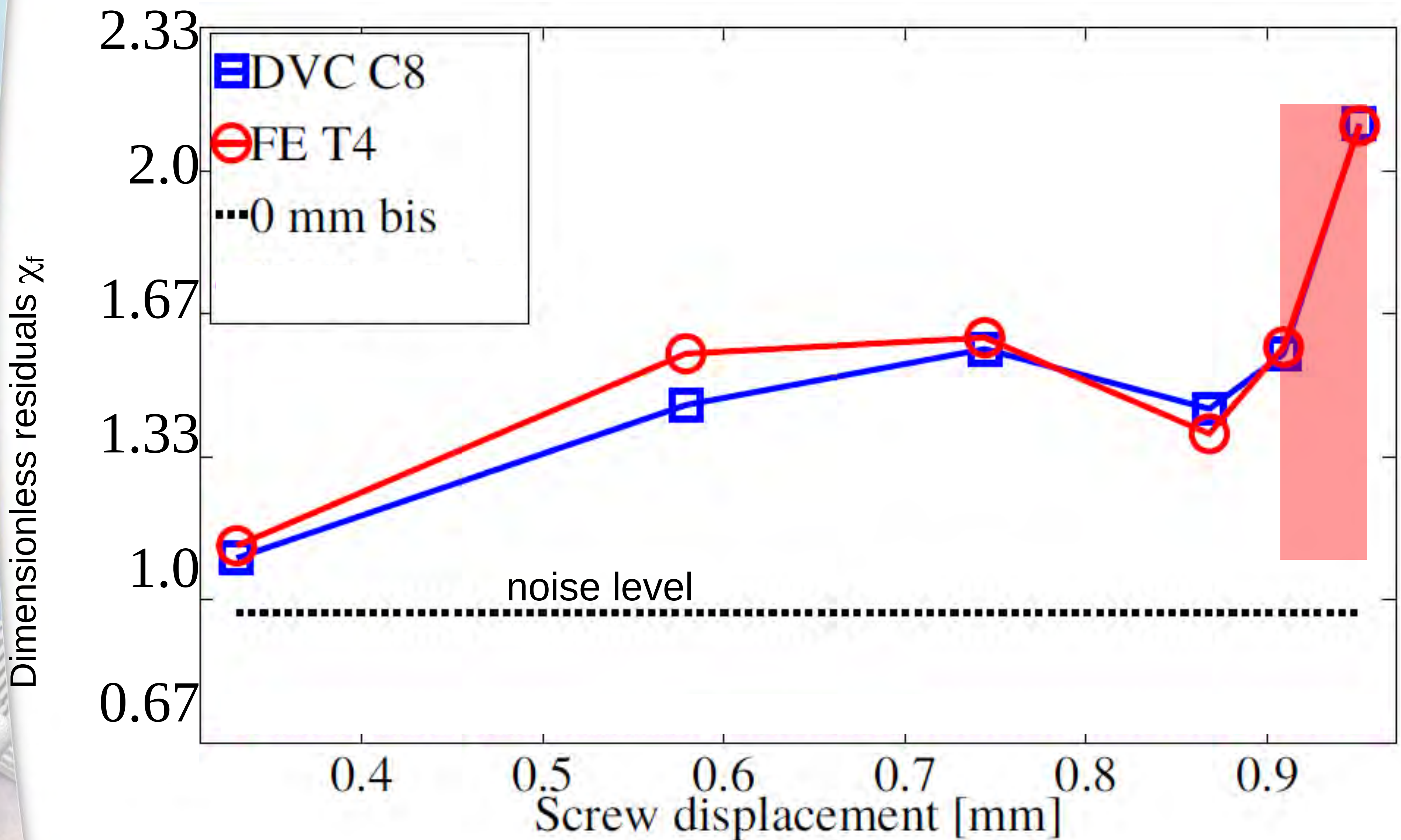



# Studied Material: NG Cast Iron

Reconstructed volume:
