



Network calculus: from theory to avionic applications

INRIA/Spades seminar

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ONERA – The French Aerospace Lab

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retour sur innovation

M. Boyer, NC: from theory to practice

Network calculus: theory

- NC: what and why?

- System modelling in network calculus

- The $(\min, +)$ dioid(s)

- From reality to contracts

- From contracts to bounds

- Aggregated and residual services

Links with other theory

- Real-Time calculus

- Event stream

- Task scheduling

- Comparison

Tools

- NC/RTC tools

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- a theory to compute *memory* and *delay* **bounds** in networks
- based on the $(\min, +)$ dioid
- used to certify A380 AFDX backbone

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Why use it?

Because it is elegant

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When to use it

- multi-hop real-time communications
- no simple analysis exists
- no (or few) cyclic dependencies
- no (or simple) feedback flow control

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Network calculus benefits

- Different accuracies
- Scalable approach

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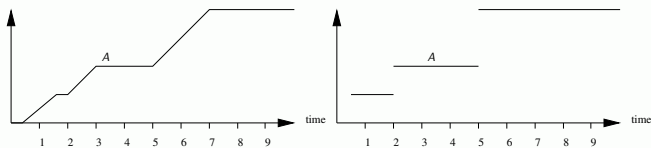
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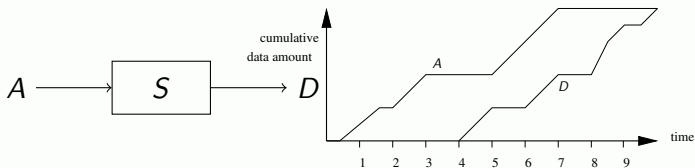
Basic object: cumulative function



- Flow: Cumulative function A
 - $A(t)$: amount of data sent up to time t
 - Properties:
 - null at 0 (and before)
 - non decreasing
 - Discrete of fluid modeling
 - Better definition than instantaneous throughput $\rho(t)$

$$A(t) = \int_0^t \rho(x) dx \quad (1)$$

Basic object: server



- Simple input/output relation:

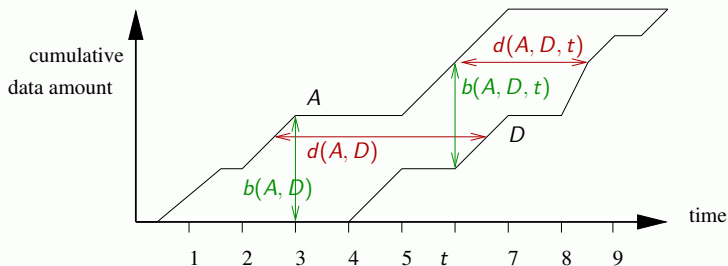
$$S \subset \mathcal{F}_0 \times \mathcal{F}_0$$

- Departure “after” arrival

$$A \xrightarrow{S} D \implies A \geq D$$

- Basic server model:
 - no loss of messages
 - infinite memory
 - no add (header, checksum, etc.)

Performance criteria: delay and backlog



$$b(A, D, t) \stackrel{\text{def}}{=} D(t) - A(t) \quad d(A, D, t) \stackrel{\text{def}}{=} \inf \{ \tau \geq 0 : A(t) \leq D(t + \tau) \}$$

$$b(A, D) \stackrel{\text{def}}{=} \max_{t \geq 0} \{ b(A, D, t) \} \quad d(A, D) \stackrel{\text{def}}{=} \max_{t \geq 0} \{ d(A, D, t) \}$$

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The $(\min, +, \mathbb{R})$ dioid

- The dioid on values: $(\wedge, +, \mathbb{R})$: $a \wedge b = \min(a, b)$
 - associativity, commutativity, distributivity:
 $a + (b \wedge c) = (a + b) \wedge (a + c)$
 - “looks like” $(+, \times, \mathbb{R})$
- The dioid on functions:

$$\text{min-plus convolution} \quad (f * g)(t) \stackrel{\text{def}}{=} \inf_{0 \leq s \leq t} \{f(t-s) + g(s)\}$$

$$\text{min-plus deconvolution} \quad (f \oslash g)(t) \stackrel{\text{def}}{=} \sup_{s \geq 0} \{f(t+s) - g(s)\}$$

$$\text{min-plus Kleene closure} \quad f^* \stackrel{\text{def}}{=} \delta_0 \wedge f \wedge (f * f) \wedge (f * f * f) \wedge \dots$$

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Why contracts?

