Large-scale and infinite dimensional dynamical systems approximation

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Directeur de thèse: Charles POUSSOT-VASSAL
Co-encadrant: Cédric SEREN
1. What about model approximation?
   - Context
   - Objectives and problem formulation

2. My contributions
   - Theoretical contribution: model approximation with time-delay structure
   - Methodological contribution: stability regions for time-delay systems
   - Industrial applications: rhin flow system

3. Conclusions and perspectives

4. Academic outputs
Large-scale dynamical systems

Large-scale systems are present in many engineering fields: aerospace, computational biology, building structure, VLI circuits, automotive, weather forecasting, fluid flow... 

- difficulties with simulation and memory management (e.g., ODE solvers);
- difficulties with analysis (e.g., frequency response, norms computation...);
- difficulties with controller design (e.g., robust, optimal, predictive, etc.);
- ... induce numerical burden;
- ... need for numerically robust and efficient tools.
Context
Large-scale dynamical models

- Highly accurate and/or flexible A/C;
- Spacecraft, launcher, satellites;
- Fluid dynamics (Navier-Stokes);
- High fidelity models.
Context
Large-scale dynamical models

- Physical system
  - Partial Differential Equations (PDEs)
    \[-\nabla \cdot \rho + \rho \, \mathbf{g} = \rho \, \mathbf{a}\]
    \[
    \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \, \mathbf{v}) = 0
    \]
  - Fluid mechanics, structure, etc.

- Fluid dynamics (Navier-Stokes);
- Spacecraft, launcher, satellites;
- Highly accurate and/or flexible A/C;
- High fidelity models.
- Discretisation
  - finite elements, finite volume, etc.

- Simulation, control, analysis, optimisation, etc.
Context
Large-scale dynamical models

- Physical system
  - Partial Differential Equations (PDEs)
    \[\n    \begin{align*}
    \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) &= 0 \\
    -\nabla \rho + \rho \vec{g} &= \rho \vec{a}
    \end{align*}
    \]

- Fluid mechanics, structure, etc.

- Differential Algebraic Equations (DAEs) or Rational Functions
  \[\dot{x}(t) = Ax(t) + Bu(t)\]

- Differential Algebraic Equations (DAEs) or Rational Functions
  \[y(t) = Cx(t) + Du(t)\]

- Discretisation
  - finite elements, finite volume, etc.

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Physical system

Differential Equations (DAEs) or Rational Functions

\[ E\dot{x}(t) = Ax(t) + Bu(t) \]
\[ y(t) = Cx(t) + Du(t) \]
\[ H(s) = H_1(s) + H_d e^{-\tau s} + \ldots \]

Partial Differential Equations (PDEs)

\[ -\nabla \cdot \rho \underline{g} = \rho \underline{a} \]
\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \]

rigid behaviour

fluid mechanics, structure, etc.

Discretisation
finite elements, finite volume, etc.
Context

Large-scale dynamical models

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Physical system

rigid behaviour

Differential Algebraic Equations (DAEs) or Rational Functions

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\dot{x}(t) = Ax(t) + Bu(t) \\
y(t) = Cx(t) + Du(t)
\]

\[
H(s) = H_1(s) + H_d e^{-\tau s} + \ldots
\]

Discretisation

finite elements, finite volume, etc.

Partial Differential Equations (PDEs)

\[
\vec{\nabla} \rho + \rho \vec{g} = \rho \vec{a}
\]

\[
\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{V}) = 0
\]

simulation, control, analysis, optimisation, etc.
**Context**

**Large-scale dynamical models**

- **Physical system**
  - Partial Differential Equations (PDEs)
    \[ \nabla \rho + \rho \mathbf{g} = \mathbf{0} \]
    \[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]

- **Fluid mechanics, structure, etc.**

- **Rigid behaviour**

- **Differential Algebraic Equations (DAEs) or Rational Functions**
  \[ E \dot{x}(t) = A x(t) + B u(t) \]
  \[ y(t) = C x(t) + D u(t) \]

- **Discretisation**
  - Finite elements, finite volume, etc.

- **Simulation, control, analysis, optimisation, etc.**

- **Large-scale**

- Highly accurate and/or flexible A/C;
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Large-scale dynamical models

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E\dot{x}(t) = Ax(t) + Bu(t) \\
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H(s) = H_1(s) + H_d e^{-\tau s} + \ldots
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Partial Differential Equations (PDEs)

\[
-\nabla \cdot \rho + \rho \mathbf{g} = \rho \mathbf{a}
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Highly accurate and/or flexible A/C;
Spacecraft, launcher, satellites;
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Simulation, control, analysis, optimisation, etc.
Large-scale dynamical models

- **Differential Algebraic Equations (DAEs) or Rational Functions**
  \[ E\dot{x}(t) = Ax(t) + Bu(t) \]
  \[ y(t) = Cx(t) + Du(t) \]

- **Partial Differential Equations (PDEs)**
  \[ \nabla p + \rho g = \rho \dot{\mathbf{a}} \]
  \[ \frac{\partial \rho}{\partial t} + \nabla (\rho \mathbf{v}) = 0 \]

- **Physical system**
  fluid mechanics, structure, etc.