1600 x 1600 x 1600 voxels
Physical size: 1 voxel = 1.1 µm



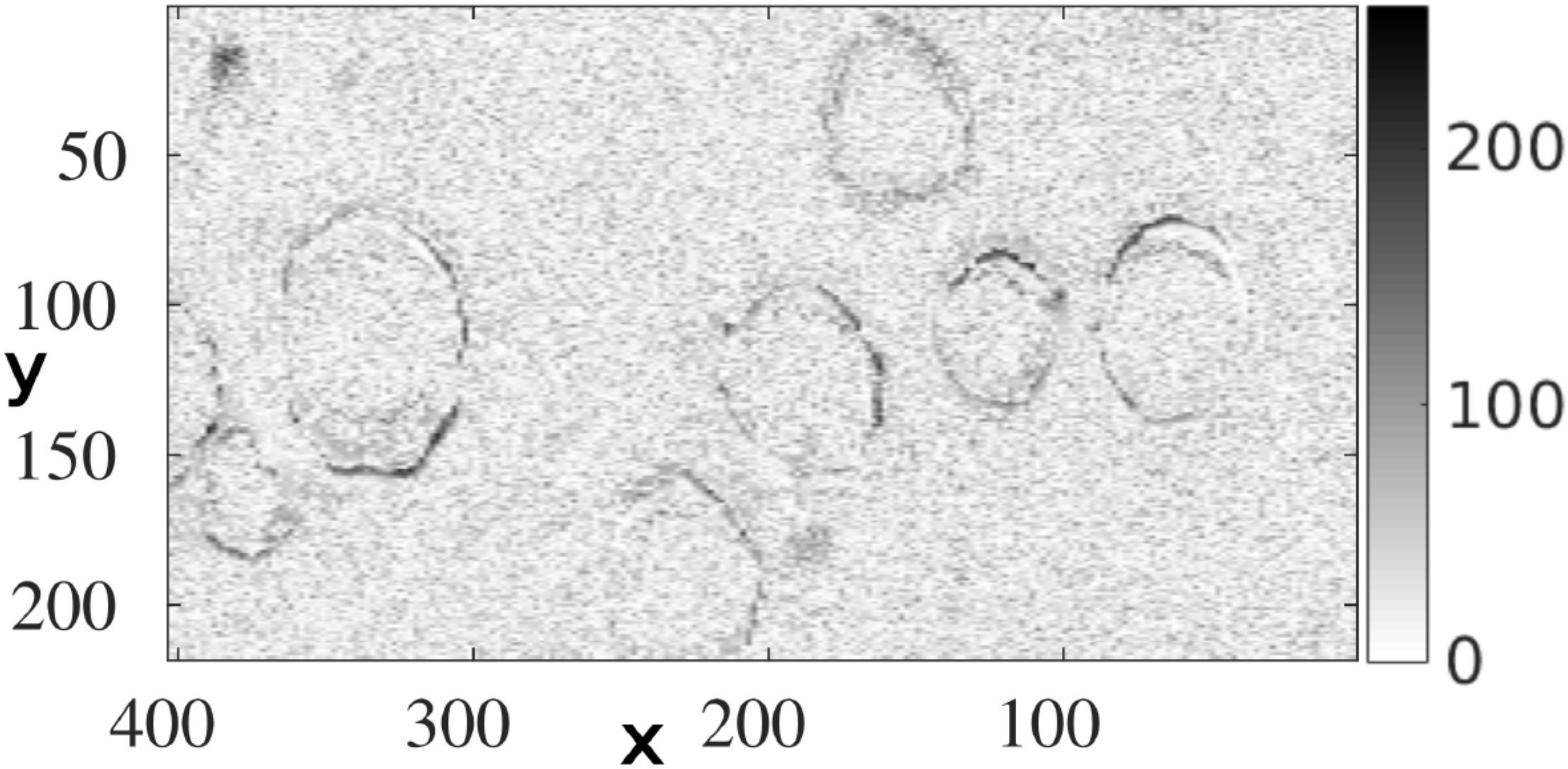
# Validation?