- cumulative functions are *real* behaviours

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Why contracts?

- cumulative functions are *real* behaviours
 - unknown at design time
 - or too complex to handle

⇒ need to handle *contracts*

- arrival curves: contracts on input traffic
- service curves: contracts on service

Arrival curve definition

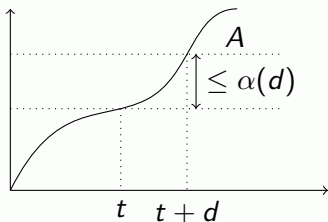
A flow A has α as (maximal) arrival curve iff

$$\forall t, \Delta \in \mathbb{R}_{\geq 0} : A(t + \Delta) - A(t) \leq \alpha(\Delta)$$

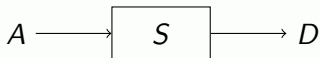
$$\iff$$

$$A \leq A * \alpha$$

- $\alpha(\Delta)$ upper bounds the amount of data send on any interval of width Δ
- minimal arrival curve also exist

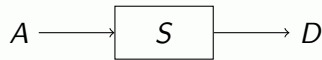


How to define a service? Engineer point of view.



- Constant service: R bits per second
 - First idea: $D(t + \Delta) - D(t) \geq R\Delta$
 - Only when there is some backlog
($\forall x \in [t, t + \Delta] : D(t) < A(t)$)
 - Generalisation to non constant: any β function
 - $D(t + \Delta) - D(t) \geq \beta(\Delta)$
 - on backlogged periods
- ⇒ minimal *strict* service

How to define a service? Mathematician point of view.



- Use $(\wedge, +)$ convolution (symmetry with arrival curve)
- Must be linked with arrival curve (more arrival, more departure, up to service capacity)

$$D \geq A * \beta$$

⇒ minimal *simple* service (or minimal *min-plus* service)

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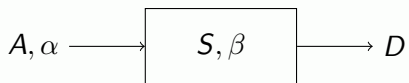
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Assume a server S with minimal service β and arrival flow A with arrival curve α , and departure D

- Bound on delay:

$$d(A, D) \leq d(\alpha, \beta) \quad (2)$$

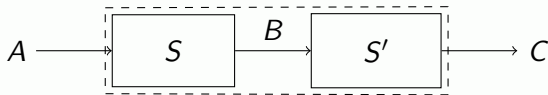
- Bound on memory usage

$$b(A, D) \leq b(\alpha, \beta) \quad (3)$$

- Arrival curve of departure D

$$(\alpha \oslash \beta) \quad (4)$$

Pay burst only once principle



Pay burst only once

The sequence S, S' can be replaced by a virtual server $S; S'$ with service curve $\beta * \beta'$.

Interest End-to-end delay is less than sum of individual delays.

$$h(\alpha, \beta * \beta') \leq h(\alpha, \beta) + h(\alpha, \beta') \quad (5)$$

Proof: $R'' \geq R' * \beta \geq (R * \beta) * \beta' = R * (\beta * \beta')$

Rq: In [20], more than 7 pages are required to prove limited version of this result.

