Embedding network calculus and event stream theory in a common model

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Track 3 : Real-Time and (Networked) Embedded Systems [RTNES]

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ONERA
THE FRENCH AEROSPACE LAB
Global context

Kind of systems

- real-time system

Kind of property

- worst case response time
Global context

Kind of systems
Distributed real-time system

Kind of property
worst case response time
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Kind of systems: flow/component

Distributed real-time system

- Components (computation node, bus, switch, etc.)
- Event flows between components
- Event reception triggers a local workload (computation, data forwarding...)

Kind of property

worst case response time
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Distributed real-time system
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Bounds on worst case response time
Global context

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Kind of property

Bounds on worst case response time
- local latency
- end-to-end latency
Two flow/component models

<table>
<thead>
<tr>
<th>Event Stream/CPA</th>
<th>Network Calculus</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E \rightarrow C \rightarrow E' )</td>
<td>( A \rightarrow C \rightarrow A' )</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Flow model</th>
<th>( E(t) ): number of events up to time ( t )</th>
<th>( A(t) ): amount of data up to time ( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contract</td>
<td>( \eta^+, \eta^- ): event arrival functions</td>
<td>( \alpha ): arrival curve</td>
</tr>
<tr>
<td>( \forall t, d \geq 0 )</td>
<td>( E(t + d) - E(t) \leq \eta^+(d) )</td>
<td>( A(t + d) - A(t) \leq \alpha(d) )</td>
</tr>
<tr>
<td>Flow transformation</td>
<td>Busy window</td>
<td>Residual service</td>
</tr>
</tbody>
</table>

- Two very close models
- No best method (depends on the system)
Toward unifying model

Goals

- more accurate results
- better understanding of each theory
- modelling of new kind of components: CAN/AFDX gateway, per block memory allocation...
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- better understanding of each theory
- modelling of new kind of components: CAN/AFDX gateway, per block memory allocation...

Success criteria
- accurate
- easy to use
  - modelling
  - proofs
Toward unifying model

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- more accurate results
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- modelling of new kind of components: CAN/AFDX gateway, per block memory allocation...

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Guidelines
- a compositional model
- an algebraic model
### Definition of the new model

<table>
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<tr>
<th>Arrival curve</th>
<th>Packet count</th>
<th>Event count</th>
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<tbody>
<tr>
<td>$A : \mathbb{R}^+ \rightarrow \mathbb{R}^+$</td>
<td>$P : \mathbb{R}^+ \rightarrow \mathbb{N}$</td>
<td>$E : \mathbb{R}^+ \rightarrow \mathbb{N}$</td>
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<tr>
<td>$A(t)$: amount of data up to $t$</td>
<td>$P(d)$: number of full packets in the $d$ first “bits”</td>
<td>$E(t)$: number of full packets up to $t$</td>
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![Diagram](image)

1. NC/CPA common model

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1. [Packet-Curves-ValueTools-2012](#)
Scenario:

- First packet: size 1, throughput 1
- Second packet: size 1, throughput 1/2
- Third packet: size 2, throughput 2
- Fourth packet: size 1, throughput 1
Scenario:

- **First packet**: size 1, throughput 1
- **Second packet**: size 1, throughput 1/2
- **Third packet**: size 2, throughput 2
- **Fourth packet**: size 1, throughput 1
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Real behaviours are unknown at design
Performance studies based on contract
Interval Bounding Pair: renaming of arrival curves/event stream
\( \phi = (\underline{\phi}, \overline{\phi}) \) is an Interval Bounding Pair (IBP) of \( f \) iff

\[ \forall t, d \geq 0 : \underline{\phi}(d) \leq f(t + d) - f(t) \leq \overline{\phi}(d) \]

Handle the contract tuple \( \langle \alpha, \pi, \eta \rangle \) where \( \alpha, \pi, \eta \) are respective IBPs of \( A, P, E \)
Taking in hand the model

- Defining a new model is easy
- Model evaluation is hard
- Taking in hand the model:
  - basic properties of the model itself
  - modelling basic component
    1 packetizer
    2 aggregation
  - model accuracy (new)
Mathematical operators

- **Min/max-plus convolution**: associative, commutative, monotonous
  \[(f \ast g)(t) = \inf_{0 \leq s \leq t} f(t - s) + g(s) \quad (f \bar{\ast} g)(t) = \sup_{0 \leq s \leq t} f(t - s) + g(s)\]

- **Composition**: associative, monotonous
  \[(f \circ g)(t) = f(g(t))\]

