Embedding network calculus and event stream theory in a common model

21st IEEE Int. Conf. on Emerging Technologies and Factory Automation [ETFA 2016]
Track 3 : Real-Time and (Networked) Embedded Systems [RTNES]

Marc Boyer   Pierre Roux

Wednesday, September 7th, 2016

ONERA
THE FRENCH AEROSPACE LAB

retour sur innovation

Outline

Outline

Global context

Kind of systems

real-time system

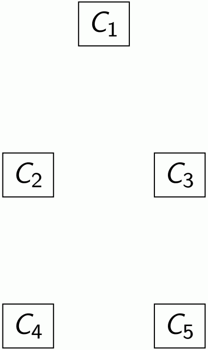
Kind of property

worst case response time

Global context

Kind of systems

Distributed real-time system



Kind of property

worst case response time

Global context

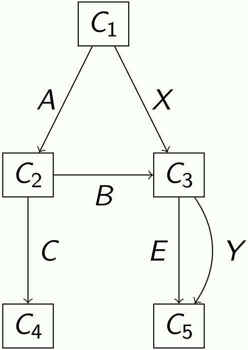
Kind of systems: flow/component

Distributed real-time system

- Components (computation node, bus, switch, etc.)
- Event flows between components
- Event reception triggers a local workload (computation, data forwarding...)

Kind of property

worst case response time



Global context

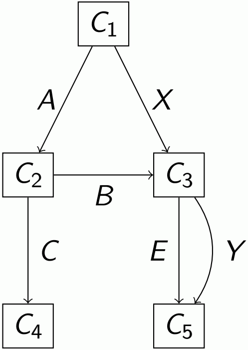
Kind of systems: flow/component

Distributed real-time system

- Components (computation node, bus, switch, etc.)
- Event flows between components
- Event reception triggers a local workload (computation, data forwarding...)

Kind of property

Bounds on worst case response time



Global context

Kind of systems: flow/component

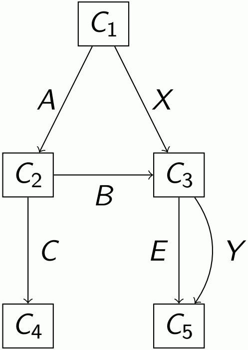
Distributed real-time system

- Components (computation node, bus, switch, etc.)
- Event flows between components
- Event reception triggers a local workload (computation, data forwarding...)

Kind of property

Bounds on worst case response time

- local latency
- end-to-end latency



Two flow/component models

	Event Stream/CPA	Network Calculus
	$E \rightarrow \boxed{C} \rightarrow E'$	$A \rightarrow \boxed{C} \rightarrow A'$
Flow model	$E(t)$: number of events up to time t	$A(t)$: amount of data up to time t
Contract $\forall t, d \geqslant 0$	η^+, η^-: event arrival functions $E(t+d) - E(t) \leqslant \eta^+(d)$ $E(t+d) - E(t) \geqslant \eta^-(d)$	α: arrival curve $A(t+d) - A(t) \leqslant \alpha(d)$
Flow transformation	Busy window	Residual service

- Two very close models
- No best method (depends on the system)

Toward unifying model

Goals

- more accurate results
- better understanding of each theory
- modelling of new kind of components: CAN/AFDX gateway, per block memory allocation...

Toward unifying model

Goals

- more accurate results
- better understanding of each theory
- modelling of new kind of components: CAN/AFDX gateway, per block memory allocation...

Success criteria

- accurate
- easy to use
  - modelling
  - proofs

Toward unifying model

Goals

- more accurate results
- better understanding of each theory
- modelling of new kind of components: CAN/AFDX gateway, per block memory allocation...

