New Belief Function Theory (BFT) based methods for multicriteria decision making

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Outlines

- Introduction
  - Multicriteria decision making in context of risk, uncertainty: e.g. natural hazards in mountains

- Three recent developments associating multicriteria decision methods and belief function theory (BFT) are described
  - ER-MCDA
  - DSmT-AHP
  - Cautious OWA

- Conclusion

AHP and BFT...
Decision under uncertainty
Example of application context: risk management related to rapid mass movements in mountains (avalanches, mountain river floods, rockfalls)

...They threaten people and infrastructures

Alternatives = risk levels, protection strategies ...

Criteria = morphology, hydrology, vulnerability...

DM = expert, community mayor...

...and we try to get protected against them

Aid-decision methods considering information imperfection are expected

The ER-MCDA* methodology

*Evidential Reasoning and Multi Criteria Decision Analysis

...to help decisions based on imperfect information provided by more or less reliable (and important) sources
Uncertainty in not the only kind of information imperfection

(unconsistency, imprecision, incompleteness, uncertainty)

p.m. Probabilities

Fuzzy sets theory (to link numerical and linguistic values)

Possibility theory (to represent both imprecision and uncertainty)

Belief function theory (to represent imprecision, uncertainty and consider heterogeneous sources)

Information fusion requires a common frame of discernment

\[ \Theta = \{ \theta_1, \theta_2 \} \quad \text{- Frame of discernment} \]

The set of exhaustive and exclusive hypotheses

\[ \theta_2 = \theta_1 \cap \theta_2 \equiv \emptyset \]

Powerset

Basic belief assignment (BBA)

\[ m(\cdot) : 2^{\Theta} \rightarrow [0,1] \]

\[ m(\emptyset) = 0 \]

\[ \sum_{\theta_i \in \Theta} m(\theta_i) = 1 \]

\[ m(\theta_1) + m(\theta_2) + m(\theta_1 \cup \theta_2) = 1 \]

Existing links

Fusion of 2 sources

\[ B_1 \]

\[ B_2 \]

Dempster fusion rule

\[ m(X) \triangleq [m_1 \oplus m_2] = \frac{\sum_{A \cap B \neq \emptyset} m_1(A)m_2(B)}{1 - \sum_{X \cap A \neq \emptyset} m_1(X)m_2(A)} \]

+ many other fusion rules (PCR...)

DSmT

Backgrounds of belief function theory

The frame of discernment: set of possible decisions, alternatives...

Decision context

\[ \Theta = \{ \theta_1, \theta_2, \theta_3, \theta_4 \} \]

Set of 4 exhaustive and exclusive hypotheses

\[
\begin{align*}
\text{(NoS)} & \quad \theta_1 \\
\text{[LS]} & \quad \theta_2 \\
\text{[MS]} & \quad \theta_3 \\
\text{[HS]} & \quad \theta_4
\end{align*}
\]

The frame of discernment: set of possible decisions, alternatives...
ER-MCDA* methodology

*Evidential Reasoning and Multi Criteria Decision Analysis

- The multi-criteria decision analysis (AHP) allows an analytic breaking down (into criteria) of the decision problem (+ traceability)

- Uncertainty theories are used:
  - to represent different kinds of information imperfection (unconsistency, imprecision, uncompletness and uncertainty)
  - to evaluate the criteria (possibility theory and belief function theory)
  - to "map" the evaluations into the frame of discernment (fuzzy sets theory)
  - To consider the multiple information sources and produce a decision (belief function theory)

(Tacnet et al., 2009) - Information Fusion for natural hazards in mountains in Advances and Applications of DSmT for Information Fusion, Vol. 3
(Tacnet et al., 2010) - A two-step fusion process for multi-criteria decision applied to natural hazards in mountains, Int. Workshop on Belief Functions theory proceedings, 1-2 april 2010, Brest, France

Decision context: assessment of a sensitivity index (sorting decision problem)

The Analytic Hierarchy Process (AHP) is used as a conceptual framework (hierarchy, weights...)