- **Pseudo-inverses**
Intrinsic properties

- IBP properties (from NC and CPA)
Intrinsic properties

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  - min/max: if $(\phi, \bar{\phi})$ and $(\phi', \bar{\phi}')$ are IBP of $f$, also is 
    $$(\max(\phi, \phi'), \min(\bar{\phi}, \bar{\phi}'))$$
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  - Kleene star closure: if $(\phi, \phi)$ is an IBP of $f$, also is $(\phi^*, \phi^*)$

where $\cdot^*$, $\cdot^*$ are Kleene-star of min/max convolutions.
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- Between IBP (contribution): from two IBPs, build the missing one
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<td>((\eta_l \circ \overline{\alpha}^{-1}, \overline{\eta}_r \circ \alpha^{-1}))</td>
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Packetizer:

- store bits, up to end-of-packet
- instantaneous packet output
- model: $E, P$ unchanged

\[ A', E', P' := P^{-1} \circ P \circ A \]
\[ E' := E \]
\[ P' := P \]
Packetizer:

- store bits, up to end-of-packet
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- model: $E, P$ unchanged

\[ A', E', P' \]

\[ A' := P^{-1} \circ P \circ A \]
\[ E' := E \]
\[ P' := P \]

\[ \alpha' := \pi^{-1} \circ \eta \]
\[ \bar{\alpha}' := \pi^{-1} \circ \bar{\eta} \]
Aggregation:

- mix of flows
- “sum” of flows is a flow
- no delay

\[
A := A_1 + A_2 \\
E := E_1 + E_2 \\
P(A_1 + A_2) := P(A_1) + P(A_2)
\]

\[
\alpha := \alpha_1 + \alpha_2 \\
\bar{\alpha} := \bar{\alpha}_1 + \bar{\alpha}_2 \\
\eta := \eta_1 + \eta_2 \\
\bar{\eta} := \bar{\eta}_1 + \bar{\eta}_2 \\
\pi := |\pi_1 \ast \pi_2| \\
\bar{\pi} := |\bar{\pi}_1 \ast \bar{\pi}_2|
\]
Case study

Two data flows, $F_1$, $F_2$, from $S$ to $C$

Using a link of throughput 1

<table>
<thead>
<tr>
<th>Flow</th>
<th>Packet size</th>
<th>Burst</th>
<th>Throughput</th>
<th>$\bar{\alpha}_i$</th>
<th>$\bar{\pi}_i$</th>
</tr>
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<tbody>
<tr>
<td>$F_1$</td>
<td>$1/2$</td>
<td>1</td>
<td>$1/4$</td>
<td>$x/4 + 1$</td>
<td>$[2x]$</td>
</tr>
<tr>
<td>$F_2$</td>
<td>1</td>
<td>1</td>
<td>$1/4$</td>
<td>$x/4 + 1$</td>
<td>$[x]$</td>
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Goal: evaluation of the packet throughput
- $F = F_1 + F_2$
- what is $\bar{\eta}$?
- challenge: modelling the link shaping
Packet throughput: no shaping

No shaping:

\[ \eta_1 = \bar{\eta}_1 \circ \bar{\alpha}_1 = \left\lfloor \frac{x}{2} \right\rfloor + 2 \]
\[ \eta_2 = \bar{\eta}_2 \circ \bar{\alpha}_2 = \left\lfloor \frac{x}{4} \right\rfloor + 1 \]
\[ \bar{\eta} \leq \bar{\eta}_1 + \bar{\eta}_2 \]
Packet throughput: with shaping

Link throughput: $\lambda(t) = t$
- Shaping reduces data throughput
  - for each flow, $\overline{\alpha}_i^s = \lambda \wedge \overline{\alpha}_i$
  - for the aggregate flow: $\overline{\alpha}_{1+2}^s = \lambda \wedge (\overline{\alpha}_1 + \overline{\alpha}_2)$
- Impact on packet throughput
  - per flow: $\overline{\eta}_i^s = \overline{\pi}_i \circ \overline{\alpha}_i^s$
  - aggregate flow:
    - $\overline{\eta}_{1+2}^s = [\overline{\pi}_1 * \overline{\pi}_2] \circ \overline{\alpha}_{1+2}^s$
    - both $\overline{\eta}_1^s + \overline{\eta}_2^s$ and $\overline{\eta}_{1+2}^s$ are packet throughput bounds
Numerical results

- The shaping only affects the start of the curve.
- The simple method has better long-term throughput.
- The new method is locally better.
Conclusion

- A new model, unifying NC and Event-Stream/CPA
- Taking the model in hand
  - algebraic results
  - some accuracy gains
- Next steps
  - composition implementation
  - aggregation improvement
  - realistic case study
Toward unifying model

**Figure:** http://xkcd.com/927/

**Situation:** There are 14 competing standards.

**14?! Ridiculous! We need to develop one universal standard that covers everyone’s use cases. Yeah!**

**Situation:** There are 15 competing standards.