Success criteria

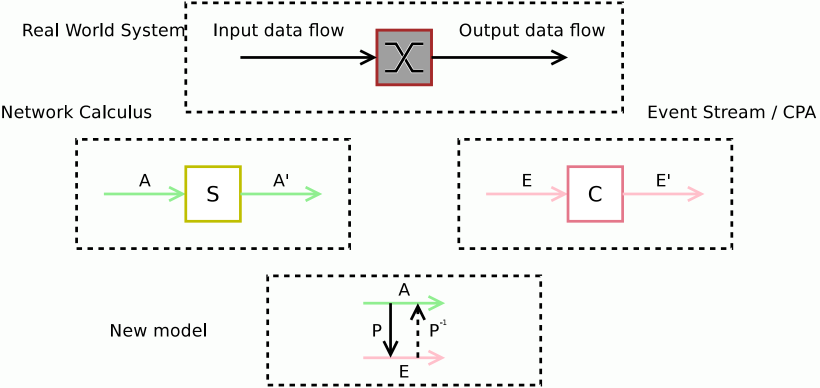
- accurate
- easy to use
    - modelling
    - proofs

Guidelines

- a compositional model
- an algebraic model

Outline

The global picture



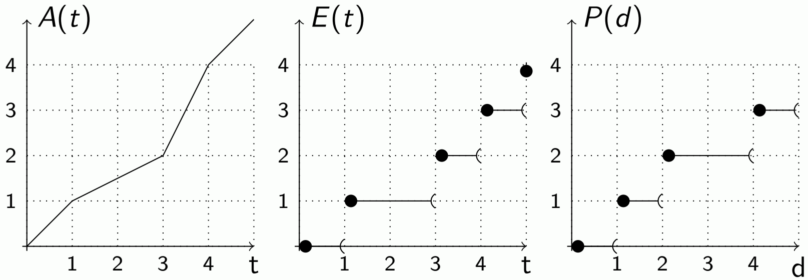
Definition of the new model

Arrival curve	Packet count	Event count
$A : \mathbb{R}^+ \to \mathbb{R}^+$	$P : \mathbb{R}^+ \to \mathbb{N}$	$E : \mathbb{R}^+ \to \mathbb{N}$
$A(t)$: amount of data up to t	$P(d)$: number of full packets in the d first "bits"	$E(t)$: number of full packets up to t

$$\underbrace{P(\overbrace{A}^{NC})}_{1} = \overbrace{E}^{CPA}$$

[1] **Packet-Curves-ValueTools-2012**.
[1] **Packet-Curves-ValueTools-2012**.
[1] **Packet-Curves-ValueTools-2012**.
[1] **Packet-Curves-ValueTools-2012**.

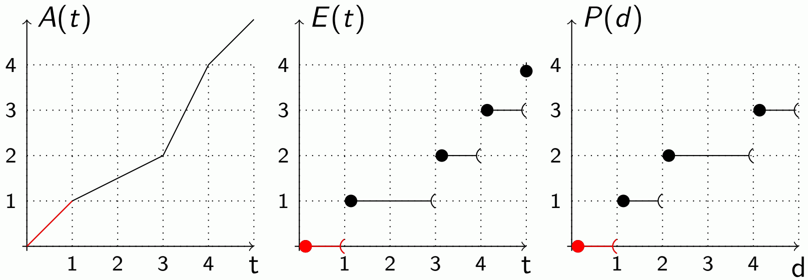
Illustration



Scenario:

- First packet: size 1, throughput 1
- Second packet: size 1, throughput 1/2
- Third packet: size 2, throughput 2
- Fourth packet: size 1, throughput 1

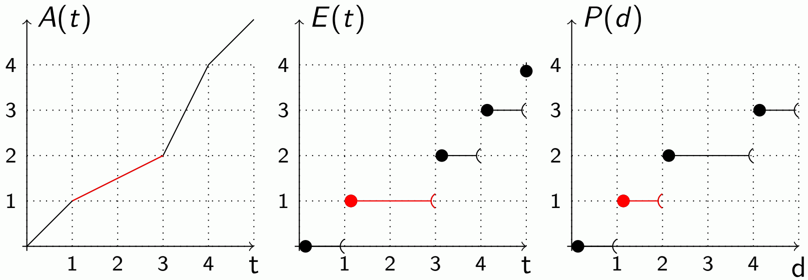
Illustration



Scenario:

- First packet: size 1, throughput 1
- Second packet: size 1, throughput 1/2
- Third packet: size 2, throughput 2
- Fourth packet: size 1, throughput 1

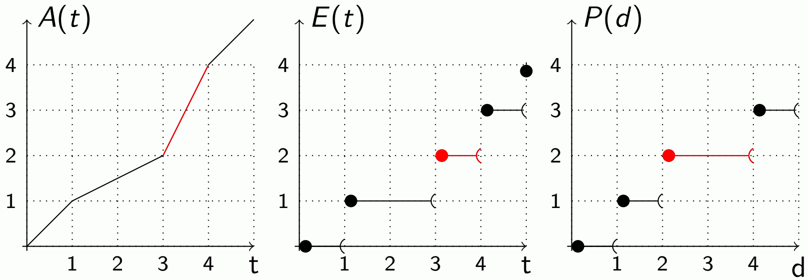
Illustration



Scenario:

- First packet: size 1, throughput 1
- Second packet: size 1, throughput 1/2
- Third packet: size 2, throughput 2
- Fourth packet: size 1, throughput 1

Illustration



Scenario:

- First packet: size 1, throughput 1
- Second packet: size 1, throughput 1/2
- Third packet: size 2, throughput 2
- Fourth packet: size 1, throughput 1

Interval Bounding Pair (IBP)

- Real behaviours are unknown at design
- Performance studies based on contract
- Interval Bounding Pair: renaming of arrival curves/event stream
  $\phi = (\underline{\phi}, \overline{\phi})$ is an Interval Bouding Pair (IBP) of f iff

  $$\forall t, d \geqslant 0 : \underline{\phi}(d) \leqslant f(t+d) - f(t) \leqslant \overline{\phi}(d)$$

- Handle the contract tuple $\langle \alpha, \pi, \eta \rangle$ where α, π, η are respective IBPs of A, P, E

Outline

Taking in hand the model

- Defining a new model is easy
- Model evaluation is hard
- Taking in hand the model:
  - basic properties of the model itself
  - modelling basic component
    1. packetizer
    2. aggregation
  - model accuracy (new)

Mathematical operators

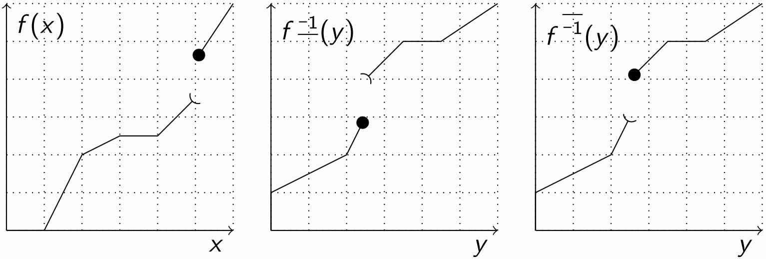
- Min/max-plus convolution: associative, commutative, monotonous

$$(f \underline{*} g)(t) = \inf_{0 \leqslant s \leqslant t} f(t-s) + g(s) \quad (f \overline{*} g)(t) = \sup_{0 \leqslant s \leqslant t} f(t-s) + g(s)$$

- Composition:associative, monotonous

$$(f \circ g)(t) = f(g(t))$$

- Pseudo-inverses



Intrinsic properties

- IBP properties (from NC and CPA)

ONERA THE FRENCH AEROSPACE LAB

Intrinsic properties

- IBP properties (from NC and CPA)
  - min/max: if $(\underline{\phi}, \overline{\phi})$ and $(\underline{\phi}', \overline{\phi}')$ are IBP of f, also is $(\max(\underline{\phi}, \underline{\phi}'), \min(\overline{\phi}, \overline{\phi}'))$

- IBP properties (from NC and CPA)
  - min/max: if $(\underline{\phi}, \overline{\phi})$ and $(\underline{\phi}', \overline{\phi}')$ are IBP of f, also is $(\max(\underline{\phi}, \underline{\phi}'), \min(\overline{\phi}, \overline{\phi}'))$
  - Kleene star closure: if $(\underline{\phi}, \overline{\phi})$ is an IBP of f, also is $(\underline{\phi}^{\overline{*}}, \overline{\phi}^{\underline{*}})$ where $\cdot^{\underline{*}}$, $\cdot^{\overline{*}}$ are Kleene-star of min/max convolutions.