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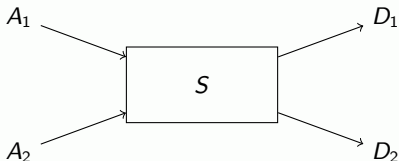
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$$A_1 \geq D_1$$

$$A_2 \geq D_2$$

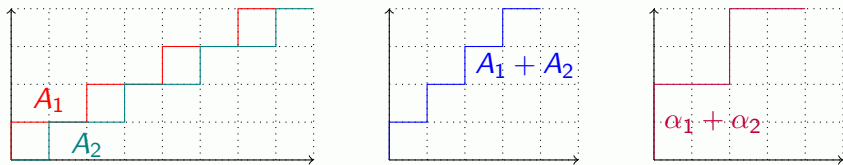
Service repartition still depends on

- server policy (FIFO, Static Priority...)
- individual flow contracts

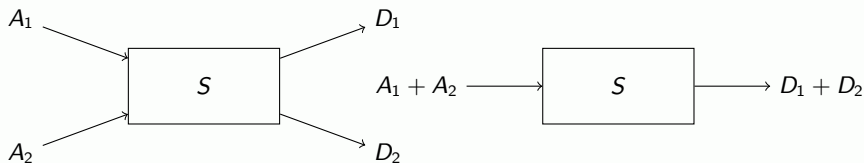
Point of view on flow aggregation

Consider two flows A_1, A_2 of arrival curves α_1, α_2 .

- $A_1 + A_2$ is an arrival curve, *i.e.* a non-decreasing function $\mathbb{R}_{\geq 0} \rightarrow \{R\}$
- $(A_1 + A_2)(t)$ is the amount of data set by both flows up to time t
- $\alpha_1 + \alpha_2$ is an arrival curve for $A_1 + A_2$
- not the best one
 - $A_1 = \nu_{2,1}, \alpha_1 = A_1, A_2(t) = A_1(t - 1), \alpha_2 = \alpha_1$



Aggregated service



$$A_1 \geq D_1$$

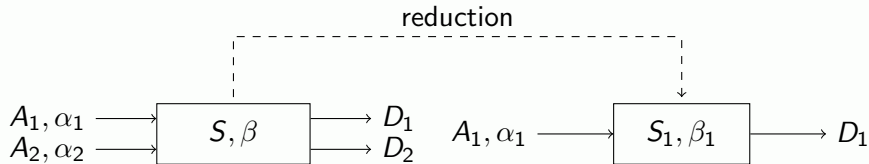
$$A_2 \geq D_2$$

$$(D_1 + D_2) \geq (A_1 + A_2) * \beta$$

Service repartition depends on

- server policy (FIFO, Static Priority...)
- individual flow contracts

Challenge: residual service



$$A_1 \geq D_1$$

$$D_1 \geq A_1 * \beta_1$$

$$A_2 \geq D_2$$

$$D_2 \geq A_2 * \beta_2$$

How to compute β_1, β_2 ?

Residual service must be defined for every scheduling policy

- Static priority: $\beta^M = [\beta - \alpha_H - l_L^{\max}]_{\uparrow}^+$
- FIFO: $\forall \theta \in \mathbb{R}_{\geq 0} : \beta_i = [\beta - \alpha_j * \delta_{\theta}]_{\uparrow}^+ \wedge \delta_{\theta}$
- GPS [17, 1]
- WFQ [11, 16]
- DRR [21, 6]
- AVB [18, 19]
- TDMA [10]
- EDF [15]

The result may depend on the kind of service (simple, strict)...

The function $[f]_{\uparrow}^+$ is the non-negative, non-decreasing closure of f .

Hierarchical scheduling

- A residual service is still a service
- Can be used to combine scheduling policies
 - SP/FIFO (AFDX)
 - SP/DRR/FIFO
 - DRR/EDF
 - ...
- Some restriction on the kind of service may exist

Heterogeneous network

- different scheduling policies may be used in a network/system
- from NC point of view, they all are service

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The same model, with another name [2]

- different presentation
- more focus on minimal arrival curves
- time domain: \mathbb{R} vs. $\mathbb{R}_{\geq 0}$

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Event stream: the trade-of between NC/RTC and scheduling

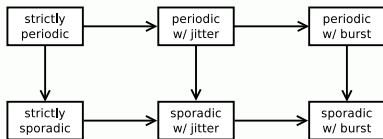
Philosophy [12]

“Furthermore, the new models [ie NC/RTC] are far less intuitive than the ones known from the classical real-time systems research, e. g. the model of rate-monotonic scheduling with its periodic tasks and worst-case execution times. A system-level analysis should be simple and comprehensible, otherwise its acceptance is extremely doubtful.”