- **Rigid behaviour**

- **Discretisation**
  finite elements, finite volume, etc.

- **Simulation, control, analysis, optimisation, etc.**

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⇒ **Objective:** alleviate numerical burden
  - allows to increase simulation speed while preserving precision.
  - allows to apply modern analyses and control techniques.
Context
Large-scale dynamical models

Example: **Cable mass model simulation**

- Full model $N = 960$ $\rightarrow$ Simulation time $\approx 23.70s$.
- Reduced model $n = 10$ $\rightarrow$ Simulation time $\approx 0.02s$.
- Approximation time $\approx 4.03s$.
Context
Realization-less model approximation

Reference modelings of interest
- [i/o] data-driven models;
- Time-Delay Systems (TDS);
- PDE-based descriptors...

Main concern
- Derive suitable low order models
Objectives and problem formulation
Model approximation \sim mathematical optimization

Objectives

Find a reduced order modeling \( \hat{H} \) for which:
- the approximation error is small;
- the stability is preserved...

... from an efficient and computationally stable procedure.

The quality of the approximation can be evaluated using some mathematical norms. Find

\[
\hat{H} := \begin{cases}
\hat{E}\dot{x}(t) = \hat{A}x(t) + \hat{B}u(t) \\
y(t) = \hat{C}x(t)
\end{cases}
\]

s.t.:

\[\|H - \hat{H}\|^2 \text{ is minimum} \rightarrow \text{optimisation problem to solve}\]
Objectives and problem formulation

\( \mathcal{H}_2 \) model approximation problem

Mathematical formulation

Find \( \hat{H} \) of order \( r << n \) which minimizes\(^a\):

\[
\hat{H} := \arg\min_{G \in \mathcal{H}_2^{n_y \times n_u}} \| H - G \|_{\mathcal{H}_2},
\]

\[
G \in \mathcal{H}_2^{n_y \times n_u},
\]

\[
\dim(G) = r \in \mathbb{N}^*
\]

\(^a\mathcal{H}_2\)-norm is the "system energy"

Tackle this problem by rational interpolation
Overview of my contributions
State of the art

Model approximation
- Data-based
- Loewner framework
- Rational interpolation
- Projection-based
  - Singular value decomposition
  - Moment-matching
- Optimal approximation
  - Modal truncation
  - Frequency-limited model reduction
  - Least meansquares
  - Gradient-based optimization
Overview of my contributions
State of the art

- Most of reduced order models considered are finite dimensional.
- But some natural phenomena have intrinsical delay behaviour, e.g., transport equation.
- Idea: Consider time-delay reduced order models.

\[ \Delta(s) = e^{-\tau s}. \]

flow into quad-copter
Overview of my contributions

List of examples

- **Example 1:** Approximation of transport phenomena by time-delay structure.

- **Example 2:** Time-delay system stability charts estimation.

- **Example 3:** Hydroelectric EDF model (Rhin river).
Full order model has **input-delay behavior**.

Finite dimensional reduced-order model **not appropriate**.

Good **input-delay** approximation.

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1 Pontes Duff, I., Poussot-Vassal, C. and Seren, C. – ”Optimal $\mathcal{H}_2$ model approximation based on multiple input/output delays systems.” – [Submitted to Automatica].
Model approximation with time-delay structure
Example 1 (Transport equation)

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Stability regions for time-delay systems
Example 2 (ETH Zürich collaboration)

Stability estimation
Exploit model reduction techniques to analyse the stability of TDS w.r.t. parameters.

Application to Robotics:

\[
\begin{align*}
10^0 \quad & 0.5 \\
0.5 & 1 \\
1 & 1.5 \\
1.5 & 2 \\
2 & 2.5 \\
2.5 & 3 \\
3 & 3.5 \\
3.5 & 4 \\
4 & 4.5 \\
4.5 & 5 \\
5 & 5.5
\end{align*}
\]

Pontes Duff, I., Vuillemin, P., Poussot-Vassal, C., Briat, C. and Seren, C.
“Approximation of stability regions for large-scale time-delay systems using model reduction techniques.” — In Proceedings of the 2015 ECC.
Stability regions for time-delay systems
Example 2 (ETH Zürich collaboration)

Perspectives:
Implement research boundary algorithm using evolutionary methods (PR GENETIC)
Example 3 \(^1\) (EDF collaboration)

⇝ PDE St-Venant fluid model

- Infinite dimensional linear parametric model;
- Modeling = relationship between outflow \(q_s\) and inflow \(q_e\) at any given nominal flow \(q_0\) s.t.:

\[
Z(q_0, s) = [G_e(q_0, s) - G_s(q_0, s)] \cdot \begin{bmatrix} q_e(s) \\ q_s(s) \end{bmatrix}
\]

\(G_e\) and \(G_s\) are rational functions of hyperbolic.