2.33
2.0
1.67
1.33
1.0
0.67
DVC C8
FE T4
0 mm bis
noise level
0.4
0.5
0.6
0.7
0.8
0.9
Screw displacement [mm]
Dimensionless residuals $\chi_f$

# Final Residual (FE)

$$\left| f(\mathbf{x}) - g(\mathbf{x} + \mathbf{u}_{FE}(\mathbf{x})) \right|$$



# The control of models - validation

RESEARCH

- Dynamic Data Driven Application Systems( DDDAS)

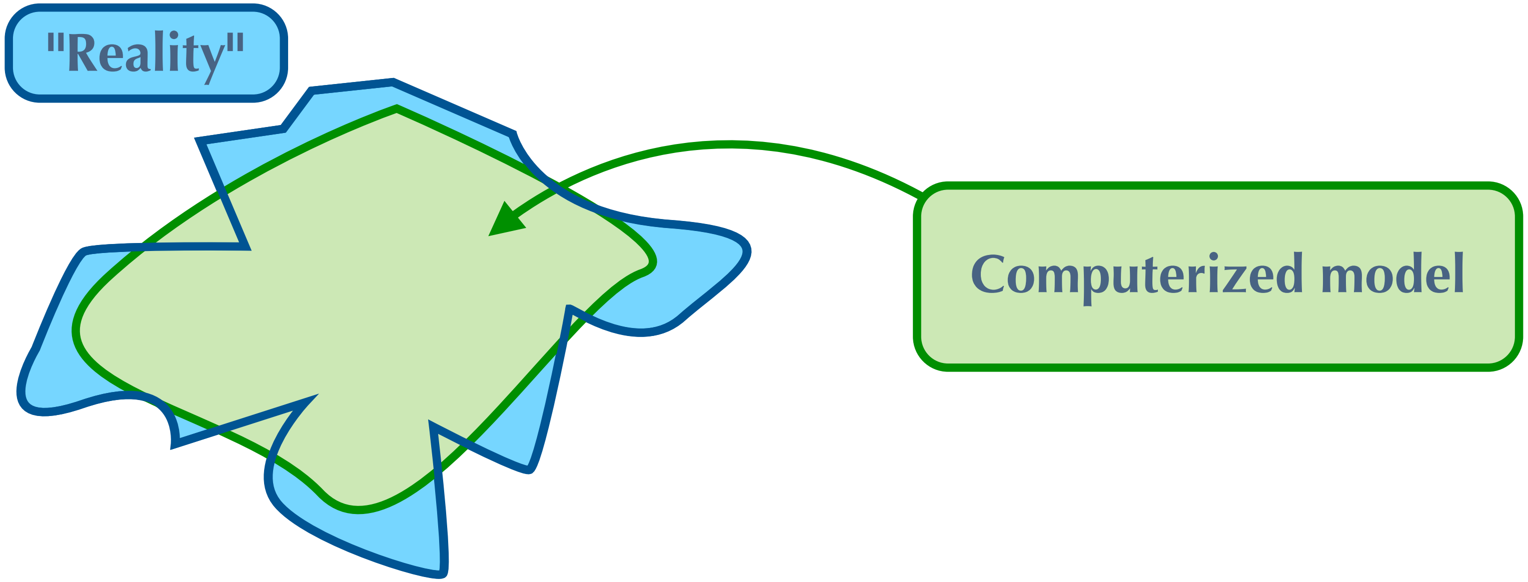
- Identification/validation of material model  (4D-images , ministructure)-Eikology

- The true validation model and the quantification of lacks of knowledge

# The control of models - validation

## The true validation problem ?

**Challenge: model validation related to the "complete" reality**




**controversial issue**

*[Popper], [Oden et al.], [Hemez et al.], [Ben-Haim] ...*

**for some people**

a model can only be invalidated but never validated

# The control of models - validation

**The true validation problem ?**

practical rules and experiments

Safety factors,margins (lack of knowledge)

The built structure performs as expected

**Model validation related to the "engineer" reality**

**Real issue and research challenge**

to go beyond the philosophical level, to elaborate practical tools?