Intrinsic properties

- IBP properties (from NC and CPA)
  - min/max: if $(\underline{\phi}, \overline{\phi})$ and $(\underline{\phi}', \overline{\phi}')$ are IBP of f, also is $(\max(\underline{\phi}, \underline{\phi}'), \min(\overline{\phi}, \overline{\phi}'))$
  - Kleene star closure: if $(\underline{\phi}, \overline{\phi})$ is an IBP of f, also is $(\underline{\phi}^{\overline{*}}, \overline{\phi}^{\underline{*}})$ where $\cdot^{\underline{*}}$, $\cdot^{\overline{*}}$ are Kleene-star of min/max convolutions.
- Between IBP (contribution): from two IBPs, build the missing one

Intrinsic properties

- IBP properties (from NC and CPA)
  - min/max: if $(\underline{\phi}, \overline{\phi})$ and $(\underline{\phi}', \overline{\phi}')$ are IBP of f, also is $(\max(\underline{\phi}, \underline{\phi}'), \min(\overline{\phi}, \overline{\phi}'))$
  - Kleene star closure: if $(\underline{\phi}, \overline{\phi})$ is an IBP of f, also is $(\underline{\phi}^{\overline{*}}, \overline{\phi}^{\underline{*}})$ where $\cdot^{\underline{*}}$, $\cdot^{\overline{*}}$ are Kleene-star of min/max convolutions.
- Between IBP (contribution): from two IBPs, build the missing one

A	P	E
$(\underline{\alpha}, \overline{\alpha})$	$(\underline{\pi}, \overline{\pi})$	

Intrinsic properties

- IBP properties (from NC and CPA)
  - min/max: if $(\underline{\phi}, \overline{\phi})$ and $(\underline{\phi}', \overline{\phi}')$ are IBP of f, also is $(\max(\underline{\phi}, \underline{\phi}'), \min(\overline{\phi}, \overline{\phi}'))$
  - Kleene star closure: if $(\underline{\phi}, \overline{\phi})$ is an IBP of f, also is $(\underline{\phi}^{\underline{*}}, \overline{\phi}^{\overline{*}})$ where $\cdot^{\underline{*}}$, $\cdot^{\overline{*}}$ are Kleene-star of min/max convolutions.
- Between IBP (contribution): from two IBPs, build the missing one

A	P	E
$(\underline{\alpha}, \overline{\alpha})$	$(\underline{\pi}, \overline{\pi})$	$(\underline{\pi} \circ \underline{\alpha}, \overline{\pi} \circ \overline{\alpha})$

Intrinsic properties

- IBP properties (from NC and CPA)
  - min/max: if $(\underline{\phi}, \overline{\phi})$ and $(\underline{\phi}', \overline{\phi}')$ are IBP of f, also is $(\max(\underline{\phi}, \underline{\phi}'), \min(\overline{\phi}, \overline{\phi}'))$
  - Kleene star closure: if $(\underline{\phi}, \overline{\phi})$ is an IBP of f, also is $(\underline{\phi}^{\underline{*}}, \overline{\phi}^{\overline{*}})$ where $\cdot^{\underline{*}}$, $\cdot^{\overline{*}}$ are Kleene-star of min/max convolutions.
- Between IBP (contribution): from two IBPs, build the missing one

A	P	E
$(\underline{\alpha}, \overline{\alpha})$	$(\underline{\pi}, \overline{\pi})$	$(\underline{\pi} \circ \underline{\alpha}, \overline{\pi} \circ \overline{\alpha})$
	$(\underline{\pi}, \overline{\pi})$	$(\underline{\eta}, \overline{\eta})$