“We don’t necessarily need to develop new local analysis techniques if we can benefit from the host of work in real-time scheduling analysis.”

Event stream model

- count events, not amount of data
 - is workload related to frame sizes?
- event stream $\langle \eta^+, \eta^-, \delta^+, \delta^- \rangle$
 - η^+, η^- maximal/minimal amount of events per interval
 - δ^+ / δ^- maximal/minimal distance between events
- event models: sub-classes of event streams



- no formal model of stream transformation
 - re-use of scheduling results
 - assume periodic-based model
 - propagation: jitter propagation + ad-hoc enhancements

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A large research area:

- Main model: periodic task (+offsets, + dependencies, +...)
- Main problem: local schedulability

Differences with network analyses:

- propagation: output of a system is input of another
- shaping: maximal throughput \implies maximal input

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Three basic objects

- system behaviour
- bounds on behaviours
- “computer friendly” sub-classes

Not all do clear distinction between these objects.

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Two main components:

- a library to handle curves (sum, minimum, convolution, etc.)
- a network analyser
- DISCO: NC analyser, in Java, Licence LGPL
 - <http://disco.informatik.uni-kl.de/index.php/projects/disco-dnc>
 - curves library and network analyser
- Real-Time Calculus (RTC) Toolbox
 - <http://www.mpa.ethz.ch/Rtctoolbox>
 - Curve Library: Java implementation (no source code) + matlab interface
 - Network analyser: Modular Performance Analysis (MPA), Matlab code
- RTaW-PEGASE
 - <http://www.realtimetowork.com/software/rtaw-pegase>
 - commercial Java tool

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Accuracy is “good enough”

Experiment on realistic configuration [5]:

- 8 switches
- 104 end-systems
- 6500 VL

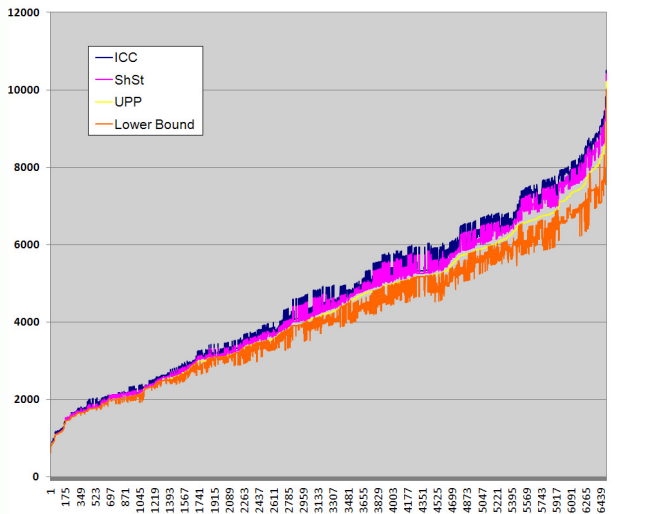
Computation time:

- Fast algorithm (ICC): 1s
- Accurate algorithm (UPP): 10s

Method accuracy:

	All Virtual Links		20% of VLs with highest WCTT	
	ICC	UPP	ICC	UPP
Min	3.74%	0%	15.2%	3.55%
Av.	31.02%	16.44%	42.08%	25.37%
Max.	82.4%	76.06%	81.53%	76.08%

Plotting accuracy



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Lot of work done from [8, 9, 13], two reference books [14, 7]...

Toward smaller upper bound for avionic systems

- either more information on system
- or more complex analyse
- but exact delay is NP-Hard [3, 4]

Next research topics:

- formal correction proof (cf Stephan Merz, Oct. 2014)
- other application domains
 - network on chip
 - from network to system

Main contributions of NC are

- clear distinction between
 - real behaviour
 - bounds on behaviours (arrival/service curves)
 - computation-friendly sub-classes
- formal definition of delay
- new point of view on real-time
- hierarchical scheduling

Drawbacks (improvement areas):

- too many definitions of service
- infimum based proofs, continuity problems

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