# The control of models - validation

- a first attempt: **the Lack-of-Knowledge Theory** *[CMAME 06]*
  ***[Ladevèze 02], [Puel], [Enjalbert], [Barthe], [Romeuf]***
  LMT-Cachan / EADS

**Classical model** + **Lacks of knowledge**

↓

**"Envelope" solution (reality: inside)**

**validation:** wideness of the "Envelope" solution

**lack-of- knowledge reduction from additional information** :
Baysian approach

***Classic tools for taking incertainties into account:***

***probabilities [Ghanem,Schueller], fuzzy sets [Zadeh 65], random sets [Dempster 68, Shafer 76], convex info-gap models [Ben-Haim, Elishakoff 90], nonparametric models [Soize 05] ...***

# LOK's illustration

**Sylda 5**
double launch
system of Ariane 5

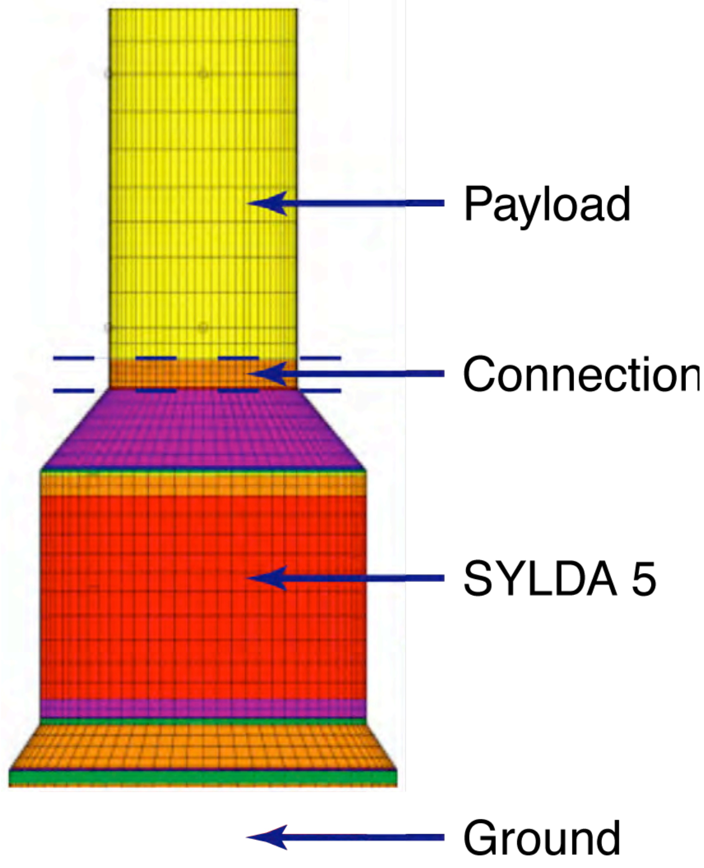
Vibration tests performed by IABG for
DASA/DORNIER under CNES contract

# LOK's illustration

First model

## SYLDA5: calculation of free vibration modes (LF)

- Studied substructures
  * $E_1$ = cylinder
  * $E_2$ = connexion
  * $E_3$ = SYLDA5
  * $E_4$ = ground
- Initial LOKs




| Substructure | Law beeing sought | $(\overline{m}_E^{-0}, \overline{m}_E^{+0})$ | Statistical moment |
|---|---|---|---|
| $E_1$ | normal | (-0.25,0.25) | ($\mu = 0.00$ / $\sigma = 0.097$) |
| $E_2$ | uniform | (-0.25,0.25) | ($\mu = 0.00$ / $\sigma = 0.097$) |
| $E_3$ | normal | (-0.25,0.25) | ($\mu = 0.00$ / $\sigma = 0.097$) |
| $E_4$ | uniform | (-0.75,0.75) | ($\mu = 0.00$ / $\sigma = 0.289$) |