\(^1\) Dalmas, V., Robert, G., Poussot-Vassal, C., Pontes Duff, I. and Seren, C. — "Parameter dependent irrational and infinite dimensional modelling and approximation of an open-channel dynamics." — [Accepted to the 15\(^{th}\) European Control Conference, 2016.]
Rhin flow system
Example 3 \(^1\) (EDF collaboration)

\[ \Rightarrow \textbf{PDE St-Venant fluid model} \]

\[ \Rightarrow \text{Result: finite dimensional parametric reduced model} \]

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Conclusions and perspectives
To sum up...

Main contributions

- Model approximation for reduced order modeling, time-delay structures: theoretical and algorithmic contributions.
- Methodological solutions for TDS stability charts estimation.
- Application to several representative industrial cases.
- Scientific collaborations:
  1. S. Gugercin/C. Beattie (Virginia Tech - 3 months stay).
  2. C. Briat (ETH-Zürich).
Conclusions and perspectives
Third (and last) year future works

Methodologies extension
Methods will be available in the MOdel REduction toolbox.

Thesis defence planned in December 2016.
New PhD position opened on model approximation.
That’s all!
Thanks for your attention, any questions?

\[ \sum_{(A, B, C, D)_i} \sum_{(\hat{A}, \hat{B}, \hat{C}, \hat{D})_i} \]

\[ AP + PAT^T + BBT^T = 0 \]

\[ \Sigma \]

More

Model reduction toolbox

Website link: http://w3.onera.fr/more/
Workshops:

- **2nd Workshop on Delay Systems, October 2013 (CNRS-LAAS, Toulouse):**
  "Model reduction for norm approximation."

- **3rd Workshop Delay Systems, October 2014 (GIPSA-Lab, Grenoble):**
  "Model reduction of time-delay systems and stability charts."

Congresses/Seminars:

- **GT MOSAR, November 2104 (ONERA, Toulouse):**
  "Model reduction of infinite dimensional systems."

- **Matrix Computation Seminar, October 2015 (Virginia Tech, USA):**
  "$H_2$ model approximation, interpolation and time-delay systems."

- **SIAM Student Chapter, November 2015 (Virginia Tech, USA):**
  "$H_2$ model approximation, stability charts and time-delay systems."

- **7th European Congress of Mathematics, July 2016, (TU Berlin):**
  "$H_2$ model approximation for time-delay reduced order systems." [invited]
Book chapter:

Pontes Duff, I., Vuillemin, P., Poussot-Vassal, C., Briat, C. and Seren, C.
*Model reduction for norm approximation: an application to large-scale time-delay systems.*
[To Appear] in Springer Series: Advances in Dynamics and Delays.

Conference papers:

Pontes Duff, I., Vuillemin, P., Poussot-Vassal, C., Briat, C. and Seren, C.
*Stability and performance analysis of a large-scale aircraft anti-vibration control subject to delays using model reduction techniques.*
[Accepted] in the Proceedings of the 2015 EuroGNC Conference.

Pontes Duff, I., Vuillemin, P., Poussot-Vassal, C., Briat, C. and Seren, C.
*Approximation of stability regions for large-scale time-delay systems using model reduction techniques.*

Pontes Duff, I., Poussot-Vassal, C. and Seren, C.
*Realization independent time-delay optimal interpolation framework.*
[Accepted] at the 54th IEEE Conference on Decision and Control, 2015.
Journal papers:

- Pontes Duff, I., Poussot-Vassal, C. and Seren, C.  
  *Optimal \( \mathcal{H}_2 \) model approximation based on multiple input/output delays systems.*  

Conference papers:

- Dalmas, V., Robert, G., Poussot-Vassal, C., Pontes Duff, I. and Seren, C.  
  *Parameter dependent irrational and infinite dimensional modelling and approximation of an open-channel dynamics.*  
  [Accepted] to the 15\(^{th}\) European Control Conference, 2016.

- Pontes Duff, I., Gugercin, S., Beattie, C., Poussot-Vassal, C. and Seren, C.  
  *\( \mathcal{H}_2 \)-optimality conditions for reduced time-delay systems of dimension one.*  
  [Accepted] to the 13\(^{th}\) IFAC Workshop on Time Delay Systems, 2016.
Academic outputs
On going works

Journal papers:

Pontes Duff, I., Poussot-Vassal, C. and Seren, C.
Model reduction and stability charts of time-delay systems.
[On going work] ➝ European Journal of Control (?)

Pontes Duff, I., Gugercin, S., Beattie, C., Poussot-Vassal, C. and Seren, C.
$H_2$-optimality conditions for structured reduced order models.
[On going work] ➝ SIAM Journals on matrix analysis and applications (?)

Technical Report:

Pontes Duff, I., Gugercin, S. and Beattie, C.
Stability and model reduction of family of TDS models.
[On going work] ➝ Event not identified.