Example of a quantitative criterion

Decision classes (elements of the frame of discernment)

- Permanent winter occupants
  - Nb. of occupants ≤ 4
  - Nb. of occupants > 20

Examples of evaluation classes

- Dwelling/facilities/Infrastructure
- Industrial Collectivity Rescue

Example of a qualitative criterion

- Morphology
- History
- Snow-climatology

Evaluated alternative: a_i

\[
\text{eval}_{AHP}(a_i) = w_{11} \cdot w_{111}(a_i) + w_{12} \cdot w_{112}(a_i) + w_{12} \cdot w_{113}(a_i)
\]

AHP Criteria-Estimator-Solution variant

Belief function theory and Multicriteria decision analysis – J.M. Tacnet and J. Dezert
ER-MCDA dissociates (imperfect) evaluation and decision

Hierarchical analysis of the decision problem

Decision
Multicriteria decision analysis

Evaluation
Numerical (quantitative) or qualitative criterion evaluation

- Criterion
- Decision classes (of the decision criterion)
- Elements of the frame of discernment

Decision making
Decision criterion

ER-MCDA methodology

ER-MCDA (A two-step fusion process based on AHP and DSmT)

1. First fusion level
   - Fusion of the criteria evaluations resulting from each source
   - Mapped and discounted masses
   - Source
   - expert(1)

2. Mass
   - Mapped Masses
   - Mapping from evaluation to decision classes

3. Imperfect evaluation
   - Reliability discounting of sources
   - Source
   - expert(2)

4. A criterion considered as a source

Fusion process

DSmT – AHP Importance (preference) discounting

DSmT-PCR rule
The DSmT-AHP methodology

...an adaptation of AHP to consider sets of alternatives evaluated by provided by more or less reliable (and important) sources

The DSmT-AHP methodology

- Original AHP is based on pairwise comparisons between solutions or alternatives
- Those comparisons can be uncertain or uncomplete
- A improvement of methods already proposed to use belief functions in the AHP framework (e.g. Beynon 2000, DS-AHP)

DSmT-AHP is an evolution that uses new fusion rules and discounting techniques
DSmT-AHP: a basic example

A simple decision problem: 
choosing the best car

between four models A, B, C, D

Criteria

Decision

Best car?

C1
Fuel economy

C2
Reliability

C3
Style

Solutions

Pairwise comparison (also called preference or knowledge) matrix

Weights

M

C1
1/4
1/3
4/1

C2
1/1
1/1
5/1

C3
1/4
1/5
1/1

= 1.0000
3.0000
0.2500

0.3333
1.0000
0.2000

4.0000
5.0000
1.0000

0.2707
0.6267
0.6936

Weights

Wc1
Wc2
Wc3

DSmT-AHP: preferences are assessed on sets of solutions
(including total ignorance)

DSmT-AHP introduces uncertainty in solutions evaluations

choosing the best car between three models A, B, C according to C1 and C2

C1
Fuel economy

C2
Reliability

M(C1) =

A
B
C

B ∪ C

1/1
0
1/3

0
1/2
1

M(C2) =

A
B

A ∪ C

B ∪ C

1/2
1/2
1

1/4
2
1/5

1
0

Priority vectors

Basic belief assignments (bba’s)

Weights

are derived into bbas

\( \hat{w}(C1) \approx [0.0889 \ 0.5337 \ 0.3774]^T \)

\( \hat{w}(C2) \approx [0.5002 \ 0.1208 \ 0.1222 \ 0.2568]^T \)
Importance and reliability are two different concepts: **specific “discounting” techniques** are proposed (transfert of bba’s on ignorance, empty set...)