Intrinsic properties

- IBP properties (from NC and CPA)
  - min/max: if $(\underline{\phi}, \overline{\phi})$ and $(\underline{\phi}', \overline{\phi}')$ are IBP of f, also is $(\max(\underline{\phi}, \underline{\phi}'), \min(\overline{\phi}, \overline{\phi}'))$
  - Kleene star closure: if $(\underline{\phi}, \overline{\phi})$ is an IBP of f, also is $(\underline{\phi}^{\overline{*}}, \overline{\phi}^{\underline{*}})$ where $\cdot^{\underline{*}}$, $\cdot^{\overline{*}}$ are Kleene-star of min/max convolutions.
- Between IBP (contribution): from two IBPs, build the missing one

A	P	E
$(\underline{\alpha}, \overline{\alpha})$	$(\underline{\pi}, \overline{\pi})$	$(\underline{\pi} \circ \underline{\alpha}, \overline{\pi} \circ \overline{\alpha})$
$(\overline{\pi}^{\underline{-1}} \circ \underline{\eta}, \underline{\pi}^{\overline{-1}} \circ \overline{\eta})$	$(\underline{\pi}, \overline{\pi})$	$(\underline{\eta}, \overline{\eta})$

Intrinsic properties

- IBP properties (from NC and CPA)
  - min/max: if $(\underline{\phi}, \overline{\phi})$ and $(\underline{\phi}', \overline{\phi}')$ are IBP of f, also is $(\max(\underline{\phi}, \underline{\phi}'), \min(\overline{\phi}, \overline{\phi}'))$
  - Kleene star closure: if $(\underline{\phi}, \overline{\phi})$ is an IBP of f, also is $(\underline{\phi}^{\underline{*}}, \overline{\phi}^{\overline{*}})$ where $\cdot^{\underline{*}}$, $\cdot^{\overline{*}}$ are Kleene-star of min/max convolutions.
- Between IBP (contribution): from two IBPs, build the missing one

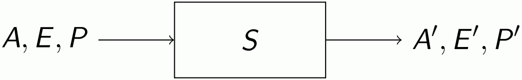
A	P	E
$(\underline{\alpha}, \overline{\alpha})$	$(\underline{\pi}, \overline{\pi})$	$(\underline{\pi} \circ \underline{\alpha}, \overline{\pi} \circ \overline{\alpha})$
$(\overline{\pi}^{\underline{-1}} \circ \underline{\eta}, \underline{\pi}^{\overline{-1}} \circ \overline{\eta})$	$(\underline{\pi}, \overline{\pi})$	$(\underline{\eta}, \overline{\eta})$
$(\underline{\alpha}, \overline{\alpha})$		$(\underline{\eta}, \overline{\eta})$

Intrinsic properties

- IBP properties (from NC and CPA)
  - min/max: if $(\underline{\phi}, \overline{\phi})$ and $(\underline{\phi}', \overline{\phi}')$ are IBP of f, also is $(\max(\underline{\phi}, \underline{\phi}'), \min(\overline{\phi}, \overline{\phi}'))$
  - Kleene star closure: if $(\underline{\phi}, \overline{\phi})$ is an IBP of f, also is $(\underline{\phi}^{\overline{*}}, \overline{\phi}^{\underline{*}})$ where $\cdot^{\underline{*}}$, $\cdot^{\overline{*}}$ are Kleene-star of min/max convolutions.
- Between IBP (contribution): from two IBPs, build the missing one

A	P	E
$(\underline{\alpha}, \overline{\alpha})$	$(\underline{\pi}, \overline{\pi})$	$(\underline{\pi} \circ \underline{\alpha}, \overline{\pi} \circ \overline{\alpha})$
$(\overline{\pi}^{\underline{-1}} \circ \underline{\eta}, \underline{\pi}^{\overline{-1}} \circ \overline{\eta})$	$(\underline{\pi}, \overline{\pi})$	$(\underline{\eta}, \overline{\eta})$
$(\underline{\alpha}, \overline{\alpha})$	$(\underline{\eta}_l \circ \overline{\alpha}^{\underline{-1}}, \overline{\eta}_r \circ \underline{\alpha}^{\overline{-1}})$	$(\underline{\eta}, \overline{\eta})$

