Separation in three-dimensional steady flows

Part 2: DETACHMENT AND ATTACHMENT SEPARATION LINES
DETACHMENT AND ATTACHMENT SEPARATION SURFACES
A separation line is a skin friction line going through a saddle point. According to the flow direction, it is associated either to a detachment or an attachment process.
Flow in the vicinity of a detachment line

Streamtube mass flow rate:
\[ q_m = \rho V n h = \text{cst} \]

Convergence of the skin friction lines when approaching the separation line entails a vertical dilatation of the streamtube: the flow springs up from the wall or detaches.

The flow detaches from the wall
The flow dives towards the wall

Divergence of the skin friction lines when streaming away from the separation line entails a vertical contraction of the streamtube: the flow dives towards the wall and attaches.
Topology in the vicinity of detachment. The separation streamsurface

Seen from upstream

- Separation surface of detachment
- Saddle point
- Node
- Separation line
- Streamline
- Skin friction line
Topology in the vicinity of detachment. The separation streamsurface

Seen from downstream
Intersection of the separation surface of detachment and of the second separation surface.
Topology in the vicinity of attachment. The separation streamsurface.

Seen from downstream.
Topology in the vicinity of attachment. The separation streamsurface

Seen from upstream

Streamline
Skin friction line
Attachment surface
Attachment line

Node
Saddle point

(Σ₂)
(S₂)
(S₁)
Rolling up of the separation surface of detachment

Section by a normal plane downstream of the structure

Formation of a horseshoe vortex
Horseshoe vortex. Detachment in front of an obstacle
Formation of a tornado like vortex

Section by a plane out of the body surface

Rolling up of the separation surface of detachment
Tornado like vortex

Tornado
Separation on a simply connected body

Non separated (or attached) flow

Separated flow with 1 saddle point and 3 nodes

Separated flow with two saddle points, 2 foci and 2 nodes
Separation (detachment type) with one saddle point and three nodes. 
Formation of a ring type vortex
Separation (detachment type) with one saddle point and three nodes. Formation of an open – or horseshoe - vortex

Details at node $N_2$
Separation with two saddle points and two foci. The separation surface of detachment
Separation with two saddle points and two foci.  
The attachment surface
Detachment with two saddle points and two foci

Crossing of the attachment surface by the tornado like vortices

Rolling up of the detachment surface
Detachment with two saddle points and two foci

Flow on the attachment surface

Attachment surface seen from downstream
Detachment with two saddle points and two foci. Variants of the surface flow pattern.

Symmetrical topology (instable ?)  Real topology
Three-dimensional separation, definitions and results (1)

A flow is separated (detached) if its skin friction line pattern contains more than two nodes.

Or equivalently:

A flow is separated (detached) if its skin friction line pattern contains at least one saddle point.

Hence:

Every separated flow contain one or several saddle points.
A detachment (attachment) line is the trace on the obstacle of a detachment (attachment) surface.

Three-dimensional separation, definitions and results (2)

Through a saddle point go two separation lines (separatrices) which are either of the attachment or detachment type.

A detachment (attachment) line is the trace on the obstacle of a detachment (attachment) surface.

The streamlines forming an attachment surface terminate at a node coincident with the attachment saddle point on the obstacle surface.

The streamlines forming a detachment surface emanate from a node coincident with the detachment saddle point on the obstacle surface.
A vortex tube (vortical structure) cannot originate or end in a fluid.

It must originate or end on a boundary (wall of free surface).

Or form a closed structure or extend to infinity (in fact closing itself at large distance).

These theorems are true for inviscid flows: In fact, viscosity tends to dissipate the motion into heat so that the vortex extends to a finite distance (which is fortunate).
Three-dimensional separation, definitions and results (4)

It originates or ends on a wall or a free surface (interface water-air):

- **Solid surface**
- **Free surface**
- **Tornado like vortices behind a bridge pier**
Or forms a closed vortical structure.

Three-dimensional separation, definitions and results (5)

Toroidal (ring) vortex
Toroidal vortex behind a circular disk facing the flow
A big toroidal vortex: smoke ring emitted by volcano Etna

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The number of critical points on an isolated body must satisfy the Poincaré formula:

\[ \sum (\text{nodes } + \text{ foci}) - \sum (\text{saddle points}) = 2 - 2p \]

Nodes and foci are topologically equivalent: they are sources or sinks for skin friction lines.

In the above formula, \( p \) is the order of the surface:

- \( p = 0 \) – first kind (order 0): simple body like a sphere
- \( p = 1 \) – second kind (order 2): body with a hole like a torus
Three-dimensional separation. Order of a surface

Order zero surface

- Nodes: 2
- Foci: 4
- Saddle points: 4

\[ \Sigma (\text{nodes+foci}) - \Sigma (\text{saddle pts}) = 2 \]

Order one surface

- Nodes: 2
- Foci: 4
- Saddle points: 6

\[ \Sigma (\text{nodes+foci}) - \Sigma (\text{saddle pts}) = 0 \]
In practice, the above formula is difficult to apply because a complete isolated body must be considered which is difficult for a wind tunnel experiment: one has to include the model + the support + the test section walls…

However it can be useful to check the topological consistency of the reconstructed flow organisation.
Concepts deduced from the critical point theory can be applied to any vector field sufficiently regular; i.e., having a finite number of critical points.

In particular, these concepts can be used to analyse the structure of the velocity field projected in a plane, which is commonly made to represent three-dimensional results.

However, one has to be aware that in this case the representation of the field is not objective since it depends on the projection method.
Construction of a three-dimensional field from experimental or computation results must follow a well defined procedure.

Firstly, one has to realise that the available information is never fine enough to allow a complete construction from the results: many details are blurred by experiment but also by the computation.

Hence, several elements of the structure have to be imagined.
The correct "strategy" is as follows:

1 - Inspect the skin friction line pattern to identify its critical points.

2 - Draw the separation lines, identifying detachment and attachment lines.

3 - Make sure that the surface pattern is topologically consistent: thus, any skin friction line must originate at a node and terminate either at a node or a focus. This operation may necessitate invention of critical points not visible on the visualisation.
4 – After this first inspection, **define the outer flow structure** by making sure that it is consistent with the skin friction line pattern.

5 – If this is not the case, **modify the surface pattern** and make a new attempt.

6 – Define the structure of the **velocity field projected in well chosen planes**. Check the consistency of the projected field with the surface pattern.
7 – Do not forget some fundamental theorems: Kelvin, Helmholtz. A vortex cannot originate nor terminate within a flow, it must closes on itself or be in contact with a material boundary, or goes to infinity (except if destroyed by viscosity!)

8 – If one arrives at an « impossible » object, this means that some topological rule has been violated: then try a new construction by starting from step one and iterate!
In complex flows, this exercise can become arduous, necessitating many trials before reaching a construction of the whole flow field which is topologically consistent.

This requires a good perception of three-dimensional objects.
Detachment vs. separation, reattachment vs. attachment

- The concept of separation implies the existence in the skin friction line pattern of at least one saddle point and associated separatrices or separation lines. It applies both to detachment and to reattachment.

- *Separation* is commonly used in English to designate detachment. The two terms are considered as equivalent but this leads to an ambiguity.

- The denomination reattachment implies existence of a previous detachment: the detached flow reattaches on the surface.

- In general, a behaviour of the attachment type is not associated to a previous detachment. Furthermore, in three-dimensions the fluid that reattaches is distinct from the fluid that detached from the wall.

- Except in special cases, the term attachment should be preferred to reattachment.