Puel et al 07

# LOK's illustration

Test results

- Vibration tests (first **8** frequencies)

$$(\tilde{\alpha}_{exp} - \overline{\alpha}) = \Delta\tilde{\alpha}_{exp}$$

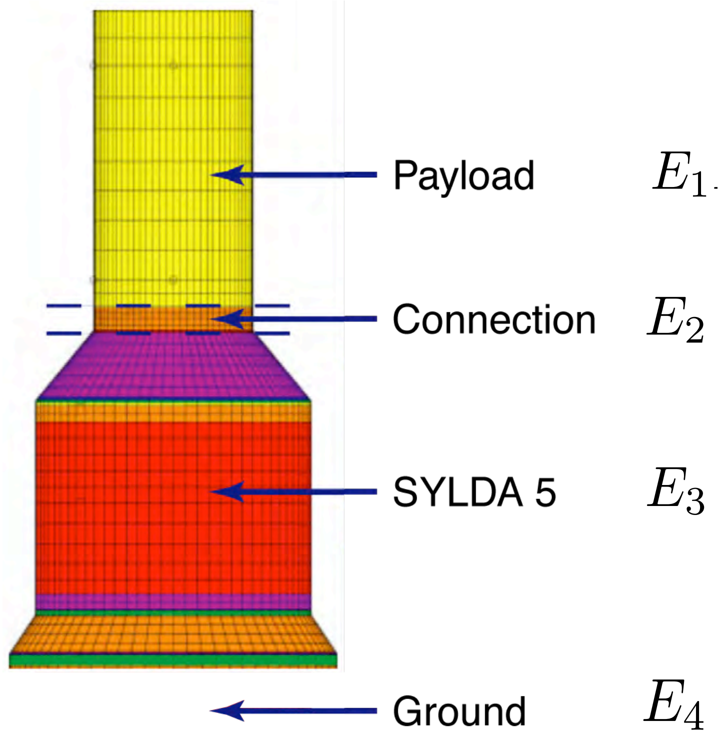
"extreme" value

$$\Delta\alpha_{exp}(\theta) \in [0, \Delta\tilde{\alpha}_{exp}]$$

- Reduced basic LOKs ( $\rho_E = 1$ ; iterative process)

# LOK's illustration

Updated model




- Reduced basic LOKs

| Substructure | Basic LOKs | | | Reduced LOKs | | |
|---|---|---|---|---|---|---|
| | Law beeing sough | $[\overline{m}_E^{-0}, \overline{m}_E^{+0}]$ | Statistical moment | Law | $[-\overline{m}_E^{-}; \overline{m}_E^{+}]$ | Statistical moment |
| $E_1$ | normal | [-0.25,0.25] | $\mu = 0.00$ / $\sigma = 0.097$ | normal | $[0; 0.144]$ | $\mu = 0.072$ / $\sigma = 0.028$ |
| $E_2$ | uniform | [-0.25,0.25] | $\mu = 0.00$ / $\sigma = 0.097$ | uniform | $[-0.060; 0]$ | $\mu = -0.030$ / $\sigma = 0.012$ |
| $E_3$ | normal | [-0.25,0.25] | $\mu = 0.00$ / $\sigma = 0.097$ | normal | $[-0.016; 0]$ | $\mu = -0.008$ / $\sigma = 0.003$ |
| $E_4$ | uniform | [-0.75,0.75] | $\mu = 0.00$ / $\sigma = 0.289$ | uniform | $[0; 0.521]$ | $\mu = 0.261$ / $\sigma = 0.150$ |

# Conclusions

- Verification and validation tools are available but not used

- Research on V&V: ↘↘↘

# The end

## thank you