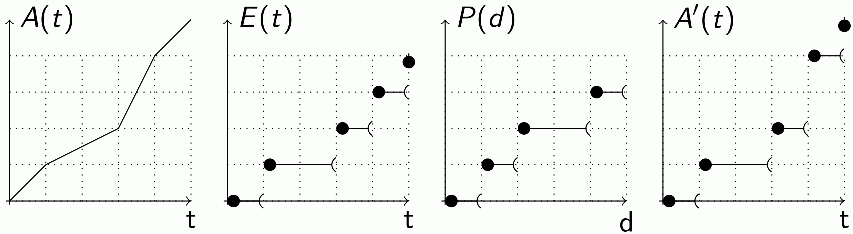
Packetizer



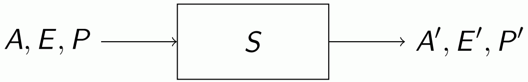
Packetizer:

- store bits, up to end-of-packet
- instantaneous packet output
- model: E, P unchanged

$$A' := P^{-1} \circ P \circ A$$
$$E' := E$$
$$P' := P$$



Packetizer



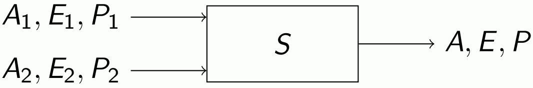
Packetizer:

- store bits, up to end-of-packet
- instantaneous packet output
- model: E, P unchanged

$$A' := P^{\underline{-1}} \circ P \circ A$$
$$E' := E$$
$$P' := P$$

$$\underline{\alpha}' := \overline{\pi}^{\overline{-1}} \circ \underline{\eta}$$
$$\overline{\alpha}' := \underline{\pi}^{\underline{-1}} \circ \overline{\eta}$$

Aggregation



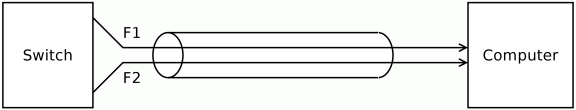
Aggregation:

- mix of flows
- "sum" of flows is a flow
- no delay

$$A := A_1 + A_2$$
$$E := E_1 + E_2$$
$$P(A_1 + A_2) := P(A_1) + P(A_2)$$

$$\underline{\alpha} := \underline{\alpha}_1 + \underline{\alpha}_2 \qquad \overline{\alpha} := \overline{\alpha}_1 + \overline{\alpha}_2$$
$$\underline{\eta} := \underline{\eta}_1 + \underline{\eta}_2 \qquad \overline{\eta} := \overline{\eta}_1 + \overline{\eta}_2$$
$$\underline{\pi} := \left\lfloor \underline{\pi_1} \,\underline{*}\, \underline{\pi_2} \right\rfloor \qquad \overline{\pi} := \left\lceil \overline{\pi_1} \,\overline{*}\, \overline{\pi_2} \right\rceil$$

Case study



- Two data flows, F_1, F_2, from S to C
- Using a link of throughput 1

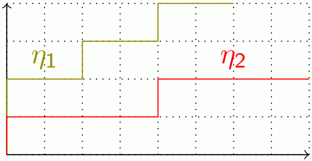
Flow	Packet size	Burst	Throughput	$\overline{\alpha}_i$	$\overline{\pi}_i$
F_1	1/2	1	1/4	x/4 +1	$\lceil 2x \rceil$
F_2	1	1	1/4	x/4 +1	$\lceil x \rceil$

- Goal: evaluation of the packet throughput
  - $F = F_1 + F_2$
  - what is $\overline{\eta}$?
  - challenge: modelling the link shaping

Packet throughput: no shaping

No shaping :