![Graph showing reliability and importance discounting factors](image)


**Conclusion DSmT-AHP**

- DSmT-AHP is an evolution of AHP to consider **uncertainty** in comparison of solutions (criteria solution context) in case of **multiple sources**

- It improves existing methods (e.g. DS-AHP) by using
  - high performance fusion rule (PCR rule, DSmT framework) for
  - New discounting methods have been proposed to consider either reliability and importance discounting
The COWA-ER (Cautious Ordered Weighted Averaging – Evidential reasoning) methodology

... a multicriteria method for decision under uncertainty
Adaptation of OWA (Ordered Weighted Averaging)

- The original OWA approach considers several alternatives evaluated in the context of different uncertain scenarios: it includes several ways (pessimistic, optimistic, hurwicz, normative) to interpret and aggregate the evaluations with respect to a given scenario.

Two evolutions are proposed:

- An OWA approach using the classical concept of expected utility based on DSmP or BetP has been proposed (p.m.).

- COWA-ER allows to take easily a decision in a context of uncertain scenarios. COWA-ER uses simultaneously the two extreme pessimistic and optimistic decision attitudes combined with an efficient fusion rule (no complex programming to get weights as in classic OWA approach): decision is more cautious.
Main steps of COWA-ER approach

Pessimistic and optimistic valuations of the expected payoffs

\[
\begin{align*}
&\text{Pessimistic choice} & \text{Optimistic choice} & \text{Combination} \\
& C & E(C) & \text{Range} \\
& A_1 & 6.2 & 10.9 & [6.2; 10.9] \\
& A_2 & 3.8 & 11.4 & [3.8; 11.4] \\
& A_3 & 4.8 & 13.2 & [4.8; 13.2] \\
& A_4 & 5.2 & & [5.2; 13.2] \\
\end{align*}
\]

Four steps of the COWA-ER* approach

* Cautious Ordered Weighted Averaging-Evidential Reasoning

1. **Step 1**: normalization of imprecise values in [0, 1]
2. **Step 2**: conversion of each normalized imprecise value into elementary bba \( m_i(.) \)
3. **Step 3**: fusion of bba \( m_i(.) \) with an efficient combination rule (typically PCR5)
4. **Step 4**: choice of the final decision based on the resulting combined bba

**COWA-ER - Step 2**

Conversion of each imprecise value into its bba (Dezert, 1991)

We use an existing method to define bba from pessimistic and optimistic strategies results

Modeling for computing a bba associated to the hypothesis \( A_i \) from an imprecise value \([a, b] \)

\[
\begin{align*}
\text{Bel}(A_i) &= \frac{1}{2} b - \frac{1}{2} a \\
\text{Pl}(A_i) &= b - a \\
\text{Bel}(\bar{A}_i) &= a - \frac{1}{2} b \\
\text{Pl}(\bar{A}_i) &= b - a \\
\end{align*}
\]

The uncertainty is represented by the length of the interval \([a, b] \)

Improvisation of the variable (expected payoff \( C_i \)) on which is defined the belief function for \( A_i \)

**Application**

<table>
<thead>
<tr>
<th>Method</th>
<th>Frame of discernment ( \Theta = { A_1, A_2, A_3, A_4 } )</th>
</tr>
</thead>
<tbody>
<tr>
<td>COWA-ER Step 2</td>
<td>We use an existing method to define bba from pessimistic and optimistic strategies results</td>
</tr>
</tbody>
</table>

Basic belief assignments of the alternatives

\[
E^{tmp}[C] = \begin{bmatrix}
0.47 & 0.82 \\
0.29 & 0.86 \\
0.36 & 0.85 \\
0.39 & 1.00
\end{bmatrix}
\]

<table>
<thead>
<tr>
<th>Alternatives ( A_i )</th>
<th>( m_i(A_i) )</th>
<th>( m_i(\bar{A}_i) )</th>
<th>( m_i(A_i \cup \bar{A}_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>0.47</td>
<td>0.18</td>
<td>0.35</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>0.29</td>
<td>0.14</td>
<td>0.57</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>0.36</td>
<td>0.15</td>
<td>0.49</td>
</tr>
<tr>
<td>( A_4 )</td>
<td>0.39</td>
<td>0</td>
<td>0.61</td>
</tr>
</tbody>
</table>
Conclusion: towards a global framework associating belief function theory and MCDA methods...

Developments are under progress to introduce BFT in outranking methods...
References