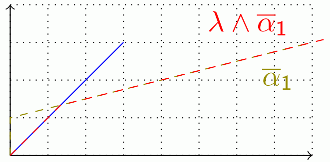
- $\overline{\eta}_1 = \overline{\pi}_1 \circ \overline{\alpha}_1 = \left\lceil \frac{x}{2} \right\rceil + 2$
- $\overline{\eta}_2 = \overline{\pi}_2 \circ \overline{\alpha}_2 = \left\lceil \frac{x}{4} \right\rceil + 1$
- $\overline{\eta} \leqslant \overline{\eta}_1 + \overline{\eta}_2$

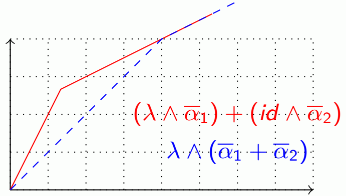


Packet throughput: with shaping

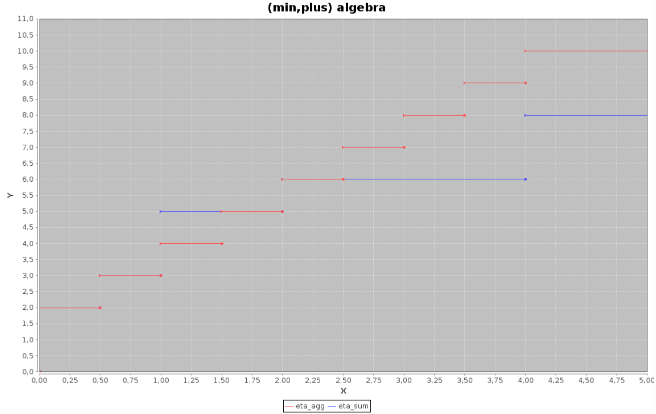
Link throughput: $\lambda(t) = t$

- Shaping reduces data throughput
  - for each flow, $\overline{\alpha}_i^s = \lambda \wedge \overline{\alpha}_i$
  - for the aggregate flow: $\overline{\alpha}_{1+2}^s = \lambda \wedge (\overline{\alpha}_1 + \overline{\alpha}_2)$
- Impact on packet throughput
  - per flow: $\overline{\eta}_i^s = \overline{\pi}_i \circ \overline{\alpha}_i^s$
  - aggregate flow: $\overline{\eta}_{1+2}^s = \lceil \overline{\pi}_1 \overline{*} \overline{\pi}_2 \rceil \circ \overline{\alpha}_{1+2}^s$
  - both $\overline{\eta}_1^s + \overline{\eta}_2^s$ *and* $\overline{\eta}_{1+2}^s$ are packet throughput bounds





Numerical results



- the shaping only affects start of curve
- the simple method has better long term throughput
- the new method is locally better

Outline

Conclusion

- A new model, unifying NC and Event-Stream/CPA
- Taking the model in hand
  - algebraic results
  - some accuracy gains
- Next steps
  - composition implementation
  - aggregation improvement
  - realistic case study

Toward unifying model

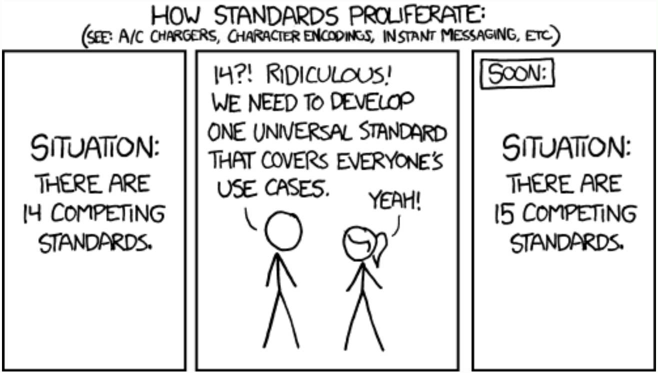


Figure: http://xkcd.com/927/

ONERA
THE FRENCH AEROSPACE LAB